Study of Backcalculated Pavement Layer Moduli from the LTPP Database

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Abstract

The main objective of this study was to investigate the fundamental principles of flexible and rigid pavement backcalculation methodologies and their potential limitations. The two-layer backcalculation approach proposed by the 1993 AASHTO Design Guide for the structural evaluation of existing pavements was also adopted. The laboratory tested (or static) layer moduli were compared with the backcalculated (or dynamic) moduli using the Long-Term Pavement Performance (LTPP) database. Relatively high variability between the relationships of the static and the dynamic moduli was observed indicating that further research study is needed to improve the current state-of-the-art backcalculation approach. In addition, it was also found that slab thickness did have significant effects on the relationship of the backcalculated subgrade elastic modulus and the backcalculated modulus of subgrade reaction. Subsequently, a revised regression model was proposed for future practical applications.

Key Words: Pavement, Backcalculation, Elastic Modulus, LTPP

1. Introduction

Nondestructive deflection testing (NDT) devices have been widely adopted to obtain surface deflection data in order to evaluate existing pavement conditions. Since the elastic moduli of pavement layers, which represent the stiffness of a pavement structure, cannot be calculated directly from surface deflection data, they are often obtained using backcalculation procedures. The fundamental principles of backcalculation procedures are based on pavement theories such as the multi-layer elastic theory and plate theory.

Traditional backcalculation procedures may be grouped into two major classifications in general: iterative method and database method. To estimate the elastic modulus of each pavement layer, an iterative backcalculation procedure has to first assume an initial trial set of modulus values, and then repetitively calculate theoretical deflections in order to match the actual surface deflection measurements within the specified ranges of error tolerance. The database approach finds a suitable set of modulus values by linearly interpreting the measured deflections with the theoretical deflections, which have already been built in a large database with pre-specified ranges of modulus values.

The study first investigates the fundamental principles of flexible and rigid pavement backcalculation methodologies and their potential limitations. The goodness of backcalculation using the current state-of-the-art approach as adopted by the Long-Term Pavement Performance (LTPP) study will be subsequently discussed [1,2].

2. Pavement Backcalculation System Based on Two-Layer Elastic Theory

Boussinesq developed closed-form solutions of a concentrated load acting on a single-layered uniform

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subgrade soil [3]. The materials are assumed to be homogeneous, isotropic, and linear elastic. Ahlvin and Ulery later provided deflection solutions at any given depth (z) and radial distance (r) for a uniformly distributed circular load acting on a single-layered system [4]. The deflection equation can be expressed as follows based on the principles of dimensional analysis:

$$\Delta_z = \frac{Pa}{E_1} f\left(\frac{r}{a}, \frac{z}{a}, \mu\right) \tag{1}$$

In which, Δ_z is the deflection at any depth z, [L]; P is the uniformly applied circular load, [F]; a is the radius of the applied load, [L]; and the layer modulus is E_1, [FL^{-2}]. Note that [F] and [L] represent the dimensions of force and length, respectively. Thus, the backcalculation problem of a single-layered system is just a simple matter of solving the unknown E_1 providing that all other parameters are known.

Through Bessel function expansion of a load function, Burmister derived a surface deflection equation for any arbitrary, uniformly distributed load which is equivalent to a concentrated load acting on a two-layer elastic pavement system [5]. Burmister further derived a maximum surface deflection equation for a uniformly distributed load acting on a two-layer elastic system. Based on the principles of dimensional analysis, the deflection equation can be simplified as follows:

$$w_c = \frac{1.5pa}{E_2} F_w\left(\frac{a}{h}, \frac{E_2}{E_1}\right) = \frac{1.5pa}{E_2} F_w \tag{2}$$

Where, w_c is the vertical surface deflection of the load center, [L]; p is the uniformly distributed vertical pressure acting on the surface, [FL^{-2}]; a is the radius of the circular load, [L]; F_w is a function of a / h and E_2 / E_1; h is the thickness of the top layer, [L]; E_1 and E_2 are the elastic moduli of the top and bottom layers, [FL^{-2}], respectively.

Scrivner analyzed the case of a Dynaflect's load configuration acting on a two-layer pavement-subgrade system [6]. To estimate the elastic moduli of surface layer and subgrade from the measured surface deflection data, Scrivner treated everything above the subgrade as a single homogeneous material to simplify the pavement as a two-layer elastic system. Since the loaded area is very small, Scrivner further treated the above uniformly distributed load as a concentrated load to simplify the mathematics. For a horizontal distance r away from the origin O, the following surface deflection w is a function of h, P, E_1, and E_2:

$$\frac{4\pi E_1}{3P} wr = \int_{x=0}^{\infty} V * J_0(x)dx = F\left(\frac{E_2}{E_1}, \frac{r}{h}\right)$$

$$x = \frac{mr}{h},\ V = \frac{1+4Nme^{-2m} - N^2 e^{-4m}}{1-2N(1+2m^2)e^{-2m} + N^2 e^{-4m}},\ N = \frac{E_1 - E_2}{E_1 + E_2} \tag{3}$$

For distance r_1 and r_3 away from the loaded center of Dynaflect, the surface deflections are w_1 and w_3, respectively. By substituting into the above deflection equation and dividing the resulting two equations with each other, the following expression is obtained:

$$\frac{w_1 r_1}{w_3 r_3} = \frac{F_1\left(\frac{E_2}{E_1}, \frac{r_1}{h}\right)}{F_3\left(\frac{E_2}{E_1}, \frac{r_3}{h}\right)} = G\left(\frac{E_2}{E_1}, \frac{r_1}{h}, \frac{r_3}{h}\right) \tag{4}$$

Where, F_1, F_3, and G are functions of E_2 / E_1, r_1 / h, and r_3 / h. For a specified NDT device (such as Dynaflect), with known r_1, r_3, and surface thickness h, one can easily find out that $w_1 r_1 / w_3 r_3$ is a function of the modulus ratio E_2 / E_1 alone from the above equation.

