Performance of a Non-Orthogonal STBC over Correlative Fading Channels

*Rainfield Y. Yen, **Hong-Yu Liu, ***Chun-Chi Lee

*Shang-Chieh Nan, and *Ming-Chih Hong

*Department of Electrical Engineering, Tamkang University
Danshuei Township, Taipei County 25137, Taiwan (R.O.C.)

**Department of Computer and Communication Engineering, Dahan Institute of Technology
Dahan Village, Sincheng Township, Hualien County 97145, Taiwan (R.O.C.).

***Department of Computer and Communication, Shu-Te University
Yanchao, Kaohsiung County 82445, Taiwan (R.O.C.)

E-mail: *rainfieldy@yahoo.com, **hongyu.liu@msa.hinet.net, *** homer1@mail.stu.edu.tw

***Abstract*—It has been recently shown that, for non-orthogonal space-time block code (STBC), the multiple-input multiple-output (MIMO) maximum-likelihood (ML) metric can also be decoupled into single-input single-output (SISO) ML metrics for decoding simplification just as for orthogonal STBC. In this work, we utilized the decoupled metrics of a non-orthogonal STBC to derive the symbol error rate (SER) in correlative fading channels and show that, when the non-orthogonal code is generated by converting an orthogonal code using proper precoding, the conversion will improve the SER performance when the MIMO channels are correlated.**

Keywords - space-time block code, MIMO, maximum-likelihood decoding, corrrelative fading channels

I. Introduction

DUE to the orthogonal structure of the orthogonal STBC, the decoding of data symbols can be made simplified by decoupling the MIMO ML metric into separate SISO ML metrics [1]-[3]. However, recently, it is found that non-orthogonal STBC can also be decoded by exactly the same simple manner [4]. This means decoupled ML decoding is not the prerogative of the orthogonal STBC. Both non-orthogonal and orthogonal STBCs belong to the class of linear STBCs [2], [4] and new codes for both are yet to be discovered, though more attention has so far been paid to the design of orthogonal STBCs [1], [5]-[7]. A non-orthogonal STBC may be derived from an orthogonal STBC by proper precoding to achieve minimum SER for correlative fading communications [8]. Since minimum error rate is the ultimate performance measure for data communications, we are motivated to investigate the SER performance of non-orthogonal STBCs so formed for correlative fading channels.

In this work, we shall derive symbol error rate (SER) expression for a non-orthogonal STBC over correlative fading channel models. This non-orthogonal code has been given in [8] and falls into a class of linear STBC specified in [4]. Comparison will be made between the SER performance for an orthogonal STBC and the corresponding non-orthogonal STBC derived from that orthogonal STBC. We will assume QAM signaling over Rayleigh fading channels in our SER derivations.

The rest of the paper is organized as follows: Section II briefly reviews the class of linear STBC introduced in [4]. Section III derives the SER of a non-orthogonal STBC for *M*-QAM transmission over correlative fading channels. Then, Section IV presents numerical examples including performance comparisons between orthogonal and non-orthogonal codes. Finally, Section V draws conclusions.

II. Non-Orthogonal STBC

We consider a wireless communication system with P transmit antennas and Q receive antennas employing linear STBC transmission. Let the equivalent baseband path gain from the pth transmit antenna to qth receive antenna be $h_{p,q}$, $p = 1,2,\ldots,P$, $q = 1,2,\ldots,Q$.

A linear STBC transmission can be described by a $P \times N$ code matrix as [2]

$$\mathbf{G} = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1N} \\ g_{21} & g_{22} & \cdots & g_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ g_{P1} & g_{P2} & \cdots & g_{PN} \end{bmatrix}. \quad (1)$$

