Performance of OSTBC Diversity for QAM over Correlated Fading Channels

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Abstract—We consider arbitrary rectangular QAM signaling in orthogonal space-time block code (OSTBC) diversity systems over correlated fading channels with Gaussian channel gains. We first decorrelate the physical branches into uncorrelated virtual branches to obtain a general moment generating function (MGF), from which closed-form symbol error probability (SEP) is then obtained for OSTBC with generalized complex orthogonal design (GCOD) and it is discovered that different information symbols may yield different SEP's.

I. Introduction

Due to its simplicity and orthogonal nature, OSTBC can transform multiple-input multiple-out (MIMO) channels into equivalent single-input single-output (SISO) channels [1]-[3]. It is now widely known that, for the same diversity order, the SEP for OSTBC systems takes the same form as that for maximal ratio combining (MRC) systems, but with a performance loss [2]. Of all the studies on SEP evaluations for QAM signaling in MRC systems [4]-[6], the SEP performance for OSTBC with generalized complex orthogonal design (GCOD) has not been treated.

The usual approach to obtain the MGF for correlated fading channels is to Laplace transform the correlative probability density function (PDF) of the received signal-to-noise ratio (SNR) for the fading model in concern [4]. A simpler approach is to first decorrelate the physical channels into uncorrelated virtual channels. Since the channels have Gaussian channel gains, the PDF of SNR for the virtual MRC channels will be a simple complex Gaussian density. Then it becomes much easier to obtain the MGF, from which the SEP can be derived. The MGF expression thus derived can have rather universal applications. First, it can be easily applied to correlated Rayleigh, Nakagami-*m*, as well as Ricean channels, etc., thus obviating the needs of calculating each MGF individually using the correlative PDF corresponding to various fading models. Second, it can be readily modified to express various scenarios of channel power distribution as well as joint fading models.

In this paper, we explore performance of OSTBC with GCOD for *arbitrary rectangular* QAM signaling over correlated fading channels. We will derive the SEP expressions for the general case of virtual channels having a combination of identical and distinct channel powers for various fading models.

II. OSTBC Diversity in Correlated Fading Channels

Consider a wireless communication system with P transmit antennas and Q receive antennas. Let the equivalent baseband path gain from transmit antenna p to receive antenna q be denoted by $\hbar_{pq}$, $p = 1,2,...,P$, $q = 1,2,...,Q$. There are $L = PQ$ terms in the set $\{\hbar_{pq}\}$. Stack these terms to form an $PQ \times 1$ vector $\mathbf{h} = [h_1, h_2, \cdots h_P, h_{P+1}, h_{P+2}, \quad \cdots, h_L]^T$ = $[\hbar_{11}, \cdots, \hbar_{P1},\ \hbar_{12}, \cdots, \hbar_{PQ}]^T$, where T denotes transpose and $h_1 = \hbar_{11}, h_2 = \hbar_{21}, \cdots$. The lth channel gain is $h_l = h_{cl} + jh_{sl}$, $l = 1,2,...,L$ with h_{cl} and h_{sl} being independent real random variables (RV) with means $\bar{h}_{cl}$, $\bar{h}_{sl}$ and variances σ_{hcl}^2 and σ_{hsl}^2 respectively. So, $E[h_l] = \bar{h}_l = \bar{h}_{cl} + j\bar{h}_{sl}$ and $V[h_l] = \sigma_{hl}^2 = E[|h_l|^2] - |\bar{h}_l|^2$ $= \sigma_{hcl}^2 + \sigma_{hsl}^2$, where $E[\cdot]$ and $V[\cdot]$ respectively denote expectation and variance. Assuming flat fading, when an information symbol x_k ($k = 1,2,...,K$, where K is the number of information symbols chosen for one OSTBC transmission block) is transmitted, the received signal and noise of the lth channel are $h_l x_k$ and n_l respectively. $n_l = n_{cl} + jn_{sl}$ is a complex Gaussian RV with n_{cl} and n_{sl} being independent, identically distributed (i.i.d.) real Gaussian RV's with zero mean and variance σ_n^2 for all l.

