TRANSFERABILITY OF ALGEBRA LANGUAGE AND SKILLS TO ALGEBRA-BASED PHYSICS: STUDENT AND INSTRUCTOR PERCEPTION AND STUDENT SKILLS

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Abstract:
This convergent parallel mixed methods study explores college instructors' and students' perceptions of transferability algebra skills and language to the study of algebra-based physics. The data analyzed included responses from on online survey completed by 31 instructors of college algebra and algebra-based physics instructors, interview transcripts from eight instructor of the survey groups, responses from survey completed by 17 students enrolled in an algebra-based physics course, results to isomorphic problems in the mathematics and physics context, and transcripts of interviews and task-based interviews of three students from the survey group. The results are organized into three distinct studies each addressing a specific set of research questions. Overall, the dissertation findings indicated that both instructors and students express concerns over transfer of both algebra skills and language to the study of physics. Results from the isomorphic problems indicated that students were not able to demonstrate an understanding of graphing concepts in either the mathematics or physics context. These students also struggle with solving quadratic equations when asked to solve for the *x* variable, creating difficulty in examining the transferability of those skills from mathematics to physics. When examining instructors' and students' perceptions of the transferability of algebra language to physics, both groups indicated similar concerns in translating the vocabulary and variables used in the algebra to physics. However, the students also indicated struggles with understanding the formulas used in physics. Both instructors and students suggested that additional application problems would assist students in making connections between algebra and physics.
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CHAPTER I

INTRODUCTION

According to the National Academics of Science (2010), "The United States ranks 27th among developed nations in proportion of college students receiving undergraduate degrees in science or engineering" (p. 8). The following year The Presidents' Office of Science and Technology (PCAST) (2011) reported that graduation rates in STEM needed to increase by 34% to meet the demand of the work force. These reports stimulated a stronger movement in education to encourage students in STEM. However, more current data in a 2015 report by the American College Testing services (ACT) indicated that although students interest in science, technology, engineering and mathematics (STEM) was 1% higher than 5 years ago the data indicated a large gap between a student's expressed interest and their measured interest at only 18% of the overall students' measuring an interest in STEM. Compounding the issue for more graduates in STEM, the 2015 ACT report indicated that those students interested in STEM were not prepared for college level courses such as math and physics.

As interest, enrollment, and degrees in STEM continue to be a concern, researchers and educators have begun to realize that students make the decision to enter the STEM fields between middle and high school (Christensen & Knezek, 2017; Holmes, Gore, Smith, & Lloyd, 2016; Maltese & Tai, 2011; Watkins & Mazur, 2013). With physics, considered one of the gate-keeper

courses for students majoring in a STEM field (Redmond-Sanogo, Angle, & Davis, 2016; Robinson, 2003; Shaw & Barbuti, 2010), educators can potential influence a student's decision to continue studying in STEM by providing students with positive experiences. However, many high school physics students find the concepts in physics to be a challenge claiming physics is difficult, irrelevant, and work intensive and not a positive experience. These students also identify physics as overall the least popular science course thus resulting in a decrease in enrollment in physics courses (Angell, Guttersrud, Henriksen, & Isnes, 2004; Barmby & Defty, 2006; Masood, 2005: Williams, Stanisstreet, Spall, Boyes, & Dickson, 2003).

A similar attitude towards physics can be observed at the college level with factors for enrollment and retention including students' high school science experience, attitudes toward application of physics in their major, and success in the general physics courses (Bergeron & Gordon, 2017; Halloun, & Hestenes, 1989; Tai, Philip, & Mintzes, 2006; Willson, Ackerman, & Malave, 2000). Focusing on student success in physics, researchers have investigated several areas that could potentially challenge students' success in physics. These areas of research include transferability of mathematics knowledge to physics studies, transferability of mathematics language to physics, and instructors' and students' perceptions of the of mathematics skill set needed for success in physics.