Here, g_{pn} is the codeword transmitted from the pth transmit antenna at the nth time slot, $n = 1,2,\ldots,N$, with N time slots constituting a block. Each codeword g_{pn} is a linear combination of information symbols $\{x_k\}$ and their conjugates $\{x_k^*\}$, $k=1,2,...,K$, $K \le N$. In other words, K information symbols over a block of N time slots are chosen for transmission through P transmit antennas. Thus, the code rate is K/N. For this work, since we will consider M-ary QAM transmission, the K symbols of $\{x_k\}$ used for a block are selected from M possible constellation points. Different block may select different K symbols from the constellation. The code matrix $\mathbf{G}$ can also be expressed alternatively by another two $P \times N$ code matrices $\mathbf{A}_k$ and $\mathbf{B}_k$ as [2]

$$\mathbf{G} = \sum_{k=1}^{K} (x_{kc}\mathbf{A}_k + jx_{ks}\mathbf{B}_k), \tag{2}$$

where $x_{kc} = \mathrm{Re}[x_k]$ and $x_{ks} = \mathrm{Im}[x_k]$, with $\mathrm{Re}[\cdot]$ and $\mathrm{Im}[\cdot]$ respectively denoting the real and imaginary part.

The received signal vector $\mathbf{r}_q = [r_{1,q}, r_{2,q}, \cdots, r_{N,q}]^T$, T denoting transposition, at the qth receive antenna over the block of N time slots is given by

$$\mathbf{r}_q = \mathbf{y}_q + \mathbf{n}_q = \mathbf{G}^T\mathbf{h}_q + \mathbf{n}_q, \tag{3}$$

where $\mathbf{y}_q = [y_{1,q}, y_{2,q}, \cdots, y_{N,q}]^T$ is the noise-free received signal vector, $\mathbf{n}_q = [n_{1,q}, n_{2,q}, \cdots, n_{N,q}]^T$ is the zero-mean additive white Gaussian noise (AWGN) vector with covariance matrix $\sigma_n^2\mathbf{I}_N$, with $\mathbf{I}_N$ being the $N \times N$ identity matrix , and $\mathbf{h}_q = [h_{1,q}, h_{2,q}, \cdots, h_{P,q}]^T$ is the channel gain vector for the qth receive antenna. For (3), we shall assume that $\mathbf{h}_q$ remains constant over one block of N time slots (quasi-static fading).

The class of the linear STBC to be discussed here as given in [4] will possess the following properties: for $k=1,2,...,K$, $i=1,2,...,K$, and $i \ne k$

$$\mathbf{A}_k\mathbf{A}_i^H = -\mathbf{A}_i\mathbf{A}_k^H, \tag{4a}$$

$$\mathbf{B}_k\mathbf{B}_i^H = -\mathbf{B}_i\mathbf{B}_k^H, \tag{4b}$$

$$\mathbf{B}_i\mathbf{A}_k^H = \mathbf{A}_k\mathbf{B}_i^H, \tag{4c}$$

where H denotes Hermitian transposition. The code class given by (4) is more general than the amicable orthogonal design (AOD) defined in [2]. Some non-orthogonal codes that do fall into the class given by (4) have appeared recently in the literature [8], [9] and we shall later use one of these non-orthogonal codes for SER performance demonstrations.

In [4], it is shown that a linear STBC satisfying (4) can be decoded by separable ML metrics (decoupled from the general ML metric) for each complex data symbol. Moreover, the individual metric can further be decomposed into two quadrature metrics, one for the real part of the data symbol and the other for the imaginary part. For square M-QAM signaling, this decomposition would further reduce the computation complexity from order $\mathrm{O}(M)$ to $\mathrm{O}(\sqrt{M})$. The two quadature metrics are given as

$$D_{ML,kc} = (R'_{k,1} - \lambda_{k,1}x'_{kc})^2, \tag{5a}$$

$$D_{ML,ks} = (R'_{k,2} - \lambda_{k,2}x'_{ks})^2. \tag{5b}$$

The mathematical symbols in (5) are illustrated below.

The kth complex data symbol is $x_k = x_{kc} + jx_{ks}$. Define

$$a_k^2 = \sum_{q=1}^{Q} \| \mathbf{A}_k^T\mathbf{h}_q \|^2 = \sum_{q=1}^{Q} \| \mathbf{a}_{k,q} \|^2, \tag{6a}$$

$$b_k^2 = \sum_{q=1}^{Q} \| \mathbf{B}_k^T\mathbf{h}_q \|^2 = \sum_{q=1}^{Q} \| \mathbf{b}_{k,q} \|^2, \tag{6b}$$