An OSTBC transmission can be described by a $N \times P$ transmission matrix as

$$\mathbf{G} = \begin{bmatrix} g_{11} & g_{21} & \cdots & g_{P1} \\ g_{12} & g_{22} & \cdots & g_{P2} \\ \vdots & \vdots & \cdots & \vdots \\ g_{1N} & g_{2N} & \cdots & g_{PN} \end{bmatrix}. \quad (1)$$

Here, g_{pn} is the codeword transmitted from the pth transmit antenna at the nth symbol instant, $n = 1,2,...,N$. The code words are sent in blocks of N symbols. Each codeword g_{pn} is a linear combination of information symbols $\{x_k\}$ and their conjugates $\{x_k^*\}$. The code rate is K/N. For QAM, the K information symbols $\{x_k\}$ for each block are selected from an M-ary QAM constellation. For GCOD, $\mathbf{G}^H\mathbf{G} = \mathbf{D}$, where $\mathbf{D}$ is an $P \times P$ diagonal matrix with the (p, p)th diagonal element of the form [1], [7]

$$l_{p,1}|x_1|^2 + l_{p,2}|x_2|^2 + \cdots + l_{p,K}|x_K|^2,\ p = 1,2,...,P, \quad (2)$$

where $\{l_{p,k}\}$ are strictly positive numbers [1]. Assume quasi-static fading so that channel gains remain constant over an N-symbol block and vary independently from block to block. We follow the squaring method used for COD in [3] to obtain the linear processor output for GCOD as

$$\hat{x}_k = \sum_{l=1}^{L}[(b_{l,k}\,|\,h_l\,|)^2 x_k + \eta_{l,k}]. \quad (3)$$

Here $b_{l,k} = b_{(q-1)P+p,k} = \sqrt{l_{p,k}}$, (see the $l_{p,k}$ definition in (2) where the subscript p cannot exceed P), and for a fixed channel realization, $\{\eta_{l,k}\}$ are independent, non- identical complex Gaussian RV's with zero means and variances $\{(b_{l,k}\,|\,h_l\,|\,\sigma_n)^2\}$ in each dimension. Define $h_{l,k} = b_{l,k}h_l$, $\mathbf{h}_k = [h_{1,k},\ h_{2,k}, \cdots h_{L,k}]^T$, and $\eta_k = \sum_{l=1}^{L}\eta_{l,k}$, (3) becomes

$$\hat{x}_k = \|\,\mathbf{h}_k\,\|^2\, x_k + \eta_k, \quad (4)$$

where η_k is now a complex Gaussian RV with zero mean and variance $\|\,\mathbf{h}_k\,\|^2\,\sigma_n^2$ in each dimension. The modified gain $\{h_{l,k}\}$ are still correlated. The equivalent SISO model of (4) shows that each information symbol is associated with its own channel gain and Gaussian noise (due to the subscript k). This simply means that, *different information symbol may yield different* SEP. This important discovery has never been brought up before.

Let $E_{av} = E[|\,x_k\,|^2]$, then the short-term ($h_{l,k}$ held fixed) received SNR at the lth channel is given by

$$\gamma_{l,k} = \frac{E[|\,h_{l,k}x_k\,|^2]}{2\sigma_n^2} = |\,h_{l,k}\,|^2\,\frac{E_{av}}{2\sigma_n^2} = z_{l,k}z_{l,k}^*, \quad (5)$$

