Effects of Channel Estimation Error on the BER Performance of OFDM Systems in Multipath Rayleigh Fading Channels

Chi-Hsiao Yih
Department of Electrical Engineering
Tamkang University
Tamsui, Taiwan, 25137
Email: chyih@ee.tku.edu.tw

***Abstract*— In this paper, we study the effects of channel estimation error on the bit error rate (BER) of orthogonal frequency division multiplexing systems in multipath Rayleigh fading channels. Due to the additive white Gaussian noise and the intercarrier interference caused by the residual carrier frequency offset, the channel estimation based on the training symbols is not perfect. We characterize the performance degradation resulting from imperfect channel state information by deriving the BER formulas for BPSK, QPSK, and 16-QAM modulation schemes. Simulation results validate the accuracy of our derived formulas.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) has become a popular transmission technique for high-data-rate wireless communications in recent years. By dividing the whole bandwidth into subchannels and transmitting data symbols in parallel, the effective data period is enlarged and the intersymbol interference (ISI) caused by the multipath fading channel can be effectively reduced [1].

The issues of channel estimation in OFDM systems have been considered in [2] where the minimum mean-square error (MMSE) and least-square (LS) channel estimators exploit the property of cyclic prefix to obtain channel estimates. The MMSE channel estimator fully uses the time and frequency-domain correlations of the frequency response of time-varying dispersive fading channels has been derived and analyzed in [3]. The authors also addressed the channel statistics mismatched problem and proposed a robust channel estimator which is insensitive to the channel statistics. The uses of pilot-symbol-aided channel estimation in both time and frequency domain have been investigated in [4] and [5].

When the OFDM-based wireless system operates in a slowly fading multiple-access environment, the use of training (preamble) symbols to facilitate the channel estimation task has been standardized in the IEEE 802.11a/g standard [6]. The effect of channel estimation error based on the long preambles in OFDM-based wireless local area networks (WLAN) was examined in [7]. The authors considered the channel estimate is not only corrupted by additive white Gaussian noise (AWGN) but also by the intercarrier interference (ICI) due to the residual carrier frequency offset (CFO). In the BER analysis, it has been assumed the channel estimate and the channel estimation error are uncorrelated when the interference power is small. As indicated in [8], the method presented in [7] may overestimate the BER in some situations.

In this paper, we consider the same scenario as in [7]. However, we perform the exact BER analysis for BPSK, QPSK, and 16-QAM modulated OFDM signals in multipath fading channels without any assumption of the correlation between the channel estimate and the channel estimation error. Therefore, our method and result are accurate even for large CFO and channel estimation error. Moreover, the expression of the derived BER formula is in a simple form and no numerical integration is needed to evaluate it.

The remainder of this paper is organized as follows. In Section II, we describe the system and channel model. Section III presents the detailed BER analysis for BPSK, QPSK, 16-QAM modulated OFDM signals with imperfect channel state information (CSI). Numerical results are described in Section IV. Finally, Section V concludes the paper.

II. System Model

We assume the multipath fading channel is fixed during one transmitted frame and the channel estimation is facilitated by the preamble (training) symbols embedded in the beginning of the data frame. In mathematical form, the mth transmitted baseband OFDM symbol in a frame can be expressed as

$$x_n(m) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k(m) e^{j2\pi nk/N}, \ m = 1, 2, \cdots, M, \quad (1)$$

where k is the index of subcarrier, N is the total number of subcarriers, $X_k(m) \in \mathcal{X}$ are the transmitted modulation symbols, and M is the number of OFDM symbols in one frame. Three types of modulation formats $\mathcal{X}$ are considered, namely, BPSK, QPSK, and 16-QAM [9]. The first P OFDM symbols are training symbols. For each OFDM training symbol, the modulation is restricted to be BPSK and the transmitted modulation symbols at each subcarrier, denoted by $X_k^{\mathsf{p}} \in \{-\sqrt{E_b}, \sqrt{E_b}\}$, are the same for the symbol index m from 1 to P. The training symbol patterns X_k^{p} for

$m = 1, 2, \cdots, P, k = 0, 1, \cdots, N-1$ are known at both the transmitter and the receiver ends.

