MULTIOBJECTIVE FUZZY OPTIMIZATION WITH RANDOM VARIABLES IN A MIX OF FUZZY AND PROBABILISTIC ENVIRONMENT

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ABSTRACT-- **The fuzzy λ-formulation is used to solve multiobjective optimization problems in which the design variables are random, the constraints as a whole has a fuzzy probability or the constraints are mixed of deterministic, stochastic and fuzzy constraints. The fuzzy optimization strategy helps one to get the highest degree of satisfaction in reaching an optimum solution. The idea of expected value and the chance constrained programming technique allows one to convert a stochastic optimization problem into its equal deterministic form. The design of a three-bar truss illustrates the proposed design optimization in detail and the design of a machine-tool spindle express the comprehensive work of this technique. Results show that the proposed formulation and method can generate a natural, well-behaved, and reliable design.**

1. INTRODUCTION

A lot of efforts have been made to approach engineering design problems as naturally as possible. In nature a lot of aspects are related to the principles of probability. The situation where one often worries about the probability of failure (or reliability) happens to the optimization process [1-4]. It is reasonable that in formulation we consider the probability point of view. The conditions for applying probability concerns is that we must have enough historical data to form the distribution. In case of a known probability density function (pdf), one can construct the associated membership function in accordance with the possibility-probability consistency principle [5].

Usually it is a high cost to obtain these pdf. However, it is relatively easy to obtain the mean and standard deviation. We introduce an approach of applying the fuzzy theory [6], and can still solve this optimum problem under probabilistic base. The fuzzy set theory which was originally developed by professor Zadeh and is a beautiful way of describing a natural optimization condition mathematically. Since the appearance of the fuzzy theory, some researches and applications have also been done such as the works of [7-11].

To deal with optimization problem which is generally recognized to be nondeterministic as well as fuzzy in nature, we present a fuzzy stochastic mathematical formulation to solve such design problem. This nondeterministic condition is not only in the design variables, but it can also be in the allowable limits as well. Using the ideas of expected value and the chance constrained programming technique [2] one can transform the stochastic problem into its deterministic form. By then one can substitute this form into the fuzzy mathematical formulation.

Here we present a design optimization method under a combination of deterministic, probabilistic, and fuzzy environment. The basic idea is to solve the problem under two different expected probabilities of occurrence and/or two extreme values of fuzzy tolerance separately. Having these two results, we can then continue the construction of membership function to fit the fuzzy optimization. An engineering optimization design often involves more than one objectives and sometimes they can be contradicting too. Accordingly, we are interested in concentrating on multiobjective problems.

The following paragraphs we will present and solve the separate pattern of fuzzy, probabilistic and fuzzy probability optimization problem. Each of them is illustrated by a well-known three-bar truss design with minimizing two objective functions. Comparison and observation are given afterwards. Eventually, we made a design problem of four objectives where a machine-tool spindle is optimized. The design process and results under a mix of fuzzy and probabilistic environment will be given subsequently. From the analysis, we conclude this study and present the characteristics between the satisfying degree and factor of safety designs.

2. PROBABILISTIC OPTIMIZATION

In a probabilistic optimization, a general stochastic optimization problems is to find $X=[x_1,x_2,...,x_n]^T$ by the following formulation:

$$\text{Minimizes } F(X) \tag{1-1}$$

subjected to:

$$P[g_j(X)-g_{j,all}(X) \leq 0] \geq P_{j,exp}, \quad j=1,2,...,k \tag{1-2}$$

where X represents a mix of random and deterministic variables, F(X) is the objective function, $P[g_j(X)-g_{j,all}(X) \leq 0]$ is the occurrence probability of constraint, $g_j(X)-g_{j,all}(X) \leq 0$, and $P_{j,exp}$ is the expectation value associated to the jth constraint. When $P[g_j(X)-g_{j,all}(X) \leq 0] \geq P_{j,exp}$, the optimization design is based on a reliability point of view. Two important variations according to the kind of allowable limits are described below.

