Fuzzy Rule Extraction for Controller Designs

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Abstract

This paper presents an innovative method for extracting fuzzy rules directly from numerical data for controller designs. Conventional approaches to fuzzy systems assume there is no correlation among features and therefore involve dividing the input and output space into grid regions. However, in most cases, it is likely that features are highly correlated. Therefore, we propose to use an aggregation of hyperspheres with different sizes and different positions to define fuzzy rules. The genetic algorithm is used to select the parameters of the proposed fuzzy systems. The inverted pendulum system is utilized to illustrate the efficiency of the proposed method for finding fuzzy control rules.

1. Introduction

Fuzzy systems have been successfully applied in many practical fields, especially in control problems. There are two major approaches for constructing fuzzy systems. One approach presumes that fuzzy rules be given by human experts. However, these initial linguistic rules are too crude for engineering purposes. Another approach often assumes input variables are independent, therefore, the membership function is assigned variable by variable. Fig.1 shows an example of conventional fuzzy rule partition. Most of the membership functions are usually assumed to be triangular, trapezoidal or bell-shape. In fact, there is no straightforward method for choosing membership functions. However it is likely that there exist correlations among input features, in this case, the fuzzy rule partition should not be divided variable by variable. To reduce requirement of large memory and minimize computation complexity, the fuzzy region should be arbitrarily shaped [1, 2]. Here we propose a new method to partition fuzzy regions. Fig.2 illustrates such a kind of fuzzy rule partition. In this paper, we also propose a method by using the genetic algorithm to fine tune the parameters of the proposed fuzzy control system to achieve a high controlled performance.

2. Fuzzy control system

As mentioned before, grid fuzzy rule partition results in the following type of fuzzy rule

IF x_1=A_1 and x_2=A_2 and ... and x_n=A_n THEN y_i is B_i

where A_i's and B_i's are fuzzy sets for inputs and outputs, respectively. Since in many applications, input variables are in some sense correlated with each other. In order to capture correlations, traditional approach is to increase the number of partitions for each variable. Therefore, a large number of fuzzy rules is required to achieve acceptable performance. However, it will result in computation complexity and memory load. This motivated us to use the aggregation of hyperspheres to approximate arbitrary fuzzy rule partition. By doing this, the correlations among input variables have been taken into account. The structure of the proposed fuzzy system is described as follows:

IF $\underline{x}$ is HS_i THEN y_i=$c_{i1}x_1+c_{i2}x_2+\ldots\ldots+c_{in}x_n+c_{in+1}$, i=1, 2,..., m,

where $\underline{x}=[x_1\, x_2 \cdots x_n]^T$ is the input vector, c_{ij}, j=1,2,..., n+1, are the consequence parameters, and HS_i is defined by the following membership function

$$HS_i(\underline{x}) = \exp(-(\frac{\sum_{j=1}^{n}(x_i - a_{ij})^2}{b_i^2})).$$

Equation $\sum_{j=1}^{n}(x_i - a_{ij})^2 = b_i^2$ defines an n-dimensional hypersphere positioned at $(a_{i1}, a_{i2}, \ldots a_{in})$ and with radius b_i which regulates how fast the hypersphere goes down. Fig.3 shows a two dimensional case for $HS_i(\underline{x})$. In the premise part, we do not construct it variable by variable as the traditional approach does. On the contrary, we regard the whole input variables as an n-dimensional pattern in the input space. Then we use a set of hyperspheres to define fuzzy sets in the input space. In the consequence part, we adopt the same approach proposed by Takagi and Sugeno [3] by taking the linear combination of the input variables.