$$2c_k = j\sum_{q=1}^{Q} \mathbf{h}_q^H(\mathbf{B}_k\mathbf{A}_k^H - \mathbf{A}_k\mathbf{B}_k^H)^*\mathbf{h}_q, \tag{6c}$$

$$R_{k,1} = \frac{1}{2}\sum_{q=1}^{Q}[\mathbf{r}_q^H\mathbf{A}_k^T\mathbf{h}_q + (\mathbf{r}_q^H\mathbf{A}_k^T\mathbf{h}_q)^H], \tag{6d}$$

$$R_{k,2} = \frac{j}{2}\sum_{q=1}^{Q}[\mathbf{r}_q^H\mathbf{B}_k^T\mathbf{h}_q - (\mathbf{r}_q^H\mathbf{B}_k^T\mathbf{h}_q)^H]. \tag{6e}$$

The first three quantities are all real positive [4]. Let $\lambda_{k,1}$ and $\lambda_{k,2}$ be the two eigenvalues of the matrix

$$\begin{bmatrix} a_k^2 & c_k \\ c_k & b_k^2 \end{bmatrix} = \mathbf{U}_k \begin{bmatrix} \lambda_{k,1} & 0 \\ 0 & \lambda_{k,2} \end{bmatrix} \mathbf{U}_k^H, \tag{7}$$

where $\mathbf{U}_k$ is the 2×2 unitary matrix whose columns consist of orthonormal eigenvectors respectively corresponding to $\lambda_{k,1}, \lambda_{k,2}$. The set of quadrature components of the data symbol and the set $(R_{k,1}, R_{k,2})$ are then transformed into new sets by

$$\begin{bmatrix} x'_{kc} \\ x'_{ks} \end{bmatrix} = \mathbf{U}_k^H \begin{bmatrix} x_{kc} \\ x_{ks} \end{bmatrix}, \tag{8a}$$

$$\begin{bmatrix} R'_{k,1} \\ R'_{k,2} \end{bmatrix} = \mathbf{U}_k^H \begin{bmatrix} R_{k,1} \\ R_{k,2} \end{bmatrix}. \tag{8b}$$

III. SER Analysis for a Non-Orthogonal STBC

We will take a non-orthogonal STBC given in [8] using M-QAM signaling over correlative fading channels. The code given in [8] satisfies all the conditions of (4) but with some relaxations, viz., $\mathbf{A}_k\mathbf{A}_k^H = \mathbf{B}_k\mathbf{B}_k^H$ (This does not imply $\mathbf{A}_k = \mathbf{B}_k$) and $\mathbf{B}_k\mathbf{A}_k^H - \mathbf{A}_k\mathbf{B}_k^H = \mathbf{0}$. With the relaxations, we easily find from (6) that $c_k = 0$,

$\lambda_{k,1} = a_k^2 = b_k^2$ (this does not imply $\mathbf{a}_{k,q} = \mathbf{b}_{k,q}$ either), and $\mathbf{U}_k = \mathbf{I}_2$. Then (5) becomes

$$D_{ML,kc} = (R_{k,1} - \lambda_k x_{kc})^2, \tag{9a}$$

$$D_{ML,ks} = (R_{k,2} - \lambda_k x_{ks})^2, \tag{9b}$$

where $\lambda_k = \lambda_{k,1} = \lambda_{k,2}$. In light of (9a) and (9b), we can picture the system as formed by equivalent channels with virtual channel gains λ_k and virtual noisy received signals $R_{k,1}, R_{k,2}$ respectively for real and imaginary signal components x_{kc}, x_{ks}. Then, (9a) and (9b) are nothing more than just the minimum distance metrics. Thus we can respectively regard $n_{kc} = R_{k,1} - \lambda_k x_{kc}$ and $n_{ks} = R_{k,2} - \lambda_k x_{ks}$ as the real and imaginary parts of a virtual complex noise. In [4], it is shown that

$$n_{kc} = \frac{1}{2}\sum_{q=1}^{Q}[\mathbf{n}_q^H \mathbf{A}_k^T \mathbf{h}_q + (\mathbf{n}_q^H \mathbf{A}_k^T \mathbf{h}_q)^H], \tag{10a}$$

$$n_{ks} = \frac{j}{2}\sum_{q=1}^{Q}[\mathbf{n}_q^H \mathbf{B}_k^T \mathbf{h}_q - (\mathbf{n}_q^H \mathbf{B}_k^T \mathbf{h}_q)^H], \tag{10b}$$

and these two noise components are identical Gaussian RV's with zero mean and variance $\sigma_{kc}^2 = \sigma_{ks}^2 = a_k^2 \sigma_n^2 / 2$.