where $z_{l,k} = z_{cl,k} + jz_{sl,k} = h_{l,k}\sqrt{E_{av}/2\sigma_n^2}$ is the scaled gain. Thus $E[z_{l,k}] = \bar{z}_{l,k} = \bar{z}_{cl,k} + j\bar{z}_{sl,k} = \bar{h}_{l,k}\sqrt{E_{av}/2\sigma_n^2}$ and $V[z_{l,k}] = \sigma_{z_{l,k}}^2 = \sigma_{z_{cl,k}}^2 + \sigma_{z_{sl,k}}^2 = E_{av}\sigma_{h_{l,k}}^2/2\sigma_n^2$. Define the scaled gain vector $\mathbf{z}_k = [z_{1,k}, z_{2,k}, ..., z_{L,k}]^T$. The combined received SNR (as in MRC) is $\gamma = \mathbf{z}_k^H\mathbf{z}_k = \sum_{l=1}^{L}\gamma_{l,k}$, where H denotes Hermitian transpose. Further define $E[\mathbf{z}_k] = \bar{\mathbf{z}}_k = [\bar{z}_{1,k}, \bar{z}_{2,k}, ..., \bar{z}_{L,k}]^T$. When channels are correlated, the covariance matrix for $\mathbf{z}_k$ is an $L \times L$ Hermitian matrix given by

$$\mathbf{C}_k = E[(\mathbf{z}_k - \bar{\mathbf{z}}_k)(\mathbf{z}_k - \bar{\mathbf{z}}_k)^H]. \quad (6)$$

$\mathbf{C}_k$ is positive definite and has L real positive eigenvalues $\{\lambda_l\}$ and hence can be unitary diagonalized as $\mathbf{C}_k = \mathbf{U}_k\mathbf{\Lambda}_k\mathbf{U}_k^H$, where $\mathbf{\Lambda}_k$ is a diagonal matrix with diagonal entries $\{\lambda_{l,k}\}$ arranged in descending order and $\mathbf{U}_k$ is a unitary matrix whose columns are corresponding eigenvectors. We thus obtain a virtual system with uncorrelated branches with uncorrelated virtual channel gain

$$\mathbf{z}'_k = [z'_{1,k}, z'_{2,k}, ..., z'_{L,k}]^T = \mathbf{U}_k^H\mathbf{z}_k \text{ and}$$

$$E[\mathbf{z}'_k] = \bar{\mathbf{z}}'_k = [\bar{z}'_{1,k}, \bar{z}'_{2,k}, ..., \bar{z}'_{L,k}]^T = \mathbf{U}_k^H\bar{\mathbf{z}}_k, \quad (7)$$

$$E[z'^*_{l,k}z'_{l',k}] = \bar{z}'^*_{l,k}\bar{z}'_{l',k} + \lambda_{l,k}\delta_{ll'}, \text{ and}$$

$$\lambda_{l,k} = E[|\,z'_{l,k}\,|^2] - |\,\bar{z}'_{l,k}\,|^2 = \sigma_{z'_{l,k}}^2,\ l,l' = 1,2,...,L. \quad (8)$$

After decorrelation, the new linear processor output is

$$\hat{x}'_k = \|\,\mathbf{h}'_k\,\|^2\, x_k + \eta'_k, \quad (9)$$

where η'_k is now a complex Gaussian noise with zero mean and variance $\|\,\mathbf{h}'_k\,\|^2\,\sigma_n^2$ in each dimension for a fixed channel realization. Since $\|\,\mathbf{h}'_k\,\| = \|\,\mathbf{h}_k\,\|$, η'_k has exactly the same PDF as η_k and hence $\hat{x}'_k = \hat{x}_k$.

In the derivations that follow, the subscript k will be dropped for convenience. The reader should keep in mind that, from here on, all $\{z_l\}$ or $\{h_l\}$ are modified correlated channel gains and should not be confused them with the original correlated physical channel gains as given in (2).