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2.1. Optimization With Deterministic Allowable Limit

Stochastic problem of this kind has the constraint as Eq. (1), where $g_j(X)$ has a nondeterministic value and $g_{j,all}(X)$ has a deterministic value. In order to solve this probabilistic optimization problem, we transform the stochastic problem by Taylor expansion into the approximately equal deterministic form [3]:

Minimize $\Psi(X) = C_1 F(X)|_{X=\bar{X}} + C_2 \sigma_{F(X)}$ (2)

subject to

$$g_j(X) + r_j(P_{j,\exp})\sqrt{\sigma_{gj(X)}^{\;2}} - g_{j,all}(X) \le 0 \quad (3)$$

$$\sigma_{g_j(X)} = [\sum_{i=1}^{n} (\frac{\partial g_j(X)}{\partial x_i})_{\bar{X}}^{2} \sigma_{x_i}^2]^{1/2} \quad (4)$$

where $r_j(P_{j,exp.})$ is the corresponding value of standard normal variate to the probability, $P_{j,exp.}$; $\sigma_{gj(x)}$ is the standard deviation of the jth constraints; σ_{xi} is the standard deviation of the ith random design variable, x_i. $C_1 \ge 0$, $C_2 \ge 0$ is the degree of importance for minimization. The stochastic optimization problem will be equal to the deterministic form only if C_2=0.

2.2. Optimization With Nondeterministic Allowable Limit

For a stochastic optimization problem with a nondeterministic allowable limit, as the similar approach above, Eq. (2) becomes:

$$g_j(X) + r_j(P_{j,\exp})\sqrt{\sigma_{gj(X)}^2 + \sigma_{gj,all(X)}^2} - g_{j,all}(X) \le 0 \quad (5)$$

$$\sigma_{g_j,all(X)} = [\sum_{i=1}^{n} (\frac{\partial g_{j,all}(X)}{\partial x_i})_{\bar{X}}^{2} \sigma_{x_i}^2]^{1/2} \quad (6)$$

where $g_{j,all}(x)$ can be a statistical data with a mean and standard deviation only. Substituting these constraints into the equal deterministic formulation, one can now search for an optimal solution.

3. FUZZY PROBABILISTIC OPTIMIZATION

3.1. The λ-Formulation Method

Designers are often caught in an optimization design situation that minimizes or maximizes more than one objective function under some fuzzy design constraints. In multiobjective fuzzy design, usually there are certain tolerance in which our final design falls in this certain acceptable range. To define this tolerance for several objective functions we must first execute the crisp optimization of each single objectives and at the same time calculate the corresponding values of the other objectives. From these values and bounds, we can construct the membership functions for the objective functions.

The mathematical λ-formulation of a multiobjective fuzzy optimization is as following [9]:

Maximize λ (7)

Subject to $\lambda - \mu f_i \le 0$, i=1,k (8)

$\lambda - \mu g_j \le 0$, j=1,m (9)

By neglecting the transition values in the constraints described by the membership function, we are constructing a formulation for the crisp multiobjective optimization.

3.2. $P_{exp.}$ is a Fuzzy Number

If $P_{j,exp}$ is fuzzy, then $P_{j,exp}$ is restricted between an upper bound, $(P_{j,exp.})^U$, and a lower bound, $(P_{j,exp.})^L$. When $P[g_j(X)-g_{j,all}(X) \le 0] = (P_{j,exp.})^U$, the membership function is said to have a degree of satisfaction of one, and when $P[g_j(X)-g_{j,all}(X) \le 0] = (P_{j,exp.})^L$, the degree of satisfaction is said to be zero.

Since the value of standard normal variate is fuzzy, then we can construct a membership function between $r_j(P_{j,exp.})^U$ and $r_j(P_{j,exp.})^L$ by changing the equal deterministic form into the following two different conditions.

3.2.1 for a deterministic allowable limit, the constraint can transform to the following:

$$r_j(P_{j,\exp.}) \le \frac{g_{j,all}(X) - g_j(X)}{\sigma_{gj(X)}} \quad (10)$$

3.2.2 for a nondeterministic allowable limit, the constraint can transform to the following::

$$r_j(P_{j,\exp.}) \le \frac{g_{j,all}(X) - g_j(X)}{\sqrt{\sigma_{gj(X)}^2 + \sigma_{gj,all(X)}^2}} \quad (11)$$

Thus, the linear membership function of standard normal variate is:

$$\mu_{rj}(P_{j,exp.}) = \begin{cases} 1 , & \text{if } r_j(P_{j,exp.}) \ge r_j(P_{j,exp.})^U \\ 0, & \text{if } r_j(P_{j,exp.}) \le r_j(P_{j,exp.})^L \\ 1-(\dfrac{r_j(P_{j,exp.}) - r_j(P_{j,exp.})^U}{r_j(P_{j,exp.})^U - r_j(P_{j,exp.})^L}), & \end{cases} \quad (12)$$

$$\text{if } r_j(P_{j,exp.})^L < r_j(P_{j,exp.}) < r_j(P_{j,exp.})^U$$

By modifying μ_{gj} in Eq. (9) into Eq. (12) and solving the multiobjective optimization problem Eqs. (7-9), one will has the solution to the fuzzy stochastic problem mentioned above.

