Inverse Problem of Multiple Conductors Buried in a Half-Space

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Abstract: Electromagnetic imaging of buried multiple conductors by using genetic algorithm has been presented. Two separate perfectly conducting cylinders of unknown shapes are immersed in one half-space and illuminated by transverse magnetic (TM) polarization plane wave from the other half-space. Based on the boundary condition and the measured scattered field, we have derived a set of nonlinear integral equations, and the imaging problem is reformulated into an optimization problem. For describing the shapes of conductors, the Fourier series is selected to expanding the shape functions. In inverse algorithms, the improved steady state genetic algorithm is employed to search for the global extreme solution of objective function. Numerical results have demonstrated that the powerful performance of the inverse algorithm. The reconstructed shapes are considerably accurate even when the initial guesses are far away from the exact ones and the buried depths of the conductors are large compared to wavelength.

1 Introduction

The inverse scattering problems have been a subject of considerable importance in nondestructive measurement, medical imaging, biological application and remote sensing. Furthermore, the inverse problems related to the underground objects are of particular importance in scattering theory. In the past 20 years, many rigorous methods have been developed to find out the shape and locations of two-dimensional (2-D) cylindrical conducting objects [1] – [8]. However, inverse problems of this type are difficult to solve because they are ill-posed and nonlinear [9]. As a result, many inverse problems are reformulated into optimization ones, and then numerically solved by several iterative methods such as the Newton-Kantorovitch method [1] – [4], the Levenberg-Marquardt algorithm [5] – [7], and the successive-overrelaxation method [8], etc. However, most of these approaches use the gradient-based searching scheme to determine the extreme of the cost function, which must highly dependent on the initial guess and usually get trapped in the local minimum. Genetic algorithm (GA) [10] is evolutionary algorithm of optimization strategy, which uses stochastic mechanism to search through the parameter space. The genetic algorithm is less prone to converge in a local extreme when compared with the gradient-based searching scheme, which in turn renders it an ideal candidate for global optimization. In the past years researchers used an improved steady state genetic algorithm (SSGA) [11], [12] to reconstruct the shape of the perfectly conducting cylinder. In general, most of objects are placed in a homogeneous space, while Chiu and Chen [13], [14] reconstruct a buried imperfectly conducting cylinder by genetic algorithm. We propose microwave imaging of a buried two conductors by the improved SSGA, in which the effects of multiple scattered fields between two conductors are investigated.

2 Theoretical Formulation

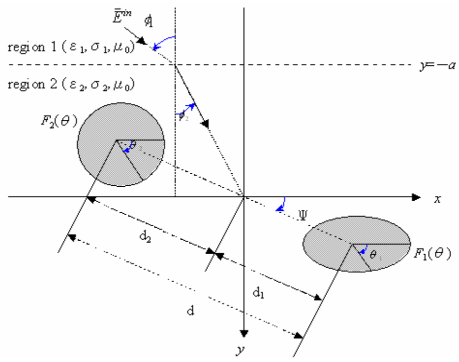


Fig. 1 Geometry of two perfectly conducting cylinders in (x, y) plane.

Let us consider two separate perfectly conductors buried in a lossy homogeneous half-space, as shown in Fig. 1. The media in regions 1 and 2 are characterized by the permittivities and the conductivities (ε_1, σ_1) and (ε_2, σ_2), respectively, while the permeability μ_0 is used for each region, i.e., only nonmagnetic media are concerned here. There are two conducting cylinders with cross section that are described by polar coordinates in the *xy* plane through shape equations $\rho=F(\theta)$. The objects are illuminated by TM plane wave with time dependence $e^{j\omega t}$ on region 1, in which the electric field is assumed parallel to the z-axis. Let the electric field E^{in} denote the incident field form region 1 to region 2 with the incident angle ϕ_1. Since the interface between region 1 and region 2, the incident plane wave generates two waves in the interface, which would exist in the absence of the conducting targets: the reflected wave in the region 1 and the transmitted wave in the region 2. Thus the unperturbed field is given by

$$\vec{E}_i = E_i(x,y)\hat{z} \tag{1}$$

where

$$E_i(x,y) = \begin{cases} E_1 = e^{-jk_1[x\sin\phi_1+(y+a)\cos\phi_1]} + R_1 e^{-jk_1[x\sin\phi_1-(y+a)\cos\phi_1]}, & y \le -a \\ E_2 = Te^{-jk_2[x\sin\phi_2+(y+a)\cos\phi_2]}, & y > -a \end{cases}$$