Background of the Problem

Investigating students' success in physics courses, researchers have examined knowledge transfer. Knowledge transfer can be used as a framework for investigating how students apply knowledge from one experience to another similar or new experience, such as applying mathematics knowledge to physics (Beach, 1999; McKeough, Lupart, & Marini, 2013; Perkins & Salomon, 1992; Singley, & Anderson, 1989). Knowledge transfer is a topic of interest for researchers of STEM since students ability to transfer mathematics knowledge is important for the study of STEM courses such as chemistry, physics, and biology (Bassok, 1990; Hoban, Finlayson, & Nolan, 2013; Menis, 1987; Ngu & Yeung, 2012; Rebello, Cui, Bennett, Zollman, & Ozimek, 2007). Researchers have identified three general types of potential knowledge transfer: classical knowledge transfer, process-based

knowledge transfer, and sociocultural-based knowledge transfer. Classical knowledge transfer examines students' success in transferring knowledge from one problem to another similar problem (Bassok, 1990; Hoban, Finlayson & Nolan, 2013; Reed, Dempster, & Ettinger, 1985; Sloutsky, Kaminski, & Heckler, 2005). Process-based knowledge transfer examines not necessarily the level of knowledge transferred but the process that students use to transfer that knowledge in hopes to find how knowledge transfer occurs (Bransford & Schwartz, 1999; Carraher & Schliemann, 2002; Rebello, Cui, Bennett, Zollman, & Ozimek, 2007). Lastly, sociocultural based knowledge transfer examines the sociocultural influence in learning and knowledge transfer for students (Beach, 1999; Evans, 1999; Pea, 1987; Terwel, van Oers, van Dijk, & van den Eeden, 2009).

Specifically, when examining the knowledge transfer of mathematics to physics researchers have primarily utilized classical knowledge transfer. Using problems of similar content but in the context of mathematics verses physics (isomorphic problems), researchers have documented students' transferability of mathematics to physics (Bollen, De Cock, Zuza, Guisasola, & van Kampen, 2016; Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012; Planinic, Ivanjek, Susac, & Milin-Sipus, 2013; Pollock, Thompson, & Mountcastle, 2007). Researchers have identified a few areas students find challenging in the transferring of mathematics knowledge to physics such as graphing, linear equations, and quadratic equations (Bollen, De Cock, Zuza, Guisasola, & van Kampen, 2016; Guttersrud, & Angell, 2010; Planinic, Ivanjek, & Susac, 2013; Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012).

Knowledge transfer offers a framework for examining the processes or struggles students may encounter when translating mathematics into the study of physics. Research in knowledge transfer of mathematics to physics has suggested one skill required for student success in physics lies in the students' mathematics skills. Data indicates that the probability of a student's success in physics is highly correlated to the student's mathematics skills set, and students who do not possess the mathematics skill set necessary for mathematics application are less likely to be successful in physics than those students who demonstrate proficiency (Champagne, Klopfer, & Anderson, 1980;

Hudson & McIntire, 1977). Interestingly, not all data indicates mathematical skill set is the entire issue and some students may have proficient mathematics skills but struggle to apply that skill set to physics (Hudson & Liberman, 1982; Hudson & McIntire, 1977; Rebello, Cui, Bennet, Zollman, & Ozimek, 2007).

All the previous research calls for examining the mathematical transfer of knowledge to physics. However, beyond comparing isomorphic mathematics and physics problems, researchers are struggling with finding possible avenues to investigate the transfer solutions. Researchers Redish and Kuo (2015) along with Pietrocola (2008) suggest investigating the transferability of mathematics to physics by examining a possible language difference between the language of mathematics and the language of physics. This line of possible research although not new has not been thoroughly investigated.

Statement of the Problem

Over the past several decades, researchers examining students' struggles with physics, along with the enrollment in the subject, have concluded that mathematics is a common variable in the struggle, regardless of the level of mathematics needed for the course (Hansson, Hansson, Juter, & Redfors, 2015; Nguyen & Rebello, 2011; Pepper, Chasteen, Pollock, & Perkins, 2012; Winegardner, 1939). As research continues to develop, an avenue to investigate is the possible inability of students to translate between the language of mathematics and the language of physics (Pietrocola, 2008; Redish, 2005; Redish & Kuo, 2015). This question has led to the object of this convergent parallel mixed methods study, to explore students, enrolled in an algebra-based college physics course, transferability of mathematics language and skills to the study of physics by examining students' skills and students' and instructors' perceptions.

Purpose and Research Questions

The overarching goal of this study was to explore the transferability of mathematics knowledge to physics. More specifically, to meet this goal, the study examined both the instructors and students' perspectives on the transferability of mathematics skills and language to physics by

examining three different perspectives. The study was divided into three separate parts all of which utilized a convergent parallel mixed methods design.