In (6a) and (6b), we can consider $\mathbf{A}_k^T \mathbf{h}_q = \mathbf{a}_{k,q} = [a_{k,1,q}\ a_{k,2,q} \cdots a_{k,P,q}]^T$ and $\mathbf{B}_k^T \mathbf{h}_q = \mathbf{b}_{k,q} = [b_{k,1,q}\ \ b_{k,2,q} \cdots b_{k,P,q}]^T$ as the equivalent virtual channel vectors for the qth receive antenna as seen by x_{kc} and x_{ks} respectively. If the original physical channels $\{h_{p,q}\}$ have complex Gaussian gains, then $\{a_{k,p,q}\}$ and $\{b_{k,p,q}\}$ will be correlated complex Gaussian gains as both $\mathbf{a}_{k,q}$ and $\mathbf{b}_{k,q}$ are linear transformations of $\mathbf{h}_q$.

For square M-QAM signaling over fading channels, x_{kc} and x_{ks} each randomly takes the value from the set $\{\ (2m-1-M_c)d,\ m = 1,2,...,\sqrt{M}\ \}$ with equal probability $1/\sqrt{M}$, where d is a constant that can be used for power control. The average transmitted energies of x_{kc} and x_{ks} are given by $E_{c,av} = E_{s,av} = \frac{M-1}{3}d^2 = \frac{d^2}{2g}$, where $g = \frac{3}{2(M-1)}$. Using (9), the received signal energies are $a_k^4 E_{c,av} = b_k^4 E_{s,av}$. Therefore, the total received signal-to-noise ratios (SNRs) after the ML linear processor are given by

$$\gamma_{kc} = a_k^2 \frac{E_{c,av}}{\sigma_n^2/2} = \sum_{q=1}^{Q}\sum_{p=1}^{P}|a_{k,p,q}|^2 \frac{E_{c,av}}{\sigma_n^2/2} = \frac{2E_{c,av}}{\sigma_n^2}\sum_{l=1}^{L}|a_{k,l}|^2 = \sum_{l=1}^{L}\gamma_{kc,l}, \tag{11a}$$

$$\gamma_{ks} = \gamma_{kc} = \sum_{l=1}^{L}\gamma_{ks,l} = \sum_{l=1}^{L}\gamma_{kc,l} = \gamma_k, \tag{11b}$$

where we have stacked $a_{k,p,q}$ and $b_{k,p,q}$ respectively into $a_{k,l}$ and $b_{k,l}$ by letting $L = PQ$, and defined $\gamma_{kc,l} = \frac{2E_{c,av}}{\sigma_n^2}|a_{k,l}|^2 = \gamma_{ks,l} = \frac{2E_{s,av}}{\sigma_n^2}|b_{k,l}|^2$ as the quadrature branch SNRs. This is equivalent to redefining $\mathbf{a}_k = [a_{k,1}, a_{k,2}, \cdots, a_{k,L}]^T$ and $\mathbf{b}_k = [b_{k,1}, b_{k,2}, \cdots, b_{k,L}]^T$ as the new virtual channel vectors respectively seen by x_{kc} and x_{ks}.

For fixed set of $\{\gamma_{kc,l}\}$ and $\{\gamma_{ks,l}\}$, the conditional SER for square M-QAM can be calculated using the moment generating function (MGF)-based approach as [10]