4. MULTIOBJECTIVE OPTIMIZATION WITH MIXED OF FUZZY AND PROBABILISTIC CONSTRAINTS

A natural optimization design problem contains random variables and consists of a mix of deterministic, nondeterministic, fuzzy and probabilistic constraints. The general mathematical formulation for such problem can be described as:

Minimizes $[f_1(X),f_2(X),\ldots,f_N(X)]^T$
subject to five types of constraints.

(A) $g_i(X)-\alpha_{i,all}(X)\leq 0, i=1,..,p$, where $\alpha_{i,all}(X)$ has a fuzzy value.

(B) $P(g_i(X)-\beta_{i,all}(X)\leq 0)\geq P_{i,exp}$, where $\beta_{i,all}(X)$ has a deterministic value.

(C) $P(g_i(X)-\beta_{i,all}(X)\leq 0)\geq P_{i,exp.}$, where $\beta_{i,all}(X)$ has a random value.

(D) $P(g_i(X)-\beta_{i,all}(X)\leq 0)\geq P_{i,exp.}$, where $\beta_{i,all}(X)$ has a deterministic value and $P_{i,exp.}$ is a fuzzy value.

(E) $P(g_i(X)-\beta_{i,all}(X)\leq 0)\geq P_{i,exp}$, where $\beta_{i,all}(X)$ has a random value and $P_{i,exp}$ is a fuzzy value.

The first step is to solve each single objective optimization under the strict environment and calculate the corresponding value of the other objective functions. Then one repeats this procedure again, only this time one relax the environment based on the allowable fuzzy range of design constraints. Choosing the maximum and minimum values of the objective functions among these results, one can construct the appropriate membership function of the objective functions. Applying the method mentioned in section 3, we can get the solution of this multiple objective in a mixed fuzzy and probabilistic constraints.

5. ILLUSTRATIVE EXAMPLES

5.1 Three-bar Truss Design Optimization

The three-bar truss shown in Fig. 1 is frequently used in describing structural optimum design [9]. We assume that the design variables are random variables. The design objectives are to minimize the weight, f_1, and the vertical deflection of the loaded joint, f_2. This problem as solved for five different environment as described in section 2 and 3:

(A) which is depicted in section 3.1. By using $f_1^{min}=2.1937$, $f_1^{max}=22.9706$, $f_2^{min}=1.3807$, $f_2^{max}=17.5737$, $g_1^{min}=g_2^{min}=20.$, $g_1^{max}=g_2^{max}=24.$, $g_3^{min}=-18.$, $g_3^{max}=-15.$, one can construct μ_{f1}, μ_{f2}, μ_{g1}, μ_{g2}, and μ_{g3} respectively. Substituting these membership functions into Eqs. (7-9), one can get the solution of this classical fuzzy problem.

(B) which is depicted in section 2.1. The probabilistic optimization with deterministic allowable limit is defined as:

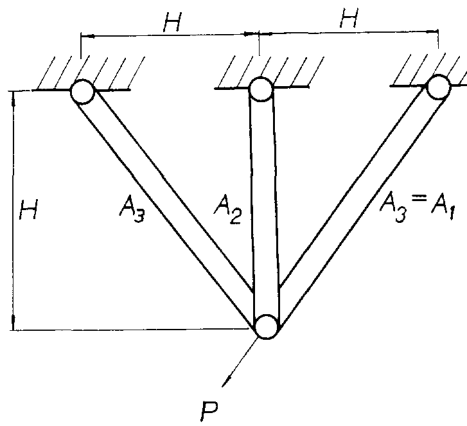

Fig. 1 Three-bar truss and its design variables.

$$\text{Minimize } F(X)=[f_1(X),f_2(X)] \tag{13-1}$$

$$\text{subject to } P[\sigma_{j,all}-\sigma_j\geq 0]\geq P_{j,exp.}, (j=1,2,3) \tag{13-2}$$

with $\sigma_{1,all}=\sigma_{2,all}=20.$, $\sigma_{3,all}=-15.$, $\sigma_{x1}=\sigma_{x2}=0.2$ and the expectation value, $P_{j,exp.}$ is 0.99997, j=1,2,3. This problem can be transformed into a deterministic form described in section 2.1.

