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Pathways for Analyzing and Responding to Student Work for Formative Assessment: The Role of Teachers' Goals for Student Learning

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This study explored how teachers interpreted and responded to their own student work during the process of formative assessment. The study involved a purposefully selected sample of 32 teachers in grades K-5 who had been trained by the Ongoing Assessment Project (OGAP) to use learning progressions to analyze and respond to evidence in student work. Since formative assessment is fundamentally an interpretive process, involving continually eliciting and interpreting evidence of student thinking from student work in order to inform teaching and learning (Black & Wiliam, 2009), the study analyzed data collected through semi-structured interviews.

The study found variations in the way teachers make sense of their student work for formative assessment that were related to their underlying goals for student learning. Teachers with an *achievement orientation* tended to focus on performance goals: giving formative assessment items to gauge student performance on problems that reflected what had recently been taught and focused on singular or multiple components of performance to make a binary judgment (i.e. students who "get it or don't get it"). Teachers with a *learning orientation* gave items to learn more about what students were able to do on different types of problems and focused on student strategies as an indicator of underlying understanding and development. These orientations also had implications for the instructional response teachers developed; as teachers looked beyond surface features of student work and binary distinctions, they developed more differentiated responses that built on students' knowledge and their ability to develop more sophisticated understanding. In between these two extremes, we found three categories of hybrid approaches to formative assessment, demonstrating a push-and-pull between achievement and learning orientations at different decision points during the steps of the formative assessment process. Those decision points – the teachers' purpose in giving an item, the evidence focused on, the interpretive framework used to analyze the evidence, and the focus of the instructional responses – offer multiple footholds in the formative assessment process where teachers can begin to try out new approaches that reflect a shift in orientation to student learning.

The study shows that using formative assessment is not simply a matter of taking up new practices and using new tools. The variations in understanding and use of the ideas that were offered in professional development, as reflected in teachers' actual practices, suggests that it is important to provide opportunities for sustained learning and supported use over time.

Keywords

Disciplines

Pathways for Analyzing and Responding to Student Work for Formative Assessment

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Pathways for Analyzing and Responding to Student Work for Formative Assessment

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Contents

Introduction

Recent research on mathematics teaching promotes a view of ambitious instruction that calls for teachers to regularly elicit, make sense of, and respond to students' mathematical thinking and problem-solving strategies (Lampert, Beasley, Ghousseni, & Kazemi, 2010; Sztajn, Confrey, Wilson & Edgington, 2012). Although there is a growing body of work in mathematics education around teacher noticing of student thinking during instruction (e.g., Schack, Fisher & Wilhelm, 2017; Sherin, Jacobs & Phillips, 2010), less attention has been paid to how teachers interpret and respond to their own student work to inform instruction.

Formative assessment is an instructional practice that involves continually eliciting and interpreting data, or evidence of student thinking, from student work in order to inform teaching and learning (Black & Wiliam, 2009). Existing studies of teachers' use of student work to inform instruction highlight that teachers tend to focus on what students are or are not able to do, rather than on what they understand (Ebby & Sirinides, 2015; Christman, et al., 2009; Goertz, Oláh, & Riggan, 2009; Kazemi & Franke, 2004; Supovitz, Ebby & Sirninides 2013), or explain student performance in terms of pre-existing beliefs or judgments about their ability (Horn, 2007; Jackson, Gibbons, & Sharpe, 2017; Wilson, Sztajn, Edgington, Webb, Myers, 2017).

This paper adds to this growing knowledge of teachers' interpretations of student thinking by exploring how teachers interpret and respond to student work during the process of formative assessment and how their interpretations reflect underlying views of learning. We examine teachers' understandings of formative assessment through the lens of how teachers sorted, interpreted, and responded to their own students' work after being introduced to a learning trajectory for formative assessment through the Ongoing Assessment Project (OGAP). Learning trajectories are "empirically supported hypotheses about levels or waypoints of thinking, knowledge, and skill in using knowledge, that students are likely to go through as they learn mathematics" (Daro, Mosher, & Corcoran, 2011, p. 12). They can enhance the formative assessment process by providing a road map for setting goals, as well as analyzing and responding to evidence of student thinking (Ebby & Petit, 2017; Heritage, 2008).

The questions guiding our analysis include:

1. **What are the variations in the way teachers make sense of their student work for formative assessment?**

2. **What do teachers' interpretations of and responses to student work reveal about their underlying goals for student learning?**

3. **What are the implications for teachers' use of learning trajectory-oriented formative assessment?**

In the following sections, we describe empirical and conceptual foundations for the OGAP formative assessment process, which teachers were introduced to during the summer prior to this study. We then describe the theoretical framework that informed our analysis of teachers making sense of, interpreting, and responding to student work during the formative assessment process, and more specifically the role played in that process by teachers' underlying goals for student learning.

Learning Trajectory Informed Formative Assessment

In recent years formative assessment has been promoted as one of the most impactful educational interventions in terms of improving student achievement, particularly in elementary mathematics (Black & Wiliam, 1998; Klute, Apthorp, Harlacher & Reale, 2017). In contrast to grading or judging student performance after learning has taken place, formative assessment involves providing feedback and adjusting instruction continuously. Effective formative assessment is a complex process, involving the following steps:

1. ***Identifying the learning goal.***

2. ***Eliciting information about what students currently know and understand in relation to the learning goal.***

3. ***Analyzing the student response for evidence of students' developing knowledge and understanding.***

4. ***Identifying the gap between the learning goal and where the learner is currently.***

5. ***Providing feedback to students and/or developing an instructional response that moves the learner closer to the learning goal.***

In distinguishing formative from summative assessments, many people erroneously focus on the frequency, timing, or format of the assessments themselves. In contrast, Black and Wiliam (2009) propose that an assessment is formative based on its function:

> Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited. (Black & Wiliam, 2009)

According to this definition, it is the interpretation and response to the evidence that is elicited by the assessment that is central to making it formative. Because formative assessment is fundamentally an interpretive process for both teachers and students, the conceptual frameworks teachers use to make sense of and respond to the evidence are of critical importance. While teachers cannot directly observe student understanding, they can draw from behaviors and evidence of students' mathematical thinking to build a model of student knowledge (Mojica & Confrey, 2009; von Glasersfeld, 1995). The teachers in this study had been introduced to research-based conceptual frameworks that spelled out a learning progression that children move through when developing understanding of core mathematics topics.

Heritage (2011) proposes that learning progressions can enhance the formative assessment process by providing a guide for teachers to formulate clear learning goals as well as to analyze and respond to evidence in student work:

> To do this effectively, teachers need to have in mind a continuum of how learning develops in any particular knowledge domain so that they are able to locate students' current learning status and decide on pedagogical action to move students' learning forward. Learning progressions that clearly articulate a progression of learning in a domain can provide the big picture of what is to be

Figure 1. OGAP Multiplication Progression (Petit, Hulbert & Laird, 2017)

OGAP Multiplication Progression (April 2016)

OGAP

Multiplicative Strategies

Multiplicative

The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problems structures and contexts.

Algorithms

Partial Products | Traditional
$$
\begin{array}{r}
16 \\
\times 42 \\
\hline
12 \\
20 \\
240 \\
400 \\
\hline
672
\end{array}
$$

$$
\begin{array}{r}
21 \\
16 \\
\times 42 \\
\hline
32 \\
640 \\
\hline
672
\end{array}
$$

Distributive Property
$$4 \times 16 = 4(10 + 6)$$
$$= 4(10) + 4(6)$$
$$= 40 + 24$$
$$= 64$$

Associative Property
$$(8 \times 2) \times 5 = 8(2 \times 5)$$
$$= 8 \times 10$$
$$= 80$$

Doubling & Halving
$$16 \times 4 = 8 \times 8$$
$$= 64$$

Known or Derived Fact
$$4 \times 6 = 24$$

Commutative Property
$$16 \times 4 = 4 \times 16$$

Powers of Ten
$$5 \times 400 = 5 \times 4 \times 10 \times 10$$

Transitional Strategies

Transitional

Open Area Model *Considers both dimensions of an array or area model*

	30 +	8
20	600	160
+		
6	180	48

$$38 \times 26 = 988$$

10	+ 5
90	45	9

$$15 \times 9 = 135$$

Area Model

6 ... 4

Considers BOTH dimensions of an array or area model, moving away from needing to see every square unit

$$6 \times 4 = 24$$

Early Transitional Strategies

Skip Counting 3, 6, 9, 12, 15

Building up
$$3+3+3+3$$
$$6+6$$
$$12$$
6 ... 6 ... 12

Skip Counting with a Model
3 6 9

Equal groups in an array
3 6 9 12 15
Considers only ONE dimension of an array or area model

Area Model – $6 \times 4 = 24$
6 ... 4 ... (24)
Considers BOTH dimensions of an array or area model

Additive Strategies

Additive

Repeated addition with or without a model – $3 \times 4 = 12$

$$3 + 3 + 3 + 3 = 12$$

3 3 3 3
Subitizing in small groups

$$3 + 3 + 3 + 3$$

Early Additive Strategies

Modeling, counting by ones

Inconsistent Grouping

Modeling, counting by subgroups
$$3 \times 4 = 12$$
2 4 6 8 10 12

Unitizes into equal groups, uses visual models, understanding of place value, and properties of operations

---

Non-Multiplicative Strategies

Adds or subtracts factors | Uses incorrect operation
Models factors incorrectly | Not enough information
Guesses | Uses procedures incorrectly

Underlying Issues and Errors

- Doesn't consider reasonableness of solution
- Error in: calculation, place value, vocabulary, property or relationship, equation, or model

learned, support instructional planning, and act as a touchstone for formative assessment (p. 1)

Similarly, Szjatin et al. (2012) propose that learning trajectories, the more commonly used term for learning progressions in mathematics education, can help teachers be "guided by the logic of the learner" in eliciting evidence of student learning and providing feedback to learners by supporting teachers in "examining the boundaries of what students do and do not understand" (p. 152).

The Ongoing Assessment Project

The notion that formative assessment can be enhanced by learning progressions is central to the Ongoing Assessment Project (OGAP) (Ebby & Petit, 2018). The OGAP Progressions describe learning trajectories in a visual format of typical strategies that students use to solve problems in several core content areas (additive, multiplicative, fractional, and proportional thinking) (See Appendices B and C). These strategies are organized into developmental levels, describing the movement from less to more sophisticated strategies as student understanding deepens. For example, Figure 1 shows how students progress in multiplication from additive to multiplicative reasoning, initially solving problems by counting equal groups by ones, then moving to repeated addition, skip counting, and using area models to eventually develop more abstract strategies based on their understanding of place value and properties of multiplication. The array and area models in the transitional level represent an important developmental bridge from additive to multiplicative thinking. Evidence suggests that the development of student strategies does not occur in a strictly linear process; rather students' use of strategies tends to move back and forth across the levels as they encounter different problem structures and contexts until ultimately stabilizing into efficient and flexible strategies (Hulbert, Petit, Ebby, Cunningham & Laird, 2017). The progressions are designed to be useful tools for teachers, helping them to identify levels of student thinking based on the strategies they use to solve particular problems, as well as offering instructional guidance for transitioning student understanding and strategies from one level to the next.

A core tenet of OGAP is that student strategies offer a window into their developing understanding. A second core tenet of the program is that teachers can learn more about the depth of student understanding by giving formative assessment items regularly and ensuring that those items represent the range of problem types and structures in the content area. For this reason, OGAP provides a bank of formative assessment items for each content area, which are intentionally engineered to elicit student understandings and common misconceptions on a range of problem types and structures. In OGAP professional development, teachers are introduced to the research on students' understanding of mathematical content they are teaching. They also practice using the progressions to analyze samples of student work on OGAP items by sorting them into different piles by strategy and common understandings.

Once teachers are introduced to the research and instructional tools in professional development, they are expected to give and analyze items regularly to inform their instruction. In addition, schools are encouraged and supported in having teachers meet regularly in grade-level groups to collaboratively analyze and respond to student work. In this study we focused on how teachers made sense of the formative assessment

process, from the administration of an assessment item to the instructional response based on the evidence. What question about student learning were teachers trying to answer when they gave a formative assessment item? How did this purpose guide the choice of the item they gave? What evidence did teachers pay attention to in the student work and how did they make sense of and interpret this evidence in relation to student learning and instruction? We were also interested in understanding whether and how these interpretations were related to teachers' underlying views about student learning. In our early stages of analysis, it became evident that the way that teachers conceptualized learning goals for their students played an integral role in shaping the interpretation process.

Theoretical Foundation: Expectations and Goals for Student Learning

In considering how teachers conceptualized goals for student learning, we draw on two bodies of research: cognitive research within mathematics education that offers detailed descriptions of children's learning trajectories in core mathematics domains (discussed above), and research on expectations, beliefs, and mindsets, which shape teachers' perceptions of students' capabilities and the learning goals they establish (e.g., Dweck, 2008; Ferguson, 1998; Jackson et al., 2017). Since all teachers in this study had been introduced to a learning-trajectory approach to student learning and formative assessment, we were interested in the extent to which teachers took up and understood or modified this approach in practice when interpreting student work. With respect to teacher expectations, there is strong evidence that teachers' beliefs about their students' abilities influence the learning opportunities they make available to them in the classroom (e.g., Diamond, Randolph, & Spillane, 2004; Jackson et al., 2017; Sztajn, 2003). Sztajn, for example, found that teachers' expectations were manifest in the types of learning goals they held for their students, based on their beliefs about students' needs, and that these were in turn associated with students' socioeconomic background and the resulting opportunities to learn.