$$\begin{aligned} P_M(e|\gamma_k) &= 1 - [1 - P_{\sqrt{M}}(e|\gamma_k)][1 - P_{\sqrt{M}}(e|\gamma_k)] \\ &= P_{\sqrt{M}}(e|\gamma_k) + P_{\sqrt{M}}(e|\gamma_k) - P_{\sqrt{M}}(e|\gamma_k)P_{\sqrt{M}}(e|\gamma_k) \\ &= \frac{4(\sqrt{M}-1)}{\sqrt{M}}Q(\sqrt{2g\gamma_k}) - \frac{4(\sqrt{M}-1)^2}{M}Q^2(\sqrt{2g\gamma_k}) \\ &= \frac{4(\sqrt{M}-1)}{\pi\sqrt{M}}\int_0^{\pi/2}\exp(-\frac{g\gamma_k}{\sin^2\theta})d\theta \\ &\quad - \frac{4(\sqrt{M}-1)^2}{\pi M}\int_0^{\pi/4}\exp(-\frac{g\gamma_k}{\sin^2\theta})d\theta, \end{aligned} \tag{12}$$

where we have utilized the following identities [10] $Q(x) = \frac{1}{\pi}\int_0^{\pi/2}\exp(-\frac{x^2}{2\sin^2\theta})d\theta$ and $Q^2(x) = \frac{1}{\pi}\int_0^{\pi/4}\exp(-\frac{x^2}{2\sin^2\theta})d\theta$.

The overall average SER for the kth data symbol is obtained by averaging (12) over γ_k as

$$\begin{aligned} P_{M,k} &= \int_0^{\infty} P_M(e|\gamma_k)p(\gamma_k)d\gamma_k \\ &= \frac{4(\sqrt{M}-1)}{\pi\sqrt{M}}\int_0^{\pi/2} M_{\gamma_k}\left(-\frac{g}{\sin^2\theta}\right)d\theta \end{aligned}$$

$$-\frac{4(\sqrt{M}-1)^2}{\pi M}\int_0^{\pi/4} M_{\gamma_k}\left(-\frac{g}{\sin^2\theta}\right)d\theta . \quad (13)$$

where $M_{\gamma}(s)=\int_0^{\infty} p(\gamma)e^{s\gamma}d\gamma$ is the MGF and we have used $\gamma_k=\sum_{l=1}^{L}\gamma_{k,l}=\sum_{l=1}^{L}\gamma_{kc,l}=\sum_{l=1}^{L}\gamma_{ks,l}$.

We now need to decorrelate the channels $\mathbf{a}_k=[a_{k,1},a_{k,2},\cdots,a_{k,L}]^T$ and $\mathbf{b}_k=[b_{k,1},b_{k,2},\cdots,b_{k,L}]^T$ to facilitate further computation. The covariance matrices of $\mathbf{a}_k$ and $\mathbf{b}_k$ can be unitary diagonalized so that a new pair of channel vectors $\mathbf{a}'_k=\mathbf{U}_{kc}^H\mathbf{a}_k=[a'_{k,1},a'_{k,2},\cdots,a'_{k,L}]^T$ and $\mathbf{b}'_k=\mathbf{U}_{ks}^H\mathbf{b}_k=[b'_{k,1},b'_{k,2},\cdots,b'_{k,L}]^T$ can be obtained, where $\mathbf{U}_{kc}$ and $\mathbf{U}_{ks}$ are respectively the unitary matrices that diagonalize the covariance matrices of $\mathbf{a}_k$ and $\mathbf{b}_k$ [15]. Now the components of $\mathbf{a}'_k$ and $\mathbf{b}'_k$, viz., $\{a'_{k,l}\}$ and $\{b'_{k,l}\}$ are uncorrelated. As a result, the new equivalent branch SNRs $\{\gamma'_{k,l}=\gamma'_{kc,l}=\gamma'_{ks,l}=2\,|a'_{k,l}|^2\,E_{c,av}/\sigma_n^2\}$ are uncorrelated [11] and independent if the original physical channels $\{h_{p,q}\}$ are Gaussian. Thus, the MGF of the combined SNRs $\gamma'_k=\sum_{l=1}^{L}\gamma'_{k,l}$ is equal to the products of MGFs of the branch SNRs. We get

$$M_{\gamma'_k}(-\frac{g}{\sin^2\theta})=\prod_{l=1}^{L}M_{\gamma'_{k,l}}(-\frac{g}{\sin^2\theta}). \quad (14)$$