From the single objective optimization, we can get $f_1^{max}=19.142$, $f_1^{min}=3.5549$, $f_2^{max}=11.0$, and $f_2^{min}=1.6569$. Therefore one can construct the linear membership function of the objective function. Thus the constraints of multiobjective formulation are:

$$\lambda-\mu_{fi(X)}\leq 0,\ i=1,2 \tag{14}$$

$$\lambda-\mu_{r_j(P_{j,exp})}\leq 0,\quad j=1,2,3 \tag{15}$$

where $\mu_{rj(Pj,exp)}$ has a degree of satisfaction equals one if $r_{j(Pj,exp.)}$ greater or equals to 3.99, and zero degree of satisfaction otherwise. Thus one can solve this probabilistic constraint problem.

(C) which is depicted in section 2.2. The probabilistic optimization problem is defined as in Eqs. (13-1,13-2). Allowable limit, $\sigma_{j,all}$ (j=1,2), is a nondeterministic value with a mean value of 20 and standard deviation of $20\%\times(\sigma_{1,all})/3$. The mean value of $\sigma_{3,all}$ is -15 and the standard deviation is $20\%\times(\sigma_{3,all})/3$. One can solve this type of problem as in former solution (B), however, the form of $r_{j(Pj,exp.)}$ is Eq. (11) instead of Eq. (10).

(D) which is depicted in section 3.1.1. The problem with deterministic allowable limits is defined as in Eqs. (13-1,13-2), but here the expected value has a fuzziness between 0.98030 and 0.99997. For $P_{j,exp}^{U}=0.99997$, the normal variate,

$r_j(0.99997)$, is 3.99; and for $P_{j,exp}{}^L=0.98030$, the normal variate, $r_j(0.98030)$, is 2.06. From Eq. (12), we obtain the membership function of the constraints. Again, solving the λ-formulation we can get the optimum results.

(E) which is depicted in section 3.1.2. The fuzzy probabilistic optimization is defined as in previous case (D), with the nondeterministic allowable limits the same as be described in case (B). One can apply former solution (D) to solve this type of problem, nevertheless, the form of $r_{j(Pj,exp)}$ is of Eq. (11) instead of Eq. (10).

Optimum solutions to these five conditions can be seen in Table 1. below.

Table 1. Final designs of three-bar truss under five types of constraints.

	Case A	Case B	Case C	Case D	Case E
λ	0.85412	0.73635	0.67460	0.81671	0.81217
f_1	5.00590	7.66450	8.69770	6.41190	6.63340
f_2	3.74300	2.65040	2.48150	3.33870	3.23470
x_1	0.57867	1.09770	1.41370	1.02580	1.06600
x_2	3.36920	4.55970	4.69930	3.51050	3.61830
FS_{g1}	1.0442	1.91660	2.40540	1.75160	1.81850
FS_{g2}	5.3434	7.54610	8.05950	5.99040	6.18300
FS_{g3}	1.0	1.97950	2.64170	1.90730	1.98490

5.2 Observations and Discussions

Since the numerical effects, the original value of FS_{g3} in case A is 0.97342, we normalize this row and obtained the above results. As shown, those five possible constraints deal with uncertainties, but they produce different results. The factor of safety, FS, in case (B) is greater than in case (A). The reason is that the probabilistic design considers the probability of failure that yields a reliable design. It is reasonable that the FS with considering a statistical allowable limit in case (C) is greater than that with a deterministic value in case (B). An allowable limit with a standard deviation σ_g will effect Eq. (5) such that the constraint of Eq. (3) becomes more strict. Thus an optimum design problem that considers a nondeterministic allowable limit will have a greater factor of safety. One also can see that the FS of case (D) is smaller than that of case (B), because of a fuzzy expectation of reliability in case (D). It is logical that the FS of case (E) is larger than the case (D) because of the same reason between case (B) and (C). In addition to case (A), the constraints of case (C) and (D) result in a design with the maximum and minimum factor of safety, respectively. One also observes in this example that the maximum FS means a maximum structual weight and minimum deflection is obtained.