> *Teachers who view intelligence and ability as fixed are likely to hold learning goals for their students that focus on performance, whereas teachers who view intelligence as malleable are more likely to view learning as a process of development.*

In our analysis of teachers' interpretations of students' work, findings from research on goal orientation also emerged as an important and explanatory factor for how teachers made sense of and used the learning progression. We found Dweck and Legget's (1988) distinction between performance goals, which focus on documenting fixed ability, and learning goals, which focus on developing or increasing ability over time, to be helpful in characterizing differences that emerged in our data in the goals teachers had for assessing student learning. Dweck's (1986) work on growth and fixed mindsets is best known for illuminating the impact of an individual's beliefs and goals on their own learning. Learners who have a fixed mindset believe that intelligence and ability are fixed, innate traits, which individuals can do little to change. From this view, performance

is an indication of one's capabilities. In contrast, individuals with a growth mindset believe that intelligence and ability are malleable and can be developed through hard work and education (Dweck, 2008). Learners with a growth mindset show persistence and seek challenge, even in the face of perceived low ability or performance.

We see a strong resonance between Dweck's (1986; 2008) conception of growth mindset and the developmental perspective underlying learning trajectories. A growth mindset is premised on the idea that learning is incremental and occurs over time, propelled forward by sustained effort, strategy development, and mentoring. Learning trajectories, which map development of understanding in particular domains, from beginning or basic understanding to increasingly more sophisticated and understanding, chart the paths this growth is likely to take (Heritage, 2011; Sztajn, 2012). Simply put, a true learning trajectory approach assumes a growth mindset.

Although Dweck's (1986) framework focused on learners' goal orientations, this work has implications for how teachers conceptualize goals for their students' learning. Teachers who view intelligence and ability as fixed are likely to hold learning goals for their students that focus on performance, whereas teachers who view intelligence as malleable are more likely to view learning as a process of development and are likely to set goals for students that map onto some sort of learning progression. The types of goals teachers set for students can impact how they interpret students' work and guide their future growth. As Elliot and Dweck (1988) put it, goals generate their "own set of concerns" and create their "own framework for processing incoming information" (p. 5). Given the fact that identifying the learning goal is the first step of the formative assessment process, it makes sense that the goals teachers hold for their students act as a framework for making sense of the evidence they collect of student learning as well as the way that they respond to that evidence. Drawing on this work, we consider how different types of goals teachers hold for their students' learning work in practice to create a unique framework for processing evidence of student thinking in the formative assessment process.

Methods

Research Context

This study was conducted in 2017-18 in the context of the second year of a three-year rollout of OGAP implementation in the School District of Philadelphia, a large economically and ethnically diverse public system. During this phase, 10 elementary schools signed up to send 116 teachers to a five-day OGAP training for the first time. An additional 19 elementary schools, whose teachers had participated in previous trainings, sent 101 new teachers to the training. Although there were additional perks for new schools who sent at least 65% of their teaching staff to training, attendance varied greatly by school. The training was comprised of five full-day workshops organized by grade band (K-2, 3-5, 6-8) and focused on conceptual understanding of the core content area, research on student learning of the core concepts, problem structures, visual models, properties of operations, understanding the Common Core standards, and using learning progressions to sort and analyze student work on formative assessment items and pre-assessments. During the school year, schools also received ongoing

support for implementing OGAP in the form of school visits by OGAP trainers and invitation to voluntary, follow-up workshops. Schools varied in the extent to which they took up these opportunities and in the leadership styles of the principals and other school leaders in supporting implementation of OGAP locally (Flack, Morrison, Hess, Kolouch & Pierce, forthcoming).

Data Collection

This study is part of a larger qualitative inquiry that explored how teachers took up and made sense of learning trajectory-oriented formative assessment in a range of school contexts. Data were collected through a series of three semi-structured interviews conducted with a purposeful sample of 32 grade K-5 teachers from nine schools who had completed their first year of OGAP training. This study drew primarily on data collected from the second of those interviews.

Participants

To identify participants for the study, all teachers who attended the OGAP summer training for the first time in the summer of 2017 were surveyed on their willingness to participate. From the subset of teachers that responded, a set of elementary schools was identified that had at least two OGAP-trained teachers at each grade band (K-2 and 3-5). Based on their responses, 32 teachers across eight schools agreed to participate in the study, with between one and six teachers at each school. Five of the eight schools were in the first year of implementing OGAP. Although all teachers in the sample had just completed OGAP training for the first time, there were three schools that were already using OGAP in grades 3-5. As a result, five teachers in our sample may have had some prior exposure to analyzing student work on OGAP items in grade-level meetings, even though it was their first time attending OGAP training. The schools represented a range of school sizes and demographics (shown in Table 1) and a range of performance profiles as determined by the district to represent student achievement, progress, and climate; out of four possible tiers, one school was in the lowest, tier 1, three in tier 2, and five in tier 3. Our selection criteria yielded a varied sample of participants from grades K-5 with teaching experience ranging between two and 36 years, with a mean of 15 years.

Teachers in the study were interviewed three times throughout the course of the school year, in the fall, winter, and spring. Each interview was designed to understand a different component of how teachers were taking up and making sense of OGAP and the learning-trajectory approach to formative assessment. The first interview focused on baseline use of formative assessment, views of students, and view of math instruction generally. The second interview focused on teachers' analysis and sense-making of a set of their own students' work on a formative assessment item. Teachers selected the item to bring to the interview and were asked to sort and analyze it as they normally would. The final interview partially mirrored the first, focusing on general formative assessment use, teachers' views of students, and their overall understanding of the trajectory-based formative assessment training they received.

Data for this study was taken primarily from the second interview as it provided evidence of the sense-making teachers were doing with their own students' work. Before the second interview, teachers were asked to administer a formative assessment item.

In the interview, they were then asked to (1) explain how they sorted their student work, (2) analyze representative examples from each pile they created, and (3) formulate instructional responses for each example as well as for the class as a whole.

Data Analysis

Analysis began by producing an analytical memo to document each teacher's categorization of student work and then looking across these memos to develop initial categories of sorting strategies. We then used a constant comparative approach to apply and continually refine these categories into a typology (Strauss & Corbin, 1990). After a typology was developed, the research team continued to refine the categories by rereading teacher interviews from each category to both better understand the commonalities of teachers in each category and the major delineations across different categories. All interviews were double-coded, and any disagreements were reconciled through discussion. This iterative engagement led to the identification of overarching foci and interpretations teachers in each category made throughout the formative assessment process. As the analysis proceeded, teachers' underlying views of learning and the guiding purposes for analyzing student work emerged as being salient dimensions that further informed our conceptual framework.

Table 1: School Demographics for Participant Sample

School	School Size **	Econ. Disad.	Black/AA	Hispanic/Latino	White	Asian	Multi-race	ELL	Trained in Additive			Trained in Multiplicative		
									K	1	2	3	4	5
A	Sm	100%	90%	1%	2%	0%	6%	0%		2		1	1	
B	Md	100%	45%	19%	4%	26%	9%	21%		1	1	1		
C	Lg	72%	17%	13%	52%	8%	10%	7%		1	1	2		
D	Sm	100%	82%	5%	3%	3%	6%	1%	1			1	1	
E	Sm	76%	21%	16%	44%	6%	12%	8%			2			2
F	Sm	100%	10%	14%	62%	1%	12%	1%				1		
G	Md	100%	7%	49%	6%	33%	6%	41%		1*		2		
H	Md	22%	9%	6%	64%	8%	13%	1%		1	1	1*		
I	Md	100%	24%	62%	5%	1%	7%	13%	1	2	2		2	
Total									**2**	**8**	**7**	**9**	**4**	**2**

*Taught a mixed class with the grade above, i.e., 1st and 2nd or 3rd and 4th

**School size determined by enrollment as of Oct. 2017. Small: less than 500; Medium: 500-1000, Large more than 1000
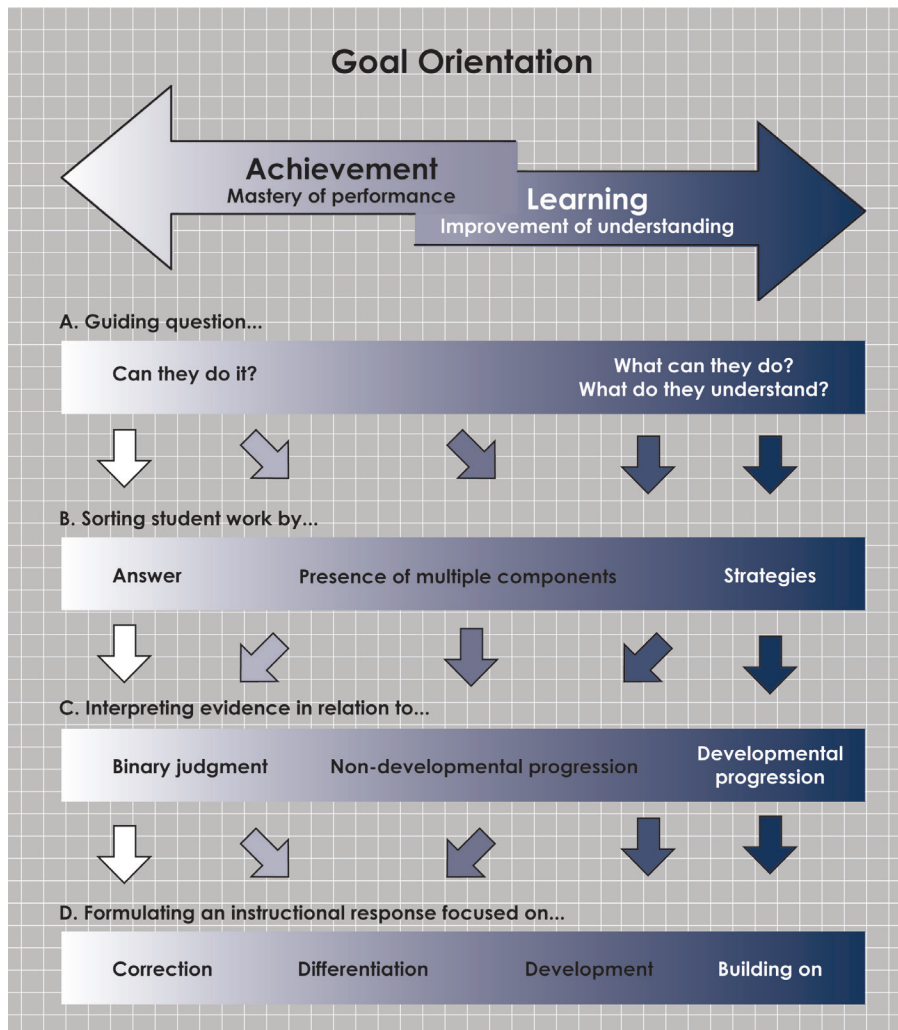
Findings: Pathways to Making Sense of Student Work

Our analysis yielded a typology of five different approaches to using student work to inform instruction that are influenced by different goals for both formative assessment and student learning. All 32 teachers in the study participated in the OGAP professional development in the summer prior to the study, which focused on developing understanding of the core content and using a learning trajectory to sort, interpret, and respond to student work in relation to a progression of understanding. Still, our analysis of their approaches to this process surfaced substantial variation in what teachers attended to and how they interpreted their own work.

We initially distinguished between two distinct and contrasting approaches: one that followed what we call an achievement orientation and one that followed a learning orientation. The learning orientation, which reflects the approach underlying the OGAP training, was characterized by teachers who used students' work on the problem as a vehicle to surface features of student understanding, focused on student strategies as a reflection of developing understanding, and situated that evidence within a developmental progression to determine an instructional response. This orientation aligns with what Dweck and Legett (1988) refer to as learning goals, as it is focused on developing or increasing ability over time. In contrast, the achievement orientation was represented by teachers who focused on performance goals, by considering student performance on the item as an end in itself, attending to accuracy and correctness to make a binary judgment (right or wrong, gets it or doesn't get it). The achievement orientation resulted in an instructional focus on correctness of student performance. As characterized by Elliot and Dweck (1988), these orientations established their

Table 2: Contrasting Characteristics of the Achievement and Learning Orientations as Applied to Student Learning and Formative Assessment

CONTEXT	DIMENSIONS	ACHIEVEMENT ORIENTATION	LEARNING ORIENTATION
Student Learning	Mindset	Fixed mindset	Growth mindset
	Goals	Performance Goals	Learning Goals
	Focus	Mastery of performance	Improvement of understanding
Formative Assessment	Purpose	Gauge mastery of skill, concept, or problem	Elicit evidence of understanding
	Analysis	Focus on accuracy and correctness	Focus on strategies
	Interpretation	Using evidence to make binary judgment	Situating evidence within developmental progression
	Instructional Response	Focus on correction or remediation to achieve desired performance	Focus on building on current understanding to get to next level of strategy or understanding

Figure 2. Five assessment pathways taken by teachers as they went through the steps of formative assessment

Goal Orientation

Achievement
Mastery of performance

Learning
Improvement of understanding

A. Guiding question...

Can they do it?