Note that the elastic moduli backcalculated from measured surface deflection data may not be unique in theory. For example, Scrivner specifically developed curves and databases for modulus backcalculation for Dynaflect tests where the radius of loaded area a and sensor locations r_1 and r_3 are fixed. From the curves of pavement thickness plotted as a function of $w_1 r_1 / w_3 r_3$ and the modulus ratio E_1 / E_2 (as shown in Figure 1), Scrivner further divided this figure into four quadrants based on lines of $w_1 r_1 / w_3 r_3 = 1$ and $h = 11.2$ in. Thus, there exists a unique solution for those two quadrants with thickness h greater than 11.2 in. However, there may be two or no solutions for the other two quadrants with thickness h less than 11.2 in. Nevertheless, this theoretical limitation is often overlooked by most traditional backcalculation programs [7].

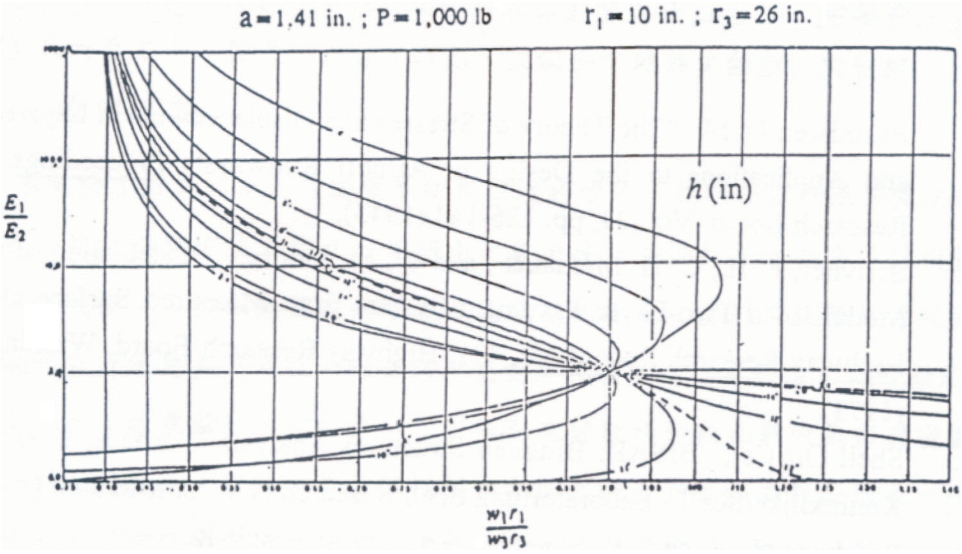

Figure 1. Scrivner's curves for two-layer backcalculation [6].

3. Rigid Pavement Backcalculation Based on the Plate Theory

Losberg [8] provided closed-form solutions for the deflection of a PCC slab resting on a dense liquid foundation (Winkler) and an elastic solid foundation under a uniformly distributed load. The following expression in terms of nondimensional deflection (w^*) was used to illustrate its relationship with normalized radial distance ($s = r/\ell_k$ or r/ℓ_e) and normalized load radius (a/ℓ_k or a/ℓ_e). These relationships (i.e., f_1 and f_2 functions) were also validated through the use of Microsoft PowerStation IMSL libraries for the integration of Bessel functions [9].

$$w^* = \begin{cases} \dfrac{wC\ell_e}{2P} = \dfrac{wD}{P\ell_e^2} = \dfrac{wE_s\ell_e}{2P(1-\mu_s^2)} = f_1\left(\dfrac{a}{\ell_e}, \dfrac{r}{\ell_e}\right) \\ \text{for Elastic Solid Foundation} \\ \dfrac{wk\ell_k^2}{2P} = \dfrac{wD}{P\ell_k^2} = f_2\left(\dfrac{a}{\ell_k}, \dfrac{r}{\ell_k}\right) \\ \text{for Winkler Foundation} \end{cases} \tag{5}$$

$$\ell_e = \sqrt[3]{\frac{E_c h^3(1-\mu_s^2)}{6(1-\mu_c)E_s}}, \quad \ell_k = \sqrt[4]{\frac{E_c h^3}{12(1-\mu_c^2)k}} \tag{6}$$

$$C = \frac{E_s}{(1-\mu_s^2)}, \quad D = \frac{E_c h^3}{12(1-\mu_c^2)} \tag{7}$$

Where, w is the surface deflection at radial distance r, [L]; ℓ_e and ℓ_k are the radius of relative stiffness for elastic solid and Winker foundations, [L], respectively; C is the modified modulus of elasticity of the subgrade, [FL^{-2}]; D is the bending stiffness of the slab, [FL]; E_c and E_s are the Young's modulus of elasticity of PCC slab and subgrade, [FL^{-2}], respectively; and μ_c and μ_s are Poisson's ratio of PCC slab and subgrade, respectively.