For simplicity of exposition, we assume the cyclic prefix is inserted at the beginning of each OFDM symbol and is removed in the demodulation process [1]. Furthermore, we assume the length of cyclic prefix is longer than the maximum delay spread of the multipath fading channel and the ISI can be completely eliminated.

The transmitted OFDM signal passes through a multipath fading channel whose impulse response is represented by the tapped-delay line model as [9]

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t - lT/N), \tag{2}$$

where $L < N$ is the number of multipaths, the path gains h_l are independently circularly symmetric complex Gaussian random variables with mean 0 and variance σ_l^2, $\delta(\cdot)$ is the Dirac-delta function, and T is the effective OFDM symbol period. The corresponding frequency response at subcarrier i is

$$H_i = \int_{-\infty}^{\infty} h(t) e^{-j2\pi it/T} dt = \sum_{l=0}^{L-1} h_l e^{-j2\pi il/N}. \tag{3}$$

Since h_l are complex Gaussian random variables, the weighted sum of them is also complex Gaussian. The mean and variance of H_i are

$$E[H_i] = \sum_{l=0}^{L-1} E[h_l] e^{-j2\pi il/N} = 0, \tag{4}$$

$$\mathrm{Var}[H_i] = E[H_i H_i^*] = \sum_{l=0}^{L-1} \sigma_l^2, \tag{5}$$

where $E[\cdot]$ denotes the probabilistic expectation and $*$ represents the complex conjugation. Without loss of generality, we assume the sum of the average power of each multipath is normalized to 1, i.e. $\sum_{l=0}^{L-1} \sigma_l^2 = 1 = \mathrm{Var}[H_i]$.

At the beginning of a frame, the CFO estimation is not accurate due to the presence of noise. The normalized CFO affecting the training symbols for channel estimation is denoted by $\epsilon = f_\Delta T$ where f_Δ is the CFO in Hertz. Then the mth received training symbol after the Fast Fourier transform (FFT) can be written as [10]

$$Y_i(m) = \alpha H_i X_i^{\mathsf{p}} + I_i + W_i(m),$$

$$m = 1, 2, \cdots, P, \; i = 0, 1, \cdots, N-1 \tag{6}$$

where the symbol $\alpha = \frac{\sin \pi\epsilon}{N \sin(\pi\epsilon/N)} e^{j\pi\epsilon(N-1)/N}$, $I_i = \sum_{k=0, k\neq i}^{N-1} H_k X_k^{\mathsf{p}} \frac{\sin(\pi\epsilon) e^{-j\pi(k-i)/N}}{N \sin[\pi(k-i+\epsilon)/N]} e^{j[\pi\epsilon(N-1)/N]}$, and $W_i(m)$ are circularly symmetric complex Gaussian random variables with mean 0 and variance $N_0/2$ in both real and imaginary components. For brevity, we simply use the notation $W_i(m) \sim \mathcal{CN}(0, N_0)$.

The channel estimate of H_i based on the received training signals $Y_i(1), Y_i(2), \cdots, Y_i(P)$ is given by

$$\hat{H}_i = \frac{1}{P} \sum_{m=1}^{P} \frac{Y_i(m)}{X_i^{\mathsf{p}}} = \alpha H_i + \hat{I}_i + \hat{W}_i,$$

$$i = 0, 1, \cdots, N-1, \tag{7}$$

where $\hat{I}_i = \frac{I_i}{X_i^{\mathsf{p}}}$, $\hat{W}_i = \frac{1}{P} \sum_{m=1}^{P} \frac{W_i(m)}{X_i^{\mathsf{p}}} \sim \mathcal{CN}(0, \frac{N_0}{PE_b})$. The training symbols can be utilized not only for channel estimation, but also for fine CFO estimation [11]. Therefore, it is reasonably assumed the CFO is perfectly estimated and compensated after the training period. In this case, the received OFDM data symbol after the FFT is

$$Y_i(m) = H_i X_i^{\mathsf{d}}(m) + W_i(m),$$

$$m = P+1, P+2, \cdots, M; \; i = 0, 1, \cdots, N-1, \tag{8}$$

where the superscript d indicates the transmitted modulation symbols are data symbols rather than preambles.