What can they do?
What do they understand?

B. Sorting student work by...

Answer | Presence of multiple components | Strategies

C. Interpreting evidence in relation to...

Binary judgment | Non-developmental progression | Developmental progression

D. Formulating an instructional response focused on...

Correction | Differentiation | Development | Building on

own "sets of concerns" (p. 5) and resulted in different frameworks through which teachers made sense of students' work. In the context of formative assessment, these frameworks included differences in the purpose of the formative assessment and how teachers analyze, interpret, and respond to their student work (summarized in Table 2).

We also identified three additional approaches that did not fit squarely into an achievement or a learning orientation, and instead reflected hybridized elements of both orientations at different steps of the formative assessment process. Figure 2 illustrates the five pathways we identified to describe the frameworks used by teachers to make sense of student work in the formative assessment process.

> *The pathways are comprised of four key decision points: a) the guiding question or purpose of giving the formative assessment item; b) sorting the student work based on evidence; c) interpreting that evidence in relation to an underlying view of learning; and d) formulating an instructional response.*

As the different pathways in Figure 3 illustrate, the purpose of giving a formative assessment item ranged from finding out whether or not students could solve a particular type of problem ("can they do it?") to a more open-ended investigation of finding out what they were able to do and wht the evidence suggested about what they understood. When looking at student work, teachers differed in whether they primarily attended to students' answers, or the presence or accuracy of various components of the work, including the answer, solution strategy, and use of visual representations. Teachers also differed in the assessment lenses they used to interpret the work, from

making a binary judgment related to correctness to considering strategies in a developmental progression where one stage of understanding builds on the next. Finally, teachers varied in how they used their interpretations of students' work to inform their instruction. Those following an achievement orientation focused on correcting or improving student performance. Those following a learning orientation tended to consider ways to improve understanding and support further development. Teachers who understood the developmental nature of the learning trajectory considered how to build on students' current knowledge and understanding to move towards more sophisticated strategies and understanding. In the middle, some teachers considered levels of student strategies in an ordered continuum but didn't show understanding of how these levels built on one another. (In Figure 3, part C, we refer this as a non-developmental progression.) Arrows illustrate these different paths taken by teachers at each step of the formative assessment process. Together these pathways comprised the analytical frame we used to make sense of our data.

The names we use to identify each pathway reflect the major focus or emphasis guiding teachers' analysis of student work in that group. Table 3 shows our categorization and names for each pathway and the frequency of each within our sample of 32 teachers. In the following sections, we describe each of the five pathways as approaches to formative assessment, using data from teachers in each group to illustrate. More detailed and descriptive case studies of a single teacher from each category can be found in Appendix A.

The Achievement Orientation to Formative Assessment

Slightly more than half of the teachers in our sample demonstrated an approach to formative assessment that was aligned with an achievement orientation, in that they focused on student performance in relation to an external standard. However, there were some important differences in what they attended to and how they interpreted the work, resulting in three variations on their approach to formative assessment, described below.

Correctness of Answer

Seven of the teachers we interviewed focused on a correct answer on the formative assessment item as the primary indicator of student achievement in relation to a specific performance goal. Most often, their objective was to determine whether students had mastered a skill or strategy recently taught. Teachers in this group sorted their students' work primarily by whether the numerical result was correct or incorrect, regardless of the strategy the child used to produce the answer or the nature of errors in the response. Some teachers also required an explanation in their consideration of correctness. In some cases, when the problem had more than one part, teachers sorted on whether the student obtained the correct answer on one or both parts of the problem, resulting in more than two piles, but in each case made binary distinctions within the piles. Sorting in this way ultimately resulted in making a binary distinction between students who "got it" and students did not.

An illustrative example can be found in the analysis of student work on a subtraction problem by a second-grade teacher who sorted into two piles by correct or incorrect answer, despite the fact that students showed a wide range of strategies. When

questioned further, she noticed and identified different strategies used by students, including: "regrouping" (US traditional algorithm), number lines, base 10 visual models, "breaking apart" (decomposing by place value), and compensation. She did not, however, attend to the sophistication of thinking or underlying understanding represented in those strategies (e.g., whether they were counting by tens or ones on the number line, whether they used a model). She marked all papers containing the correct answer with a large C. As she said, "they either got it right or they didn't get it right."

For some, like the following third grade teacher, sorting by correctness was a conscious decision to not use the OGAP progression:

> I was spending more time figuring out what level they were at and what that meant and it was frustrating to me… So I just do it where, "Are they right? Are they wrong? What did they do right? What did they do wrong?" And that informs my instruction from there.

For others, sorting by correctness reflected their interpretation of OGAP. One teacher, for example, recalled that sorting by a "common theme" of understanding had been presented as an option at the OGAP training. She explained, "My theme was correct answer versus not correct answer."

In general, teachers who sorted primarily by correctness said that it helped inform their small group instruction, identify common mistakes, and determine if an intervention was needed. One teacher explained, "For small groups, I can see which group, if they totally didn't get it at all, I know that I'm going to have to reteach that skill in a small group." Teachers also described how it helped them get an overall sense of the class performance. Said one, "It shows me where the bulk of my kids are and whether they knew something or not." Another said, "It shows me that even though we are. . . past this, they still aren't completely grasping it."

In their proposed instructional responses, teachers focused on those students who were not demonstrating correct performance. When asked what she was planning to do as a result of giving a formative assessment, a second-grade teacher focused on the work in the incorrect pile:

> Maybe put them into small groups with all the same similar issues, like all these ones that subtracted up and just show them like on a whiteboard or with base 10 blocks, the correct way of getting it and the correct answer, and, "Now look at your answer, what do you see here, can you tell why yours is wrong?" You know, just to get them to stop doing that.

Underneath this sorting strategy, we found a view of learning as a process of improving performance by helping students learn how to solve problems correctly. Getting the correct answer provided an indication that students were ready to move on to the next topic, while unsuccessful performance required an intervention, most often in the form of small group instruction. The teachers in this group did not make use of the OGAP progressions and tended to use OGAP items in ways that seemed at odds with their intent. They treated the formative assessment items as a way to measure performance in relation to a specific learning goal (e.g., being able to solve a word problem involving two-digit subtraction) and saw a correct answer as the primary indicator that performance had been achieved. The thinking process that lay behind the use of the strategy or production of the answer was less important to them.

Table 3. Distribution of praticipating teachers by pathway

ORIENTATION	PATHWAY	NUMBER OF TEACHERS
Achievement	Correctness of Answer	7
	Components of Performance	4
	Progression of Strategies	6
Learning	Progression of Strategies	6
	Progression of Understanding	9
	Total	**32**

Components of Performance

Four teachers who demonstrated an achievement orientation went beyond focusing on a single correct answer to consider multiple components or features of the response, including whether students could model the problem, show their thinking, or explain their answers. Like the previous group, teachers in this group wanted to determine whether students could successfully perform a previously taught skill or strategy to solve the problem correctly. At the same time, their approach demonstrated an appreciation of the complexity of solving problems and the range of steps and strategies required. As a result, they did not look solely for the presence of a single correct answer, but considered multiple components, including some evidence of student understanding, resulting in multiple piles or categories of proficiency. For example, one second-grade teacher explained that she wanted students to practice "figuring out what the problem was talking about" and to "try to show a model." Other teachers talked about whether students' "reasoning was incorrect," and tried to determine "their level of explaining" or created their own categories that indicated some focus on understanding. "I sort it into three piles, like right, wrong, and like close but confused," said one. These examples illustrate that teachers in this group considered various dimensions of student work that went beyond simply producing the correct answer.

Like the previous group, teachers in this group eventually made binary judgments (e.g., correct/incorrect, present/absent) for each component, resulting in piles that showed differences among the students but were not sequenced in any way and often attended to superficial aspects of student work instead of those revealing of student understanding. For example, one second-grade teacher focused on whether students used the correct operation, whether they used visual models or drawings to support their solution, and whether they had made errors in the use of the model. She felt that using models was important to understanding but did not focus on how students used the models or their underlying understanding of concepts. In addition, she often overlooked evidence of understanding (e.g., a student who added up from the starting number was characterized as using the wrong operation) or misconceptions that the strategies

revealed (e.g., students who were using models to represent the answer rather than the problem situation).

These teachers were also similar to the Correctness-of-Answer group in how they thought about the instructional implications of students' work, in that they tended to focus on students who were not performing well in an effort to correct errors or show students the correct way to solve the problem. However, given the number of variations their sorting produced, their instructional responses tended to be more differentiated in relation to targeting student needs. They also formulated instructional responses that began to acknowledge the role of understanding, such as increased attention to visual models.

In general, teachers who sorted by the presence of multiple components of performance said that it helped them identify who had not reached the desired level of performance so that they could intervene or gauge the readiness of the class to move on in the curriculum. As one teacher in this group explained:

> If overall, everyone is doing what I need, then I know, okay, great, we can move on to something different. And if I only have a few kids that are struggling, that's an easy fix. If most of them got it wrong, then I need to reteach that skill or there's something that maybe they just didn't understand.

In this case, "what I need" included various aspects of student performance that went into producing a correct response. Successful performance in relation to these criteria was an indication that it was okay to move on to the next topic or skill, while unsuccessful performance required targeted intervention.

This group was our smallest group (n=4) and could be considered an offshoot of the Correctness-of-Answers group. They brought similar goals to formative assessment and thought about learning in a similarly binary process of mastery of performance. The key difference between these two groups appears to be a recognition of the complexity involved in solving problems. Sorting the student work by multiple components helped these teachers to identify what was needed in a more precise way than by simply looking at correct or incorrect answers, but this practice did not necessarily reveal what students already knew or show how to build on that developing understanding. Unlike the groups of teachers we discuss next, these teachers were not inclined to sequence different components of student performance in a progression that signaled different levels of achievement or development.

Progression of Performance

Much like the first two approaches, the six teachers who followed a Progression-of-Performance pathway saw formative assessment as a means of determining successful performance. Teachers in this category, however, attended to qualitative differences in student work and viewed performance on a spectrum; rather than simply identifying work as correct or incorrect, they categorized student work as more and less correct and incorrect, demonstrating an increasingly complex view of performance. Instead of viewing all correct responses (or incorrect ones) as the same, they differentiated by how close each response was to the most desired solution. For example, one teacher described an incorrect piece of student work as follows: "So they have a concept and understanding on how to arrive at the answer, and they set their problems up, but they were off computationally, by like two numbers, they just simply added or subtracted wrong… So the answer was close." Thus, this teacher attended to the proximity to being

correct, rather than simply identifying the work as "incorrect" as in previous approaches.

Like the previous group, teachers in this group attended to multiple components of performance in student work, but when sorting they ordered their piles of student work by perceived distance from the most desired performance. In describing her piles, a first-grade teacher explained: "they kind of get varying difficulties as they go. And there's like, one, two, three... nine piles ...This is the furthest from the correct, closest to the correct."

Two teachers in this category assigned levels of performance to the work that were adapted from the four-level scoring guidelines used for state testing. Commonly referred to by teachers as a rubric, the score levels are used to designate "understanding of mathematical concepts and procedures required by the task" as either thorough (4), general (3), partial (2), or minimal (1) (Pennsylvania Department of Education, 2015). When asked how she determined that work was a 4 or thorough understanding, one second-grade teacher explained, "They showed all the work, and they both, they got the correct answer and they showed how they got the correct answer. So they had a very good understanding of how to solve it, how to set the problem up, and the answers were correct." In contrast, students who demonstrated understanding of how to solve the problem but had a calculation error were given a 3 for general understanding, and students who left out a step in the process were given a 2 for partial understanding.

Two additional teachers in this group sorted student work on a topic that had not been part of their summer training (e.g., fractions), and so found themselves applying the idea of looking for a progression of thinking and understanding to a content area for which they had no framework. As a result, these teachers had some focus on looking for gradations of understanding but did not know what evidence to look for and ended up creating progressions that reflected gradations of performance. For example, a third-grade teacher who was analyzing student work on a problem that asked students to compare visual models showing 1/4 and 2/9, sorted it in relation to the justification the student provided ("no reason or confusing reason" "all different reasons" and "connecting the idea that they know one fourth is equal to two eighths"). Acknowledging that she was unsure what to look for, she described asking herself, "What was the perfect answer? Maybe I should start there." She felt that the use of equivalent fractions was an important component of the justification but was unsure how to evaluate other explanations that students offered and was not able to construct a progression of developing understanding.

Most teachers in this category talked about instructional responses that were focused on getting students to the most desired performance, but the intensity of the response was in proportion to the proximity to the desired performance. As such, they did not treat all incorrect solutions as representing a complete lack of mastery. Instead, they thought about more intense responses for those who were far from the desired performance (e.g., needing to go back and learn foundational skills) and less intense responses for those who were very close to the desired solution (e.g., encouraging students to be more careful or check their work).