The constraint with fuzzy sense enables the result to have a higher degree of satisfaction, λ, than nonfuzzy (crisp or probabilistic) constraints. Therefore, the trended value of λ is oppsite to the trend of factor of safety. The maximum λ in this example is the same as a minimized structural weight and increased deflection. Thus, one can clearly see the characteristics of fuzzy, nondeterministic, and probabilistic design optimization.

6. ENGINEERING DESIGN EXAMPLE

Machine-tool Spindle Design

The design of a machine-tool spindle adopted from [12] with a few modifications is used to illustrate an engineering optimum fuzzy design under mixed constraints environment. The design variable, τ_{all} and σ_{all} are assumed to be random variables with deviations of $\sigma_{D1}=\sigma_{D2}=\sigma_{L1}=\sigma_{L2}=\sigma_{L3}=0.2$, $\sigma_{\tau all}=15\%\times(\tau_{all})/3$ and $\sigma_{\sigma all}=20\%\times(\sigma_{all})/3$. The model of a machine-tool spindle is shown in Fig. 2. The design objectives are minimizing the total weight of the spindle, W_t(kg), the maximum twist per unit applied torque, ϕ(deg), the maximum deflection per unit applied load, δ(m), and the fundamental natural frequency, ω(Hz). Therefore the optimization problem can be mathematically described as to find $X=[D_1,D_2,L_1,L_2,L_3]^T$ by the formulation as:

Minimizes $F(X)=[Wt(X),\phi(X),\delta(X),\omega(X)]$
where

$$W_t=\frac{\pi}{4}\rho\,(L_1+L_2+L_3)\,(D_1^2-D_2^2)\quad (Kg) \tag{16}$$

$$\phi=\frac{180M_t}{\pi}\,\frac{32(L_1+L_2+L_3)}{\pi G(D_1^4-D_2^4)}\quad \text{(Degree)} \tag{17}$$

$$\delta=F\left[\frac{64L_1^2(L_1+L_2)}{3\pi E(D_1^4-D_2^4)}+\frac{1}{k_1}\left[\left(\frac{L_1}{L_2}\right)^2+1\right]^2+k_2\left(\frac{L_1}{L_2}\right)^2\right] \tag{18}$$

$$\omega=\left[\frac{G(D_1^4-D_2^4)}{64(L_1+L_2+L_3)}\left(\frac{1}{I_{p1}}+\frac{1}{I_{p2}}\right)\right]^{1/2}\quad (H_z) \tag{19}$$

subject to two deterministic constraints are:

$$2t_{min}-(D_1-D_2)\leq 0 \tag{20}$$

$$\frac{\phi\pi}{180(L_1+L_2+L_3)}-4.6E10^{-3}\leq 0 \tag{21}$$

The two fuzzy constraints are of the form:

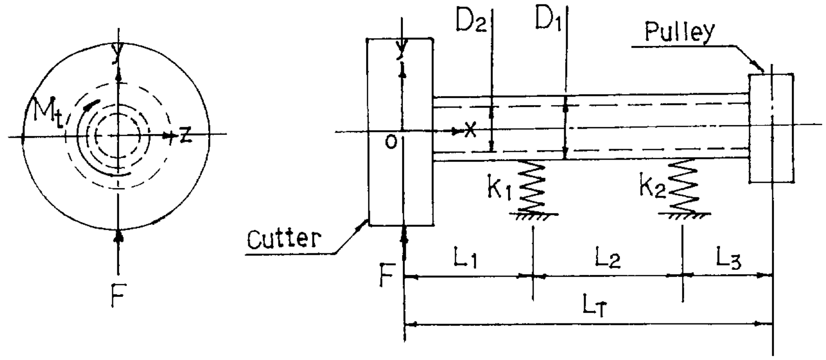

Fig. 2 Structure of the machine-tool spindle.

$$L_1+L_2-L_a \le 0 \quad (22)$$

where L_a is a fuzzy value between 0.745m and 0.755m in which 0.75m has the highest degree of satisfaction.
Therefore membership function for this fuzzy constraint has a degree of satisfaction equals to one when L_1+L_2 equals to 0.75 and when L_1+L_2 is less / equals to 0.745 or greater / equals to 0.755 the degree of satisfaction is zero, this can be seen in Fig. 3 below.