III. BER Analysis

First, we need the following lemma to characterize the effects of channel estimation error on the BER of various modulation schemes in OFDM systems.

Lemma 1[9]: Let X and Y be zero mean, correlated complex-valued Gaussian random variables and $D_1 = \mathrm{Re}[XY^*]$, $D_2 = \mathrm{Im}[XY^*]$ where $\mathrm{Re}(x)$ and $\mathrm{Im}(x)$ denote the real part and the imaginary part of x, respectively. Then

$$P(D_1 < 0) = \frac{1}{2} \left\{ 1 - \frac{\mathrm{Re}[\mu_{XY}]}{\sqrt{\mu_{XX}\mu_{YY} - (\mathrm{Im}[\mu_{XY}])^2}} \right\} \tag{9}$$

$$P(D_2 < 0) = \frac{1}{2} \left\{ 1 - \frac{\mathrm{Im}[\mu_{XY}]}{\sqrt{\mu_{XX}\mu_{YY} - (\mathrm{Re}[\mu_{XY}])^2}} \right\} \tag{10}$$

where

$$\mu_{XY} = E[XY^*], \; \mu_{XX} = E[XX^*], \; \mu_{YY} = E[YY^*]. \tag{11}$$

□

For notational simplicity, we omit the index of OFDM symbol m in the following discussion.

A. BPSK

For BPSK modulation, the FFT outputs of the OFDM data symbol are

$$Y_i = H_i X_i^{\mathsf{d}} + W_i, \; i = 0, 1, \cdots, N-1, \tag{12}$$

where $X_i^{\mathsf{d}} \in \{-\sqrt{E_b}, \sqrt{E_b}\}$. For the ith subcarrier, the decision statistics for the BPSK modulated OFDM signal with imperfect CSI is $\mathrm{Re}[Y_i \hat{H}_i^*]$ and the corresponding BER is

$$P_b(i) = P\left\{ \mathrm{Re}[Y_i \hat{H}_i^* < 0] | X_i^{\mathsf{d}} = \sqrt{E_b} \right\}, \tag{13}$$

where $\hat{H}_i$ is in (7). Conditioning on the transmitted symbol X_i^{d}, the received signal Y_i and channel estimate $\hat{H}_i$ are both zero mean complex Gaussian random variables. To utilize

Lemma 1 to obtain $P_b(i)$, we first compute $\mu_{Y_i\hat{H}_i|X_i^{\mathsf{d}}}$, $\mu_{Y_iY_i|X_i^{\mathsf{d}}}$, and $\mu_{\hat{H}_i\hat{H}_i|X_i^{\mathsf{d}}}$ as follows.

$$\mu_{Y_i\hat{H}_i|X_i^{\mathsf{d}}} = E[Y_i\hat{H}_i^*|X_i^{\mathsf{d}}] = \alpha^* X_i^{\mathsf{d}} + E[H_i\hat{I}_i^*]X_i^{\mathsf{d}}. \quad (14)$$

The expectation $E[H_i\hat{I}_i^*]$ can be computed as

$$E[H_i\hat{I}_i^*] = \sum_{k=0,k\neq i}^{N-1} E[H_iH_k^*]\frac{X_k^{\mathsf{p}}}{X_i^{\mathsf{p}}}\frac{\sin(\pi\epsilon)e^{j\pi(k-i-\epsilon(N-1))/N}}{N\sin[\pi(k-i+\epsilon)/N]}. \quad (15)$$

Since $H_i = \sum_{l=0}^{L-1} h_l e^{-j2\pi il/N}$, the expectation $E[H_iH_k^*]$ for $i \neq k$ in (15) is given by

$$E[H_iH_k^*] = \rho_{i,k} = \begin{cases} 1, & i = k \\ \sum_{l=0}^{L-1}\sigma_l^2 e^{j2\pi(k-i)l/N}, & i \neq k \end{cases}. \quad (16)$$