This group of teachers appeared to have an overall view of learning as performance, but at the same time seemed to straddle achievement and learning orientations. While teachers attended to correctness and components of student answers, they no longer used a binary view when interpreting the student work. Teachers viewed the formative assessment process as a mechanism to assist in getting students to mastery, but by being more differentiated in their sorting process, they were able to more precisely tailor the intensity and focus of the instructional response. Thus, while there was some

focus on learning and understanding, getting to the most correct or desired solution or performance was still the ultimate goal of the formative assessment process.

The Learning Orientation to Formative Assessment

In contrast to the teachers and approaches described above, the remaining teachers in our sample sorted their student work by focusing primarily on the strategies students used to solve the problems, and then considered the correctness of answer and other components. Importantly, this focus placed emphasis on students' developing understanding, regardless of whether the student had demonstrated the desired performance. However, we saw variation in the degree to which teachers understood and drew upon the developmental progression to analyze and respond to their student work. In describing these two variations, we begin with the approach that is most aligned with a learning orientation and reflects the approach that teachers were introduced to during the OGAP training. We then describe a variation we found on this approach that included some elements of an achievement orientation.

Progression of Understanding

This approach was characterized by placing primary emphasis on the strategies students used to solve the problem, using that strategy as evidence of the current level of understanding, situated within a developmental progression. The nine teachers in this group selected formative assessment items to learn more about what their students could do on different types of problems, rather than to assess what had been recently taught. As one teacher explained:

> [Students'] strategies will change depending on the question and how they understand the question. [Giving OGAP items] helps me understand, it helps me see maybe the type of question that they're not necessarily understanding to the point where they go back to the additive strategies or they become in the non-multiplicative stage. It helps open my eyes to some of the specific things that they might be struggling with that I'm not able to see elsewhere.

To sort and make sense of their student work, these teachers consulted the OGAP progression and sorted by the strategies that students used to solve the problem, creating multiple piles containing both correct and incorrect responses. In contrast to teachers previously described, they paid primary attention to how students were solving the problem as an indication of students' developing understanding of number and operations. For example, they looked at whether students used ones, tens, multiples of ten, or base-10 understanding to count, add, or subtract to solve the problem. A few teachers in this category sorted student work on a topic not covered by the OGAP progression (e.g., interpreting data), but created their own developmental progression of strategies. The focus of their analysis was on how students got to the answer and what that suggested about students' developing understanding. One first-grade teacher explained the benefits of sorting by strategy:

> I mean, it forces you to think about their way of thinking, rather than just right or wrong. What's their thought process? What strategy are they using? Who's higher on the thinking process, to be more quick and efficient than everybody else... like rather than the specific black and white, right or wrong answer.

In thinking about instructional implications, teachers drew upon their understanding of the development of that concept to build upon what students knew and move all students forward. A fourth-grade teacher talked about how he would respond to students who were solving a multiplication problem by drawing out equal groups and counting by ones:

> I would try to move them from... If they already have the circles, I would maybe have them put it in an array since they already feel comfortable drawing the circles. I would have them draw the same amount of circles if they want to in an organized array, and then once they understand the grouping of that array, I'd turn it into an area model and go from there.

Like this teacher, teachers who used the OGAP progression often formulated instructional responses that focused on using specific visual models highlighted in professional development and on the progression, such as area models or open number lines, or focused on developing more advanced strategies. Some teachers described thinking about instructional implications as an inherent part of the sorting and analysis, rather than being a separate step. As one teacher commented, "I tried to group it... based on where they fell so I could see where they needed to go next."

Teachers who sorted student work by strategy as a way to discern developmental levels of understanding talked not only about how it helped them determine what students needed, but also how to help those students who seemed to be "getting it" continue to improve.

Several teachers talked about using an instructional strategy learned in training ("select and sequence") where they would select examples of student work from different levels on the progression to project in class and then have students make sense of and discuss similarities between the strategies. One teacher explained, "I'll put one or two pieces of work under the document camera and display it on the board and have the students tell me what they see, what they notice about it, and have them compare the different strategies."

The intent of this routine is to help students build on what they already know to understand and make connections to more sophisticated strategies. This focus on building on what students knew and could do to move their thinking and strategies forward incrementally was a distinguishing characteristic of this approach to formative assessment. Many teachers found this approach empowering, as it gave them a concrete way to think about supporting students. One fourth-grade teacher reflected:

> Just seeing the progression and understanding the progression and knowing that if a student is not there, where do I need to go in order to get him to the next step? I don't have to jump him all the way up to get him to where I want him. I could just focus on the next step and then slowly work him to there because the goal is at the end of the year where they should be, sooner, great, but a little at a time... At their progress, what they can, can't do... But I think the one thing that I have learned is that it's okay. If they didn't get it, then that's okay. I can figure out a way to get them to get it.

Teachers who sorted student work by strategy as a way to discern developmental levels of understanding talked not only about how it helped them determine what students needed, but also how to help those students who seemed to be "getting it" continue to

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improve. They saw improvement, rather than performance, as the goal of the formative assessment process, and differentiation as a way to benefit all students' learning rather than only those who weren't meeting the expectations.

Progression of Strategies

Six of the teachers we interviewed used the OGAP progression and/or labels from the progression to describe student work; at the same time, their analysis of student work revealed a partial understanding of learning occurring in a progression. Like the teachers who sorted with a progression of understanding, these teachers gave formative assessment items to learn more about student thinking and/or students' ability to do different types of problems. As one teacher described, "I was really interested in seeing that, diving deeper into how my students were thinking, so that I could help move them forward."

The focus on improvement and student thinking led teachers to put primary attention on the strategies students used, rather than the answers and components of the answers. For example, one fourth-grade teacher described sorting into two piles based on two levels of strategies from the multiplication progression (shown in Figure 1), multiplicative and early transitional, and then within those piles looking for "underlying issues," which provided evidence of misunderstandings or errors.

When sorting, these teachers connected evidence of students' strategies to specific levels on the OGAP progression to make sense of the work. However, at times they demonstrated some misunderstandings in their use of the progression or difficulty integrating multiple forms of evidence in their interpretations. For example, one fourth-grade teacher characterized two incorrect solutions to the problem 118 x 4 as multiplicative: one that showed the use of the distributive property with a minor error in the last step and one that showed the first step of a multiplicative strategy by multiplying 100 x 4. This second student was ultimately unable to use 100 x 4 to solve 118 x 4 and ended up misusing the standard algorithm to get an answer of 1,132, suggesting significant misconceptions about multiplication and place value. The teacher focused on the strategy as the primary indicator of the level of thinking, but did not consider the nature of the errors in relation to whether the solution showed multiplicative thinking:

> If they used a particular strategy, and you can tell, you have evidence that they're trying to use that strategy, but it didn't quite work out, or they started out using one particular strategy, and then they kind of got it mixed up a little... at least they're thinking multiplicatively. At least they're thinking that way, but they just had some underlying issues, which is one of the parts of the progression.

In this case, by focusing on the strategy as the primary indicator of understanding, the teacher seemed to miss other aspects of student work, which revealed important misconceptions.

As in this example, teachers who sorted using a progression of strategies were primarily focused on identifying and creating piles based on identified strategies but were not necessarily able to consider the larger holistic picture of the evidence of multiplicative thinking in the student work. They did not always understand the learning progression well enough to see how strategies reflected developing understanding and/or how the strategies built upon each other to build procedural fluency with understanding. Thus, while they may have ordered those strategies sequentially, guided by the progressions, it was not a truly developmental progression. Other teachers in this

category only partially or inconsistently used the progression to make distinctions among strategies. These teachers seemed to be making the transition to focus on strategy over answer, but in doing so were not able to focus on and integrate all aspects of the evidence in relation to the development of student thinking.

The focus on strategies as an indicator of student thinking also led these teachers to use the location of the strategy on the progression to think about next instructional steps in terms of moving forward or addressing conceptual issues. Teachers who sorted using a progression of strategies viewed learning as a progression but sometimes had notable misunderstandings, which led to mischaracterizations of student understanding given the work presented. In some cases, they understood the need to move all students towards higher level strategies, but not necessarily how to build on what students already knew to get there.

Despite drawing incompletely on the developmental progression of student thinking, the focus on evidence of student thinking was more important to these teachers' analyses and instructional responses than the product or correctness of the answer. Teachers who sorted in this way also sometimes described how it changed their initial assumptions about student work. One third grade teacher remarked, "It shows me exactly where some of my kids are instead of where I thought they were."

Detailed case studies of teachers sorting and making sense of their own student work for instructional purposes, illustrating each of the five pathways, can be found in Appendix A.

Discussion

In this section, we discuss our findings in light of our research questions. First, we characterize the variations we found in how teachers made sense of their students' work during the formative assessment process. We then consider what teachers' interpretations and responses revealed about their underlying goals for student learning. Finally, we consider the implications of these findings for supporting teachers' use of learning-progression-oriented formative assessment.

Despite the fact that all teachers had been introduced to the OGAP progressions and a learning-trajectory approach to formative assessment, we found substantial variation in the ways they made sense of their students' work and used it to inform their instruction. Most of the teachers in this sample looked beyond correct or incorrect answers when analyzing their own student work, and they looked at the work with an eye towards what they might do next to improve student learning or performance. However, as illustrated by Figure 3 and the descriptions above, we saw teachers taking different approaches at the different steps of the formative assessment process. Some teachers gave formative assessment items to see if their students could solve particular types of problems that reflected what had been taught, while others gave it to see what students could do on new and different types of problems. Teachers varied in what they paid attention to and prioritized in students' responses as well as how they interpreted these features in terms of student learning. Sometimes their analysis resulted in a binary judgement, while other times it resulted in multiple categories that could be placed in a continuum. Finally, the focus of their intended instructional responses ranged from correction (showing them how to solve correctly) to differentiation (addressing specific needs) to development (moving to the next stage of level of understanding) to building on (using evidence of what students could do to continuously improve understanding).

It is important to note that all teachers in the study were drawing assessment problems from the same bank of items, provided by the program, and had access to the same content specific learning progressions. In many cases, the actual student work the teachers were analyzing reflected similar strategies and common errors. The stark differences and degree of variation found across the 32 teachers illustrates that simply providing teachers with similar training and resources will not necessarily lead them to take up formative assessment in the same way.

Our analysis of the different pathways teachers followed to interpret student work illustrates the significant role that teachers' goals for student learning play in the formative assessment process. Our findings surfaced two contrasting views of learning, reflecting the distinctions in learning goals offered by Dweck and Legget (1988). Teachers adopting an achievement orientation tended to focus on performance by prioritizing whether the solution met certain requirements, regardless of the strategy the child used or other hints the solution revealed about the student's understanding. Teachers with a learning orientation instead focused on their students' developing understanding by attending to the strategies students were using and the underlying concepts reflected in the use of those strategies. As Elliot and Dweck (1988) proposed, these indicators took on their own meaning and importance in the process of interpreting student work.

The different pathways that surfaced from our analysis suggest that the formative assessment process involves making a series of decisions that establish, shift, or reinforce a learning or achievement orientation. At the same time, the three hybrid pathways that fall between the two extremes in Figure 3 reveal some dynamic aspects of teachers' views or the possibility of some fluidity in these orientations at each decision point. For example, some teachers, at times, seemed to be moving toward a learning orientation, by looking beyond just the correct answer at multiple components in the student solution, but then were pulled back to an achievement orientation when interpreting those components in a way that involved a binary judgement. Limited understanding of the developmental nature of the learning progressions (i.e., how strategies build in sophistication as concepts develop and deepen) sometimes pulled teachers who were sorting by strategies back toward the more familiar achievement orientation, resulting in categorical distinctions that were differentiated but not developmental.

Interestingly, the variations we describe here were not related to the self-reported frequency of use of OGAP items for formative assessment. Future analysis of the data collected for this study will focus on other internal and external factors that may be related to learning orientations and approaches to analyzing and responding to student work.

Implications

Our findings have several implications for teachers' use of learning progressions for formative assessment. Our study shows that using formative assessment is not simply a matter of taking up new practices and using new tools. Teachers need opportunities to make explicit and critically examine their views about student learning in relation to new practices and goals for formative assessment. They may also need more time to make sense of the developmental trajectory underlying the OGAP progression, both in theory and in practice. The variations in take-up and understanding of the ideas that were offered in training, as reflected in teachers' actual practices, suggests that it is important to provide opportunities for sustained learning and supported use over time.

The variety of pathways that resulted from decision-making during the formative assessment process highlights the complexity of analyzing student work for the purposes of informing one's instruction, as there are several steps in the process at which teachers make interpretations: conceptualizing what they hope to learn from an assessment, what they focus on when looking at the student work, how they interpret the evidence in student work, and how they draw on that evidence to make instructional decisions or adjustments. Each of these decision points opens up or closes opportunities to view student learning as a process of achievement (performance) or continual growth (progression). At the same time, these decision points offer multiple footholds or points in the formative assessment process where teachers can begin to try out new approaches that reflect a shift in orientation to student learning. As Dweck (2015) is careful to point out, people tend to demonstrate a mixture of fixed and growth mindsets, and simply stating that one believes in a growth mindset is only the first step in a journey towards shifting beliefs and practices. Moving from an achievement orientation to a learning orientation involves a significant shift that goes against commonly held beliefs and practices of assessment.

