# Modeling and Control for a Thermal Barrel in Plastic Molding Processes

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## Abstract

In this paper, a new approach is proposed to model and control the temperature of a thermal barrel of a plastic molding machine. Usually the thermal barrel behavior is expressed in terms of a parameterized linear model to be used in control strategies design. We establish a new model based on the structure of a Takagi-Sugeno fuzzy system, and utilize the clustering method to generate the rule base of the fuzzy system. The proposed methodology is shown to be more effective than a conventional method in constructing system models. Meanwhile, the developed fuzzy model may provide a more accurate output prediction than conventional linear models suggested in the literature. In order to evaluate the control performance, the thermal models are integrated into the Internal Model Controller to control the temperature of a thermal barrel. The system is subjected to a step input and the responses depict the control performance of the models. The fuzzy model shows excellent performance in the step response, while the linear model has an oscillatory output at steady state. The proposed fuzzy model has the capability of application to control temperature in a plastic molding process.

***Key Words***: Fuzzy Systems, Thermal Control, Multivariable Control

## 1. Introduction

Precision temperature control is a key factor in a plastic molding process in order to manufacture high valued-add products, for example, optical glasses, CDs, and DVDs etc. In the literature, many researchers have modeled the thermal processes of plastic extrusion and injection molding by a first-order discrete equation [1−4], and the temperature distribution was usually expressed in terms of a parameterized linear model to be used in control strategies design; Kochhar and Parnaby [5] established a time series equation to model the relationship between the heat rate of the heater, speed of the rotating screw, and temperature and pressure of the melted polymer at the outlet of mold. Tsai and Lu [4] created a linear discrete model for the thermal barrel and identified the coefficients of the model using the Recursive Least-Squared Error (RLSE) method. Due to the properties of nonlinearity and complexity in a plastic molding process, the linear thermal model provides accurate outputs only in some application range and is not appropriate for precision process control. On the other hand, a molding process with a nonlinear model costs too much time in calculation and is not applicable for on-line control. The paper focuses on the analysis and modeling of thermal behavior in the thermal barrel, and a simple and effective model is developed for the thermal control in plastic molding processes.

Fuzzy systems are simple and approximate models, which can highly represent nonlinear systems and easily integrate *a priori* knowledge of data obtained from the processes. This research investigates the input-output data of the thermal barrel, and proposes an approach based on fuzzy theory to model the thermal behavior of the barrel. The Takagi-Sugeno (TS) fuzzy system [6,7] is employed

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to represent the system output as a linear combination of the input data. The number of fuzzy rules in the rule base of the TS fuzzy model decides the performance of the system. If the fuzzy system contains too many rules than required, the system becomes a complex system, and spends too much time on calculation during on-line control. On the other hand, if the system has too few rules, it would not have enough information to model the real system. Many researchers have proposed techniques to generate rule bases of fuzzy systems [8]. Most of the procedures first decide the number of rules, and then determine the system model using a method such as RLSE, gradient descent, or table look-up. However, these methods strongly depend on the experience and knowledge about the system. The paper utilizes another method named clustering-based method [8,9] to determine the number of rules using the input and output data. The basic concept is to divide the input and output data into several clusters, then assign a fuzzy rule to each cluster.

Several researchers had proposed algorithms for control of an injection molding process. Seaman *et al.* [10] used the concept of multiobjective optimization to tune a PID controller and meet multiple objectives in injection molding processes. Fang and Yao [11] designed a fuzzy controller to eliminate the couple influence of variables and showed that the controlled system has a good set-point characteristic. Zheng *et al.* [12] employed a feed-forward compensator and a fuzzy controller to reduce the coupling between system variables. Tsai and Lu [4] developed a self-tuning predictive control for improving set-point tracking performance and disturbance rejection. In order to compare the proposed fuzzy model with other approaches in the literature [4,13], the thermal models are integrated into Internal Model Control (IMC) [14] for temperature control of a thermal barrel in plastic molding processes. The system is subjected to a step input and the response will depict the dynamic performance of the models. The concept of IMC is to generate the command based on the inversion of the system model. If the mathematical model is accurate enough, the difference between the outputs of the mathematical model and the physical system will vanish. In this case, the IMC performs as an open loop controller and the transfer function of the system is a unity function. Otherwise, if the model is slightly different from the physical system, a large difference will exist between the IMC output and the reference input. Therefore, the IMC is an appropriate tool that can be used to evaluate the accuracy of a mathematical model.

This paper is organized as follows: In the second section, the theoretical basis of the modeling methods will be discussed. The developed method is applied to model a thermal barrel of plastic molding processes in the third section. In section IV, performance evaluation of the developed methodology is investigated. Some experimental results are shown to verify the proposed thermal model and control algorithm. The paper concludes with recommendations for future research work in the last section.

## 2. Modeling of Thermal Barrel

In plastic extrusion and injection molding processes, the particle polymer is fed into the thermal barrel and heated continuously. The configuration of a thermal barrel in plastic extrusion and injection molding machines are as shown in Figure 1. Several pairs of heaters are equipped around the surface of the thermal barrel to supply energy for the system.

The thermal barrel is divided into four different temperature zones and the thermo behavior of each zone is analyzed separately. The configuration of the thermal barrel with four temperature zones is depicted in Figure 2. The thermodynamic equation for each zone can be derived by the concept of thermal equilibrium of the system,