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If the weighted average defuzzifier is used in the final stage of our fuzzy system, and the firing strength of the premise part in the i-th rule is defined by

$$w_i = HS_i(\underline{x}),$$

then the output y inferenced from the fuzzy system can be calculated by

$$y = \frac{\sum w_i y_i}{\sum w_i}$$

Since the contour of the membership function $HS_i(\underline{x})$ is defined by the values $\{a_{i1}, a_{i2}, \ldots, a_{in}, b_i\}$ and the value of y_i is determined by the values $\{c_{i1}, \ldots, c_{in+1}\}$, these parameters affect the control performance of the fuzzy system to a large extent. If there are m fuzzy rules we want to construct, then there will be total mx(n+1)x2 parameters $\{a_{i1}, a_{i2}, \ldots, a_{in}, b_i, c_{i1}, \ldots, c_{in+1}, 1 \le i \le m\}$ which are going to be found to make the fuzzy system achieve acceptable performance. In this paper, the genetic algorithm is used to fine tune these parameters. It is discussed in the following section.

3. Application in the inverted pendulum control problem

The goal of this experiment is to train the proposed fuzzy system as a controller to produce an appropriate control signal u to balance the inverted pendulum system by using the genetic algorithm. The schematic of the inverted pendulum system is shown in Fig.4. Let $x_1(t)=\theta(t)$ (angle of the pole with respect to the vertical) and $x_2(t)=\dot{\theta}(t)$ (angular velocity of the pole), then the state equation can be express as [4,5]

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = H(x_1, x_2, F) = \frac{g\sin(x_1)+\cos(x_1)\left(\frac{-F-mlx_2^2\sin(x_1)}{m+m_c}\right)}{l\left(\frac{4}{3}-\frac{m\cos^2(x_1)}{m+M}\right)}$$

where g (acceleration due to the gravity) is 9.8 meter/sec^2, m_c (mass of cart) is 1.0 kg, m (mass of pole) is 0.1 kg, l (half length of pole) is 0.5 meter, and F is the applied force in newton.

In this experiment, we assume that the inverted pendulum system can be efficiently balanced by four fuzzy rules, therefore, 24 parameters are required to be coded as a chromosome in the initial population, which is randomly chosen from the problem space. We use 121 numerical data uniformly distributed in the square region [-10 10] * [-10 10] to train the fuzzy system. The time step is 0.01 second. The block diagram of the control system is shown in Fig.5, where the control scheme is consisted of 3 time stages, that is, we apply all training data to each chromosome in the population and simulate a period of time of 3*0.01 second in order to select the chromosome with high performance. In our simulations the settings of the genetic algorithm parameters are: population size = 200; bit length for each parameter =10; crossover rate=0.9; mutation rate = 0.5; and generation = 500. The control objective is to balance the inverted pendulum system under the condition that the force applied on the cart can not be too large in practical, therefore the degree of the pole angle θ and the scale of the force F must be considered simultaneously. Taking the merit of the genetic algorithm, we define the fitness function by

$$f_i(x,y) = \lambda_1 \exp(-x) + \lambda_2 \exp(-y), \quad \lambda_1 + \lambda_2 = 1$$

$$x = \frac{\sum_{i=1}^{121} \theta_i^2}{121}$$

$$y = \frac{\sum_{i=1}^{121} (|f_i^1| + |f_i^2| + |f_i^3|)}{121},$$

where θ_i is the angle of the last time stage for the i-th trainning data and f_i^1, f_i^2, f_i^3 are the applied force corresponding to each time stage for the i-th training data, and all the fitness values are in the range [0,1]. The parameters λ_1, λ_2 are the scaling factors corresponding to the angle and the force. If λ_1 is much larger than λ_2, which means we take more consideration about the final angle, the searching process will be guided to a larger force result, while when λ_2 is much larger than λ_1, the force will be reduced but it takes more time to balance the pole. After 500 generations of training by considering $\lambda_1 = \lambda_2 = 0.5$, the four extracted control rules are represented as