Hoffman and Thompson [10] proposed the following concept to backcalculate the modulus values of a rigid pavement system. The area of the deflection basin using four deflection sensors was defined by equation (8). Higher AREA values indicate stiffer slabs relative to the foundation; whereas lower values are indicative of some serious slab weakening problem. Where, AREA is the normalized area of deflection basin, ranging from 11.1 to 36 inches; w_0 is the measured maximum deflection at the center of the load, [L]; and w_1, w_2, w_3 are the measured deflections at distance 12, 24, 36 in. from the load center, [L].

$$AREA(in.) = 6 * \left[1 + 2\left(\frac{w_1}{w_0}\right) + 2\left(\frac{w_2}{w_0}\right) + \left(\frac{w_3}{w_0}\right)\right] \tag{8}$$

ERES consultants, Inc. [11] and Foxworthy [12] conducted several hundreds of ILLI-SLAB finite element runs to model pavement response produced by the loadings of a Falling Weight Deflectometer and a Road Rater. For a given slab thickness, the elastic modulus of the slab and the modulus of subgrade reaction are varied over a practical range and the "AREA" and maximum center deflection are calculated and plotted. The actual measured AREA and maximum center deflection are used to determine the in situ slab modulus and the modulus of subgrade reaction using linear interpretation.

Ioannides indicated that Westergaard's maximum deflection and the surface deflections at a given radial distance (for sensor *i*) of a PCC slab resting on a Winkler foundation can be expressed as follows [3,13]:

$$d_0 = \frac{w_0 k \ell_k{}^2}{P} = f_0\left(\frac{a}{\ell_k}\right) \tag{9}$$

$$d_i = \frac{w_i k \ell_k{}^2}{P} = f_i\left(\frac{a}{\ell_k}\right) \tag{10}$$

Where, *a* is the radius of the applied load; d_0 or d_i is the normalized deflection under the center of the load or at any sensor *i* ($i = 0, 1, 2$, or 3); w_0 or w_i is the surface deflection under the center of the load or at any sensor *i*, [L]. If two deflections (w_0 and w_1) are measured, the following equation indicating that ℓ_k may be determined from the deflection ratio (w_1/w_0) for a given load radius:

$$\frac{d_1}{d_0} = \frac{w_1}{w_0} = \frac{f_i(a/\ell_k)}{f_0(a/\ell_k)} = f(\ell_k) \tag{11}$$

If four sensors are used, modulus values are often backcalculated using the average deflection as indicated by the AREA concept. Thus, there exists a unique relationship between AREA and the radius of relative stiffness (ℓ_k) for a given load radius and pre-specified sensor locations. A unique relationship between AREA and the radius of relative stiffness (ℓ_e) for a PCC slab resting on elastic solid foundation can also be obtained using similar approach.

As implemented in the ILLI-BACK program [3], a fixed load radius (a = 5.9 in.), and the deflection w_i at four radial distances of 0, 12, 24, 36 in. as well as the area of deflection basin AREA were calculated for the backcalculation of a concrete pavement resting on a dense liquid or elastic solid foundation under an interior circular load. After the radius of relative stiffness (ℓ_k or ℓ_e) is determined, four k or E_s values (one for each sensor) are obtained and their average is taken based on the following expression.

$$k = \frac{Pd_i}{\ell_k^2 w_i} \tag{12}$$

$$E_s = \frac{2(1-\mu_s^2)Pd_i}{\ell_e w_i} \tag{13}$$

Based on the averaged k or E_s value and the equation (6), the elastic modulus of the slab (E_c) is subsequently determined.

4. Multi-Layered Backcalculation Programs and Their Limitations

The fundamental principles of backcalculation procedures are based on pavement theories such as the multi-layer elastic theory and plate theory. The most often used multi-layer elastic theory was simplified using Odemark's equivalent thickness assumptions [14]. Basically, materials are assumed to be homogeneous, isotropic, and linear elastic, even though they are often far from reality. Various programs were developed to facilitate the layer moduli backcalculation of a more practical multi-layered system.

A backcalculation procedure often assumes there exists a unique combination of elastic moduli which will result in the same measured deflection data as those calculated from pavement theory, when a dynamic load is applied to a pavement system. Thus, if the thickness of each pavement layer, load configuration, loaded area, and Poisson's ratios are known, a specific set of layer moduli may be chosen to calculate corresponding theoretical deflections from pavement theory and compared to the measured deflections. If their differences are not within the specified error tolerance, it is necessary to choose a new set of moduli and repeat the previous process until such condition is met. The resulting final set of layer moduli represents the stiffness of the pavement system. Thus, there exist unlimited sets of layer moduli which may satisfy the specified error tolerance criteria for a particular set of measured deflection data. Different specified error tolerance, initial trial modulus values and ranges may all affect the results of backcalculation in different way.

Hall [15] summarized the basic assumptions and limitations of various analytical models for modulus backcalculation. Examples of iterative backcalculation programs including BISDEF, CHEVDEF, WESDEF, ELSDEF, and BOUSDEF were discussed. The capabilities of some database backcalculation programs such as COMDEF and MODULUS were also investigated. Efforts made in the proper use of analytical models, reduction of the need to arbitrarily guess input modulus values, and calibration of temperature effects, and nonlinear behavior of materials will all contribute to assure the reasonableness of backcalculation results.