As shown in [11], $\sum_{l=1}^{L}\gamma_{k,l}=\gamma_k=\gamma'_k=\sum_{l=1}^{L}\gamma'_{k,l}$. Thus, $M_{\gamma'_k}(-\frac{g}{\sin^2\theta})=M_{\gamma_k}(-\frac{g}{\sin^2\theta})$. Using this along with (14), we can rewrite (13) as

$$P_{M,k}=\frac{4(\sqrt{M}-1)}{\pi\sqrt{M}}\int_0^{\pi/2}\prod_{l=1}^{L}M_{\gamma'_{k,l}}\left(-\frac{g}{\sin^2\theta}\right)d\theta -\frac{4(\sqrt{M}-1)^2}{\pi M}\int_0^{\pi/4}\prod_{l=1}^{L}M_{\gamma'_{k,l}}\left(-\frac{g}{\sin^2\theta}\right)d\theta . \quad (15)$$

If the original physical channels $\{h_{p,q}=h_{p,q,c}+jh_{p,q,s}\}$ are zero mean Gaussian RVs, so will be the virtual channels seen by x_{kc} and x_{ks}, i.e., $\{a'_{k,l}=a'_{kc,l}+ja'_{ks,l}\}$ and $\{b'_{k,l}=b'_{kc,l}+jb'_{ks,l}\}$. Then both x_{kc} and x_{ks} will experience Rayleigh fading. The MGF is given by [11]

$$M_{\gamma'_k}(s)=\frac{1}{\prod_{l=1}^{L}(1-\bar{\gamma}'_{k,l}s)}, \quad (16)$$

where $\bar{\gamma}'_{k,l}=E(\gamma'_{k,l})$ is the average SNR of the lth virtual branch with $E(\cdot)$ denoting expectation.

It is important to note that, since $\mathbf{A}_k$ and $\mathbf{B}_k$ are different for different k, so are a_k and b_k, and hence, different data symbols may experience different SER $P_{M,k}$.

We now briefly describe the generation of the non-orthogonal code as given in [8] used for the above SER derivation. The non-orthogonal code that fits the description here can be derived from an orthogonal STBC with code matrix $\mathbf{G}$ by precoding as

$$\mathbf{C}=(\mathbf{D}_f\mathbf{U}_h^H)^T\mathbf{G}, \quad (17)$$

where $\mathbf{U}_h$ is the unitary matrix that diagonalizes the channel covariance matrix $\mathbf{R}_{hh}=E[\mathbf{hh}^H]$ (for Rayleigh fading, $E[\mathbf{h}]=\bar{\mathbf{h}}=\mathbf{0}$), with $\mathbf{h}=[h_1,h_2,...,h_L]$ being the $L\times 1$ vector formed by stacking $\{h_{p,q}\}$, $L=PQ$, and $\mathbf{D}_f$ (a function of SNR and $\mathbf{R}_{hh}$) is a diagonal matrix whose diagonal elements are obtained by optimum power loading [8, eq. (30)]. For a given $\mathbf{U}_h$ and $\mathbf{D}_f$, it can be readily verified that $\mathbf{CC}^H$ (see example given in the next section) is no more diagonal. Thus a non-orthogonal code is generated and this non-orthogonal code will have $\mathbf{A}_k\mathbf{A}_k^H=\mathbf{B}_k\mathbf{B}_k^H$ and $\mathbf{B}_k\mathbf{A}_k^H-\mathbf{A}_k\mathbf{B}_k^H=\mathbf{0}$.

IV. A NUMERICAL EXAMPLE

Our example uses a non-orthogonal STBC that is obtained by precoding a rate 3/4 COD code in [5] given by

$$\mathbf{G}=\begin{bmatrix}-x_3 & 0 & x_1^* & x_2^* \\ -x_2 & x_1^* & 0 & -x_3^* \\ x_1 & x_2^* & x_3^* & 0\end{bmatrix}. \quad (18)$$