III. The General MGF

We want to find the MGF of $\gamma = \mathbf{z}^H\mathbf{z}$. Note that $\gamma' = \mathbf{z}'^H\mathbf{z}' = \mathbf{z}^H\mathbf{U}\mathbf{U}^H\mathbf{z} = \gamma$. So, equivalently, we are to find the MGF of γ'_l. This is much easier to handle since the components in $\mathbf{z}'$ are independent for Gaussian channel gains. The PDF of $\mathbf{z}'$ is

$$f(\{z_l'\}) = f(\{z_{cl}', z_{sl}'\}) = \frac{1}{\prod_{l=1}^{L} 2\pi\sigma_{z'cl}\sigma_{z'sl}} \times$$

$$\exp\left\{-\sum_{l=1}^{L}\left[\frac{(z_{cl}' - \bar{z}_{cl}')^2}{2\sigma_{z'cl}^2} + \frac{(z_{sl}' - \bar{z}_{sl}')^2}{2\sigma_{z'sl}^2}\right]\right\}, \quad (10)$$

where $\sigma_{z'cl}^2 = V[z_{cl}']$, $\sigma_{z'sl}^2 = V[z_{sl}']$. So the MGF of γ is

$$M_\gamma(s) = E\{\exp[s\sum_{l=1}^{L}(z_{cl}'^2 + z_{sl}'^2)]\}$$

$$= \frac{\exp\left\{\sum_{l=1}^{L}\left[s\left(\frac{\bar{z}_{cl}'^2}{1-2\sigma_{z'cl}^2 s} + \frac{\bar{z}_{sl}'^2}{1-2\sigma_{z'sl}^2 s}\right)\right]\right\}}{\prod_{l=1}^{L}(1-2\sigma_{z'cl}^2 s)^{1/2}(1-2\sigma_{z'sl}^2 s)^{1/2}}. \quad (11)$$

The MGF of (11) is a general expression that can be applied to OSTBC diversity systems in any fading environment having Gaussian channel gains. If $\sigma_{z'cl}^2 = \sigma_{z'sl}^2 = \sigma_{z'l}^2/2$ (circularly symmetric Gaussian channel), then (11) reduces to, using (8),

$$M_\gamma(s) = \frac{\exp\left[\sum_{l=1}^{L}\frac{|\bar{z}_l'|^2 \lambda_l s}{1-\lambda_l s}\right]}{\prod_{l=1}^{L}(1-\lambda_l s)}. \quad (12)$$

Both (11) and (12) apply to both correlated and uncorrelated channels, the $\{\lambda_l\}$ in (12) takes correlation into account.

Replacing proper values for λ_l and $|\bar{z}_l'|$, (12) can be readily used to deduce the MGF for Rayleigh, Nakagami-*m*, or Ricean fading channels, etc.

IV. Rectangular QAM Performance for OSTBC

Consider rectangular *M*-QAM transmission in OSTBC systems over fading channels. Let $M = M_1 M_2$, where M_1 and M_2 are the symbol numbers used respectively in the horizontal and vertical *M*-QAM dimension.

Assuming quasi-static fading where the SNR's $\{\gamma_l'\}$ remain unchanged over one *N* symbol block. Then, using $\bar{\gamma} = \sum_{l=1}^{L}\bar{\gamma}_l = \sum_{l=1}^{L}\bar{\gamma}_l' = \bar{\gamma}'$ (generally, $\bar{\gamma}_l \neq \bar{\gamma}_l'$), the SEP for *M*-QAM conditioned on $\{\gamma_l'\}$ can be shown to be [4]

$$P_M(E) = \left[\frac{2(M_1-1)}{M_1} + \frac{2(M_2-1)}{M_2}\right]I(\frac{\pi}{2}) - \frac{4(M_1-1)(M_2-1)}{M_1 M_2}I(\frac{\pi}{4}), \quad (13)$$

where

$$I(\phi) = \frac{1}{\pi}\int_0^{\phi}\prod_{l=1}^{L}M_l(-\frac{g}{\sin^2\theta})d\theta. \quad (14)$$