Hall further presented closed-form solutions for rigid pavement backcalculation which greatly enhanced the effectiveness of in situ pavement evaluation. Furthermore, Crovetti [16] indicated that finite slab size, the locations of loading plate (interior, edge and corner of the slab), and the presence of adjacent slabs or a tied concrete shoulder, etc. may all affect pavement surface deflection measurements. Lee [17] and Bair [18] proposed a modified deflection ratio backcalculation procedure, which was implemented in a user-friendly backcalculation program (TKUBAK) to expand its applicability for any different NDT loading radius, sensor locations, finite slab sizes, and different locations of loading plate [9]. Sheu [19] further investigated the effects of adjacent slabs and temperature curling on rigid pavement backcalculations. Nevertheless, it is still a very challenging task in validating the backcalculated results using field NDT deflection measurements.

5. AASHTO's Two-Layer Backcalculation Approach

5.1 Flexible Pavement Backcalculation

In the AASHTO overlay design procedure, nondestructive deflection testing is strongly recommended for the structural evaluation of existing pavements [20]. Due to the fact that at sufficiently large distance from the load center, the deflection measured at the pavement surface is almost entirely due to subgrade deformation only. Thus, a subgrade M_R may be backcalculated using the following expression regardless of the number of layers above the subgrade:

$$M_R = \left(\frac{0.24P}{d_r r} \right) \quad r \geq 0.7a_e \tag{14}$$

In which, M_R is the backclculated subgrade resilient modulus (psi); P is the applied load (lbs); d_r is the deflection at a distance *r* from the center of the load (in.); *r* is the distance from the center of the load (in.). Also note that no temperature adjustment is needed in determing M_R since the deflection is only due to subgrade deformation. a_e is the radius of the stress bulb at the subgrade-pavement interface (in.) determined by the following expression:

$$a_e = \sqrt{a^2 + \left(D \times \sqrt[3]{\frac{E_P}{M_R}} \right)^2} \tag{15}$$

Where, a is the radius of NDT load plate (in.); D is the total thickness of pavement layers above the subgrade (in.); E_P is the effective modulus of all pavement layers above the subgrade (psi). The temperature of the AC mix during deflection testing must be measured or estimated from surface or air temperature. The deflection (d_0) measured at the center of the load plate is adjusted to a standard temprature of 68 °F (20 °C). The effective modulus of all layers above the subgrade may be determined using the following expression:

$$d_0 = 1.5Pa \left[\frac{1}{M_R \sqrt{1 + \left[(D/a) * (\sqrt[3]{E_P / M_R}) \right]^2}} + \frac{1 - \frac{1}{\sqrt{1 + (D/a)^2}}}{E_P} \right] \tag{16}$$

For a load plate radius of 5.9 inches, the ratio of E_P / M_R may be determined by the above equation if the maximum deflection (d_0) and the total thickness of all layers above the subgrade (D) are known. Thus, E_P may then be determined providing that subgrade resilient modulus M_R is known.

Figure 2 depicts the backcalculated resilient modulus versus laboratory tested results [20]. The recommended approach for the determination of design M_R from NDT backcalculation requires an adjustment factor (C) to make the backcalculated (dynamic) modulus consistent with the laboratory tested (static) subgrade modulus. Thus, a value of C of no more than 0.33 is recommended using the following equation:

$$M_R = C \left(\frac{0.24P}{d_r r} \right) \tag{17}$$

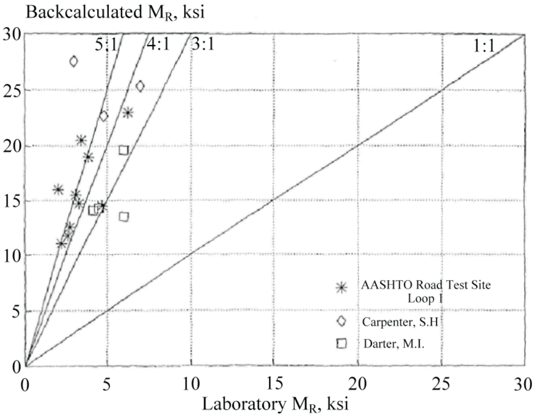

Figure 2. Comparison of backclaculated resilient modulus versus laboratory tested results [20].

5.2 Rigid Pavement Backcalculation

Hall [15] solved Losberg's deflection equation through direct integration of Bessel functions for radial distances of 0, 30.5, 61.0, 91.4 cm (0, 12, 24, and 36 inches) and for ℓ_k and ℓ_e values from 38.1 to 203.2 cm (15 to 80 inches) using the IMSL library. Consequently, the following regression models were developed using SAS statistical software package:

$$\ell_k = \left[\frac{\ln\left(\dfrac{36-AREA}{1812.279}\right)}{-2.559340}\right]^{4.387009} \quad \text{or} \quad \ell_e = \left[\frac{\ln\left(\dfrac{36-AREA}{4521.676303}\right)}{-3.645555}\right]^{5.334281} \tag{18}$$

With AREA calculated from the four measured deflections, the radius of relative stiffness (ℓ_k or ℓ_e) in inches may be obtained from the above equations. The effective k-value or the elastic modulus (E_s) of subgrade may be obtained by rearrangement of Westergaard's or Losberg's maximum interior deflection equation as follows. The elastic modulus of PCC slab (E_c) can then be determined using the appropriate ℓ_k or ℓ_e equation as defined by equation (6). This backcalculation approach is also adopted by the AASHTO overlay design procedure for the evaluation of existing concrete pavements [20].