Next, we compute $\mu_{Y_iY_i|X_i^{\mathsf{d}}}$ as

$$\mu_{Y_iY_i|X_i^{\mathsf{d}}} = E[Y_iY_i^*|X_i^{\mathsf{d}}] = |X_i^{\mathsf{d}}|^2 + N_0. \quad (17)$$

Finally, the $\mu_{\hat{H}_i\hat{H}_i|X_i^{\mathsf{d}}}$ is given by

$$\begin{aligned}\mu_{\hat{H}_i\hat{H}_i|X_i^{\mathsf{d}}} &= E[\hat{H}_i\hat{H}_i^*|X_i^{\mathsf{d}}] = E[\hat{H}_i\hat{H}_i^*] \\ &= E\left[\left(\alpha H_i + \hat{I}_i + \hat{W}_i\right)\left(\alpha^* H_i^* + \hat{I}_i^* + \hat{W}_i^*\right)\right] \\ &= |\alpha|^2 + \frac{E[|I_i|^2]}{|X_i^{\mathsf{p}}|^2} + \frac{N_0}{PE_b} + 2\mathrm{Re}(\alpha E[H_i\hat{I}_i^*]). \quad (18)\end{aligned}$$

The expectation $E[|I_i|^2]$ in (18) is $E[|I_i|^2] =$

$$\sum_{k_1=0,k_1\neq i}^{N-1}\sum_{k_2=0,k_2\neq i}^{N-1}\frac{\rho_{k_1,k_2}X_{k_1}^{\mathsf{p}}X_{k_2}^{\mathsf{p}}\sin^2(\pi\epsilon)e^{-j\pi(k_1-k_2)/N}}{N^2\phi(k_1,i)\phi(k_2,i)} \quad (19)$$

where $\phi(k,i) = \sin[\pi(k-i+\epsilon)/N]$.

For subcarrier i, the BER of BPSK modulated OFDM signal with imperfect channel estimate can be evaluated by Lemma 1 as

$$P_b(i) = \frac{1}{2}\left[1 - \frac{\mathrm{Re}[\mu_{Y_i\hat{H}_i|X_i^{\mathsf{d}}}]}{\sqrt{\mu_{Y_iY_i|X_i^{\mathsf{d}}}\mu_{\hat{H}_i\hat{H}_i|X_i^{\mathsf{d}}} - (\mathrm{Im}[\mu_{Y_i\hat{H}_i|X_i^{\mathsf{d}}}])^2}}\right], \quad (20)$$

where $\mu_{Y_i\hat{H}_i|X_i^{\mathsf{d}}}$, $\mu_{Y_iY_i|X_i^{\mathsf{d}}}$ and $\mu_{\hat{H}_i\hat{H}_i|X_i^{\mathsf{d}}}$ are given in (14), (17), and (18) with $X_i^{\mathsf{d}} = \sqrt{E_b}$. Finally, the average BER over all subcarriers is

$$P_b = \frac{1}{N}\sum_{i=0}^{N-1}P_b(i). \quad (21)$$

B. QPSK

The constellation of QPSK is denoted by

$$\mathcal{X} = \left\{\frac{[(2i-1)+(2q-1)j]\sqrt{E_s}}{\sqrt{2}}, i = 0,1,\ q = 0,1\right\}, \quad (22)$$

where E_s is the symbol energy. Two information bits are mapped into a QPSK constellation symbol by the Gray encoding [9] as shown in Fig. 1(a).

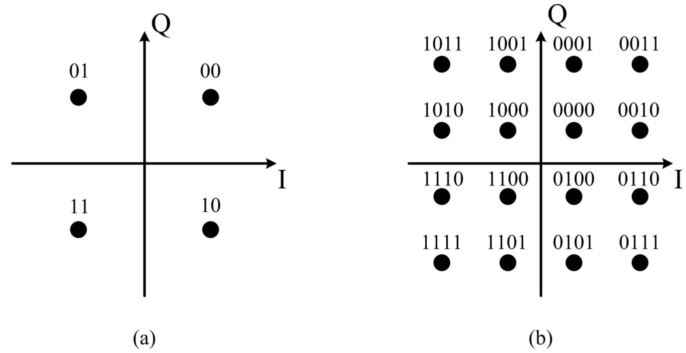

Fig. 1. (a) QPSK constellation with Gray encoding. (b) 16-QAM constellation with Gray encoding.