A final implication of our findings is to acknowledge that the achievement orientation is currently the default view of the most commonly used assessments in the U.S. educational system. In mathematics, both classroom-based and standardized assessments tend to focus on short, multiple-choice answers that can be used to calculate "percent correct" and make binary or normative judgments about student performance. Performance is communicated to students and parents as percentages and grades that encourage comparison to an externally derived standard and/or comparison with other students. The fact that these same test scores are used to make judgments about teacher performance reinforces the focus on ensuring that students demonstrate the desired performance rather than develop and grow over time. While most teachers may recognize the reasons why it is important to adopt a learning orientation, in practice they are pulled back into an achievement orientation.

Our study also raises questions for further exploration. Are the approaches to sorting student work described here developmental? When teachers move from a singular focus on correct answers to consider multiple components of student work, does this set the groundwork for adopting a progression-oriented approach? Is creating a progression of performance a beginning stage of constructing a progression of understanding? These two pathways reflect significant differences in orientations—are we missing a stage that occurs in between that could help teachers make the shift towards prioritizing student strategies over performance?

Finally, our data come from a single use of formative assessment, and it is important to note that we are not suggesting that this one instance reflects a complete picture of a teachers' orientation to learning or learning goals. Although looking at this one moment in time helps us to highlight differences in approaches to formative assessment, we suspect that many teachers might show us a different approach with a different problem on a different day. Further study can illuminate whether and how teachers' sorting practices are situated within their practice over time in a range of contexts, and also how those practices are influenced by both internal and contextual factors.

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Appendix A: Case Studies

In this appendix, we present case studies to illustrate the five pathways for analyzing students' work that emerged from our analysis. We begin with two ends of the continuum, shown in the shaded rows in Table A.1 below: the correctness of answer and progression of understanding pathways. In many ways, these two pathways are antithetical to one another in almost every respect: the information teachers paid attention to in the student work, the implicit framework they used for classification of the work, and the implications they drew from the evidence for future instruction. We first explore these differences and then turn to examples of the three hybrid pathways to explore the push and pull between achievement and learning goal orientations to mathematics in the context of formative assessment.

Table A.1 Characteristics of Case Studies Representing Five Pathways

PATHWAY	CASE	ASSESSMENT PURPOSE	SORTING	INTERPRETATION	RESPONSE
Achievement					
Correctness of Answer	Anne	Can they do it?	Answer	Binary Judgment	Correction
Components of Performance	Bethany	Can they do it?	Multiple Components	Binary Judgment	Differentiation
Progression of Performance	Catherine	Can they do it?	Multiple Components	Continuum	Differentiation
Progression of Strategies	Diana	What can they do?	Strategies	Progression	Development
Progression of Understanding	Elena	What can they do?	Strategies	Developmental Progression	Building On

Case Study A: Correctness of Answer

We begin with the case study of Anne, who focused on the correctness of student answers as the primary source of evidence that her students were "getting it" in relation to the instructional goal. While her students used a variety of strategies to solve the given problem, Anne interpreted the use of those strategies as evidence that students were demonstrating what they had been taught to solve the problem successfully.

Giving an OGAP Item "To see how they're doing"

Anne was a second-grade teacher with 30 years of teaching experience. She was in her seventh year of teaching at a small, relatively high-performing, K-5 school with a diverse student population (44% White, 21% Black/African American, 16% Hispanic/Latino, 6% Asian). For the interview, Anne brought her students' work on a result-unknown subtraction problem with two-digit numbers: "There were 91 apples at the store. 68 apples were sold. How many apples are left? Show or explain how you know." (91 – 68 = x) She explained that she had chosen the problem because her class had been working on two-digit subtraction strategies, and she "thought it would be a good one to see how they're doing." She also liked using OGAP items because the textbook would direct students towards a specific strategy to use, but "this allows them to choose what strategy they want to use, which is better for me to see what they're strong and weak on."

Figure A.1 Student Work

STUDENT A

STUDENT B

Sorting Student Work by Correctness

The student work that Anne brought reflected a wide variety of strategies for solving the problem, including algorithms and visual models. In sorting the work, however, she focused primarily on whether or not the work showed the correct answer, resulting in two large piles. The student work in the "correct" pile had all been marked with a large "C" on the paper while the work in the "incorrect" pile had been marked with a large "X." As Anne looked through the piles and talked with the interviewer, she noted the different strategies they were using: "regrouping" (US standard algorithm), number lines, base 10 visual models, "breaking apart" (decomposing by place value parts), and compensation. However, she did not attend to the sophistication of thinking represented by those different strategies in the OGAP Additive Progression. See Appendix B for how the progression distinguishes, for example, between decomposing by tens and by multiples of ten, or the use of a visual model versus an efficient algorithm.

　　The three examples of student work in Figure A.1 were all placed in the "correct" pile because they had the correct
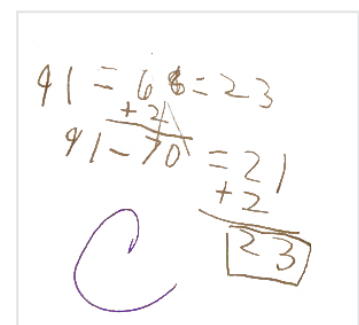

STUDENT C

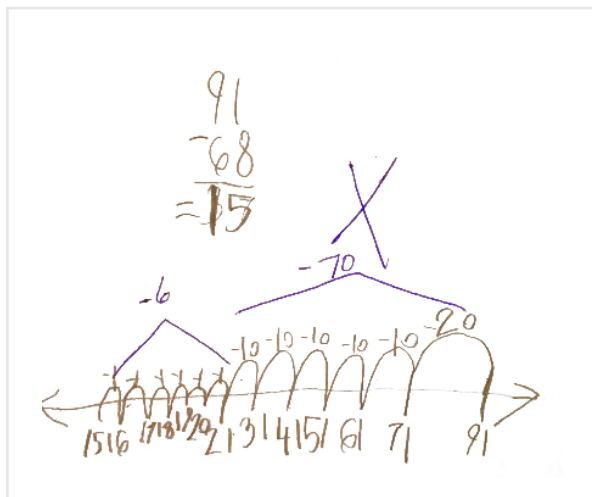*Figure A.2. Student D's Incorrect Solution*

STUDENT D

numerical answer of 23. Anne did not seem concerned about whether the answer was labeled to reflect the context of the problem (23 apples).

Each of these strategies shown in Figure A.1 can be located at different levels on the OGAP Subtraction Progression. Student A's strategy of counting back 68 from 91 by tens and then ones on a number line is an early transitional strategy, while Student B's strategy of finding the distance between 68 and 91 using multiples of ten would be a slightly more advanced transitional strategy and Student C's use of compensation to change the problem into 91 minus 70 and adjust the answer would be an additive strategy. However, Anne valued these strategies equally because the students obtained the correct numerical answer by using strategies that were taught in the curriculum. "They all used some strategy... The ones that got it right used different strategies, but I don't think that they're in a different hierarchy. Like, they are all strategies we've been dealing with, so, they just – Yeah, I didn't get deep with it."

For the work in the incorrect pile, Anne identified whether the students were trying to use an appropriate strategy and then determined where the errors were. In the example shown in Figure A.2, Student D subtracted 76 on the number line instead of 68. Anne noted that student set it up correctly (91 – 68) and may have been trying to use a strategy she had taught (the number line) but noting the errors in his strategy she concluded that he was not "paying attention." She remarked, "and it's a shame, because he's got it set up properly". In this case the use of the number line to subtract was an indicator that he was setting the problem up correctly.

Instructional Implications: "Show them the correct way of getting it and the correct answer"

When asked what she was planning to do as a result of giving this formative assessment, Anne focused on the work in the incorrect pile and the need to correct student errors.

> Maybe put them into small groups with all the same similar issues. Like all these ones that subtracted up and just show them like on a whiteboard or with base 10 blocks, the correct way of getting it and the correct answer, and 'Now look at your answer, what do you see here, can you tell why yours is wrong?' You know, just to get them to stop doing that.

Anne also explained that sorting in this way helped her determine whether students were "getting it" and whether she needed to go back and spend more time on a topic or strategy. As she said:

> I think it's good. I mean, it's really diagnostic. It's a good way of just ... You know, the topic test at the end of a chapter is a long time coming, so this is more open ended to the extent that you get to see where the students are strong and where they're weak or you know, you get to see what their thought processes are for how to solve problems.

She also explained that giving formative assessment items helped to inform her pacing and preparation for the state test, which students would take in third grade. "Because you know, we don't have a lot of time, and some of them just aren't getting the number thing and it's like, okay, how do we fix it or how do we make them better," she said.

Case Study E: Progression of Understanding

Elena offers a contrasting case to that of Anne. Rather than focusing on correct or incorrect answers, Elena sorted her student work by strategy, using the OGAP multiplicative reasoning progression to interpret those strategies in terms of developing understanding, and then noting correct answers, labels, and other issues in student work.

Giving an OGAP Item to See "What do students remember?"

Elena is a teacher with over 20 years of experience. For the past four years she had been teaching fourth grade at a small K-4, low-performing school in a very economically distressed neighborhood with a primarily Hispanic/Latino population (62%). For the interview, Elena brought student work on a measurement conversion problem involving multiplication by a factor of 12: "Farmer Brown donated 7 dozen eggs to the senior center. How many eggs did he donate? Show your work" (7 x 12 = x). She explained that she and her grade partners had chosen to give this problem for two reasons: they wanted to see what students remembered about multiplication, and students were going to need to be able to use a given measurement conversion on the state test.

Sorting Student Work by Strategy

In contrast to Anne, Elena focused primarily on student strategies when sorting her student work. She referred to the *OGAP Multiplication Progression* (see Appendix C) to create three piles of student work in relation to the level of the strategy the student used, and then looked for errors or issues in the work. This resulted in work with correct and incorrect answers being in each pile.

   The first pile that Elena created was made up of student work that reflected the use of multiplication (7 x 12) to solve the problem. She noted that within this pile, some students had correctly labeled the answer as 84 eggs, but others had just written 84 or 84 dozen eggs. She pulled out the example shown below in Figure A.3 and remarked it was "one of the best ones" because the student "multiplied correctly, got the right answer, then they wrote the answer . . . and they actually wrote everything out perfectly." Although Elena valued student explanation, she considered the work that contained strong explanations as a subset of the multiplicative strategy pile.

Figure A.3. A multiplicative strategy

Her second pile she described as "additive" because the students had "added 12 seven times and got their number." In both examples shown in Figure A.4, students added up 12 seven times. Student E had the incorrect label (84 dozens) and Student F had an incorrect answer of 74, which she described as "a simple error, but it's still additive with errors."

The third pile she made was characterized by work that did not show evidence of understanding the multiplicative situation in the problem:

> It just represents students that didn't seem to have any understanding of what the problem was asking them. They didn't understand that they had to multiply and they didn't understand that they had to subtract or add or anything. They didn't know what operation to use, and just did all kinds of I don't know what… So they just didn't seem to have grasped the concept, or the understanding of the question.

Figure A.4. Two additive solutions

STUDENT E   STUDENT F

As she talked about each of these examples, she looked for student sensemaking and understanding. In one example, the student had tried to represent 12 dozen eggs with representations of base 10 blocks but showed several misconceptions. Elena worked hard to make sense of what the student had done, noting "it was interesting what she was getting confused with. I think she was trying to picture something, but her picture was totally off."

When asked about what she learned from this assessment across the whole class, Elena focused on each of the three groups she had created from her sort. In reference to the non-multiplicative pile, she noted that there were "several that were really confused" and talked specifically about what they might need to develop understanding. For example:

> What she needs is to be able to see a dozen. I would love to be able to, for her it would be good to have even an egg carton that has 12 and then for her to actually see or even draw the 12 in each one, but I think seeing, visualizing the actual egg carton with the 12 and say you have seven of these. I think that in

itself would give her a better picture and better understanding and not get it confused.

For the additive pile she remarked that "they struggled with the multiplication, so I see that they've resorted back to what they were comfortable with, and what they're usually comfortable with is addition and subtraction." And for the students whose work showed multiplicative thinking, she talked about next steps for students who had not correctly labeled the answer ("I would probably work on making sure that they wrote 84 what?")

Instructional Implications: "Where I need to go in order to get him to the next step?"

After analyzing the student work by sorting it by strategy, Elena explained how she had selected two examples of student work, one that was additive and one that showed multiplicative reasoning (Figure A.5), and then facilitated a whole class discussion about the work, both in terms of strategy and the meaning of the quantities:

In reference to Student E's work, she noted that there were two issues she wanted to highlight, the incorrect label and moving towards a more efficient strategy, so she asked, "What's a quicker way? How many twelves did she put on there? What would've been a faster way?"

With Student G's work she chose to illustrate a more efficient strategy as well as to highlight the same issue with the label. "So it reinforced it twice instead of just the one time." She explained that she liked to show and discuss examples of student work to help build their confidence and sensemaking:

A lot of them are like 'Oh!' Because they do know, and they know how to do it but it didn't click, and so it was a good thing to see their response and their reaction when they realized that they did know it. [For] a lot of them it's lack of confidence too, and that's some of the ones that said 'Oh.' Because they did know it, but maybe because when they read it they didn't understand it.