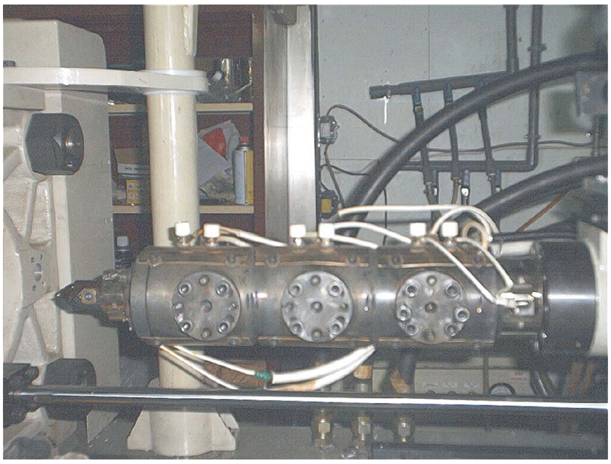

**Figure 1.** Thermal barrel of plastic product processes.

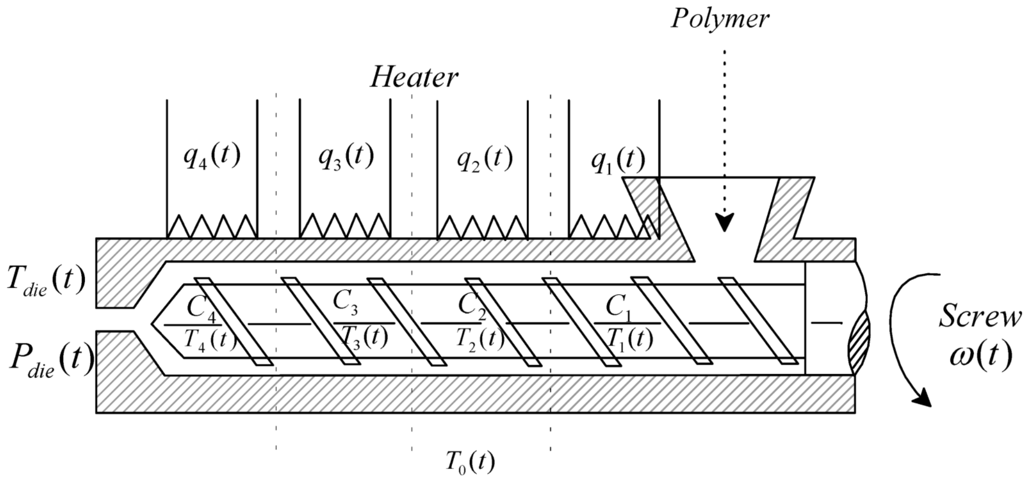


**Figure 2.** Model configuration of a thermal barrel.

$$C_i \frac{dT_i(t)}{dt} = q_i(t) - q_o(t) \tag{1}$$

where $C_i$ is the thermal capacitance of zone $i$; $T_i(t)$ is the temperature of zone $i$ in centigrade (C); $q_i(t)$ and $q_o(t)$ represent the input and output power, respectively. Note that Eq. (1) is a nonlinear function in general, so it is necessary to develop a simple and efficient model for the purpose of on-line control. In the literatures [4,13], a linear approximate model is frequently utilized for Eq. (1). However, the linear model has the disadvantage of inaccuracy and is not appropriate for precision process control. A new approach will be developed based on fuzzy systems to overcome this problem.

### 2.1 Linear Thermal Model

A linear discrete model for Eq. (1) can be derived as the following expression, and the details were shown in the literature [13].

$$T(k) = AT(k-1) + B_0U(k-d) + B_1U(k-1-d) + E(k) \tag{2}$$

where $T(k)$ represents the temperature at time $t = k$; $U(k)$ is the power transferred to the thermal barrel, which includes the power transferred from the heater to the barrel and that transferred from the barrel to the environment; $d$ is the delay time of the power transferred; $E(k)$ is the error term. For all four temperature zones in matrix form, Eq. (2) can be expressed as

$$\begin{bmatrix} T_1(k) \\ T_2(k) \\ T_3(k) \\ T_4(k) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} T_1(k-1) \\ T_2(k-1) \\ T_3(k-1) \\ T_4(k-1) \end{bmatrix} + \begin{bmatrix} b_{01} & 0 & 0 & 0 \\ 0 & b_{02} & 0 & 0 \\ 0 & 0 & b_{03} & 0 \\ 0 & 0 & 0 & b_{04} \end{bmatrix} \begin{bmatrix} u_1(k-d) \\ u_2(k-d) \\ u_3(k-d) \\ u_4(k-d) \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{12} & 0 & 0 \\ 0 & 0 & b_{13} & 0 \\ 0 & 0 & 0 & b_{14} \end{bmatrix} \begin{bmatrix} u_1(k-1-d) \\ u_2(k-1-d) \\ u_3(k-1-d) \\ u_4(k-1-d) \end{bmatrix} + \begin{bmatrix} e_1(k) \\ e_2(k) \\ e_3(k) \\ e_4(k) \end{bmatrix} \tag{3}$$

For the linear thermal model, the coefficients $a_{ij}(k)$ and $b_{ij}(k)$ in matrices, $A$, $B_0$, and $B_1$, can be determined by a parameter estimation method. Two methods can be utilized to identify the parameters, namely, pseudo-inverse matrix method [15] and RLSE method [16]. For the pseudo-inverse matrix method, it defines input, output, and coefficient vectors and matrices of the system as the following expressions:

$$Y = T(k) = \begin{bmatrix} T_1(k) & T_2(k) & T_3(k) & T_4(k) \end{bmatrix}^T$$

$$h = [a_{11} \; a_{12} \; a_{21} \; a_{22} \; a_{23} \; a_{32} \; a_{33} \; a_{34} \; a_{43} \; a_{44} \; b_{01} \; b_{02} \; b_{03} \; b_{04} \; b_{11} \; b_{12} \; b_{13} \; b_{14}]^T$$

$$W = \begin{bmatrix} T_1(k-1) & T_2(k-1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_2(k-1) & 0 & u_1(k-d) & 0 & 0 & 0 & u_1(k-1-d) & 0 & 0 & 0 \\ 0 & 0 & T_1(k-1) & T_2(k-1) & T_3(k-1) & 0 & 0 & 0 & 0 & 0 & T_3(k-1) & 0 & 0 & u_2(k-d) & 0 & 0 & 0 & u_2(k-1-d) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_4(k-1) & 0 & 0 & 0 & u_3(k-d) & 0 & 0 & 0 & u_3(k-1-d) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_3(k-1) & T_4(k-1) & 0 & 0 & u_4(k-d) & 0 & 0 & 0 & u_4(k-1-d) \end{bmatrix}$$

Eq. (3) can be rearranged as

$$Y = Wh \tag{4}$$

The coefficient vector, $h$, can be determined by using the pseudo-inverse matrix method

$$h = W^T (WW^T)^{-1} Y \tag{5}$$

On the other hand, the RLSE method also can be used to determine the system coefficients. Assume that the estimated values of parameters, $a_{ij}(k)$ and $b_{ij}(k)$, are denoted as $\hat{a}_{ij}(k)$ and $\hat{b}_{ij}(k)$, respectively. The RLSE method is described by the following equations [16],

$$\hat{\Phi}_i(k) = \hat{\Phi}_i(k-1) + \frac{P_i(k-1)\zeta_i^T(k)\left[T_i(k) - \zeta_i(k)\hat{\Phi}_i(k-1)\right]}{1+\zeta_i(k)P_i(k-1)\zeta_i^T(k)} \tag{6}$$

where $\hat{\Phi}_i(k)$ are the parameter vectors of Zone $i$ to be identified; $\zeta_i(k)$ are the matrices which contain input and output variables of Zone $i$; $P_i(k)$ are diagonal weight matrices,

$$P_i(k) = P_i(k-1) - \frac{P_i(k-1)\zeta_i^T(k)\zeta_i(k)P_i(k-1)}{1+\zeta_i(k)P_i(k-1)\zeta_i^T(k)}$$