Since the channel estimation is not perfect, the constellation of the demodulated signal is scaled and rotated. To compute the BER of the most significant bit (MSB) of the constellation symbol, we consider two constellation symbols $\frac{(1+j)\sqrt{E_s}}{\sqrt{2}}$ and $\frac{(-1+j)\sqrt{E_s}}{\sqrt{2}}$ were sent since they have different BERs due to imperfect CSI. This is different from the perfect CSI case where usually only one constellation symbol is considered to be sent due to the symmetry of constellation and decision boundary.

From Fig. 1(a), it is obviously the decision boundary for the MSB of the QPSK symbol is the real axis and the BER of the MSB of the subcarrier i is

$$\begin{aligned}P_{b1}(i) &= \frac{1}{2}\left(P\left\{\mathrm{Im}[Y_i\hat{H}_i^*] < 0|X_i^{\mathsf{d}} = \frac{(1+j)\sqrt{E_s}}{\sqrt{2}}\right\}\right. \\ &+ \left.P\left\{\mathrm{Im}[Y_i\hat{H}_i^*] < 0|X_i^{\mathsf{d}} = \frac{(-1+j)\sqrt{E_s}}{\sqrt{2}}\right\}\right) \quad (23)\end{aligned}$$

Similarly, to compute the BER of the least significant bit (LSB) of the constellation symbol, we consider two constellation symbols $\frac{(1+j)\sqrt{E_s}}{\sqrt{2}}$ and $\frac{(1-j)\sqrt{E_s}}{\sqrt{2}}$ were sent since they have different BERs due to imperfect CSI. The decision boundary for the LSB of the QPSK symbol is the imaginary axis and the BER of the LSB of the subcarrier i is

$$\begin{aligned}P_{b2}(i) &= \frac{1}{2}\left(P\left\{\mathrm{Re}[Y_i\hat{H}_i^*] < 0|X_i^{\mathsf{d}} = \frac{(1+j)\sqrt{E_s}}{\sqrt{2}}\right\}\right. \\ &+ \left.P\left\{\mathrm{Re}[Y_i\hat{H}_i^*] < 0|X_i^{\mathsf{d}} = \frac{(1-j)\sqrt{E_s}}{\sqrt{2}}\right\}\right). \quad (24)\end{aligned}$$

Finally, the average BER over all subcarriers is

$$P_b = \frac{1}{2N}\sum_{i=0}^{N-1}[P_{b1}(i) + P_{b2}(i)]. \quad (25)$$

Conditioning on X_i, the random variables Y_i and $\hat{H}_i$ are also Gaussian because they are the weighted sum of Gaussian random variables. Therefore, we can utilize Lemma 1 to compute $P_{b1}(i)$ and $P_{b2}(i)$ where $\mu_{Y_i\hat{H}_i|X_i^{\mathsf{d}}}$, $\mu_{Y_iY_i|X_i^{\mathsf{d}}}$, and $\mu_{\hat{H}_i\hat{H}_i|X_i^{\mathsf{d}}}$ are given in (14), (17), and (18) with possible X_i^{d} in (23) and (24).

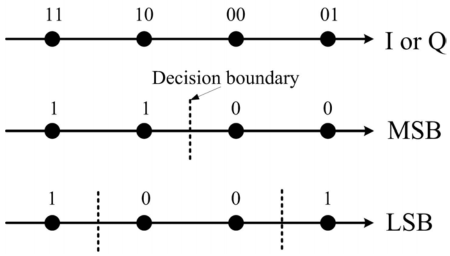

Fig. 2. 16-QAM bit-by-bit demapping.