Figure A.5. Two solutions selected by Elena for discussion
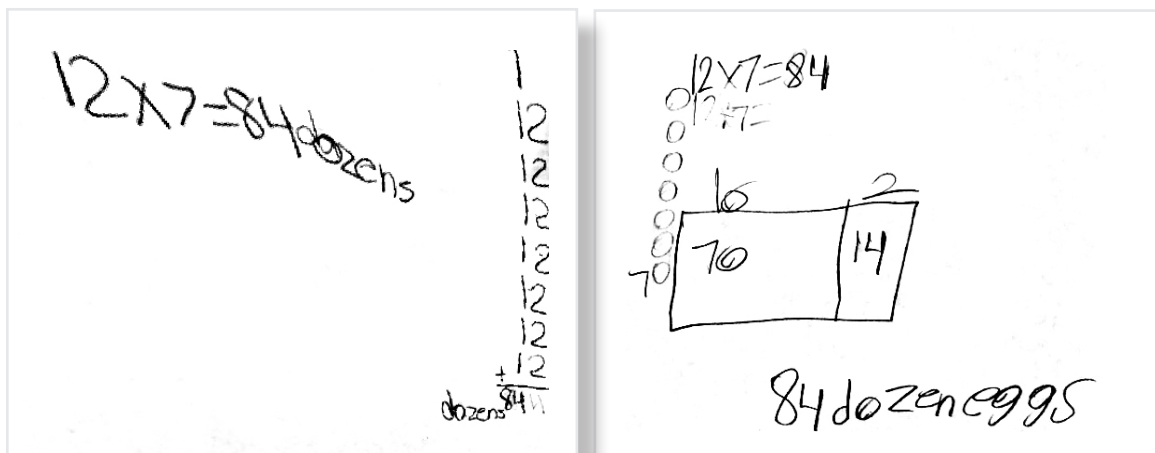
STUDENT E   STUDENT G

This focus on helping students recognize what they did know and understand was a theme across Elena's responses in the interview. As she reflected on the benefits of sorting student work with the progression and then using the progression to select and sequence student work for discussion, she talked about how it had changed the way she was looking at students who "didn't get it":

> Just seeing the progression and understanding the progression and knowing that if a student is not there, where I need to go in order to get him to the next step. I don't have to jump him all the way up to get him to where I want him. I could just focus on the next step and then slowly work him to there because the goal is at the end year where they should be, sooner, great, but a little at a time... At their progress, what they can, can't do... But I think the one thing that I have learned is that it's okay. If they didn't get it, then that's okay. I can figure out a way to get them to get it.

As Elena describes, as a teacher, thinking about learning in this way and with the support of the learning progression gave her a greater sense of agency in terms of knowing how to help students move forward. She also reflected on the benefit of this approach for students in terms of supporting growth and improving their attitudes towards math:

> I think it's very beneficial, especially to the students because they see, they do, know something. You give them credit for what they know because they got the right answer, it is 84, you just clarify it was eggs not dozens. So you praise them for what they do know just try to encourage them to improve or grow.

Thus, Elena's focus was on helping students recognize what they knew and understood as well as how they could continually improve. She saw learning math as a continual progression that included, but was not limited to, performance on a single item.

Performance and Progression Orientations

The approach taken by Anne and Elena differ in several respects. Both teachers gave a word problem; Anne chose one that was parallel to what they had been doing in class and she gave it to see "what they're strong and weak on," while Elena gave a problem about something they hadn't focused on in a while "to see what students remembered about multiplication." Both mentioned the fact that the knowledge they gained would be important for knowing how prepared students were for state testing.

   In sorting the student work, Anne placed primary importance on the correct numerical answer while Elena paid attention to the strategy students were using to solve the problem. In both classes, students used a range of strategies along the respective additive and multiplicative progressions, but for Anne, student use of strategies was part of the desired performance while for Elena they offered a window into the level of student understanding. For Anne, the sophistication of the strategy did not matter as long as it led to the correct answers.

   Both teachers paid attention to incorrect answers and tried to ascertain what caused the error. Elena's sorting by strategy resulted in correct and incorrect answers being in all

three piles. The treatment of errors had implications for the kind of instructional responses these teachers developed as a result of looking at the student work. Anne's response focused on students who were not meeting the standard for performance (which in her case was getting the correct numerical answer), and the emphasis was on reviewing or showing them correct strategies. Elena's response was to think about the needs of each group in terms of the appropriate next step on the progression, moving them forward from their current level of understanding towards more sophisticated strategies.

Hybrid Pathways: The Push and Pull of Achievement Orientation

The two pathways characterized above illustrate achievement (performance) and learning (progression) orientations that we observed in teachers' analysis of student work, which in many ways are diametrically opposed. We also identified three hybrid approaches that incorporate aspects of both the progression and performance orientations. We see these hybrid pathways through the formative assessment process as illustrating ways that teachers might take up elements of a learning orientation, and in some cases begin to move towards using developmental progressions to analyze and respond to student work. In many cases, however, movement toward a developmental approach appears to be constrained by a view of learning not fully compatible with this perspective, and/or a limited understanding of the developmental nature of learning progressions. In the case studies that follow, we present the three hybrid pathways – components of performance, progression of performance, and progression of strategies—beginning with the one most closely aligned with an achievement orientation.

Case Study B: Components of Performance

Bethany is a case of a teacher who sorted her student work to determine who was "getting it" and who needed more support, but she also had an appreciation of the complexity of challenging problems and multiple facets in students' responses. As a result, she did not look only for the presence of a single correct answer, but also for whether students showed their thinking and explained their answers.

Giving an OGAP Item to "practice figuring out" and "show a model"

Bethany was in her ninth year of teaching and sixth year of teaching second grade. She taught at the same school as Elena from Case 2, a small K-4 school in a primarily economically disadvantaged Hispanic population. For the interview, Bethany discussed her students' work on the following problem: "Serena walked from her house to the grocery store. Then Serena walked 7 blocks from the grocery store to her grandmother's house. Serena walked 28 blocks altogether. How many blocks did Serena walk from her house to the grocery store? Show or explain how you know" ($x + 7 = 28$). Bethany explained that the second-grade teachers had chosen this start unknown problem because they wanted to have students practice "figuring out what the problem was talking about" and "try to show a model." She also described how she discussed the problem with students as a whole class first, in order to help them identify the important information and then model how to write it as a subtraction number sentence ($28 - 7 =$ __). Rather than analyzing how they interpreted the problem (which could be solved with addition or subtraction), she focused on how they performed the subtraction that she had modeled.

Sorting Student Work According to the presence of Multiple Components

When she sorted the work, Bethany checked for a series of components in relation to her perception of their importance. First, she considered whether they used the correct operation (subtraction); then she considered whether they used visual models or drawings to support their solution; and finally she considered whether they had made errors in the use of the model. This process resulted in four piles as shown in Figure A.6.

The first pile included student work where the students used addition, which she considered to be the wrong operation to use for this problem. Importantly, one of the two solutions in this first pile was correct, as the student had added 21 and 7 to get 28, but Bethany had not noticed this until the interview. The second pile included students who used only the U.S. standard subtraction algorithm to obtain the answer but did not show a visual model. The third pile included students who were trying to use a visual model – such as a representation of tens and ones blocks – to support their work, but had errors or misconceptions. The final and largest pile (n=20) included students who used correct visual models, whether it was to support their work or to figure out the answer.

In some cases, the model used by students was correct but did not necessarily relate to their solution strategy. Figure A.7 shows an example of student work that Bethany

Figure A.6. Bethany's Sorting Strategy
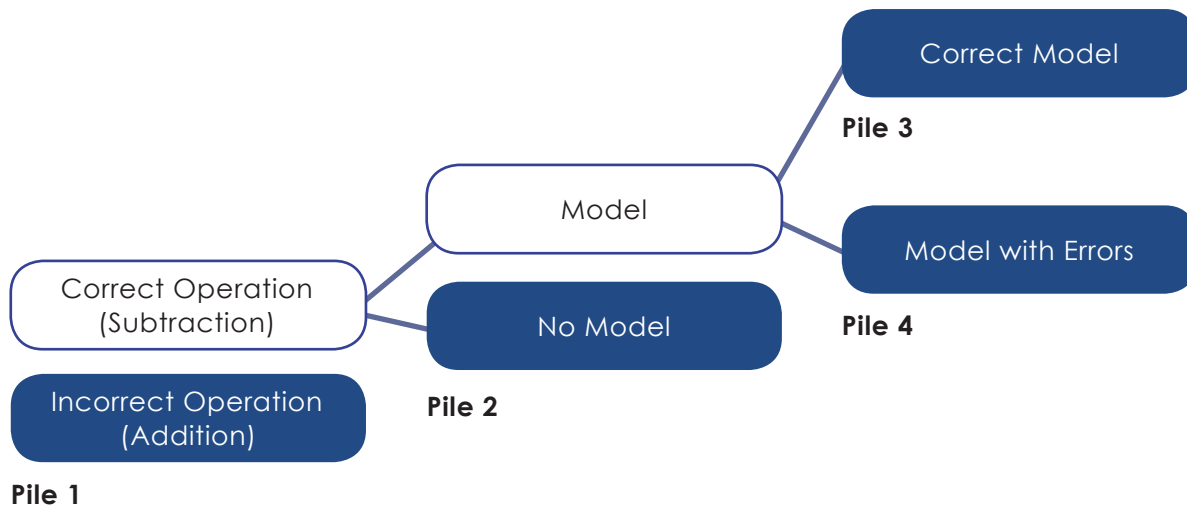
initially put in the pile of "correct answers with a model." The student drew 28 tallies under the tens column and seven in the ones, but then crossed out 18 from the tens and one from the ones, perhaps attempting to model the standard algorithm, which was written next to it. In describing the work, Bethany realized that the model was problematic and moved it to the pile with errors in the model. "Her tens and ones are off, so she would be someone else I would focus on a model with even though she has the right answer."

In her initial sorting of student work, Bethany seemed more focused on whether there was a model than how the student used the model to determine the answer. To her, the use of a visual model was an important component and an indication that students could explain their solutions, rather than an indication of what they understood about the problem.

As she talked about this pile, she further divided the work by the type of model students used: number lines, tens and ones, or a picture where they modeled by ones. When questioned, she mentioned that number lines and tens and ones were more advanced models, but these distinctions did not appear to play a role in her sorting strategy. Further she also did not pay attention to how they were using those models,

Figure A.7. An exampe of a student who used an algorithm and a model
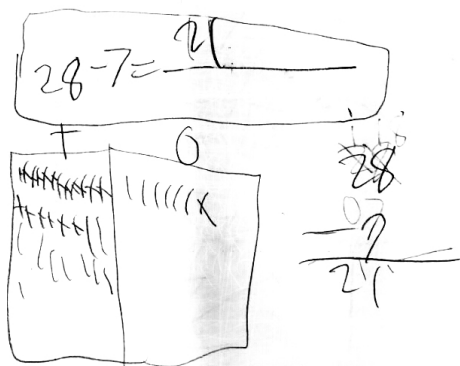
for example, whether they made jumps of one or ten on the number line, which would put them at different levels on the OGAP Additive Progression (See Appendix B).

They all have the same answer, but they all kind of did it in a different way. So, whatever you are comfortable with as your model. So to show them that it can be done with tens and ones. It can be done with a picture. And then it can be done with a number line, but then we would even talk further about what way would be more... efficient for you.

Instructional Implications: "Let's set it up with a model"

Bethany's sorting illustrates her belief that visual models were an important means to understanding the problem situation, rather than evidence of developing understanding. Her instructional implications also focused on the use of visual models. She did not feel that any further instruction was needed for those students who had used an algorithm only to successfully solve the problem. "I think they understand the traditional algorithm, so I don't know if I would even go back and say you have to do a model because they did understand what they had to do, so probably just continue regular instruction," she explained.

For the students who had made errors, she talked about having students "go back and figure out a model first":

> Let's look at a problem. Let's set it up with a model, and they could even be able to pick, like, do you want to do the open number line? Okay, let's see how it would look. If you want to do base ten blocks, let's see how it would look. So just focusing on getting a model, so they can understand, at least, what they're supposed to be doing to help explain their thinking.

For the students who had correctly used models, she described having them "talk about how they decided to solve it" and "think about what could be another way, another model they could use." Her primary focus on the use of models as a component of students' solutions appeared to help her determine whether students were "getting it" and whether she needed to go back and spend more time on a concept or topic. As she said:

> We have a pacing guide here in the district, so it's like, 'Gotta get it done,' but then you look at the work, and it's like, 'Wow. I thought they were getting it, but this kind of shows that they didn't, so how about we take a step back?' So definitely [it] lets you see clearer.

Bethany's focus on the use of models along with the correctness of the answer may have helped her to put some focus on developing conceptual understanding, but this was not always focused on an accurate diagnosis of what students already understood.