Here, $M_l(s) = \int_0^\infty f_l(\gamma_l')e^{s\gamma_l'}d\gamma_l'$ is the MGF of γ_l', and $g = 3/(M_1^2 + M_2^2 - 2)$. Consider only circularly symmetric Gaussian channel gains. Then, using (12), we have

$$\prod_{l=1}^{L}M_l(-\frac{g}{\sin^2\theta}) = M_\gamma(-\frac{g}{\sin^2\theta}) = \frac{\exp\left[-\sum_{l=1}^{L}\frac{|\bar{z}_l'|^2 g/\sin^2\theta}{1+\lambda_l g/\sin^2\theta}\right]}{\prod_{l=1}^{L}(1+\lambda_l g/\sin^2\theta)} \quad (15)$$

We now consider SEP performance for Rayleigh fading channels. Results for Nakagami-*m* and Ricean fading can be readily extended. However, due space limitation, we will not produce these results here. As stated earlier, we will treat the general case of the uncorrelated virtual channels with a combination of identical and distinct power values (i.e., after decorrelation). Results for special cases of virtual channels with identical or distinct channel powers can be readily deduced from the general results.

If, before decorrelation, the normalized complex channel gain z_l is Gaussian with zero mean and identical variances in each real dimension, then we have Rayleigh fading. The decorrelated gain z_l' will also be Gaussian with zero mean and identical variances $\sigma_{z'cl}^2 = \sigma_{z'sl}^2 = \sigma_{z'l}^2/2$. Then, using (8), (15) reduces to the Rayleigh MGF as

$$\prod_{l=1}^{L}M_l(-\frac{g}{\sin^2\theta}) = M_\gamma(-\frac{g}{\sin^2\theta}) = \frac{1}{\prod_{l=1}^{L}(1+\bar{\gamma}_l' g/\sin^2\theta)}. \quad (16)$$

For channels having combination of identical and distinct powers, let L_1 channels have identical SNR $\bar{\gamma}_1'$, L_2 channels have identical SNR $\bar{\gamma}_2'$, …, L_R channels have identical SNR $\bar{\gamma}_R'$, where $\sum_{r=1}^{R}L_r = L$. Then, (16) becomes

$$M_\gamma(-\frac{g}{\sin^2\theta}) = \frac{1}{\prod_{r=1}^{R}(1+\bar{\gamma}_r' g/\sin^2\theta)^{L_r}}. \quad (17)$$

Using variable change $z = \tan\theta$ and formulas in [8, eq. (9.1.1), eq. (45.3.6.1), eq. (45.3.6.11), eq. (45.3.6.13), eq. (45.3.6.22)], we can get

$$I(\phi) = \sum_{r=1}^{R}\sum_{s=1}^{L_r}G(r,s)I_3(\phi, s, g\bar{\gamma}_r'), \quad (18)$$

where

$$G(r,s) = \lim_{x \to (-g\bar{\gamma}'_r)} \frac{1}{(L_r - s)!} \frac{d^{L_r - s}}{dx^{L_r - s}} \Big[\frac{x^{L-1}}{\prod_{\substack{p=1 \\ p \neq r}}^{R} (x + g\bar{\gamma}'_p)^{L_p}} \Big]. \quad (19)$$

When $R = 1$ (identical channel powers), we must replace $\prod_{\substack{p=1 \\ p \neq r}}^{R} (x + g\bar{\gamma}'_p)^{L_p}$ in (19) by 1. And in (17), (18),

$$I_3(a,n,b) = \frac{1}{\pi} \sum_{i=0}^{n-2} \binom{n-2}{i} \sum_{k=0}^{i} \binom{i}{i-k} \Big(-\frac{b}{1+b}\Big)^{i-k} \Big(\frac{1}{1+b}\Big)^{k}$$

$$\times I_4(\tan a, b, 1+b, n-k), \ a,b \text{ real}, \ n \geq 2 \text{ integer}, \quad (20)$$