$$k = \left(\frac{P}{8d_0\ell_k^2}\right)\left\{1+\left(\frac{1}{2\pi}\right)\left[\ln\left(\frac{a}{2\ell_k}\right)+\gamma-1.25\right]\left(\frac{a}{\ell_k}\right)^2\right\} \tag{19}$$

$$E_s = \left[\frac{2P(1-\mu_s^2)}{d_0\ell_e}\right]\left[0.19245-0.0272\left(\frac{a}{\ell_e}\right)^2+0.0199\left(\frac{a}{\ell_e}\right)^2\ln\left(\frac{a}{\ell_e}\right)\right] \tag{20}$$

Also note that an adjustment factor of 0.5 is recommended to convert the effective dynamic (backcalculated) k-value into effective static k-value for use in design. However, no specific guideline is provided for the adjustment of backcalculated elastic modulus of PCC slab (E_c). Similar approach was also adopted to develop a backcalculation procedure for bare concrete and composite pavements using different loading plate configurations (such as the deflection measured at a distance of 8, 12, 18, 24, 36, 60 inches away from the load center). More detailed information can be found in the literature [21].

6. Study of LTPP Backcalculation Results

6.1 Database Preparation

Starting from 1987, the LTPP program has been monitoring more than 2,400 asphalt and Portland cement concrete pavement test sections across the North America. Very detailed information about original construction, pavement inventory data, materials and testing, historical traffic counts, performance data, maintenance and rehabilitation records, and climatic information have been collected. There are 8 general pavement studies (GPS) and 9 specific pavement studies (SPS) in the LTPP program. Of which, only those GPS (1 to 2 for asphalt concrete and 3 to 5 for portland cement concrete) pavements were used for this study.

Initially, the DataPave 3.0 program was used to prepare the database. However, in order to obtain additional variables and the latest updates of the data, the Long-Term Pavement Performance database retrieved from http://www.datapave.com (or LTPP DataPave Online, Release 18.0) [22] became the main source for this study. This database is currently implemented in an information management system (IMS) which is a relational database structure using the Microsoft Access program. Automatic summary reports of the pavement information may be generated from different IMS modules, tables, and data elements. The thickness of pavement layers was obtained from the IMS Testing module rather than the IMS Inventory module to be consistent with the results of Section Presentation module in the DataPave 3.0 program.

6.2 Comparison of Laboratory Tested and Backcalculated Moduli of AC Pavements

The static (or laboratory tested) elastic modulus data was recorded in the IMS Testing module. In the LTPP database, the dynamic moduli of AC layers were backcalculated using the MODCOMP4 program [2] and the data could be retrieved from the IMS Monitoring module. Thus, it would be interesting to compare the laboratory tested layer moduli versus the backcalculated dynamic Young's moduli so as to have a better understanding of their associated variability. As shown in Figure 3, the variability of the relationship between the dynamic and the static (or laboratory tested) moduli could not be ignored [23]. The average ratios of which are approximately 2.6, 2.7, 7.3, and 3.4 by eliminating some apparent outliers for AC surface, base, subbase, and subgrade layers, respectively.

In this study, the NDT deflections (d_8, d_{12}, d_{18}, d_{24}, d_{36}, d_{60}) measured at a distance of 8, 12, 18, 24, 36, 60 inches away from the load center under an applied load of 9,000 pounds were retrieved and used to backcalculate the subgrade M_R using equation (14). The comparison of backclaculated resilient modulus versus laboratory tested results is also shown in Figure 4. The mean values and the ratios of backclaculated versus laboratory tested subgrade M_R are summarized in Table 1. Except for the case of NDT deflections measured at 8 inches away from the load center (d8), the average ratios of the backcalculated versus laboratory tested subgrade M_R were ranging from 2.8 to 3.4. These results also indicated that the recommendation of an adjustment factor (C) of about 0.33 may be appropriate, though more research is needed to reduce the variations.

6.3 Comparison of Laboratory Tested and Backcalculated Moduli of PCC Pavements

The modulus of each pavement layer backcalculated using the ERESBACK 2.2 program [1] was retrieved