Case Study C: Progression of Performance

Similar to Bethany, Catherine also attended to multiple components or dimensions of student performance. But unlike Bethany, Catherine also adopted some aspects of a progression orientation by sequencing these components in a continuum.

Giving an OGAP Item "To see what they could do"

Catherine was a first-grade teacher with nine years of experience. She had been teaching first grade for six years at the same K-4 school where Elena and Bethany taught. For the interview, Catherine brought student work on a put-together/take-apart problem with three single-digit addends: "Nadia went to the store to buy vegetables. She spent 8 dollars on lettuce, 3 dollars on tomatoes, and 2 dollars on carrots. How much money did she spend altogether? Show or explain how you know" ($8 + 3 + 2 = x$).

Catherine explained that she had intentionally chosen this item because it was aligned with the work they had done on three addends but was slightly more advanced than the tasks they had done thus far. She said, "I wanted to see what they could do and where to go from there." Her purpose for giving the item had some elements of a progression orientation, in that she was not only focusing on whether they were getting it but seeing how far they could extend their understanding.

Catherine sorted the student work into nine piles, arranged in a continuum of performance, beginning with "furthest from correct" and moving towards "closest to correct." Although correctness was a priority, she looked at multiple components of student performance to categorize the work, including whether they used the correct operation, added all three quantities, used a model, and wrote an equation. Further, she arranged these components into a continuum towards what she felt to be the ideal performance. Of the nine piles she created, four contained work with incorrect solutions and five with correct solutions. She described and ordered the piles according to the continuum show in Figure A.8.

As shown in Figure A.8, Catherine sorted student work in relation to a continuum that moved from incorrect modeling of the problem, to using a visual model to represent the quantities in the problem, to solving the problem correctly with addition (without a model). She explained that the large number of piles was related to her thinking about instructional implications of their work: "Where I could go with that specific group of kids, what would be the next step."

Sorting by What to Do Next with Each Child

In the example she selected from her first pile, at the lowest end of the continuum, the student had drawn 10 circles but after counting and labeling them correctly up to seven, wrote nine in the next circle and nine as the final answer. Catherine used this work to diagnose precisely what the student needed to work on: identifying the quantities in the problem, drawing a model to represent the quantities, numeral writing, and understanding the value of eight.

> I could see that he understood that he had to draw a model. To me, in my opinion, he understood that he had to put them together, because he did the circle. He didn't separate them and put them in different piles... I put him by himself in this group because I knew that I wanted to work on writing. Here's the

number eight, let's draw a model for eight. And then being able to combine those three models within, into one.

In another example from the fourth pile (correct visual model, but incorrect answer/ equation), the student had drawn 13 circles but then wrote the equation 1 + 2 = 13 on the page. Catherine categorized this solution as "incorrect" but noted both strengths and weaknesses in the work.

> I see that this student did again circle all three numbers. He had an equation of 13 circles. Although they're in one straight line, they're not in an organized, like eight, three, and two. There's no way to tell where eight ended and three started and two started. He didn't have any method to check off what he counted as he went, to keep himself on track. [He] has an equation with two addends and a plus sign, but has the answer of 13. But the numbers are one plus two, and one was not one of our numbers.

She concluded that this student needed to work on "the organization of the model. Being able to show the eight, and then the three, and then the two. And then doing an equation that would have three addends with the correct numbers as the addends."

Finally the last pile, where students had added 8 + 3 + 2 = 13, she described as "just having the equation" and noted, "they knew it without the modeling. They knew the abstract."

The continuum created by Catherine's sorting had some similarities to the OGAP Addition Progression (Appendix B), in that it moved from the use of a model to the use of more abstract reasoning. However, in her analysis, she was prioritizing performance, sorting by "what they were able to show me" rather than looking for the developing understanding that was represented by the strategy (e.g., counting by ones or using numerical reasoning).

Instructional Implications: Differentiated Support

Catherine's emergent continuum seemed to help her to see what each group needed to work on to improve their performance and proved useful for Catherine in developing instructional implications based on the evidence in the work. She explained that she worked with the whole class to compare solutions and emphasize what she was looking for in their performance:

> We looked at the ones with no equation versus an equation, or no model versus a model. And we talked about how just to be more efficient, or to show your work, better than not showing your work. Because it helps us check it. It makes us do the right thing. It just makes it clearer when you're checking it to understand where the thinking was going. So we did talk about making models and making equations.

She also described how she worked with a small group to address the underlying conceptual issues in modeling the problem situation:

> As a small group, I've talked to the students who were in pile three. The [students who used only] two addends, instead of three. We sat down and kind of worked a little bit deeper at getting three addends and having a model for all three, instead of just two.

Notably, her instructional response is targeted at specific student needs, but lacks specificity in terms of how to develop that understanding.

Figure A.8. Catherine's sorting strategy
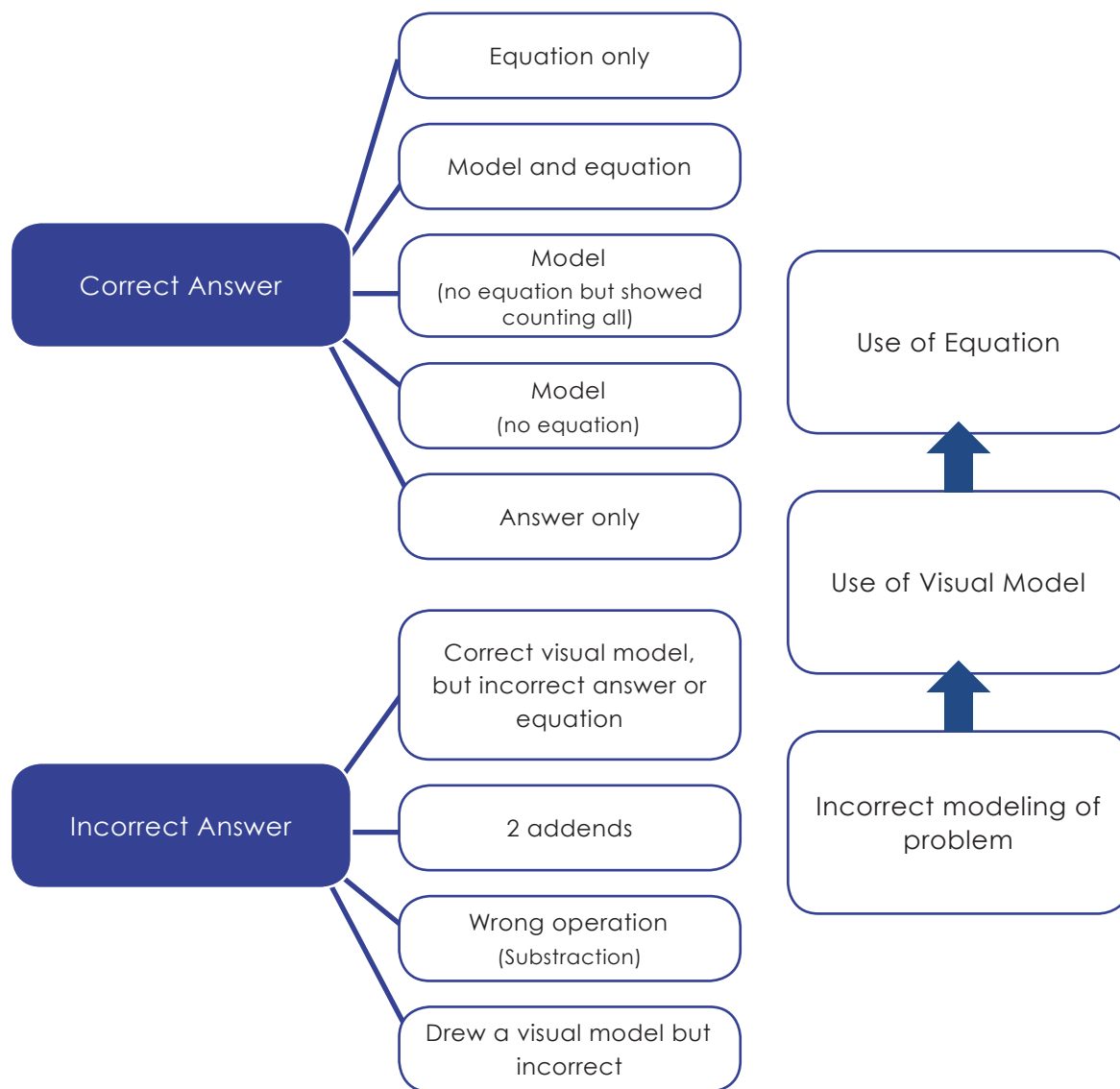
Case Study D: Progression of Strategies

Diana's approach to analyzing and sorting student work incorporated many aspects of a progression pathway, including a focus on examining students' strategies and their understanding. At the same time, she did not have a solid understanding of the progression from counting by ones to counting by tens, and she seemed unsure about how to use this information in her teaching. In some respects, she appeared to be learning through the process of articulating her thinking during the interview.

Giving an OGAP Item: "I wanted to see if they could do it"

Diana was one of two first-grade teachers at a small, low-performing K-8 school, where she had taught for more than 21 years in a 23-year career. The student population was 100% economically disadvantaged and 90% Black/African American. In preparation for this interview, she had given her students the following start unknown item: "Some people were on the train. Then 12 more people got on. Now there are a total of 25 people on the train. How many people were on the train to begin with? Show or explain how you know" ($x + 12 = 25$). She explained that she gave her first-graders this item because she "wanted to see if they could do it with the missing addend. How many people were there to begin with? That's always like a trickier thing." Notably her analysis of students' work seemed focused on determining whether they understood this aspect of the problem, as well as other information their work showed her about their understanding.

### Sorting Student Work by Strategy and Understanding

Diana sorted the work into three categories as shown in Figure A.9. Each category indicated either something about students' understanding of the problem situation or the strategy they used to solve it. Her description of student work in the first two piles illustrates a way that her analysis differentiated between errors of precision and deeper misconceptions.

Figure A.9. Diana's Sorting Strategy
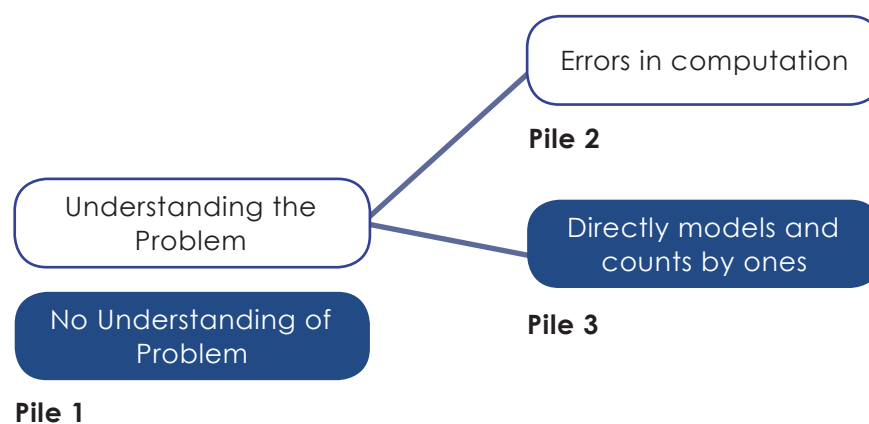
One pile of student work that Diana created represented students who "didn't get it." These nine students made errors in how they set the problem up. Diana pulled out Student H's work as an example (Figure A.10). "She drew circles and labeled them up to 12, which is the 12 more people got on the train. Then she drew 25 circles here, and the 25 was the total, but she did not realize that. She added these 2 together and got 38. She missed the idea of a missing part." It is worth noting that Diana did not comment on Student H's minor miscalculation (12 + 25 = 38) but seemed more concerned that the student did not understand that the problem asked what number to add to 12 to get a sum of 25.

Figure A.10. Student H's work showing misunderstanding of the problem situation
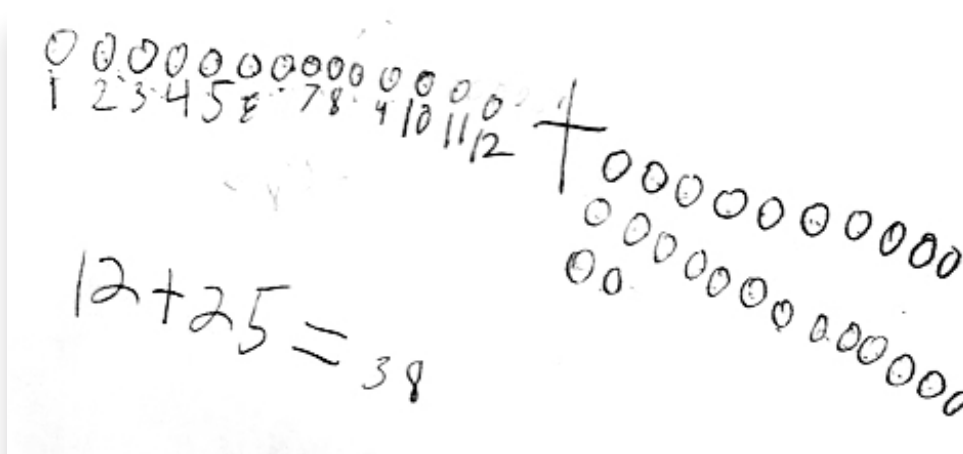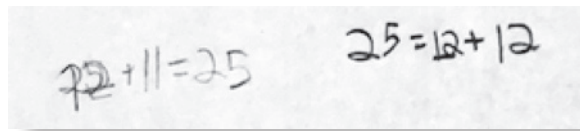
Eight additional pieces of student work were also in this pile. Several were harder to interpret than Student B's. For some, Diana said the students' strategy did not make sense to her. Others made detectable errors in interpreting the problem.