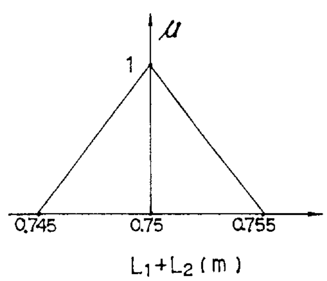

Fig. 3 Representation of linear membership function of L_1+L_2

$$L_1+L_2+L_3-L_T \le 0 \quad (23)$$

where L_T is a fuzzy value between 1.0m and 1.02m in which 1.0m has the highest degree of satisfaction. Thus if $L_1+L_2+L_3$ equals 1.0 the degree of satisfaction is equals to one where as a value of $L_1+L_2+L_3$ greater/equals to 1.02 has a zero degree of satisfaction. Fig. 4 below depicted this situation.

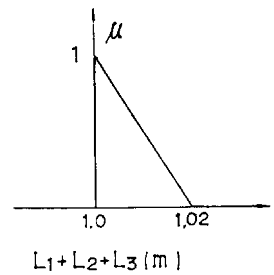

Fig. 4 Representation of linear membership function of $L_1+L_2+L_3$.

The three fuzzy probabilistic constraints are expressed as:

$$P\left(\frac{16 M_t D_1}{\pi (D_1^4-D_2^4)} - \tau_{all} \le 0\right) \ge P_{\exp} \quad (24)$$

$$P\left(\frac{M_1 D_1}{2I} - \sigma_{all} \le 0\right) \ge P_{\exp} \quad (25)$$

$$P\left(\frac{M_2 D_1}{2I} - \sigma_{all} \le 0\right) \ge P_{\exp} \quad (26)$$

where P_{exp} is a fuzzy value between 0.97 and 0.9999 in which 0.9999 has the greatest satisfaction. Some parameters in the above equations are represented as $M_1=F_1L_1$, $M_2=|(F+F_2)-F_1|L_2$ and $I=\pi(D_1^4-D_1^4)/64$, in which F is the applied force acting on the grinder or cutter; F_1 and F_2 are the reaction force at the left bearing and right bearing, respectively. The five side constraints are:

$$0.05(m)\leq D_i\leq 0.1\ (m),\ i=1,2 \qquad (27\text{-}1)$$

$$0.2(m)\leq L_1 < 0.3\ (m) \qquad (27\text{-}2)$$

$$0.4(m \leq L_2\leq 0.6\ (m) \qquad (27\text{-}3)$$

$$0.1(m)\leq L_3\leq 1.5\ (m) \qquad (27\text{-}4)$$

Some numerical data used here are F=250N, M_t=1500N.m, τ_{all}=200Mpa, σ_{all}=350Mpa, $\rho=7.85\times10^3$kg/m^3, $G=8.04\times10^{10}$N/m^2, $E=2.06\times10^{11}$N/m^2, $k_1=k_2=9.8\times10^7$N/m, I_{P1}=8.0kg.m^2, and I_{P2}=2.25kg.m^2. To solve this mixed fuzzy probabilistic optimization problem, we first solve those four optimization objectives in a strict environment one by one. The strict environment will cause several modifications to the above constraints. Here Eqs. (24-26) has only one deterministic P_{exp} value which is 0.9999, and the fuzzy constraints in Eqs.(22-23) become $L_1+L_2=0.75$ and $L_1+L_2+L_3=1.0$. Therefore we will have four optimized objective values with the corresponding values of the other unoptimized objectives.

The next step is to solve the relaxed single objective optimization. This is done by repeating the above procedure, only this time Eqs. (22-23) become $L_1+L_2\geq0.745$, $L_1+L_2\leq0.755$ and $L_1+L_2+L_3\leq1.02$, $L_1+L_2+L_3\geq1.0$, and P_{exp} in Eqs. (24-26) has a value of 0.97. From this two procedures we now have the total of 32 values of the objective functions shown in Table 2. and Table 3.

We choose the best and worst values among these objective functions for the construction of membership function of these objective as described in section 3.1. Completing the membership function constructions for the design constraints and substituting these membership function into the formulations in section 3.1., we can obtain the solutions to this problem in a mixed fuzzy and probabilistic environment. The results are presented in Table 4.

Because of relaxing the constraints, one can see that the optimized value of individual objective function in Table 3 is better than the value in Table 2. The final designs of Table 4 as shown in Fig. 5 which logically falls between the lower bound and the upper bound of the individual optimum result. This optimum design illustrates a natural approach of an optimization problem. Therefore a fuzzy probabilistic optimization does produce a better and reasonable natural solution.