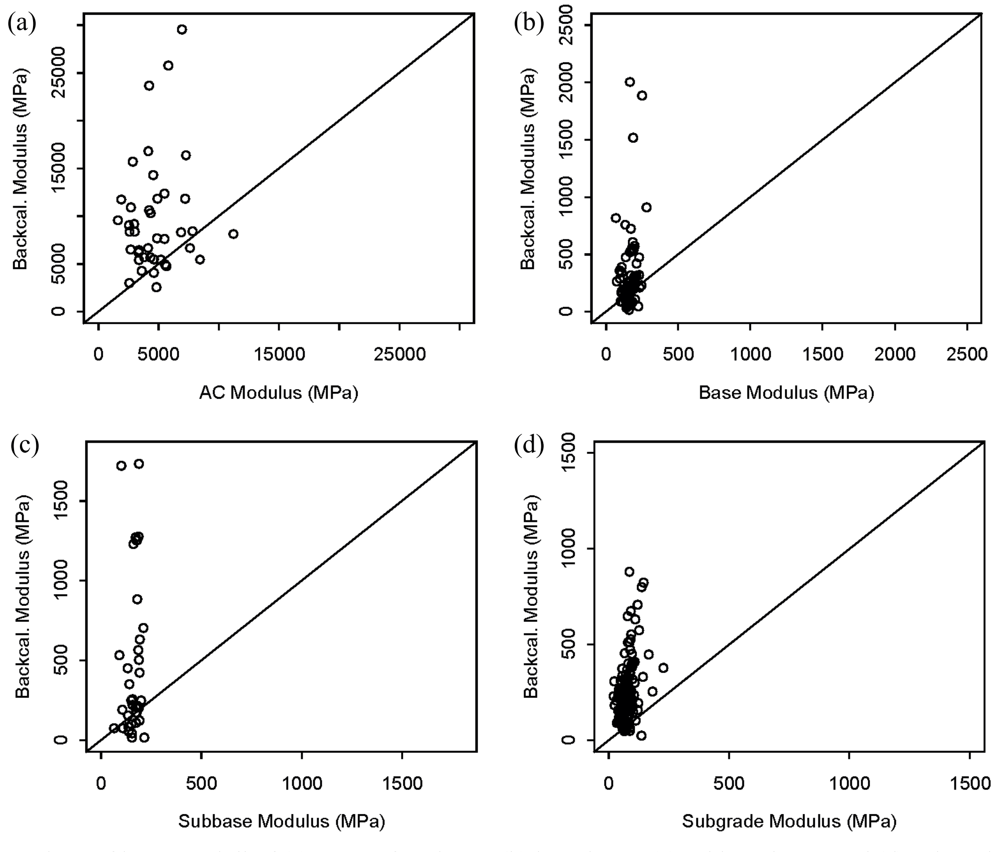

Figure 3. Comparison of layer moduli of (a) AC surface layer; (b) base layer; (c) subbase layer; and (d) subgrade obtained from laboratory testing and backcalculation program.

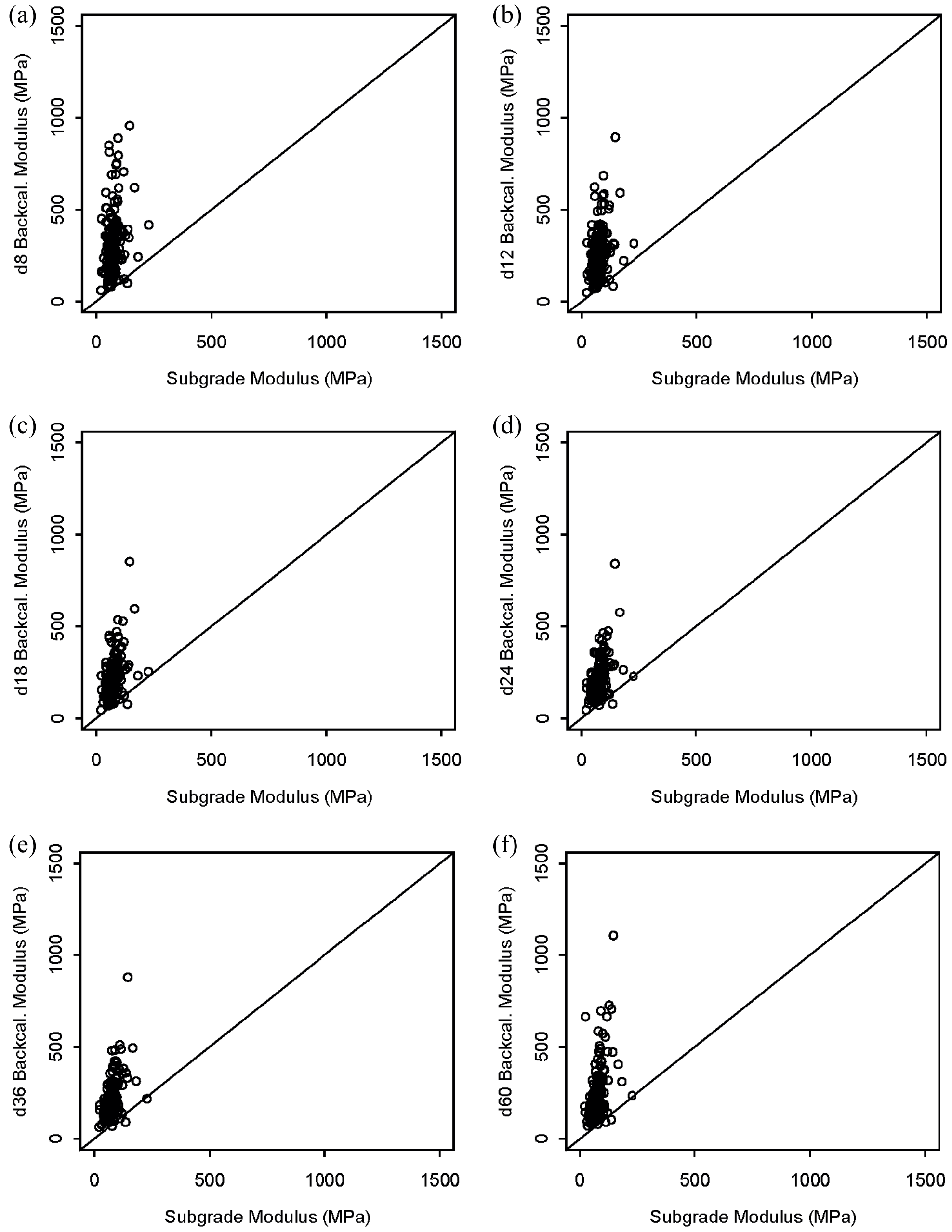

Figure 4. Comparison of backclaculated resilient modulus versus laboratory tested results using equation (14) and the deflections measured at a distance of (a) 8; (b) 12; (c) 18; (d) 24; (e) 36; and (f) 60 inches away from the load center.