$$R_1 = \frac{1-n}{1+n}, T = \frac{2}{1+n}, n = \frac{\cos\phi_2}{\cos\phi_1}\sqrt{\frac{\varepsilon_2 - j\frac{\sigma_2}{\omega}}{\varepsilon_1 - j\frac{\sigma_1}{\omega}}}$$

$$k_1\sin\phi_1 = k_2\sin\phi_2,\ k_i^2 = \omega^2\varepsilon_i\mu_0 - j\omega\mu_0\sigma_i, \quad \mathrm{Im}(k_i) \le 0$$

i=1, 2.

The scattered field in each region can be expressed by

$$E_S(x,y) = \begin{cases} -\int_0^{2\pi} G_1(x,y,F_1(\theta'),\theta')J_1(\theta')d\theta' - \int_0^{2\pi} G_1(x,y,F_2(\theta'),\theta')J_2(\theta')d\theta', & y \le -a \\ -\int_0^{2\pi} G_2(x,y,F_1(\theta'),\theta')J_1(\theta')d\theta' - \int_0^{2\pi} G_2(x,y,F_2(\theta'),\theta')J_2(\theta')d\theta', & y > -a \end{cases} \tag{2}$$

In (2), the current density and Green's functions are given by

$$J_h(\theta_h') = -j\omega\mu_0\sqrt{F_h^{\,2}(\theta_h') + (F'_h(\theta_h'))^2}\,J_{sh}(\theta_h')$$

$$G(x,y,F_h(\theta'),\theta') = \begin{cases} G_1(x,y,F_h(\theta'),\theta'), & y \le -a \\ G_2(x,y,F_h(\theta'),\theta') = G_f(x,y,F_h(\theta'),\theta') + G_S(x,y,F_h(\theta'),\theta'), & y > -a \end{cases} \tag{3}$$

where h=1, 2.

$J_s(\theta)$ is the induced surface current density which is proportional to the normal derivative of electric field on the conductor surface. $G(x,y,x',y')$ is the Green's function in terms of Fourier transform [3]. Note that G_1 and G_2 denote Green's function for line source in region 1 and region 2, respectively. In (3b), $H_0^{(2)}(k_2 r)$ is the Hankel function of second kind of order zero. According to boundary condition on conductor surface, the total tangential electric field must be zero, which is shown as follows:

$$\hat{n} \times \vec{E} = 0 \tag{4}$$

where $\hat{n}$ is the outward unit vector normal to the surface of the conducting objects. In (4), the boundary condition on these two conductors leads to an integral equation for $J_1(\theta)$ and $J_2(\theta)$

$$E_2(x_1,y_1) = -E_S(x_1,y_1) = \int_0^{2\pi} G_2(x_1,y_1,F_1(\theta'),\theta')J_1(\theta')d\theta' + \int_0^{2\pi} G_2(x_1,y_1,F_2(\theta'),\theta')J_2(\theta')d\theta' \tag{5}$$

$$E_2(x_2,y_2) = -E_S(x_2,y_2) = \int_0^{2\pi} G_2(x_2,y_2,F_1(\theta'),\theta')J_1(\theta')d\theta' + \int_0^{2\pi} G_2(x_2,y_2,F_2(\theta'),\theta')J_2(\theta')d\theta' \tag{6}$$

$(x_1, y_1)=(d_1\cos\Psi+F_1(\theta)\cos\theta,\ d_1\sin\Psi+F_1(\theta)\sin\theta)$

$(x_2, y_2)=(-d_2\cos\Psi+F_2(\theta)\cos\theta,\ -d_2\sin\Psi+F_2(\theta)\sin\theta)$

The direct problem is to compute the scattered field in region 1 when the shapes and positions are given. This can be achieved by first solving for J_1 and J_2 from (5) and (6). Thus, the scattered field in (2) can be solved in each region. Then the inverse problem is to determine the shapes and positions of the conducting cylinders when the scattered electric field E_S is measured outside the scatterer.