This research proposed to answer the following questions in the three different studies:

Study 1: College Instructors' Perceptions of Transferability of Algebra Language and Skills to Studies in Physics.

- How do instructors of college algebra and algebra-based physics perceive the transferability of algebra language and skills to the study of algebra-based physics courses?
- Is there a significant difference between instructors of college algebra and instructors of algebra-based physics perceptions of transferability of algebra language and skills to physics?

Study 2: Transferability of Algebra Language and Skills to Physics: Students Perceptions of Mathematics Ability.

- How do students perceive the transferability of their algebra language and skills from mathematics courses to physics courses?

Study 3: Students' Transferability of their Algebra Knowledge to Physics

- Does a student's knowledge of linear equations, quadratic equations, and graphing differ between a mathematics context and a physics context?

Significance of the Study

The impact of this study could potentially offer instructors of both college algebra and algebra-based physics a set of mathematics skill necessary to transfer to physics studies. This knowledge would allow college algebra instructors to emphasize the application of the mathematics skills in physics while offering physics instructors the knowledge to aid students in a review of mathematics skills and translation of those skills to physics. This study could also potentially offer basic research on the possible transferability of language between mathematics and physics that can be applied to other physical science courses along with potentially offering evidence of the importance of setting up communication between instructors of mathematics and physics allowing for

the easier transfer of mathematics skills to physics. Finally, the data could also potentially offer students' knowledge of the translation needed between mathematics and physics so that they are aware of the skills needed to be successful in physics.

Assumptions, Limitations, and Delimitations

The first assumption of this study was that all participants responded honestly to all parts of the study and that the questions were presented in a manner that was understandable so that the participants could answer them to the best of their ability. Also, for the students enrolled in algebra-based physics, it was assumed that they had met the institution's minimal mathematics requirement to enroll in the course.

The limitations of the study included that the study was limited by the number of instructors in the state willing to participate in the study although every effort was made to encourage participation that would provide a sample size to represent the population. Finally, due to self-reporting and opinions, the study was limited by response bias.

The study was delimitated to students enrolled in algebra-based physics in one institution in the state. The study was also delimitated to only instructors in the state of Oklahoma that taught college algebra and algebra-based physics.

Summary

For this study, Chapter I offers the background, purpose, and significance of this study. Chapter II gives an overview of the literature related to the transfer of mathematics knowledge to physics along with instructors' and students' perceptions of knowledge transfer. Chapter III offers the methodology and results to part I of the study. Chapter IV and Chapter V, respectively, examine the methodology and results to study 2 and study 3.

CHAPTER II

REVIEW OF LITERATURE

This paper reviews the professional literature related to the transferability of mathematics knowledge to physics. First, the paper provides an overview of knowledge transfer and the processes of knowledge transfer. Next, an examination of both teachers' and students' perceptions of skills and use of mathematics in physics has been presented. Finally, a discussion on the students' skills in interpreting mathematics symbolism in comparison to physics and the possibility of a language barrier between the two will be discussed.

Knowledge Transfer

Knowledge transfer examines how knowledge gained from one experience is applied to another similar or new experience (Beach, 1999; McKeough, Lupart, & Marini, 2013; Perkins & Salomon, 1992; Singley, & Anderson, 1989). Knowledge transfer is a topic of interest for researchers of STEM since students ability to transfer mathematics knowledge is important for the study of STEM courses such as chemistry, physics, and biology (Bassok, 1990; Hoban, Finlayson, & Nolan, 2013; Menis, 1987; Ngu & Yeung, 2012; Rebello, Cui, Bennett, Zollman, & Ozimek, 2007). The concept of knowledge transfer in education and psychology has developed into a collection of frameworks for studying learners' application of knowledge. Researchers have explored and offered a variety of processes through which to examine knowledge transfer along with factors related to improving knowledge transfer. To further clarify the ways in which

knowledge transfer is researched, it can potentially be divided into three types of transfer: classical knowledge transfer, process-based knowledge transfer, and sociocultural based knowledge transfer (see Figure 2.1).

Knowledge Transfer		
Classical Knowledge Transfer	→	Examination of the transfer of knowledge from one experience to a similar or new experience.
Process-based Knowledge Transfer	→	Examination of the process of knowledge transfer from one experience to a new experience.
Sociocultural Knowledge Transfer	→	Examination of social and cultural factors such as a topic's worth on knowledge transfer to new situations.