Table 1. Comparison of the backcalculated versus laboratory tested subgrade M_R using equation (14)

	Laboratory Tested	MODCOMP4 Backcalculated	Backcalculation using d_r (deflection at a distance *r* from the center of the load, in.) and equation (14)					
			d8*	d12	d18	d24	d36	d60
Mean Subgrade M_R (MPa)	73	250	315	251	217	207	211	241
Ratio	1	3.4	4.3	3.4	3.0	2.8	2.9	3.3

Note: * indicates that d8 is not sufficiently away from the load center and is not appropriate for backcalculation.

from the IMS Monitoring module. The laboratory tested layer moduli were compared with the backcalculated moduli so as to have a better understanding of their associated variability in this study. The variability of the relationship between the laboratory tested (or static) and backcalculated (or dynamic) moduli could not be ignored. Figures 5(a)-(c) depicts the average ratios are approximately 1.4, 1.5, and 1.5 for surface, subbase, and subgrade layers for dense liquid foundation, respectively [24]. Note that very few laboratory tested modulus of subgrade reaction are available in the database. Likewise, Figures 5(d)-(f) depicts the average ratios are roughly 1.0, 1.1, and 3.0 for surface, subbase, and subgrade layers for elastic solid foundation, respectively [24]. It is noted that the recommendation of dividing the backcalculated modulus of subgrade reaction (or k-value)

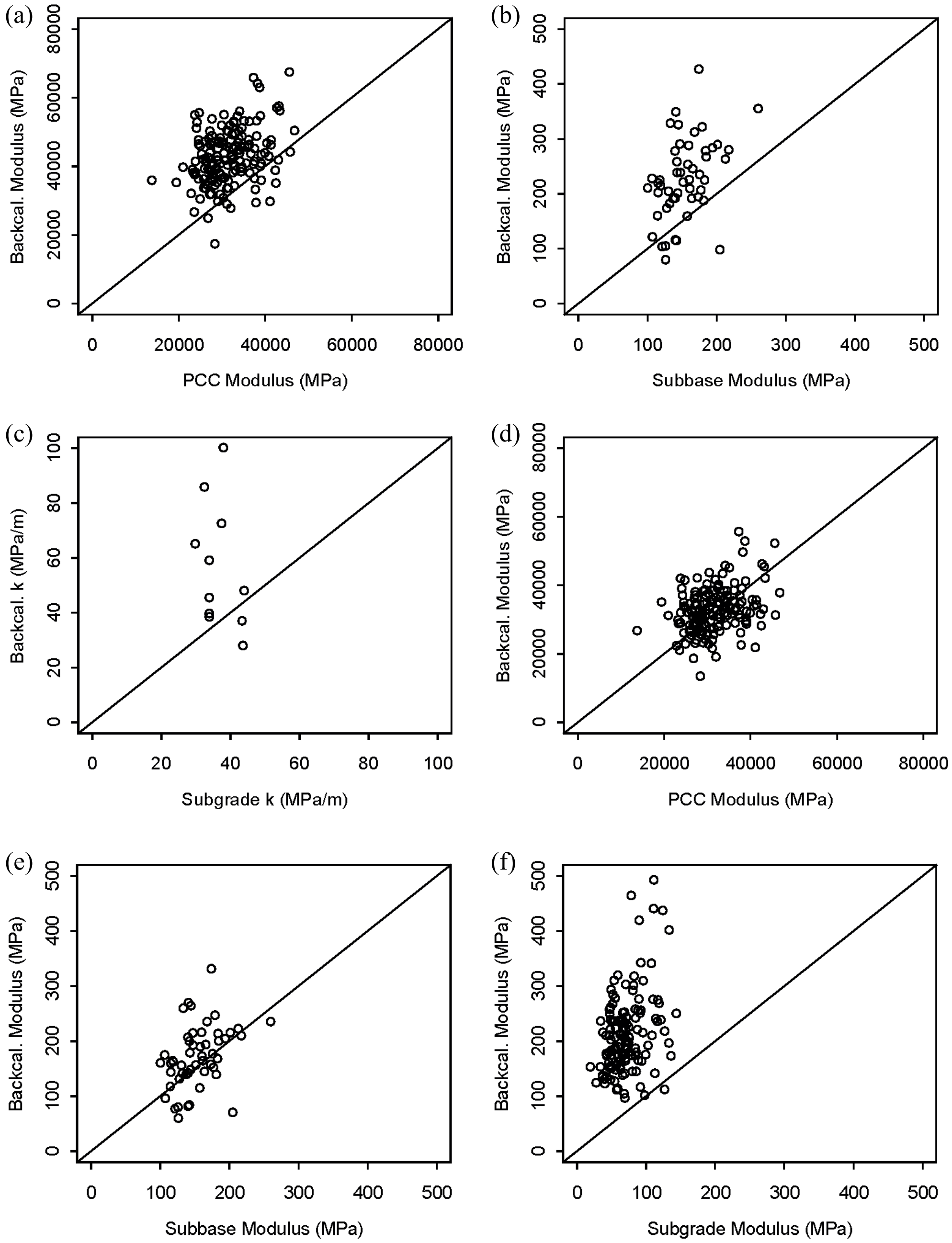

Figure 5. Comparison of laboratory tested and backcalculated layer moduli of (a) surface, (b) subbase, and (c)subgrade for dense liquid foundation; and (d), (e), (f) for elastic solid foundation, respectively.

by 2 as the static k-value by AASHTO [20] may be a reasonable choice, though more research study is still needed to reduce the variability.