3 Numerical Results

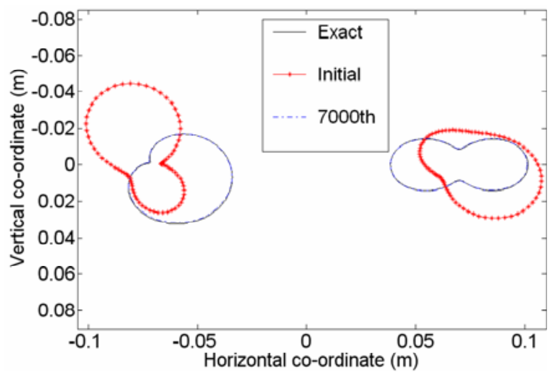
Let us consider two separate perfectly conducting cylinders which are immersed in a lossless half-space. The permittivities in region 1 and region 2 are characterized by $\varepsilon_1=\varepsilon_0$ and $\varepsilon_2=2.56\,\varepsilon_0$, respectively, and the permeability μ_0 is used for each region which are only nonmagnetic media. A TM plane wave of unit amplitude is incident from region 1 to region 2 as shown Fig. 1. The frequency of the incident wave is chosen to be 3GHz. We set the position of the interface at $y=-a$, and the two objects are buried at the same depths ($a\cong 5\lambda_0$) or at the different depths (the buried depths of the objects are larger than $5\lambda_0$ and less than $5\lambda_0$, respectively). The scattered fields are measured on a probing line along $y=-(a+0.01)$ m in region 1. The two targets are illuminated by three waves with incident angles ϕ_1 that are 0°, 45° and 315°, respectively, and then the 8 measurements at equally separate points are used along $y=-(a+0.01)$ for each incident angle. Therefore there are totally 24 measurements in each simulation. Here, the parameter a is set to 0.5 m in all simulation.

In parameters of GA, the number of unknowns is set to be 10. The population size is chosen as 150. The maximum number of generation is set to be 7000. The search range of unknown coefficients is chosen from 0 to 0.1. Note that the binary string length of the unknown coefficients is set to be 14 bits (i.e. L=14). The extreme values of the coefficients of the shape functions can be determined by prior knowledge of the objects. The crossover probability p_c and the mutation probability p_m are set to 0.1 and 0.08, respectively.

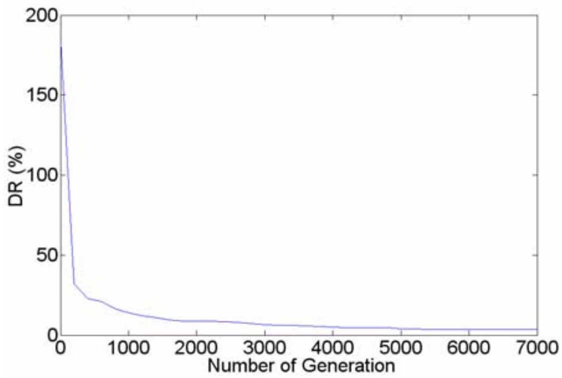
For the simulation, the shape functions are chosen to be $F_1(\theta)=[0.02+0.0115\cos(2\theta)]$ m and $F_2(\theta)=[0.02+0.015\cos(\theta)+0.01\sin(\theta)]$ m, respectively, The d_1, d_2 and Ψ are 0.1 m, 0.1 m and 0°, respectively. The reconstructed shape functions are plotted in Fig. 2(a). Moreover, the relative error of the shapes shown in Fig. 2(b), it is seen that the relative error reduces as the generations increase. The relative error is about 3.5% in final generation. The reconstructed results are good although the reconstructed shapes of the initial generation are far away from exact ones.

4 Conclusions

This paper proposes an imaging problem for buried multiple conductors. The targets are illuminated by TM plane wave in region 1. Based on the boundary condition and measured scattered field, a set of nonlinear integral equations have derived and reformulated the inversion problem into an optimization problem. Here, we have employed Fourier series to describe the shapes. The shapes of the objects can be reconstructed by the SSGA from the measured scattered fields in region 1. In this study, the main difficulties for applying the SSGA to this problem are how to choose the parameters such as population size, crossover probability, mutation probability and bit length of string. Finally, the results show that the reconstructed errors are almost convergent when the number of generation is about 1500th generation. It is seen that the reconstructed results are good even though the buried depths of the objects are more than eight times of wavelength, and the multiple scattering between two conductors are serious.



(a)



(b)

Fig. 2 (a) The reconstructed shapes. The solid curves represent exact shapes, and other curves are the calculated shapes in third generation and in final generation, respectively. (b) The error of the reconstructed shapes in each generation.

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