The weight matrices have the initial values,

$$P_i(0) = I_n$$

where $\lambda$ is a very large positive constant, and $I_n$ is an identity matrix with rank $n$. $\hat{\Phi}_i(k)$ and $\zeta_i(k)$ represent the parameter vectors of the coefficients and input-output data, respectively. The parameter vectors for all the four temperature zones can be expressed as

$$\zeta_1(k) = [T_1(k-1) \quad T_2(k-1) \quad u_1(k-d) \quad u_1(k-1-d)]$$

$$\zeta_2(k) = [T_1(k-1) \; T_2(k-1) \; T_3(k-1) \; u_2(k-d) \; u_2(k-1-d)]$$

$$\zeta_3(k) = [T_2(k-1) \; T_3(k-1) \; T_4(k-1) \; u_3(k-d) \; u_3(k-1-d)]$$

$$\zeta_4(k) = [T_3(k-1) \quad T_4(k-1) \quad u_4(k-d) \quad u_4(k-1-d)]$$

$$\hat{\Phi}_1(k) = \left[\hat{a}_{11}(k) \quad \hat{a}_{12}(k) \quad \hat{b}_{01}(k) \quad \hat{b}_{11}(k)\right]^T$$

$$\hat{\Phi}_2(k) = \left[\hat{a}_{21}(k) \quad \hat{a}_{22}(k) \quad \hat{a}_{23}(k) \quad \hat{b}_{02}(k) \quad \hat{b}_{12}(k)\right]^T$$

$$\hat{\Phi}_3(k) = \left[\hat{a}_{32}(k) \quad \hat{a}_{33}(k) \quad \hat{a}_{34}(k) \quad \hat{b}_{03}(k) \quad \hat{b}_{13}(k)\right]^T$$

$$\hat{\Phi}_4(k) = \left[\hat{a}_{43}(k) \quad \hat{a}_{44}(k) \quad \hat{b}_{04}(k) \quad \hat{b}_{14}(k)\right]^T$$

Based on a group of input and output data, the model coefficients are estimated by using Eqs. (5) or (6). The estimated coefficients $\hat{a}_{ij}(k)$ and $\hat{b}_{ij}(k)$ can be substituted into Eq. (3) to get a linear approximate model for a nonlinear physical system. However, the linear approximate model provides accurate model output only in some range of the application. We will show the limitation of the linear model in the later section.

### 2.2 Fuzzy System Model

In order to improve the disadvantage of inaccuracy in the linear approximation model, A TS fuzzy system model is proposed based on the clustering method [8].

#### 2.2.1 Takagi-Sugeno Fuzzy System

Assume that there are a group of input-output data pairs, $\{x = (x(1), x(2),\ldots,x(p-1)); y(p) = x(p)\}$, in which $x(1), x(2),\ldots,x(p-1)$ represent ($p$-1) input variables of the system, and $x(p)$ is the output variable. For a TS fuzzy system with $c$ fuzzy rules, the rule base of the system can be described as the following expression [8]:

Rule $m$: if $x$ is $A_m(x)$ then

$$y_m = b_{m0} + \sum_{j=1}^{p-1} b_{mj} x(j) \quad m = 1,2,...,c \tag{7}$$

where $A_m(x)$ is the antecedent of the fuzzy rule; $b_{mj}$ are coefficients; $y_m$ is the consequent of the fuzzy rule. The multidimensional membership function $A_m(x)$ can be viewed as the fuzzification from the original input data; while the consequent of the fuzzy rule, $y_m$, is set to be a linear combination of the input variables. If the fuzzy system has $c$ fuzzy rules, the output of the TS fuzzy system can be obtained by the weighted average method [8],

$$y = \frac{\sum_{m=1}^{c} A_m(x) \cdot y_m}{\sum_{m=1}^{c} A_m(x)} = \sum_{m=1}^{c} g_m \cdot \left[ b_{m0} + \sum_{j=1}^{p-1} b_{mj} x(j) \right] \tag{8}$$

$$g_m = \frac{A_m(x)}{\sum_{j=1}^{c} A_j(x)} \quad , \quad m = 1,2,...,c$$

### 2.2.2 Clustering Method

As mentioned in the preceding section, the number of rules, $c$, in the rule base of the TS fuzzy model affects the performance of the system. If the fuzzy system contains more rules than it needs, the system becomes a complex system. On the contrary, if the system has too few rules, it would not have enough information to model the real system. By the clustering method, the number of rules is decided by adjusting the relational grade of the clusters. It will be defined latterly, the relational grade as a Gaussian function, which is based on the distance between two vectors. In the procedure of clustering methods, the collected data is divided into several subsets or strings of data according to the relational grades between these data. Therefore, each string of data has its own characteristics that can be distinguished from other data strings. The procedures to determine the structure of the fuzzy system are described as follows.

Assume a set of $n$ vectors in a $p$-dimensional space,

$$X = \{x_1, x_2, \ldots, x_n\}$$

where $x_i = (x_i(1), x_i(2), \ldots, x_i(p))$ is a vector with $p$ variables, in which $(x_i(1), x_i(2), \ldots, x_i(p-1))$ and $x_i(p)$ are input and output variables of $i^{th}$ data point, respectively. Among these $n$ vectors, the vectors which have a high relational grade can be collected to be a string named cluster. In this paper, the concept of similarity proposed by Wong [9] is utilized to determine the relational grade. According to the method, first select a data point as a reference vector, and then find a comparative vector that has a high relational grade with the reference vector. Further, choose the comparative vector with high relational grade as a new reference vector, and repeat the procedure. By this recursion method, the reference vector can be replaced during each cycle of the procedure, and eventually converges to the center of a cluster. The procedure is summarized in five recursive steps [9]:

**Step 1.**

Define $n$ movable vectors $v_i$ ($i$ = 1,2,…., $n$) and let $v_i = x_i$, where $x_i$ is the initial value of $v_i$;