6.4 Relationship between Elastic Modulus and Modulus of Subgrade Reaction

For practical concerns, a relationship between the elastic modulus and the modulus of subgrade reaction is often needed. According to the literature [1], the following empirical relationship was developed from the GPS and SPS data analysis:

$$k = 0.296E_s \qquad (21)$$
$$\text{Statistics: } R^2 = 0.872,\ SEE = 9.37,\ N = 596$$

In which, k is the modulus of subgrade reaction (MPa/m), E_s is the subgrade elastic modulus (MPa), R^2 is the coefficient of determination, SEE is the standard error of estimates, and N is the number of observations. According the available GPS data, very good agreements have been achieved using the above relationship.

Nevertheless, Barenberg [25] has indicated the theoretical difference using elastic solid foundation or dense liquid foundation for having same maximum deflections in backcalculation analysis. Assuming a Poisson ratio of 0.5 for subgrade, a Poisson ratio of 0.15 for concrete slab, and the elastic modulus of the slab is 4 Mpsi (27.6 GPa), the following relationship was derived after some simplification process.

$$E_s^{4/3} = 283.7 * h * k \qquad (22)$$

In which, k is the modulus of subgrade reaction (pci), E_s is the subgrade elastic modulus (psi), and h is the slab thickness (in). As shown in Figure 6(a), the effect of slab thickness has to be considered in such a relationship.

The aforementioned relationship was further verified by comparing the backcalculated subgrade elastic moduli with the backcalculated modulus of subgrade reaction from the LTPP database. Slab thickness did have significant effects on this relationship as shown in Figure 6(b). Consequently, the following relationship is developed using regression techniques. In which, k is the modulus of subgrade reaction (MPa/m), E_s is the subgrade elastic modulus (MPa), and h is the slab thickness (cm).

$$E_s = 0.9015(k * h)^{3/4} \qquad (23)$$
$$\text{Statistics: } R^2 = 0.9524,\ SEE = 15.87,\ N = 138$$

7. Conclusion and Recommendation

The fundamental principles of flexible and rigid pavement backcalculation methodologies and their potential limitations were first investigated. The laboratory tested (or static) layer moduli and the backcalculated (or dynamic) moduli were retrieved from the Long-Term Pavement Performance (LTPP) database. Relatively high variability was observed indicating that further research is needed to improve the current state-of-the-art backcalculation approach. An adjustment factor of about 0.33 for converting the dynamic subgrade M_R to the static M_R may be appropriate for AC backcalculations. Generally speaking, the recommendation of dividing the effective

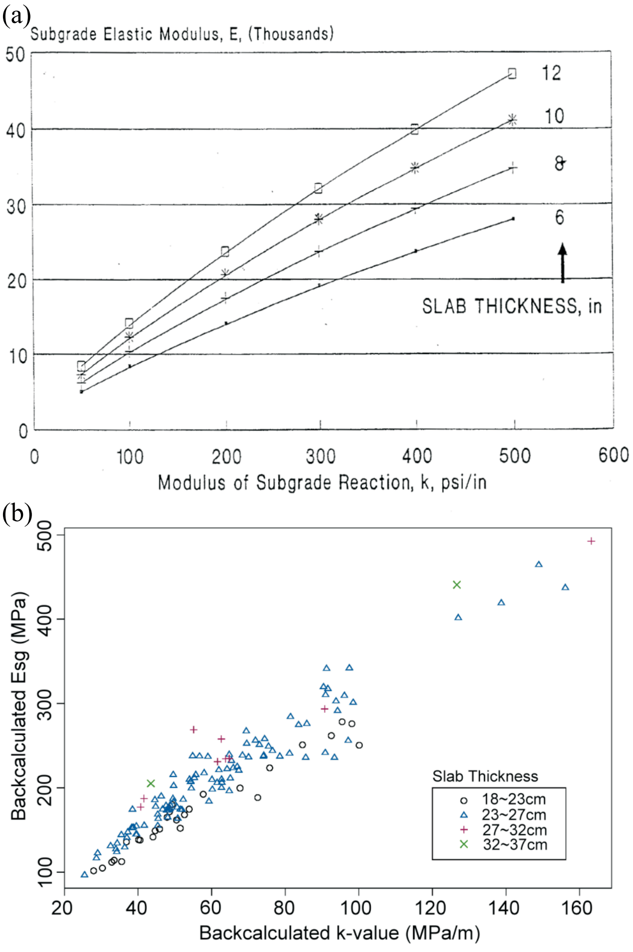

Figure 6. Comparison of elastic solid foundation versus dense liquid foundation based on: (a) theoretical comparison [25]; and (b) backcalculated results.

dynamic k-value by 2 as the effective static k-value by AASHTO [20] is a reasonable choice, though more research study is still needed to reduce the variability as well as to study other possible adjustments. In addition, it was also found that slab thickness did have significant effects on the relationship of the backcalculated subgrade elastic modulus and the backcalculated modulus of subgrade reaction. Subsequently, a revised regression model was proposed for future practical applications.

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