Table 2 Single objective optimum design for machine-tool spindle under strict environment.

	Minimize W_t	Minimize ϕ	Minimize δ	Minimize ω
W_t (kg)	18.030650	38.717230	32.811630	19.017680
ϕ (deg)	0.3227200	0.1263700	0.1635300	0.3272800
δ (m)	0.98419E-05	0.71722E-05	0.43502E-05	0.77093 E-05
ω (Hz)	155.35717	248.26556	218.24642	154.27129
D_1 (m)	0.0854280	0.1000000	0.0944270	0.0834130
D_2 (m)	0.0662770	0.0609940	0.0599550	0.0622940
L_1 (m)	0.2831400	0.2801600	0.2000000	0.2524900
L_2 (m)	0.4668600	0.4698400	0.5500000	0.4911900
L_3 (m)	0.2566400	0.2500000	0.2500000	0.2587600

Table 3 Single objective optimum design for machine-tool spindle under relaxed environment.

	Minimize W_t	Minimize ϕ	MInimize δ	Minimize ω
W_t (kg)	14.871940	46.193810	47.165140	15.054530
ϕ (deg)	0.3978700	0.1160200	0.1184600	0.4318500
δ (m)	0.10044E-04	0.61092E-05	0.41188E-05	0.67876E-05
ω (Hz)	139.91751	259.10027	256.41843	134.29929
D_1 (m)	0.0823620	0.1000000	0.1000000	0.0812770
D_2 (m)	0.0659420	0.0500000	0.0500000	0.0649730
L_1 (m)	0.2742100	0.2619400	0.2000000	0.2251700
L_2 (m)	0.4805800	0.4928100	0.5550000	0.5214500
L_3 (m)	0.2357700	0.2442500	0.2650000	0.2773900

Table 4 Multi-objective optimum design for machine-tool spindle under mixed fuzzy and probabilistic environment.

W_t (kg)	20.895910	D_1 (m)	0.0947950
ϕ (deg)	0.2205700	D_2 (m)	0.0748290
δ (m)	0.70934E-05	L_1 (m)	0.2600600
ω (Hz)	187.91780	L_2 (m)	0.4922100
		L_3 (m)	0.2484300

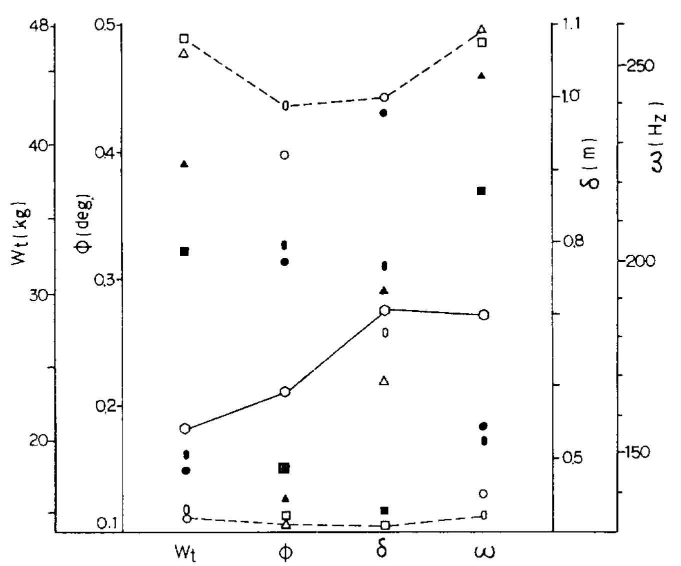

Fig. 5 Optimum points of single and multiobjective optimization in machine-too spindle design.

7. CLOSING

The degree of satisfaction and the factor of safety designs under uncertainties are presented and compared in this study. Both of them can neatly be a design index or design criterion. The higher degree of satisfaction means the design reaches a level of compromising and optimizing all objective functions in a common fuzzy design space. A fuzzier design space can generate a higher satisfying degree, but a lower factor of safety. However, the traditional factor safety design is too conservative to reach a reasonable design. Probabilistic-based optimization does provide a much reliable design and a stronger structure, nonetheless, it demands higher cost and is not usually used in all kind of design. To formulate a design problem with mixed fuzzy, probabilistic, and deterministic information can contribute a natural, well-behaved and reliable design, yet it is not the only way to achieve the design target. Other researches on this subject can be widely done.

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