**Step 2.**

Calculate the similarity by the following equation,

$$r_{ij} = \exp\left( -\frac{\left\| v_i - v_j \right\|^2}{2\sigma^2} \right), \; i = 1,2,...,n; \; j = 1,2,....,n \tag{9}$$

where $r_{ij}$ represents the relational grades between the reference vector $v_i$ and the comparative vector $v_j$; $\left\| v_i - v_j \right\|$ is the Euclidean distance between $v_i$ and $v_j$; and $\sigma$ is the width of the Gaussian function in Eq. (9);

**Step 3.**

Modify the relational grades between the reference vector $v_i$ and the comparative vector $v_j$ according to the following rule,

$$r_{ij} = \begin{cases} 0 & if \;\; r_{ij} < \xi \\ r_{ij} & \text{otherwise} \end{cases}$$

where $\xi$ is a small constant set up to be 0.01 in this paper;

**Step 4.**

Calculate a new vector set $v_i^{'} = (v_i^{'}(1), v_i^{'}(2), \ldots v_i^{'}(p))$, $i$ = 1,2,....,n,

$$v_i'(k) = \frac{\sum_{j=1}^{n} r_{ij} v_j(k)}{\sum_{j=1}^{n} r_{ij}} \quad , \; k = 1,2,...., p$$

**Step 5.**

If all the vectors $v_i^{'}$ are the same as $v_i$, $i = 1, 2, \ldots, n$, then stop; otherwise let $v_i = v_i^{'}$, $i = 1, 2, \ldots, n$, and go to Step 2.

By this procedure, the data points with high relational grades are collected as a cluster. The relational grade is modified in Step 3 to prevent the movable vector from being affected by the vectors with low relational grades. The movable vectors will gradually converge to a vector of values. Therefore, the number of convergent vectors is the number of clusters, and the convergent vector is viewed as the center of the corresponding cluster. By the clustering method, $n$ input-output data are divided into $c$ clusters,

$$c_m = \{c_m(1), c_m(2), \ldots, c_m(p)\},\ m = 1,2,\ldots,c$$

Once the cluster centers are determined, the antecedent proposition, $A_m(x_i)$, of the $i^{\text{th}}$ input, $x_i$, can be arranged according to the relationship between $i^{\text{th}}$ input data and $m^{\text{th}}$ cluster center. A Gauss function [8] is chosen to represent the membership function,

$$A_m(x_i) = \exp\left( -\frac{\sum_{k=1}^{p-1}(x_i(k) - c_m(k))^2}{2\delta_m^2} \right), \quad m = 1,2,\ldots,c; i = 1,2,\ldots,n \tag{10}$$

$$\delta_m = \sqrt{\frac{-\sum_{k=1}^{p-1}(x_m^*(k) - c_m(k))^2}{2\ln(\alpha)}}, \quad m = 1,2,\ldots,c$$

where $c_m$ is the $m^{\text{th}}$ clustering center; $\delta_m$ indicates the width of the Gaussian function in Eq. (10); $x_m^*$ is the most far away data point of the $m^{\text{th}}$ clustering data; $\alpha$ is a constant between 0 and 1.

### 2.2.3 Coefficient Estimation

Using the procedure of the clustering method, $n$ data points are distributed into $c$ clusters. According to Eq. (8), the output of the TS fuzzy system with $n$ input-output data points and $c$ cluster centers can be expressed as the following equations,

$$y_i = \sum_{m=1}^{c}\left[ g_{im}b_{m0} + g_{im}\sum_{j=1}^{p-1} b_{mj}x_i(j) \right], m = 1,2,\ldots,c; i = 1,2,\ldots n \tag{11}$$

$$g_{im} = \frac{A_m(x_i)}{\sum_{j=1}^{c} A_j(x_i)},\ m = 1,2,\ldots c;\ i = 1,2,\ldots,n$$

Define the vectors and matrices containing input and output variables as the following equations,

$$Y = [y_1\ \ y_2\ \ \cdots\ \ y_n]^T$$

$$h = [b_{10}\ \ b_{11}\ \ \cdots\ \ b_{1p-1}\ \ \cdots\ \ b_{c0}\ \ b_{c1}\ \ \cdots\ \ b_{cp-1}]^T$$

$$W = \begin{bmatrix} g_{11} & g_{11}x_1(1) & \cdots & g_{11}x_1(p-1) & \cdots & g_{1c} & g_{1c}x_1(1) & \cdots & g_{1c}x_1(p-1) \\ g_{21} & g_{21}x_2(1) & \cdots & g_{21}x_2(p-1) & \cdots & g_{2c} & g_{2c}x_2(1) & \cdots & g_{2c}x_2(p-1) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n1}x_n(1) & \cdots & g_{n1}x_n(p-1) & \cdots & g_{nc} & g_{nc}x_n(1) & \cdots & g_{nc}x_n(p-1) \end{bmatrix}$$

The coefficient vector, $h$, can be determined using the pseudo-inverse matrix method in Eq. (5) or the RLSE method in Eq. (6). Once the coefficients are determined, the fuzzy system in Eq. (11) can be used to model a nonlinear system.

## 3. Comparison of Model Accuracy

Output prediction of the mathematical models in Eqs. (3) and (11), namely the linear model and TS fuzzy model, are compared in this section. A representative model is established by measuring the temperature distribution with respect to the input power. These measured input-output data, i.e. the sampled temperature and the corresponding input power, are utilized to estimate the coefficients of models in Eqs. (3) and (11). By the definition of model accuracy, the difference between a mathematical model and the representative model should be as small as possible.