$$I_4(\beta,c,e,m) = \frac{-\beta}{2(m-1)e(c+e\beta^2)^{m-1}}$$

$$+ \delta(m-2) \frac{\tan^{-1}(\beta\sqrt{e/c})}{2(m-1)\sqrt{e^3 c}}$$

$$+ \frac{[1-\delta(m-2)]}{2(m-1)e} \Big[\frac{\tan^{-1}(\beta\sqrt{e/c})}{c^{m-3/2} e^{1/2}} C(m-2)$$

$$+ \sum_{k=1}^{m-2} \frac{\beta F_k(m-2)}{c^k (c+e\beta^2)^{m-k-1}} \Big], \ c,e \text{ real}, \ m \geq 2 \text{ integer}, \quad (21)$$

where

$$C(u) = \frac{(2u-1)(2u-3)\cdots 1}{2^u u!} = \frac{(2u-1)!!}{2^u u!},$$

$$F_k(u) = \frac{\delta(k-1) + [1-\delta(k-1)](2u-1)(2u-3)\cdots(2u-2k+3)}{2^k u(u-1)(u-2)\cdots(u-k+1)},$$

$$\delta(k): \text{discrete-time unit impulse.} \quad (22)$$

Substituting (17) and (18) into (13), we have the desired expression of SEP. Note that, by setting $L_r = 1$ for all *r*, we have the result for distinct power channels. Then, by setting $R = 1$, we have the result for identical power channels .

V. Numerical Results

We adopt the rate 3/5 GCOD code using $P = 6$ transmit antennas as given in [7]. For simplicity, we use one receive antenna. Thus, we have equivalently an order-6 diversity system ($L = 6$). The transmission matrix is of size 30×6 [7, (6)]. The code uses $K = 18$ and $N = 30$. There are three groups (group A, B, C) yielding three different SEP's. Assuming exponential correlation model, i.e., correlation coefficient between the *l*th and l' th channel gain is $\rho_{ll'} = \rho^{|l-l'|}$ [5]. Beginning with correlated physical channels having identical channel powers and using $\rho = 0.8$, we find all 6 eigenvalues of $\mathbf{C}_k$ are distinctive. Thus, after decorrelation, the virtual channels will have distinct channel powers. The SEP vs. $\bar{\gamma}/L$ performance curves for groups A, B, C for 2×8 QAM signaling over Rayleigh fading are presented in Fig. 1. Monte Carlo simulated results are found in excellent agreement with the theoretical result.

VI. Conclusion

By channel decorrelation, we derive a general MGF expression from which closed-form SEP is obtained for arbitrary rectangular QAM signaling in OSTBC-GCOD diversity systems over correlated Rayleigh fading channels. We discover that different information symbols may yield different performance. A GCOD example is given for demonstration. Theoretical SEP results are found in excellent agreement with Monte Carlo simulations.

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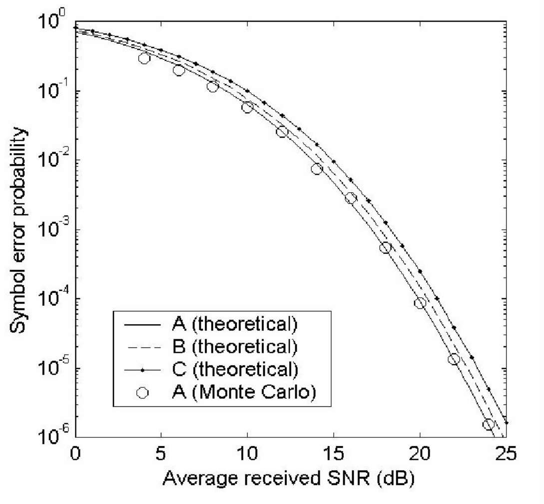

Figure1. Performance of rate 3/5 OSTBC with GCOD using 6 transmit antennas for 2x8 QAM over Rayleigh fading channels.