If $\underline{x}$ is HS_1 Then force=$0.024\theta+0.08\dot{\theta}+1.186$
If $\underline{x}$ is HS_2 Then force=$0.839\theta+0.127\dot{\theta}-1.186$
If $\underline{x}$ is HS_3 Then force=$0.08\theta+0.003\dot{\theta}-1.159$
If $\underline{x}$ is HS_4 Then force=$0.084\theta+0.006\dot{\theta}+1.159$

where $\underline{x} = [\theta, \dot{\theta}]^T$, and HS_1, HS_2, HS_3, HS_4 are the linguistic labels characterized by {0.41, 28.494, 15,347}, {18.533, -0.097, 89.149}, {-7.859, 8.357, 2.443}, {-10.244, -5.034, 65.982}. The performance of the constructed controller have been evaluated by starting from several different initial points. Some of simulation results are illustrated in Fig.6. These results show that the inverted pendulum system can be balanced by the four rules within a short time. Furthermore, the proposed system can also balance the pole from some initial points that don't fall into the region of the training data set.

4. Conclusion

In this paper, we proposed a method by using the genetic algorithm for extracting fuzzy rules directly from numerical data for constructing a fuzzy controller. We use a set of hyperspheres to define fuzzy sets. The inverted pendulum system is used to test the validity of the proposed method. From the simulation results, we find that the parameters of the fuzzy system can be efficiently selected so that the controlled system achieves a high performance based on the direction of the proposed fitness function.

References

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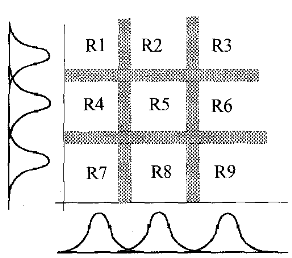

Fig.1 An example of traditional fuzzy rule partition.

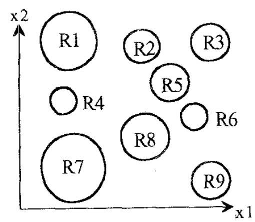

Fig.2 An example of modified fuzzy rule partition.

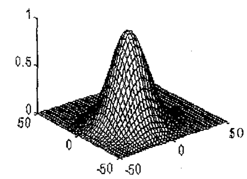

Fig.3 Membership function for $HS_i(\underline{x})$.

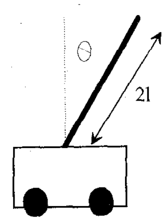

Fig. 4 The inverted pendulum system.

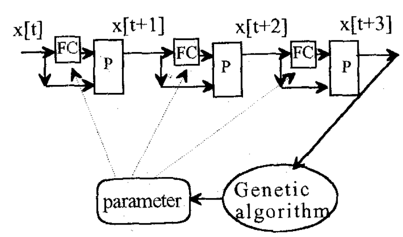

Fig.5 The control scheme (FC=fuzzy controller,P= plant).

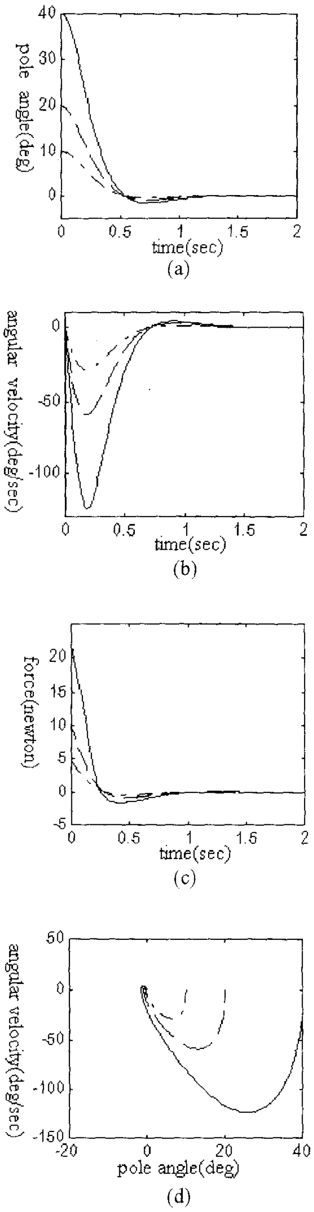

Fig.6 Results from three initial states (10,0) (20,0) (40,0).