Figure 2.1. Types of knowledge transfer.

Knowledge transfer research began in the early 1900s when the desire to improve the learning process and expand existing knowledge on the science of learning, moved Thorndike, a psychologist, to begin exploring knowledge transfer. Thorndike's interest in changing the philosophy of learning beyond generalized transfer, knowledge transferred between subjects that contain no similar content, began researching knowledge transfer. Thorndike and Woodworth (1901) researched knowledge transfer by examining the ability of individuals to transfer learned "functions" to new situations. In this study, the researchers defined "functions" as the ability of a person to become efficient at estimating the areas of paper shapes. The participants were given a series of paper rectangles in which they practiced guessing the areas. Once the participants were efficient at estimating areas of rectangles, the participants were then given multiple shaped pieces of paper such as circles, triangles and irregular shapes to repeat the testing of area estimation. Examining the ability of individuals to transfer the learned "functions" to a new situation, the researchers found that there was too large a difference between types of situations to observe any substantial transition of knowledge. Thorndike theorized that transfer is applicable between

activities that share common elements and these shared common elements are rarely identical making transfer limited (Singley & Anderson, 1989).

Thorndike's research set up the framework of classical transfer of knowledge that sparked decades of research. In knowledge transfer, researchers are looking for the students' success in transferring knowledge from one problem to another analogous problem similar to Thorndike's idea of common elements (Bassok, 1990; Hoban, Finlayson & Nolan, 2013; Reed, Dempster, & Ettinger, 1985; Reed, Ernest, & Banerji, 1974; Sloutsky, Kaminski, & Heckler, 2005). Continuing research along the lines of analogous knowledge transfer or classical knowledge transfer, Reed, Dempster, and Ettinger (1985) examined students' transferability of algebra knowledge in two situations. The first situation examined algebra knowledge transferred to problems that were similar to the originally learned problem, and the second situation examined algebra knowledge transferred to problems that were unrelated to the original problem learned. The two different types of problems were designed to determine the level of knowledge transfer beyond Thorndike's theory. In the first of two experiments, Reed, Dempster, and Ettinger examine whether sample problems with solutions would help students transfer knowledge later to test questions containing both related and unrelated problems (transfer problems). In the second study, the students utilized the solutions to the sample problems during the testing situation. The results of both of these studies suggested a lack of knowledge transfer to the unrelated problem. Although, the second study's data show more correct answers to related problems.

Reed, Dempster, and Ettinger (1985) investigated a third scenario where students utilized more elaborate solutions to the practice problems along with explanations for the solutions during the same testing scenario as before. Results indicated an improvement of knowledge transfer to related problems, but again knowledge transfer is not evident in unrelated problems. For a final experiment, the researchers offered the students more complex practice problem with continued access to solutions for the same testing scenario. Data indicated that the students had an even

higher level of knowledge transfer to similar problems than in previous studies, and there was some evidence of knowledge transfer to unrelated problems. Reed, Dempster, and Ettinger (1985) concluded that students utilize a syntactic approach to solving problems and the students were trying to replace numbers in the solutions instead of creating an understanding that can be transferred, verifying the original theory of Thorndike that knowledge transfer tends to happen between related problems.

Some researchers have found knowledge transfer to be unproductive or non-informative enough to change academic practices, so they began to examine process knowledge transfer. These researchers have striven to examine not necessarily the level of knowledge transferred but the processes that students use to transfer knowledge (Bransford & Schwartz, 1999; Carraher & Schliemann, 2002; Rebello, Cui, Bennett, Zollman, & Ozimek, 2007).

Approaching this more contemporary view of knowledge transfer processes, Rebello, Cui, Bennett, Zollman, and Ozimek (2007) suggested a transfer theory of associations where the transfer was a dynamic process of knowledge transfer of mathematics to physics. The researchers adapted existing theories on knowledge transfer to create two types of associations, horizontal and vertical. Horizontal association demonstrated the transfer of knowledge in situations such as the end of the chapter problems or those considered "plug and chug" where the students only need to apply mathematical skills to a formula. This type of knowledge transfer was already an association for the learners and did not require the learner "to critically examine the situation or the assumptions underlying the model that they use to solve it" (p. 9). Vertical association, on the other hand, demonstrated the knowledge transfer applied to a new situation. In this type of knowledge transfer, the student had no information on the situation and had to draw upon multiple previous experiences to create a solution to the experience. In the latter case, researchers are more interested in the process students use to provide a solution than the particular knowledge that was used since solutions can follow multiple paths using various information.