C. 16-QAM

The 16-QAM constellation with Gray encoding is shown in Fig. 1(b). The first and third bits correspond to the inphase (I) bits, while the second and fourth bits correspond to the quadrature (Q) bits. The I and Q components of the 16-QAM symbols are Gray encoded by assigning the bits 11, 10, 00, and 01 to the levels $-3d$, $-d$, d, and $3d$ where $d = \sqrt{E_s/10}$.

Since we are interested in the evaluation of BER, we need to determine the decision boundary for each bit first. In Fig. 2, the decision boundaries for the MSB and LSB of the I/Q components are depicted [12]. Due to the symmetry of I and Q components, we only need to calculate the BER for I components.

Let $\mathcal{X}$ be the constellation of 16-QAM, $\mathcal{X}_1$ be the set of the four 16-QAM constellation symbols having d as their I-component, i.e. $\mathcal{X}_1 = \{x \in \mathcal{X} : \mathrm{Re}[x] = d\}$. Similarly, let $\mathcal{X}_2$ be the set of the four 16-QAM constellation symbols having $3d$ as their I-component, i.e. $\mathcal{X}_2 = \{x \in \mathcal{X} : \mathrm{Re}[x] = 3d\}$. Since the decision boundary for the MSB bit is the imaginary axis, for subcarrier i, the BER of the MSB bit of I components is given by

$$P_b^{\mathrm{MSB}}(i) = \frac{1}{8} \sum_{X_i^{\mathsf{d}} \in \mathcal{X}_1 \cup \mathcal{X}_2} P\left\{ \mathrm{Re}[Y_i \hat{H}_i^*] < 0 | X_i^{\mathsf{d}} \right\} \tag{26}$$

On the other hand, the decision boundaries for the LSB bit are $I = -2d$ and $I = 2d$ on the I-Q plane. For subcarrier i, the BER of the LSB bit of I components is

$$P_b^{\mathrm{LSB}}(i) = \frac{1}{8} \sum_{X_i^{\mathsf{d}} \in \mathcal{X}_1} \left[1 - P\left(-2d < \frac{\mathrm{Re}[Y_i \hat{H}_i^*]}{|\hat{H}_i|^2} < 2d | X_i^{\mathsf{d}} \right) \right] + \frac{1}{8} \sum_{X_i^{\mathsf{d}} \in \mathcal{X}_2} P\left(-2d < \frac{\mathrm{Re}[Y_i \hat{H}_i^*]}{|\hat{H}_i|^2} < 2d | X_i^{\mathsf{d}} \right) \Bigg]. \tag{27}$$

The average BER of 16-QAM with imperfect CSI is

$$P_b = \frac{1}{2N} \sum_{i=0}^{N-1} \left[P_b^{\mathrm{MSB}}(i) + P_b^{\mathrm{LSB}}(i) \right]. \tag{28}$$

Conditioning on the transmitted data symbol X_i^{d}, Y_i and $\hat{H}_i$ are both Gaussian. Therefore, we can use Lemma 1 to compute $P_b^{\mathrm{MSB}}(i)$ directly where $\mu_{Y_i \hat{H}_i | X_i^{\mathsf{d}}}$, $\mu_{Y_i Y_i | X_i^{\mathsf{d}}}$, and $\mu_{\hat{H}_i \hat{H}_i | X_i^{\mathsf{d}}}$ are given in (14), (17), and (18) with $X_i^{\mathsf{d}} \in \mathcal{X}_1 \cup \mathcal{X}_2$. However, the BER formula of $P_b^{\mathrm{LSB}}(i)$ is not in the exact form of Lemma 1.