### 3.1 Sampled Temperature Data

In order to construct a representative model for the real process, an experiment was prepared for measuring the temperature in the thermal barrel of the plastic molding process as shown in Figure 3. In the setup,

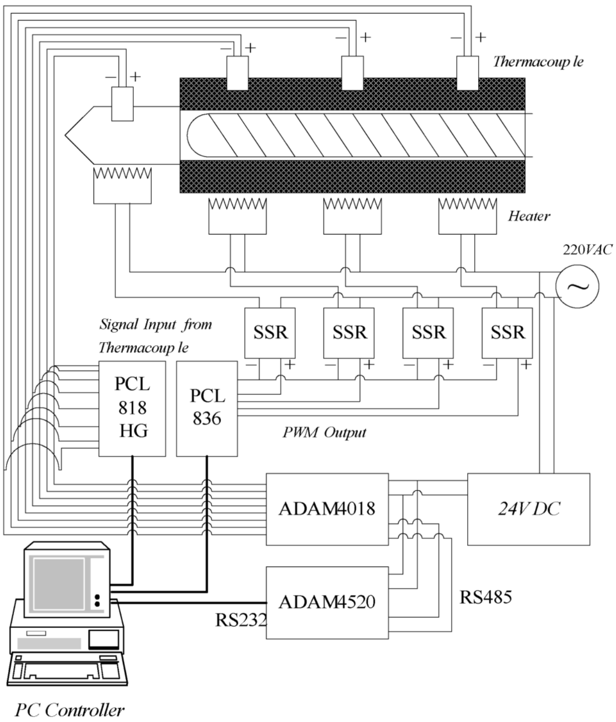


**Figure 3.** Temperature control setup.

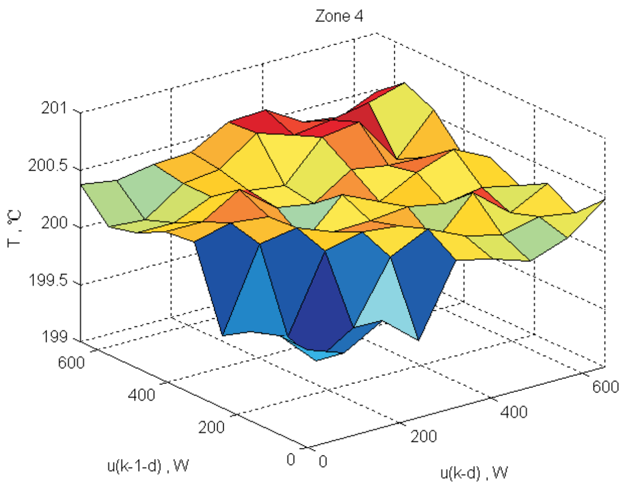


**Figure 4.** Distribution of sampled temperature output.

four thermocouples are equipped to measure temperature at each temperature zone of the thermal barrel. Then the measured signal is converted to a digital signal by an ADC interface, PCL-818HG [17], and fed back to the PC-based controller. In order to increase the efficiency of the heating device, the heat rate command is sent to the heater using a pulse width modulation (PWM) format through a PWM interface PCL-836 [18]. In this experiment, the sampling time of this system is set to be 5 seconds, and the delay time of heat transferred is measured to be eight sampled periods ($d = 8$).

The temperature increment of the barrel is measured according to each specified input power, $u(k\text{-}d)$ and $u(k\text{-}1\text{-}d)$, which are ranged from 0 to 1400 *Watts* at the first three temperature zones and ranged from 0 to 640 *Watts* at Zone 4. The reference temperature of the barrel ranges from room temperature (25 °C) to 235 °C. The range of the input power is divided into 8 sampling intervals, and the range of reference temperature is partitioned into 21 sampling intervals. Therefore, there are totally 1782 (= 9 × 9 × 22) samples of measurements. To display the results of measurement, the sampled temperature data at Zone 4 is plotted in Figure 4, in which the initial temperature, $T(k\text{-}1)$, is set to be 200 °C. This figure depicts the temperature variation corresponding to a pair of specified input power, $u(k\text{-}d)$ and $u(k\text{-}1\text{-}d)$. It exhibits that the temperature drops when both $u(k\text{-}d)$ and $u(k\text{-}1\text{-}d)$ are closed to zero *Watts*. The temperature drop is caused by heat transferred from the system to the environment. It is noted that the maximum temperature increment occurs both $u(k\text{-}d)$ and $u(k\text{-}1\text{-}d)$ approach 640 *Watts*. The result expresses that the relationship between the temperature and the power supply rate is a nonlinear function.

### 3.2 Linear Thermal Model

Now the linear approximate model in Eq. (3) is utilized to model the physical system depicted in Figure 4. The sampled temperature data and the corresponding input power are known quantities in the preceding subsection, and can be used to identify the coefficients of the model. By using the method in Eq. (5) or (6), the coefficients of the linear approximate model are estimated [13]. These coefficient matrices determined by the RLSE method are listed as the following expression:

$$A = \begin{bmatrix} 1.0022 & 0.0008 & 0 & 0 \\ -0.0179 & 1.0036 & 0.0164 & 0 \\ 0 & 0.0109 & 1.0049 & -0.0156 \\ 0 & 0 & -0.0014 & 1.0044 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0.3491 & 0 & 0 & 0 \\ 0 & 0.3728 & 0 & 0 \\ 0 & 0 & 0.4020 & 0 \\ 0 & 0 & 0 & 0.7671 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.3504 & 0 & 0 & 0 \\ 0 & 0.3702 & 0 & 0 \\ 0 & 0 & 0.4109 & 0 \\ 0 & 0 & 0 & 0.7716 \end{bmatrix}$$

Note that the initial constant of the weight function in the RLSE method is set to be λ = 10000. The complete linear thermal model for the thermal barrel of a plastic molding process can be written as

$$\begin{bmatrix} \hat{T}_1(k) \\ \hat{T}_2(k) \\ \hat{T}_3(k) \\ \hat{T}_4(k) \end{bmatrix} = \begin{bmatrix} 1.0022 & 0.0008 & 0 & 0 \\ -0.0179 & 1.0036 & 0.0164 & 0 \\ 0 & 0.0109 & 1.0049 & -0.0156 \\ 0 & 0 & -0.0014 & 1.0044 \end{bmatrix} \begin{bmatrix} T_1(k-1) \\ T_2(k-1) \\ T_3(k-1) \\ T_4(k-1) \end{bmatrix} + \begin{bmatrix} 0.3491 & 0 & 0 & 0 \\ 0 & 0.3728 & 0 & 0 \\ 0 & 0 & 0.4020 & 0 \\ 0 & 0 & 0 & 0.7671 \end{bmatrix} \begin{bmatrix} u_1(k-d) \\ u_2(k-d) \\ u_3(k-d) \\ u_4(k-d) \end{bmatrix} + \begin{bmatrix} 0.3504 & 0 & 0 & 0 \\ 0 & 0.3702 & 0 & 0 \\ 0 & 0 & 0.4109 & 0 \\ 0 & 0 & 0 & 0.7716 \end{bmatrix} \begin{bmatrix} u_1(k-1-d) \\ u_2(k-1-d) \\ u_3(k-1-d) \\ u_4(k-1-d) \end{bmatrix} + \begin{bmatrix} e_1(k) \\ e_2(k) \\ e_3(k) \\ e_4(k) \end{bmatrix} \quad (12)$$

where $\hat{T}_i(k)$ indicate the estimated values of temperature. The temperature distribution of the linear model in Eq. (12) for Zone 4 is plotted in Figure 5. The figure depicts the temperature distribution of Zone 4 with respect to corresponding input power $u(k\text{-}d)$ and $u(k\text{-}1\text{-}d)$, and the initial temperature of Zone 4 is set to be 200 °C. There is a temperature increment when both $u(k\text{-}d)$ and $u(k\text{-}1\text{-}d)$ approach to zero value. This phenomenon violates the thermal behavior of the real model as shown in Figure 4. It concludes that the linear model can predict the temperature approximately only in some range but not in the whole spectrum.