Continuing to research process knowledge transfer, Rebello, Cui, Bennett, Zollman, and Ozimek (2007) studied two separate situations to assess the transfer of knowledge from mathematics to physics. In the first study considered by the researchers to examine classical knowledge transfer, physics and calculus equations similar to the students' homework or exam equations were used to assess transfer of calculus knowledge. The researchers verified that the students did, in fact, have calculus skills ruling out mathematical skills as an issue to knowledge transfer. However, the researchers observed that students demonstrated difficulty in transferring calculus knowledge to the physics problems and were not able to set up the problems to solve.

The impact of examining the process of the knowledge transfer and not just the transfer itself indicates that the prior theories on knowledge transfer research may contain limitations in the scope of the assessment of the transfer and not the transfer of knowledge itself. Rebello, Cui, Bennett, Zollman, and Ozimek (2007) summarize the impact of their research, "Our results appear to indicate the main difficulty that students appear to have does not lie in their lack of understanding of mathematics per se, rather it lies in their inability to see how mathematics is appropriately applied in physics problems" (p. 30).

The addition of procedural knowledge transfer research in addition to classical knowledge transfer offered researchers expanded data in which to examine knowledge transfer. However, as studies in learners' cognitive process expanded, researchers began to examine the sociocultural influence in learning and knowledge transfer (Beach, 1999; Evans, 1999; Pea, 1987; Terwel, van Oers, van Dijk, & van den Eeden, 2009).

Continuing to expand on Thorndike's classical knowledge transfer to include social and cultural aspects, Pea (1987) included not only classical transfer (situational transfer) in his theoretical framework for examining knowledge transfer but also included selective transfer, a sociocultural approach. For situational knowledge transfer, knowledge transfer is linked to a previous experience of the individual learner. For example, the common element, the language/vocabulary, in the situation resembles the language of the previous experience such as

the variables in a mathematics class. Pea, however, expressed the need for researchers and educators to look beyond the individual learner to also include the social and cultural effects on the transfer of knowledge on the individual learner. Pea defined this inclusion of social and cultural effects on transfer as selective transfer. Selective transfer examines such cultural and social factors as the individual's views on the worth of the transfer or their evaluation of the effectiveness of the knowledge transfer. Pea theorizes that these individual's view affects the level of knowledge transfer.

Following a similar framework as Pea, Evans (1999) approached situational knowledge transfer issues by incorporating and expanding the selective transfer suggested by Pea to affective issues. Evans examined the mathematical evaluation skill of nontraditional students by creating a setting that allows the students to provide a language that would bridge the mathematics to their personal experience. Theorizing that linguistics and mannerisms were both potential transfer issues, Evans created a setting for the mathematics problem by asking the participant if the problems resembled anything in their daily life. Using the participant's response, the problems were set up using the language of the participant's connection to daily life such as business practices. Evans concluded that knowledge transfer can be benefited by building bridges between mathematics to learners' experiences by examining the relationship of transfer and the discourse between mathematics and outside activities.

Specifically, with more of research in the transferability of mathematics to physics following classical knowledge transfer, one method researchers incorporated into their studies was to observe knowledge transfer between isomorphic problems (Bollen, De Cock, Zuza, Guisasola, & van Kampen, 2016; Guttersrud, & Angell, 2010; Hoban, Finlayson, & Nolan, 2012; Planinic, Ivanjek, Susac, & Milin-Sipus, 2013; Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012). Isomorphic problems consist of two or more problems of the same content but different contexts. Hoban, Finlayson, and Nolan (2012) studying classical knowledge transfer between mathematics and chemistry utilized isomorphic problems to examine knowledge transfer. They

used two questions that were similar in structure but different in context with one relating to chemistry and the other to mathematics to determine if students are able to transfer knowledge from mathematics to chemistry. The first set of questions, mathematics questions, offered the students a linear graph asking for the calculation of the slope of the line. Whereas the second set of questions, chemistry questions, also offered the students a linear graph asked for the calculation of the rate of change of a concentration of a reactant. Both questions required the students to utilize the same mathematical knowledge of linear equations and slope with the only difference in the questions being the meaning of the slope in terms of mathematics or chemistry. The use of isomorphic questions provided Hoban, Finlayson, and Nolan (2012) the opportunity to observe the difficulty of students in translating the mathematical knowledge to the chemistry context when data indicate that the students were able to correctly answer the questions in the context of mathematics.

