Correlation between ICI and the Carrier Signal in OFDM under Doppler Spread Influence

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Abstract

In this paper, we derive an exact expression for the power correlation between the inter-carrier interference (ICI) and the carrier signal and show that the correlation is non-negligible for normal Doppler spread values. However, the correlation tends to vanish as Doppler spread approaches infinity. Then, the symbol error rates (SERs) for *M*-QAM OFDM systems over frequency-selective Ricean fading channels based on both the correlated and the uncorrelated ICI models are given for comparison.

1. Introduction

Unlike the additive white Gaussian noise (AWGN), the ICI in an orthogonal frequency division multiplexing (OFDM) over fading channels is correlated with the desired carrier term hence also is the ICI power correlated with the desired carrier power. However, many researchers assume the ICI to be uncorrelated with the carrier term [1]-[3]. In [4], symbol error rate (SER) performance for QAM OFDM systems in frequency-selective fast Ricean fading channels is given for both the correlated and uncorrelated ICI models. It is shown that the difference in SER for the two models will widen as the specular component is increased. However, in [4], no detailed correlation analysis is given. In this paper, we shall carry out such an analysis and derive an exact expression for the covariance between the ICI power and the desired carrier power as a function of Doppler spread. We reach the conclusion that *the ICI power will only become uncorrelated with the desired carrier power as Doppler spread approaches infinity. In the normal practical range of Doppler spread, the ICI and the carrier terms are actually correlated.*

The paper is organized as follows. Section 2 presents the detailed ICI analysis. Then, Section 3 gives numerical results. Finally, Section 4 makes the conclusion.

2. ICI Analysis

Assuming the OFDM system uses *N*-point discrete Fourier transform (DFT) and ignoring the cyclic prefix, then the transmitted baseband data sequence is given as

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi nk/N}, \quad 0 \le n \le N-1, \quad (1)$$

where X_k is the *k*th sub-carrier data symbol in the frequency domain.

For frequency-selective channels, the baseband discrete channel frequency response is given by

$$H_k = \sum_{m=0}^{L-1} h_m(n) e^{-j2\pi mk/N}, k = 0,1,\ldots,N-1, \quad (2)$$

where $h_m(n)$ is the discrete-time channel impulse response or tap gain. The time index *n* is used to account for time variation. For the Ricean fading model, h_0 is a non-zero mean Gaussian random variable (RV) and the amplitude $|h_0|$ is a Rice RV. Define the expectation $E[h_0] = \overline{h}_0$. Then, $h_0'(n) = h_0(n) - \overline{h}_0$ is a zero mean complex Gaussian RV or $|h_0'(n)|$ is a Rayleigh RV. The rest of the taps $\{h_m, m \ne 0\}$ are zero mean Gaussian RVs and $\{|h_m|, m \ne 0\}$ are Rayleigh RVs.

At the receiver, the noiseless received signal is given by

$$y_n = \sum_{m=0}^{L-1} h_m(n) x_{n-m}, \quad n = 0,1,\ldots,N-1. \quad (3)$$

The convolution sum in (3) is circular, i.e., the index $n-m$ for x_{n-m} is of modulo *N*. Taking the DFT of (3), we obtain the frequency-domain output as

$$\begin{aligned} Y_k &= \sum_{n=0}^{N-1} y_n e^{-j2\pi nk/N} \\ &= \frac{1}{N}\sum_{n=0}^{N-1} H_k(n) X_k \\ &\quad + \frac{1}{N}\sum_{n=0}^{N-1}\sum_{\substack{p=0\\p\ne k}}^{N-1} H_p(n) X_p e^{j2\pi n(p-k)/N}, \\ &\qquad k = 0,1,\ldots,N-1. \end{aligned} \quad (4)$$