### 3.3 Fuzzy Thermal Model

In this subsection, the concept of TS fuzzy system is utilized to model the physical system as shown in Figure 4. According to Eq. (7), the rule base of TS fuzzy system

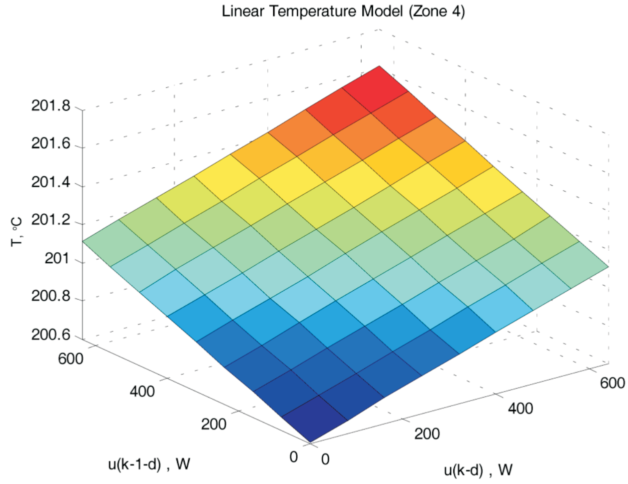


**Figure 5.** Temperature distribution of linear model at 200 °C.

for the thermal barrel is defined as the following statement:

Rule $m$: if $x$ is $A_m(x)$ then

$$T_m(k) = b_{m0} + b_{m1}u(k-d) + b_{m2}u(k-d-1) + b_{m3}T_m(k-1)$$

As expressed in Eq. (11), the outputs of the fuzzy system are determined by using the average weighting method,

$$\hat{T}(k) = \sum_{m=1}^{c} g_m \left[ b_{m0} + b_{m1}u(k-d) + b_{m2}u(k-d-1) + b_{m3}T_m(k-1) \right] \quad (13)$$

To complete the fuzzy model, it proceeds to identify the rule base, $C_m$, and estimate the coefficient vector, $h$. By the clustering method with the selected width of Gaussian function, σ, those 1782 sampled input-output data are divided into $c$ clusters. For different values of σ, the numbers of fuzzy rules are determined and listed in Table 2. Also, those 1782 input-output data are utilized to estimate the coefficient vector by the RLSE method with λ = 10000. The resultant temperature distribution of the fuzzy system model of Eq. (13) is plotted in Figure 6. The

**Table 2.** Comparison of model results

| Models | σ, λ Values | Rules | MSE |
|---|---|---|---|
| Linear model [13] | λ = 104 | 1 | 1.287 |
| Fuzzy model | σ = 0.45 | 9 | 0.028 |
| | σ = 0.35 | 16 | 0.026 |
| | σ = 0.25 | 60 | 0.021 |

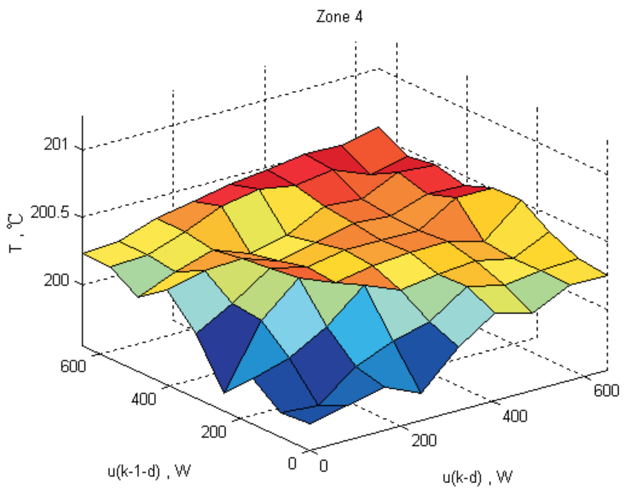


**Figure 6.** Clustering-based fuzzy system model.

figure depicts the temperature distribution of Zone 4 according to the corresponding input power $u(k\text{-}d)$ and $u(k\text{-}1\text{-}d)$, and the initial temperature is set to be 200 °C. In this case, temperature drops as both $u(k\text{-}d)$ and $u(k\text{-}1\text{-}d)$ approach zero values. This phenomenon matches the thermal behavior of the real model. Therefore, the fuzzy model provides a better temperature prediction than that by the linear approximate model.

### 3.4 Model Comparison

In order to evaluate the accuracy of these models, the paper defines Mean Squared Error (MSE) by the following equation,

$$\text{MSE} = \frac{\sum_{i=1}^{n}(T_i - \hat{T})^2}{n} \tag{14}$$

where $T_i$ are the sampled temperature outputs of the real model, and $\hat{T}_i$ are the estimated outputs of a model. The MSE value can be used to evaluate the accuracy of the corresponding model. In this section, the proposed fuzzy model is compared with the linear approximate model, and the results are listed in Table 2. In the second column of the table, λ represents the initial value of estimated data in RLSE method; σ is the width of Gaussian function in relational grades. The number of rules in the third column represents the number of clusters and indicates the complexity of the model. The fuzzy models have multiple rules and a complex function, while the linear model has one single rule. On the point of model accuracy, the fuzzy model with 60 rules has a MSE as low as 0.021, while the linear model with single rule involves with a large MSE value of 1.287.