For demonstration, we shall use three transmit antennas and a single receive antenna and the constant channel correlation model [12]. Constant correlation can be obtained by a circularly symmetric three-element antenna array with close spacing between elements [12]. The channel covariance matrix is then given by

$$\mathbf{R}_{hh}=\begin{bmatrix}\sigma_{h_1}^2 & \sigma_{h_1}\sigma_{h_2}\rho & \sigma_{h_1}\sigma_{h_3}\rho \\ \sigma_{h_1}\sigma_{h_2}\rho & \sigma_{h_2}^2 & \sigma_{h_2}\sigma_{h_3}\rho \\ \sigma_{h_1}\sigma_{h_3}\rho & \sigma_{h_2}\sigma_{h_3}\rho & \sigma_{h_3}^2\end{bmatrix}, \quad (19)$$

where $\sigma_{h_l}^2 = E[|h_l|^2]$, $l = 1,2,3$ and ρ is the constant correlation coefficient between any pair of the three channels. In our simulations, we select three values for this coefficient, $\rho = 0.3,\ 0.6,\ 0.9$ and the ratios between channel variances as $\sigma_{h_1}^2 : \sigma_{h_2}^2 : \sigma_{h_3}^2 = 1:1:0.6$. For the case of $\rho = 0.6$, the unitary matrix $\mathbf{U}_h$ can be readily calculated as

$$\mathbf{U}_h = \begin{bmatrix} -0.6335 & 0.7071 & -0.3141 \\ -0.6335 & -0.7071 & -0.3141 \\ -0.4442 & 0 & 0.8959 \end{bmatrix}. \quad (20)$$

For a 20 dB average received SNR per channel and $\rho = 0.6$, the $\mathbf{D}_f$ can be computed as

$$\mathbf{D}_f = \begin{bmatrix} 0.6935 & 0 & 0 \\ 0 & 0.5561 & 0 \\ 0 & 0 & 0.4580 \end{bmatrix}, \quad (21)$$

while the average received SNR per channel is defined as $\frac{1}{L}\sum_{l=1}^{L} E(|h_l|^2)E(|x_k|^2)/\sigma_n^2$. Using (18), (20), and (21), we can verify that $\mathbf{CC}^H = (\mathbf{D}_f\mathbf{U}_h^H)^T\mathbf{GG}^H(\mathbf{D}_f\mathbf{U}_h^H)^* = \sum_{k=1}^{K}|x_k|^2\mathbf{U}_h^*\mathbf{D}_f^2\mathbf{U}_h^T$ is no more diagonal. The SER vs. average received SNR performance for square 16-QAM over correlative Rayleigh fading channels using the above data is presented in Fig. 1. Theoretical curves using (15) and Monte Carlo simulated curves are both given. It is seen that theoretical results are in excellent agreement with Monte Carlo simulated results. Note that small correlation yields better performance resulting from larger diversity. Then, Fig. 2 presents the comparisons between the SER curves for the orthogonal code of (18) and the corresponding non-orthogonal code for $\rho = 0.6$ and $\rho = 0.9$. Clearly, we see that the non-orthogonal code outperforms the orthogonal counterpart.

V. Conclusion

We derive the SER for a non-orthogonal STBC coupled with QAM signaling in correlative fading channels. The non-orthogonal code is formed by precoding an orthogonal STBC using optimum power loading. MIMO ML metric decoupling into SISO metrics is still applicable to the non-orthogonal code. Simulation results of SER performances for a code example in excellent agreements with theoretical results show that non-orthogonal codes outperform the orthogonal counterparts when MIMO channels are correlated.

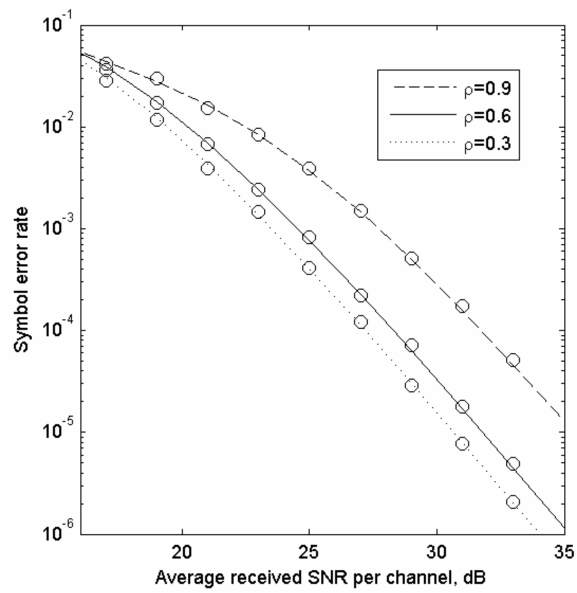

Figure 1. SER performance for a rate 3/4 non-orthogonal STBC using square 16-QAM over Rayleigh fading with constant channel correlations. Monte Carlo simulations are marked by circles.