The first term in (4) is the contribution from the *k*th subcarrier and is the desired carrier term. The second term is ICI. For a fixed channel realization, the desired carrier power can be readily obtained from the first term of (4) as

$$P_{\mathrm{C},k}=\frac{\sigma_X^2}{N^2}\left|\sum_{n=0}^{N-1}H_k(n)\right|^2, \tag{5}$$

and the ICI power from the second term of (4) is

$$P_{\mathrm{ICI},k}=\frac{\sigma_X^2}{N^2}\sum_{\substack{p=0\\p\neq k}}^{N-1}\left|\sum_{n=0}^{N-1}H_p(n)e^{j2\pi n(p-k)/N}\right|^2, \tag{6}$$

where $\sigma_X^2=E[|X_k|^2]$ is the transmitted signal power. From (2), both $P_{\mathrm{C},k}$ and $P_{\mathrm{ICI},k}$ contains $\{h_m(n)\}$, they are obviously correlated when the channels are fading.

Averaging over all channel realizations, the average received power of the *k*th sub-carrier can be calculated as

$$\begin{aligned}E[P_{\mathrm{C},k}]&=E[P_{\mathrm{C}}]\\&=\sigma_X^2|\bar{h}_0|^2+\frac{\sigma_X^2}{N^2}\Big[\sum_{m=0}^{L-1}\sigma_m^2\Big]\Big[N+2\sum_{i=1}^{N-1}(N-i)\rho_i\Big]\end{aligned} \tag{7}$$

where $\sigma_0^2=E[|h_0'|^2]$ and $\sigma_m^2=E[|h_m|^2]$, $m\neq 0$, and ρ_i is the classical correlation coefficient for Rayleigh fading given as [5]

$$\begin{aligned}\rho_i&=J_0(2\pi i f_M T/N)\\&=\begin{cases}E[h'_0(n)h'^*_0(n-i)]/\sigma_0^2, & m=0,\\ E[h_m(n)h_m^*(n-i)]/\sigma_m^2, & m=1,2,\ldots,L-1.\end{cases}\end{aligned} \tag{8}$$

Note that $\rho_0=1$ always. In (8), $\sigma_0^2=E[|h_0'|^2]$, f_M is maximum Doppler frequency (in Hz), T is one OFDM block length (in sec.), and $J_0(x)$ is the zeroth order Bessel function of the first kind [5]. Similarly, the average ICI power can be also calculated as

$$\begin{aligned}E[P_{\mathrm{ICI},k}]&=E[P_{\mathrm{ICI}}]\\&=\sigma_X^2\Big[\sum_{m=0}^{L-1}\sigma_m^2\Big]\Big\{1-\frac{1}{N^2}\Big[N+2\sum_{i=1}^{N-1}(N-i)\rho_i\Big]\Big\}.\end{aligned} \tag{9}$$

When $\rho_i=1$ (slow fading or zero Doppler), (9) becomes zero implying zero ICI. Then, as $\rho_i\to 0$, $i\neq 0$, (very fast fading or $f_M\to\infty$), it is easily shown that the carrier power of (7) and the ICI power of (9) will both level to a constant (different constant values for different specular component values).

To find the correlation between the ICI power and the desired carrier power, it suffices to find the power covariance defined as

$$\begin{aligned}&Cov[P_{\mathrm{C},k},P_{\mathrm{ICI},k}]\\&=E[P_{\mathrm{C},k}P_{\mathrm{ICI},k}]-E[P_{\mathrm{C},k}]E[P_{\mathrm{ICI},k}].\end{aligned} \tag{10}$$

Using two formulas derived from the moment theorem for complex Gaussian processes [6] given as

$$E[h_m(n_1)h_m^*(n_2)h_m(n_3)]=0, \tag{11a}$$

$$\begin{aligned}&E[h_m^*(n_1)h_m^*(n_2)h_m(n_3)h_m(n_4)]\\&=E[h_m^*(n_1)h_m(n_3)]E[h_m^*(n_2)h_m(n_4)]\\&\quad+E[h_m^*(n_2)h_m(n_3)]E[h_m^*(n_1)h_m(n_4)],\end{aligned} \tag{11b}$$

then with great patience, we can show that (10) can be calculated to yield the result as