## 4. Internal Model Control

In order to evaluate the control performance of the developed thermal models, an internal model controller (IMC) [14] is constructed as shown in Figure 7. In the block diagram, $T^*(k)$ and $\hat{T}(k)$ represent the reference temperature input and the estimated temperature, respectively. The controller calculates the temperature difference, $\Delta T$, between the temperature measured from the thermal barrel and that estimated from model. This temperature error represents the inaccuracy of the temperature model. This error value is fed back to the controller to compensate the command. In the controller, an inverse model is utilized to generate the command of the heater, and the command is sent into the heater of the thermal barrel and the mathematical model simultaneously. A new temperature output can be obtained at the outlets of the system and the model. If the temperature difference

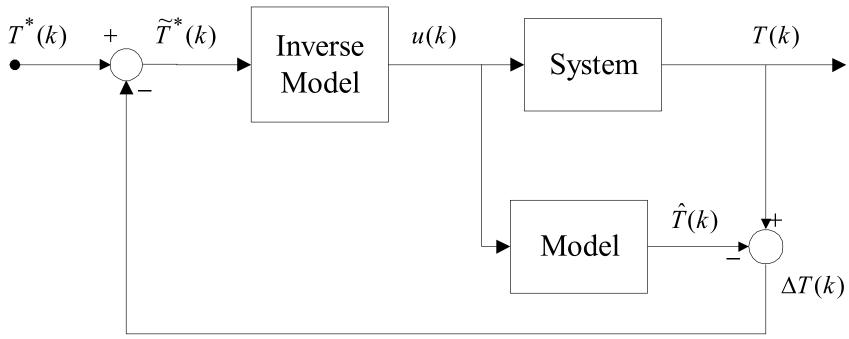


**Figure 7.** Internal Model Control.

between thermal barrel and the model vanishes, i.e. the mathematical model matches the real system exactly, there is no signal feedback, and the IMC become an open-loop feedforward controller. On the other hand, if the model and the system are slightly different, the temperature difference, $\Delta T$, will not disappear. According to the characteristic of IMC, the temperature output, $T(k)$, will not follow the reference temperature exactly. In the following subsections, the inverse models are derived for the linear and fuzzy models, which are needed in the IMC control loop.

### 4.1 Inverse Linear Model

Rearranging Eq. (3), the inverse model of the linear approximate model is derived as the follows,

$$\begin{bmatrix} u_1(k-d) \\ u_2(k-d) \\ u_3(k-d) \\ u_4(k-d) \end{bmatrix} = \begin{bmatrix} b_{01} & 0 & 0 & 0 \\ 0 & b_{02} & 0 & 0 \\ 0 & 0 & b_{03} & 0 \\ 0 & 0 & 0 & b_{04} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} T_1(k) \\ T_2(k) \\ T_3(k) \\ T_4(k) \end{bmatrix} - \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} T_1(k-1) \\ T_2(k-1) \\ T_3(k-1) \\ T_4(k-1) \end{bmatrix} - \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{12} & 0 & 0 \\ 0 & 0 & b_{13} & 0 \\ 0 & 0 & 0 & b_{14} \end{bmatrix} \begin{bmatrix} u_1(k-d-1) \\ u_2(k-d-1) \\ u_3(k-d-1) \\ u_4(k-d-1) \end{bmatrix} - \begin{bmatrix} e_1(k) \\ e_2(k) \\ e_3(k) \\ e_4(k) \end{bmatrix} \right\} \quad (15)$$

For a consistent expression in time, the sampling time is shifted $d$ steps in advance. Meanwhile the initial temperature, $T_i(k+d)$, is replaced by the temperature command, $T_i^*(k)$. If the model is accurate, the temperature difference $\Delta T(k) = 0$ and $T_i^*(k) = \tilde{T}_i^*(k)$. Eq. (15) is changed to be

$$\begin{bmatrix} u_1(k) \\ u_2(k) \\ u_3(k) \\ u_4(k) \end{bmatrix} = \begin{bmatrix} b_{01} & 0 & 0 & 0 \\ 0 & b_{02} & 0 & 0 \\ 0 & 0 & b_{03} & 0 \\ 0 & 0 & 0 & b_{04} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} \tilde{T}_1^*(k) \\ \tilde{T}_2^*(k) \\ \tilde{T}_3^*(k) \\ \tilde{T}_4^*(k) \end{bmatrix} - \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} T_1(k+d-1) \\ T_2(k+d-1) \\ T_3(k+d-1) \\ T_4(k+d-1) \end{bmatrix} - \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{12} & 0 & 0 \\ 0 & 0 & b_{13} & 0 \\ 0 & 0 & 0 & b_{14} \end{bmatrix} \begin{bmatrix} u_1(k-1) \\ u_2(k-1) \\ u_3(k-1) \\ u_4(k-1) \end{bmatrix} - \begin{bmatrix} e_1(k+d) \\ e_2(k+d) \\ e_3(k+d) \\ e_4(k+d) \end{bmatrix} \right\} \quad (16)$$

The PWM command in the IMC block diagram of linear approximate model can be generated by Eq. (16).

### 4.2 Inverse Fuzzy Model

The inverse fuzzy model can be derived directly from Eq. (13). Considering the estimated temperature after $d$ sampling periods, Eq. (13) is expressed as

$$\hat{T}(k+d) = \sum_{m=1}^{c} g_m \begin{bmatrix} b_{m0} + b_{m1}u(k) + b_{m2}u(k-1) \\ +b_{m3}T_m(k+d-1) \end{bmatrix} \quad (17)$$

Rearrange Eq. (17) and replace the estimated temperature, $\hat{T}(k+d)$, by the reference temperature, $\tilde{T}^*(k)$,

$$u(k) = \frac{\tilde{T}^*(k) - \sum_{m=1}^{c} g_m \left[ b_{m0} + b_{m2}u(k-1) + b_{m3}T_m(K+d-1) \right]}{\sum_{m=1}^{c} g_m b_{m1}} \quad (18)$$

This provides the inverse fuzzy system model. In the experiment example, $T_m(k+d\text{-}1)$ is a future data and replaced by the temperature, $T_m(k\text{-}1)$, at previous time step.