To apply Lemma 1 to compute $P_b^{\mathrm{LSB}}(i)$, we need to transform the random variable Y_i into a new random variable $\hat{Y}_i$ so that Lemma 1 is applicable for $\hat{Y}_i$ and $\hat{H}_i$. To be more specific, we consider to compute the following probability

$$f(X_i^{\mathsf{d}}, D) = P\left\{ \mathrm{Re}[Y_i \hat{H}_i^*] < |\hat{H}_i|^2 D | X_i^{\mathsf{d}} \right\}. \tag{29}$$

Let $\hat{Y}_i = Y_i - \hat{H}_i D = H_i X_i^{\mathsf{d}} + W_i - \hat{H}_i D$, then

$$f(X_i^{\mathsf{d}}, D) = P\left\{ \mathrm{Re}[\hat{Y}_i \hat{H}_i^*] < 0 | X_i^{\mathsf{d}} \right\}. \tag{30}$$

To apply Lemma 1, we first compute

$$\mu_{\hat{Y}_i \hat{H}_i | X_i^{\mathsf{d}}} = E[\hat{Y}_i \hat{H}_i^*] = E[H_i \hat{H}_i^*] X_i^{\mathsf{d}} - E[\hat{H}_i \hat{H}_i^*] D, \tag{31}$$

where

$$E[H_i \hat{H}_i^*] = \alpha^* + E[H_i \hat{I}_i^*], \tag{32}$$

and $E[\hat{H}_i \hat{H}_i^*]$ has been given in (18). Then we compute

$$\mu_{\hat{Y}_i \hat{Y}_i | X_i^{\mathsf{d}}} = E[(Y_i - \hat{H}_i D)(Y_i - \hat{H}_i D)^* | X_i^{\mathsf{d}}] = |X_i^{\mathsf{d}}|^2 + N_0 + E[\hat{H}_i \hat{H}_i^*] |D|^2 - 2\mathrm{Re}(E[\hat{H}_i D H_i (X_i^{\mathsf{d}})^*]). \tag{33}$$

Once the computation of $f(X_i^{\mathsf{d}}, D)$ based on Lemma 1 is done, we can express $P_b^{\mathrm{LSB}}(i)$ in terms of $f(X_i^{\mathsf{d}}, D)$ as

$$P_b^{\mathrm{LSB}}(i) = \frac{1}{8} \left(\sum_{X_i^{\mathsf{d}} \in \mathcal{X}_1} \left[1 - f(X_i^{\mathsf{d}}, 2d) + f(X_i^{\mathsf{d}}, -2d) \right] + \sum_{X_i^{\mathsf{d}} \in \mathcal{X}_2} \left[f(X_i^{\mathsf{d}}, 2d) - f(X_i^{\mathsf{d}}, -2d) \right] \right). \tag{34}$$

IV. Numerical Results

In this section, we present some simulation results to validate our theoretical analysis.

A. Simulation Setup

We consider an OFDM system with $N = 64$ subcarriers. The effective OFDM symbol period is $T = 3.2$ us and the subcarrier frequency spacing f_s is 312.5 kHz. The received signal is sampled at the rate of 20 MHz. The power delay profile of the multipath Rayleigh fading channel is exponentially decaying and the root mean square (rms) delay spread is equal to 100 ns. We also assume the channel is fixed for whole frame and is independent from frame to frame. These parameters and assumptions are typical for the indoor WLAN applications.

The OFDM training symbol consists of 64 subcarriers, which are modulated by the BPSK symbol of the sequence $S = \sqrt{E_b}[S_{0,15}\ S_{16,31}\ S_{32,47}\ S_{48,63}]$, where

$$S_{0,15} = [-1,-1,1,1,-1,1,1,-1,1,1,-1,1,-1,1,-1,1],$$

$$S_{16,31} = [1,1,-1,-1,1,-1,1,1,-1,1,1,-1,-1,1,-1,1],$$

$$S_{32,47} = [1,1,1,1,1,-1,-1,-1,-1,1,-1,1,-1,1,1,-1],$$

$$S_{48,63} = [-1,-1,-1,1,1,1,1,1,1,-1,-1,-1,1,-1,1,-1].$$

The training pattern S is selected randomly under the peak-to-average power constraint, we do not try to optimize the training pattern S at this moment.