$$\begin{aligned}&Cov[P_{\mathrm{C},k},P_{\mathrm{ICI},k}]=\\&\frac{\sigma_X^4}{N^3}\sum_{m_1=0}^{L-1}\sum_{m_2=0}^{L-1}\sum_{n_1=0}^{N-1}\sum_{n_2=0}^{N-1}\sum_{n_3=0}^{N-1}\sum_{n_4=0}^{N-1}\sigma_{m_1}^2\sigma_{m_2}^2\rho_{n_1-n_4}\rho_{n_2-n_3}\Big|_{m_1-m_2-n_4+n_3=-N,0,N}\\&-\frac{\sigma_X^4}{N^4}\Big[\sum_{m_1=0}^{L-1}\sigma_{m_1}^2\Big]^2\Big[N+2\sum_{n=1}^{N-1}(N-n)\rho_n\Big]^2.\end{aligned} \tag{12}$$

Equation (12) is for frequency-selective channels where $L>1$. For frequency-nonselective channels where $L=1$, the first term of (12) must be modified to be conditioned only on $m_1-m_2-n_4+n_3=0$ as this term cannot possibly equal $\pm N$.

We also can see that (12) is independent of *k*. If $f_M\to\infty$, thus $\rho_i\to 0$, $i=1,2,\ldots,L$ (Note again that $\rho_0=1$ always), (12) can be shown to reduce to

$$\begin{aligned}&Cov[P_{\mathrm{C},k},P_{\mathrm{ICI},k}]=\\&\frac{\sigma_X^4}{N^4}\sum_{\substack{p=0\\p\neq k}}^{N-1}\left|\sum_{m=0}^{L-1}e^{j2\pi m(p-k)/N}\sigma_m^2\right|^2\left|\sum_{n=0}^{N-1}e^{-j2\pi n(p-k)/N}\right|^2=0.\end{aligned} \tag{13}$$

Equation (13) tells us that, as Doppler spread approaches infinity ($\rho_i\to 0$), the ICI power becomes uncorrelated with the desired carrier power.

3. Simulation Results

Normalizing the transmitted signal power and the channel power, and further assuming an exponential channel power, simulation result for $N=128$ and $L=16$ shows that the covariance of (12) has a maximum value $Cov[P_{\mathrm{C},k},P_{\mathrm{ICI},k}]_{\max}$ at $f_MT=0.7$. The normalized covariance $Cov[P_{\mathrm{C},k},P_{\mathrm{ICI},k}]/Cov[P_{\mathrm{C},k},P_{\mathrm{ICI},k}]_{\max}$ is plotted against normalized Doppler spread f_MT in Fig. 1. A magnified plot will show that, after about $f_MT=1.9$, the normalized covariance will go down below 0.1 and eventually decays to zero as Doppler spread goes to infinity. Taking an IEEE 802.11a standard with $f_c=5$ GHz and $\Delta f=1.25$ MHz, the value of $f_MT=1.9$ corresponds to a vehicular speed of 515km/hr. Thus for a normal vehicular speed (usually less than 515km/hr), the Doppler spread f_MT is within 1.9. From Fig. 1, we see that ICI is indeed correlated with the carrier term in the region $f_MT<1.9$.

Take a fixed transmitted signal-to-noise ratio SNR $\sigma_X^2 / \sigma_Z^2 = 25$ dB, where σ_Z^2 is the additive noise power. Then using a frequency-selective channel having exponential power profile and dispersion length $L = 4$, we present in Fig. 2 the simulation results of SER P_M vs. average received carrier-to-ICI plus noise ratio (CINR) $\bar{\gamma} = \frac{1}{N}\sum_{k=0}^{N-1}\bar{\gamma}_k$ for 16-QAM OFDM with $N = 64$ and $K_R = -\infty$ (Rayleigh fading), 9, and 17 dB. Both correlated and uncorrelated ICI models are considered. For correlated model, the *k*th subcarrier average CINR $\bar{\gamma}_k$ is defined as

$$\bar{\gamma}_k = E\left[\frac{P_{\mathrm{S},k}}{P_{\mathrm{ICI},k} + \sigma_Z^2}\right], \qquad (14)$$

while for the uncorrelated model as

$$\bar{\gamma}_k = \frac{E[P_{\mathrm{S},k}]}{E[P_{\mathrm{ICI},k}] + \sigma_Z^2}. \qquad (15)$$