### 4.3 Control Results

The IMC algorithms developed in the preceding sections are applied to temperature control for the experiment. One example of a step response is exhibited and the performance between the linear approximate model and the fuzzy model is compared in this subsection. In the example, four temperature zones are subjected to specified temperature inputs, which are 100 °C, 165 °C, 210 °C, and 210 °C for temperature zones from Zone 1 to Zone 4. The step responses are depicted in Figures 8-11. The dashed line and solid line represent the responses of the linear model and fuzzy model, respectively. From the results, the IMC with fuzzy model has shorter settling time than that with linear model in Figure 8. Meanwhile, the control with linear model also involves oscillatory outputs as shown in the plots for Zone 2 and 3. It con-

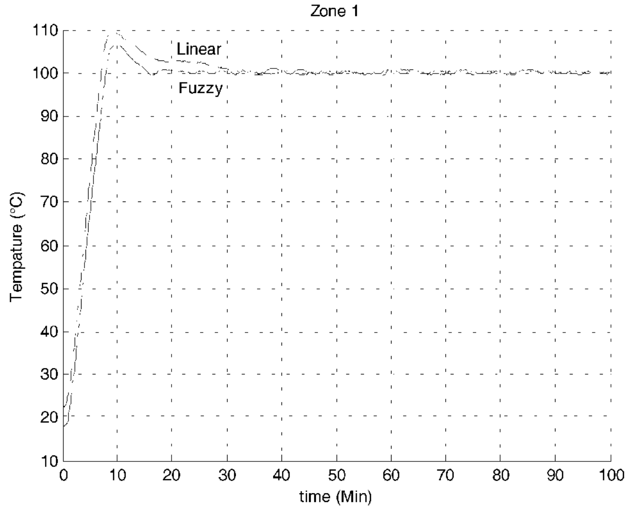


**Figure 8.** IMC control for Zone 1.

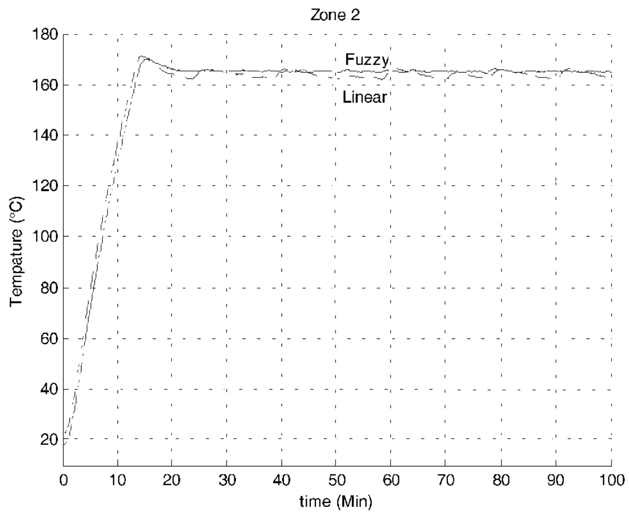


**Figure 9.** IMC control for Zone 2.

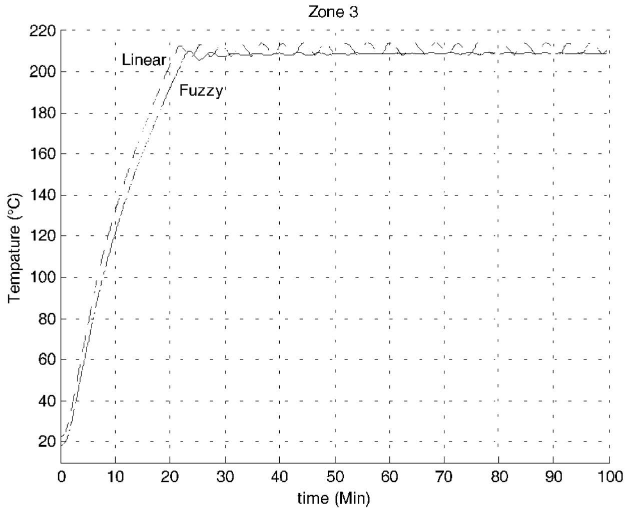


**Figure 10.** IMC control for Zone 3.

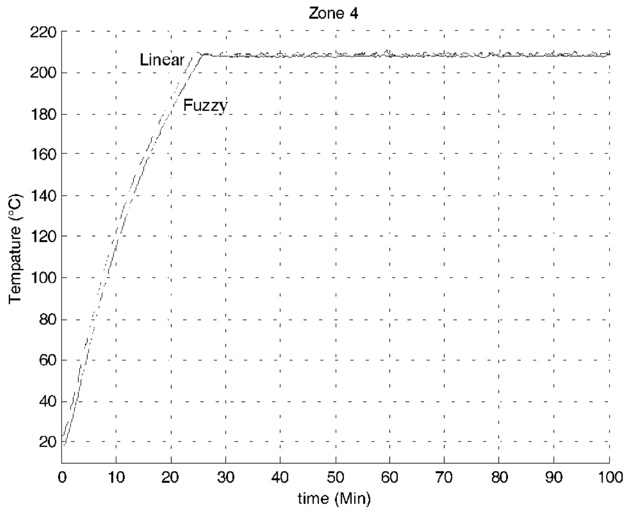


**Figure 11.** IMC control for Zone 4.

cludes that the IMC with fuzzy model provides more accurate temperature prediction and have better step response than that with the linear model. Therefore, the fuzzy models are more appropriate than the linear model in control strategies design.

## 5. Conclusion Remarks

In this research, a thermal model is established based on Takagi-Sugeno fuzzy system and applied to model and control the temperature distribution of a thermal barrel in plastic molding processes. In conventional industry applications, the thermal barrel behavior is expressed in terms of a parameterized linear model to be used in control strategies design. However, the linear model is only applicable in some range of the application and is not appropriate for precision process control. A simple and effective fuzzy model is proposed to overcome the disadvantage of the linear model. The fuzzy model can represent the nonlinear behavior of the thermal barrel and provide accurate output in all application ranges. The clustering method is employed to generate the rule base of the fuzzy system by dividing the original input-output data into several clusters according to the similarity of the data. The proposed methodology is shown to be more effective in constructing the model than the methods in the literatures [8].

In order to evaluate the control performance, the thermal models are integrated into the internal model controller to control the temperature of a thermal barrel. The system is subjected to a step input and the response depicts the control performance of the models. The controller with fuzzy models shows an excellent performance in step response, while the controller with linear model has a large settling time and an oscillatory output for 2 out of the 4 temperature zones. Note that, the paper evaluates the control performance of the proposed model by using an internal model controller, which is more like a feedforward controller or an inverse dynamic controller. We believe that, in the case of large temperature range, an accurate system model and an IMC will be more easily implemented in injection molding processes than other controllers [4,10,11] which have complicated computation algorithms. The proposed fuzzy model can provide accurate input-output data in a large temperature range and has the capability of applications in precision process controls in plastic molding processes.

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