B. Results

Figs. 3 and 4 show the effects of channel estimation error on the BER performance of BPSK and 16-QAM modulated OFDM signals in multipath Rayleigh fading channels, respectively. The number of training symbols P used for channel estimation is 1. The solid lines are obtained from computer simulation and the plot symbols are computed from our theoretical results. The horizontal axis represents the modulated data symbol SNR E_s/N_0. We assume the number of the OFDM data symbols are much greater than that of the OFDM training symbols in one frame, hence the loss of power in the OFDM training symbols is negligible. Since the subcarriers of training OFDM symbols is BPSK modulated, the bit SNR E_b/N_0 and symbol SNR E_s/N_0 is related by $\frac{E_b}{N_0}\log_2|\mathcal{X}| = \frac{E_s}{N_0}$ where $|\mathcal{X}|$ is the size of the constellation.

From Figs. 3 and 4, it is evident the theoretical analysis exactly matched with the simulation results for different normalized CFO ϵ. When the CFO is perfectly compensated in the channel estimation stage (i.e. $\epsilon = 0$), the performance loss due to imperfect CSI is about 3 dB and 7 dB for BPSK and 16-QAM, respectively. As the normalized CFO ϵ increases, the channel estimate becomes less reliable and the BER performance becomes worse. Due to the effect of ICI created by the CFO, there exist error floors when E_b/N_0 is large. Finally, by examining those three figures closely, we find the performance degradation due to channel estimation error is more severe in high-order modulation than in BPSK. That implies the high-order modulation like 16-QAM needs more accurate CFO and channel estimation to avoid performance loss.

V. Conclusions

We investigated the effects of channel estimation error on the BER performance of OFDM systems in multipath fading channels. For BPSK, QPSK, and 16-QAM modulated OFDM signals, we derived the BER formula characterizing the performance degradation due to imperfect channel estimation. Computer simulation was conducted to verify the accuracy of our theoretical results. From the BER expression, we learn the BER is dependent on the patterns of training symbols. The design of optimal training sequence in the sense of minimal BER is left for further study.

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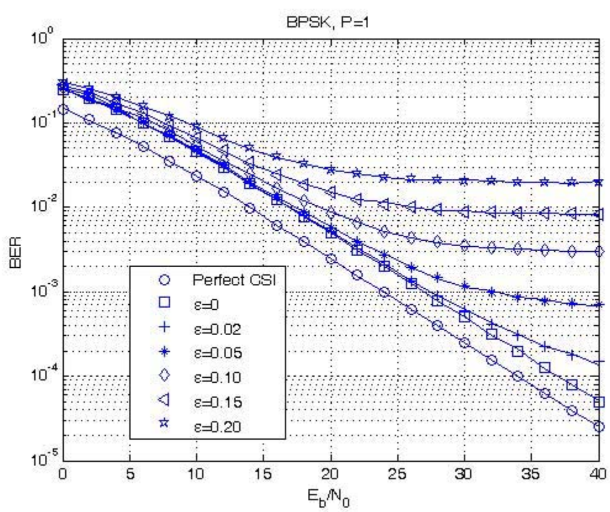

Fig. 3. Effect of channel estimation error on the BER of BPSK modulated OFDM signals in multipath Rayleigh fading channels.

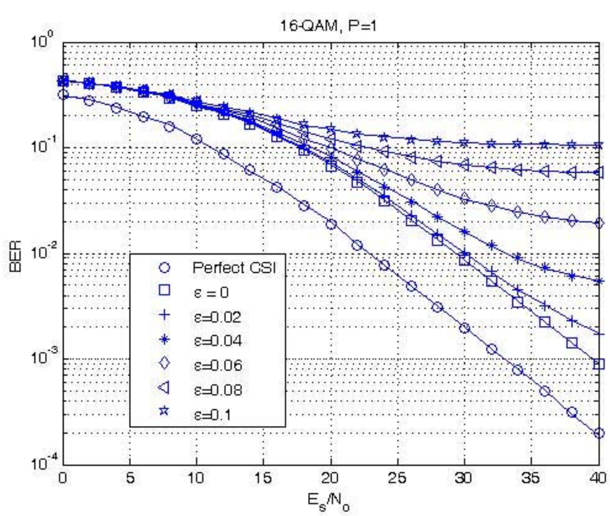

Fig. 4. Effect of channel estimation error on the BER of 16-QAM modulated OFDM signals in multipath Rayleigh fading channels.