Equation (15) is the model used in [2] and [3]. For a given Doppler spread f_M (given $\bar{\gamma}_k$), over 100,000 channel realizations (equivalently 100,000 realizations of $\bar{\gamma}_k$) have been used for averaging. In Fig. 2, we see that the correlated and uncorrelated ICI models do yield different results. The difference or gap widens as K_R is increased. For Rayleigh fading ($K_R = -\infty$ dB), the gap is minimal and hence the use of the uncorrelated ICI approximation can be considered justifiable. However, for Ricean fading, especially when the K_R factor is large (large specular component), the uncorrelated ICI approximation becomes poor. For example, in Fig. 2, at CINR$= 20$ dB for the large $K_R = 17$ dB, the correlated ICI model shows an SER around 2×10^{-3} while the uncorrelated ICI model shows an SER around 10^{-4}, a difference about an order of magnitude. It is conceivable that the uncorrelated ICI model should yield better but inaccurate results (lower SER) since the ICI has been treated like AWGN in (15) thus equivalent to no fading effect.

4. Conclusion

The power correlation between the ICI and the desired carrier term is derived for OFDM systems in frequency-selective fast Ricean fading channels under the influence of Doppler spread. It is found that, for Doppler spreads arising from normal mobile speeds, the power correlation is non-negligible. The correlation will approach zero as the Doppler spread approaches infinity. The SER performances based on correlated and uncorrelated ICI models are shown to yield different results.

5. References

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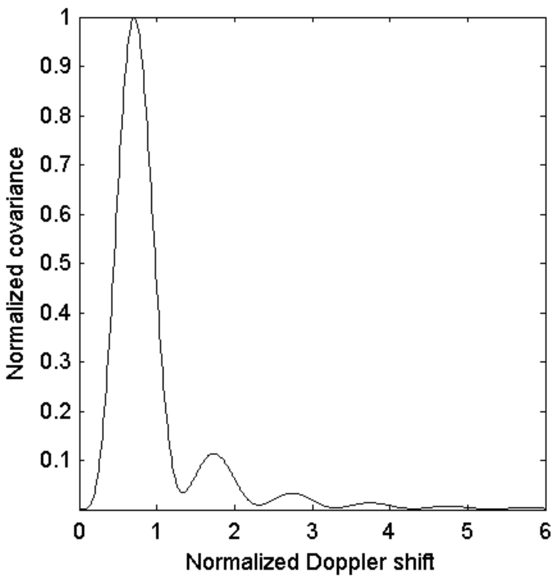

Figure 1: Covariance between carrier and ICI power vs. Normalized Doppler spread. Exponential channel power profile, $N = 128$, $L = 16$.

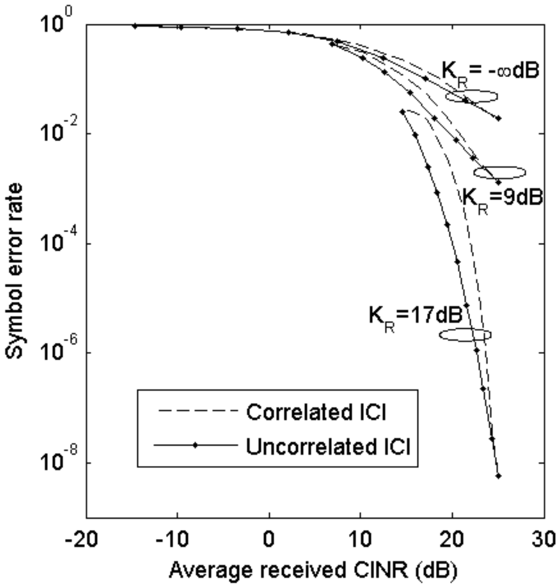

Figure 2: SER vs. average received CINR for 16-QAM OFDM for both correlated and uncorrelated ICI models. $N = 64$, exponential channel power profile with $L = 4$, fixed SNR 25 dB.