In her second pile, Diana categorized the two students' work as "gets concept but made a mistake." Student J's work in Figure A.11 shows two incorrect number sentences totaling 25. Instead of focusing on his errors, Diana noted that his equations showed an understanding of the start unknown situation: "This one he ended up with the wrong, but. . . not in the way off, because this shows that he has an understanding of math. This is more like a precision problem." The evidence in this work suggested to Diana that Student J understood that 12 plus the start value should add up to 25 and was trying out different start values.

Looking at a checklist that was created from the OGAP Addition Progression Diana stated, "He's closer to additive, and he just made a mistake." (See Appendix B)

Diana's third pile, which she characterized as "kids who all drew it... they all used circles" illustrates her attention to both strategy and the progression. Although the student work in this pile showed understanding of the problem and a correct solution, Diana expressed some concern that they all drew circles and counted by ones rather than using a more advanced strategy, such as a number line. When discussing Student K (Figure A.12), she referred to a recent test she had given. Student K had successfully used an open number line to show jumps of 10 and 1, to solve problems like 25 + 30. As a result, the student's strategy of drawing 25 circles, crossing off 12, and counting the remaining

Figure A.11. Student J's incorrect solution that shows understanding of the problem situations

Figure A.12. Student's K's strategy of representing 25 circles and crossing out 12
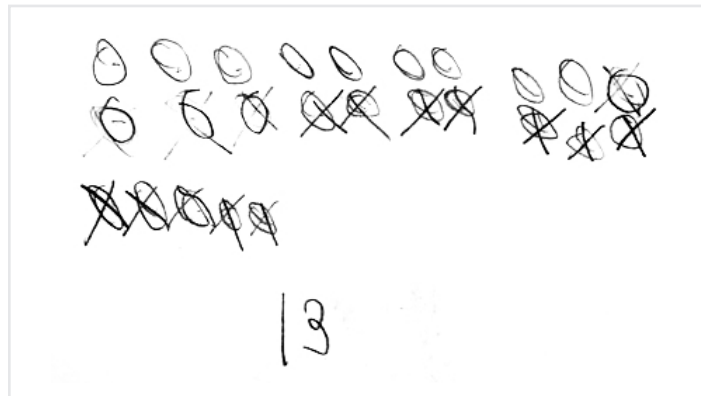
circles by one puzzled and concerned her.

This attention to strategy as an indicator of the level of sophistication of understanding is an important component of applying the developmental progression of additive thinking to formative assessment.

Instructional Implications: "I want to move them up out of here"

When asked to think about next steps for the whole class, Diana pointed to early counting level of the OGAP progressions for addition and subtraction and stated: "I want to move them up out of here." Looking at the progression, she went on to explain, "Counting on from the first would be the next step . . .That's still low but count on from the first. I think some kids can do that." Although she wasn't always referring to the progression in her sorting of student work, Diana was able to use it to identify the appropriate next level of strategy for most of her students. She went on to talk about the fact that in a recent test she had given her students, they were more successful with counting on, but she recognized that may have been a result of her telling them to count on from one of the numbers. At first, she found this to be conflicting information, but then concluded, "Maybe the reason that they didn't do as well because they weren't told... Then telling them this was enough to make it work."

Diana was drawing sophisticated conclusions from multiple data points. On the one hand, she recognized that students were performing better on her own test because she told them to count on a number line. On the other hand, she recognized that this may have pushed them to try a more sophisticated strategy: "That's like a legitimate

next step. Right? Telling them that number. Then the next step would be leaving that number off and seeing if they can do it on their own." She also recognized that not all her students were ready for this push, as she talked about the students who hadn't shown understanding of the problem situation needing to start at a lower level: "I don't know that these kids who came up with 54 and 17 and 39 are there... They need the circles, or they need actual counters."

Diana focused on whether students understood the ideas or could use particular strategies but had not yet mapped these strategies onto a discernable developmental progression. She demonstrated some familiarity with OGAP additive levels of understanding (i.e., recognizing that counting by ones was less sophisticated than making jumps of ten on a number line), but did not use them consistently to analyze her students' work. She may have been developing her own tentative views of learning as a developmental progression, but she struggled to embrace this view to guide her teaching. At the end of the interview, Diana observed the following:

> I know that the OGAP was showing where a child's number sense is. Right? The things they're saying for developing number sense take time. The problem is for them to go on to second grade with the number sense... it takes time to develop that number sense... I think it's developing, but then I see this and I get scared. I think, wait a minute, it's not developing like I thought... The only thing I can think of is maybe they'll go on to second [grade] and it will continue to develop.

As this quote suggests, Diana appreciated the developmental nature of learning and she was beginning to think about how it could inform her instruction. She gave the formative assessment item to learn more about what students "could do on their own" and to uncover their developing understanding, rather than checking to see if they got it right. Although not completely adopting the OGAP approach, she was beginning to show a learning orientation towards formative assessment.

Appendix B. Additive Reasoning Progression—Addition

OGAP Additive Reasoning Progression – Addition

As students learn new concepts or interact with new problem situations and problems structures, they may move up and down along this progression.

Subitizing, Unitizing, Commutativity, Number Composition, Place value understanding

Additive

Additive Strategies

Traditional US algorithm

$$
\begin{array}{r}
{}^{1}\ \\
18 \\
+27 \\
\hline
45
\end{array}
$$

Transparent algorithms
Decomposes by place value and adds

18 + 27 = ?

18 = 10 + 8   18 + 20 = 38
27 = 20 + 7   38 + 7 = 45
30 + 15 = 45

Partial Sums

$$
\begin{array}{r}
18 \\
+27 \\
\hline
30 \\
+15 \\
\hline
45
\end{array}
$$

Uses properties

Associative Property
7+2+8=7+ (2 + 8)

Commutative Property
6+7=7+6

Flexible compensation
18 + 27
20 + 25
45

Fact Fluency

Fact Recall
6+7=13

Derived Facts
6+6=12
6+7=13

Using 10s
6+7
6+4=10 and 10+3=13

Transitional (Tens)

Transitional Strategies

Efficient Use of a Model

18   +   27
10 + 8   20 + 7
30 + 15
45

+20   +2   +5
18   38 40   45
Jumps by multiples of 10 on a number line

+10   +7   +1
63   73   80 81
Jumps by a 10 and efficient groups of ones

Inefficient decomposition to derive facts

6+7
"I know 5+5 is 10 and 3 more is 13"

Early Transitional Strategies

Adding inefficiently with or without a model

Adding on by tens
18+10=28
28+10=38
38+2=40
40+5=45

+10   +10
18   28   38 39 40 41 42 43 44 45
Makes jumps of 10 on a number line

Unitizes on a model

Combines or counts by 10s using base 10 representations

18 + 27 = 45

Counting (Ones)

Counting Strategies

Mental counting strategies:
• Count on from first
• Count on from larger

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Counting on with physical or visual model: 6+7

6
7 8 9 10   11   12 13
"7,8,9,10,11,12,13"

Early Counting Strategies

Direct modeling and counting from 1 with model (count 3 times)

1 2 3 4 5 6
1 2 3 4 5 6 7
"1,2,3,4,5,6,7,8,9,10,11,12,13"

Non-Additive Strategies
• Uses incorrect operation
• Models problem situation incorrectly
• Guesses
• Not enough information
• Uses procedures incorrectly

Underlying Issues/Errors
• Does not consider reasonableness of solution
• Error in counting, calculation, place value, property, equation, or model
• Units inconsistent or missing

Appendix C. Multiplicative Reasoning Progression—Multiplication

OGAP Multiplication Progression (April 2016)

OGAP

Multiplicative Strategies

Multiplicative

Algorithms

Partial Products
$$16$$
$$\times 42$$
$$12$$
$$20$$
$$240$$
$$400$$
$$672$$

Traditional
21
$$16$$
$$\times 42$$
$$32$$
$$640$$
$$672$$

Distributive Property
$$4 \times 16 = 4(10 + 6)$$
$$= 4(10) + 4(6)$$
$$= 40 + 24$$
$$= 64$$

Associative Property
$$(8 \times 2) \times 5 = 8(2 \times 5)$$
$$= 8 \times 10$$
$$= 80$$

Doubling & Halving
$$16 \times 4 = 8 \times 8$$
$$= 64$$

Known or Derived Fact
$$4 \times 6 = 24$$

Commutative Property
$$16 \times 4 = 4 \times 16$$

Powers of Ten
$$5 \times 400 = 5 \times 4 \times 10 \times 10$$

Transitional Strategies

Transitional

Open Area Model *Considers both dimensions of an array or area model*

	30 +	8
20	600	160
+ 6	180	48

$$38 \times 26 = 988$$

10 + 5 : 90 , 45 — 9

$$15 \times 9 = 135$$

Area Model
6 / 4

$$6 \times 4 = 24$$

Considers BOTH dimensions of an array or area model, moving away from needing to see every square unit

Early Transitional Strategies

Skip Counting 3, 6, 9, 12, 15

Building up
$$3 + 3 + 3 + 3$$
$$6 + 6$$
$$12$$

Skip Counting with a Model
3 6 9

Equal groups in an array
3 6 9 12 15

Considers only ONE dimension of an array or area model

Area Model – 6 x 4 = 24
6 / 4 (24)

Considers BOTH dimensions of an array or area model

Additive Strategies

Additive

Repeated addition with or without a model – 3 x 4 = 12
$$3 + 3 + 3 + 3 = 12$$

3 3 3 3
Subitizing in small groups

$$3 + 3 + 3 + 3$$

Early Additive Strategies

Modeling, counting by ones

Inconsistent Grouping

Modeling, counting by subgroups
3 x 4 = 12
2 4 6 8 10 12

Non-Multiplicative Strategies

Adds or subtracts factors
Models factors incorrectly
Guesses

Uses incorrect operation
Not enough information
Uses procedures incorrectly

Underlying Issues and Errors

- Doesn't consider reasonableness of solution

- Error in: calculation, place value, vocabulary, property or relationship, equation, or model

The strategies students use move back and forth across the levels as they learn new concepts and/or interact with new problems structures and contexts.
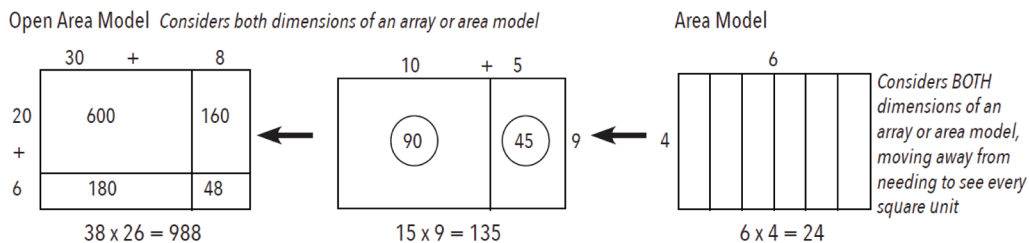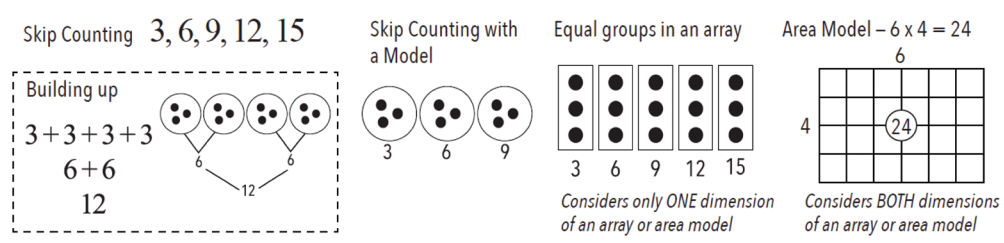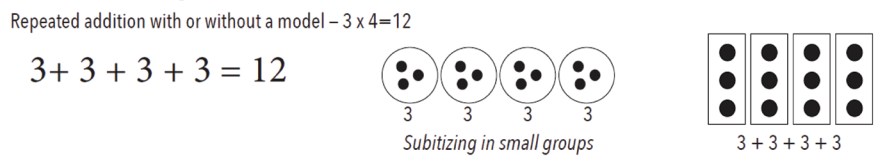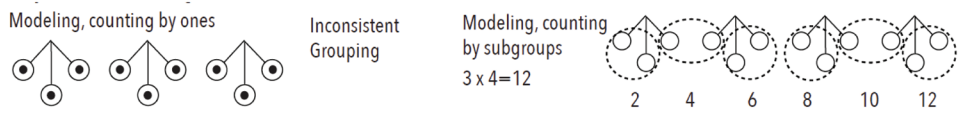
Unitizes into equal groups, uses visual models, understanding of place value, and properties of operations

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