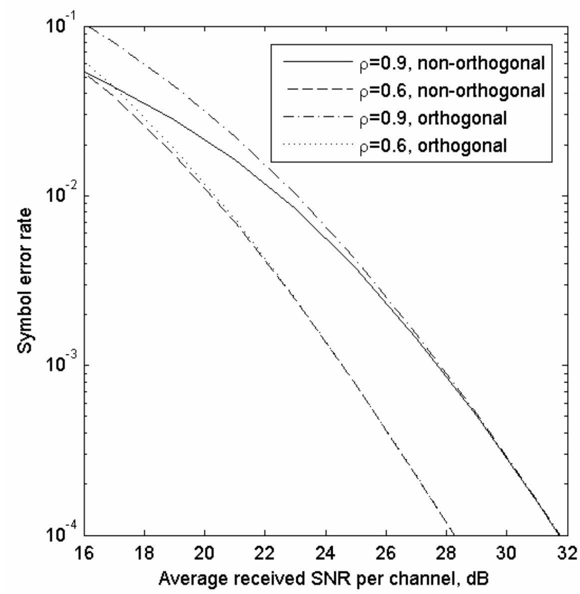

Figure 2. SER performance comparisons between the rate 3/4 orthogonal and corresponding non-orthogonal STBC over Raleigh fading with constant channel correlations.

References

[1] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time codes from orthogonal designs', *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456-1467, July 1999.

[2] E. G. Larsson and P. Stoica, *Space-Time Block Coding for Wireless Communications*. New York: Cambridge Univ. Press, 2003.

[3] C. Xu and K. S. Kwak, “On decoding algorithm and performance of space-time block codes,” *IEEE Trans. Wireless Commun*. vol. 4, no. 3, pp. 825-829, May 2005.
[4] H.-Y. Liu and R. Y. Yen, “Maximum-Likelihood decoding for non-orthogonal and orthogonal linear space-time block codes” *IEEE Trans. Veh. Technol.* To be published.
[5] B. Hochwald, T. L. Marzetta, and C. B. Papadias, “A transmitter diversity scheme for wideband CDMA systems based on space-time spreading,” *IEEE J. Select. Areas Commun.*, vol. 19, no. 1, pp. 48-60, Jan. 2001.
[6] L. C. Tran, J. Seberry, Y. Wang, B. J. Wysocki, T. Xia, and Y. Zhao, “Two complex orthogonal space-time codes for eight transmit antennas,” *Electronic Lett.*, vol. 40, no. 1, pp. 55-57, Jan. 2004.
[7] W. Su and X.-G. Xia, “Two generalized complex orthogonal space-time block codes of rates 7/11 and 3/5 for 5 and 6 transmit antennas,” *IEEE Trans. Inf. Theory*, vol. 49, no. 1, pp. 313-316, Jan. 2003.
[8] S. Zhou and G. B. Giannakis, “Optimal transmitter eigen-beamforming and space-time block coding based on channel correlations,” *IEEE Trans. Inf. Theory*, vol. 49, no.7, pp.1673-1690, July 2003.
[9] A. Hjorungnes, D. Gesbert, and J. Akhtar, “Precoding of space-time block coded signals for joint transmit-receive correlated MIMO channels,” *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp.492-497, Mar. 2006.
[10] M. K. Simon and M.-S. Alouini, *Digital communication over fading channels*, Wiley, New York, 2000.
[11] R. Y. Yen and H.-Y. Liu, “A general moment generating function for MRC diversity over fading channels with Gaussian channel gains,” *Int. J. Commun. Syst.,* vol. 20, no. 3 pp. 265-272, Mar. 2007.
[12] V. A. Aalo, “Performance of maximal-ratio diversity systems in a correlated Nakagami-fading environment,” *IEEE Trans. Commun.*, vol. 43, no. 8, pp.2360-2369, Aug. 1995.