When using isomorphic problems to examine how students of both algebra-based and calculus-based physics approaches kinematics graphing based on context, Bollen, De Cock, Zuza, Guisasola, and van Kampen (2016) found that students in algebra-based physics struggled more in transferring knowledge from mathematics to physics than the calculus-based physics students. Additionally, the researchers indicated that students in algebra-based physics were better able to answer questions concerning graphs that were context-free as compared to those with context. Similarly, Planinic, Ivanjek, Susac, and Milin-Sipus (2013), examining students' level of transferability of graphing knowledge using isomorphic problems, found that students struggled in transferring knowledge of slope between several different contexts (mathematics, physics, and other areas). The authors explained the problems with slope in contexts other than math required "one more step in solving: interpretation and translation of context into mathematical language" (p. 7).

Earlier, Planinic, Milin-Sipus, Katic, Susac, and Ivanjek (2012), examining students' transfer of mathematics knowledge to physics, developed two sets of isomorphic graphing

problems for both mathematics and physics. The researchers found that the students were more likely to answer graphical problems in the mathematics context correctly as compared to the physics context. The researchers also suggested that the students found little similarity between problems in the same content when presented in different contexts, and the students used different skills to answer the questions based on the context. Even earlier, research by Bassok and Holyoak (1989) examined the transfer of isomorphic series and sequence problems from algebra to physics. Data from this study indicated that knowledge transfer was evident between isomorphic problems in physics and algebra when similar variables were used. However, data did not show the same knowledge transfer on isomorphic equations that use different quantities. The researchers suggested more time may be needed in connecting concepts.

To summarize the knowledge transfer concept, The National Research Council (2000) published a statement that included the examination of the key components of learning and transfer that are important concepts for educators that can be used to improve instruction for knowledge transfer. The National Research Council established four components that are key to transfer including: 1) understanding that learners must first learn the knowledge, 2) the learner needs to have an abstract representation of the knowledge, 3) transfer is dynamic, and 4) transfer depends on previous learning. These components suggest that students must master knowledge before they are able to transfer the knowledge to a different experience. That would include that the students understand the subject and have not simply memorized the subject. The learner also needs ample time to learn the subject so that it can be mastered. Motivation can influence the learners to spend more time on the subject thus increasing understanding. Learner motivation leads to the importance of context and problem representation to improve so that a learner is more connected to the subject. Finally, metacognition is a component that can improve knowledge transfer by having learners reflect and improve on their own transferability. The National Research Council reminds, “One aspect of previous knowledge that is extremely important for understanding learning is cultural practices that support learners’ prior knowledge. Effective

teaching supports positive transfer by actively identifying the relevant knowledge and strengths that students bring to a learning situation and building on them" (p. 78).

Knowledge transfer, a framework through which to examine students' learning, offers several different views through which to examine students' transfer of mathematical knowledge. For example, a researcher can examine a student's transfer of knowledge based on elements that are common such as isomorphic problems or can examine the process through which a student transfers knowledge from one problem to another. Knowledge transfer also sets up a framework to examine how social interaction affects the learning and transfer of mathematical knowledge.

Perceptions and Skills of Mathematics Use in Physics

When investigating the issues of mathematics in physics it is important to examine both the educators and students' perceptions of teaching and use of mathematics in the physics classroom along with the required mathematical skills. Understanding these perceptions is one tool for identifying areas of concern in students' application of mathematics in the physics classroom. Researchers have documented that the perceptions (attitudes) of both the instructor and the students affect the performance of students in mathematics (Archambault, Janosz, & Chouinard, 2012; Kim, Damewood, & Hodge, 2000; Lamb & Daniels, 1993; Rice, Barth, Guadagno, Smith, & McCallum, 2012; Siskandar, 2013; Wilkins and Ma, 2003). Minimal research has been conducted on students and instructors' perceptions of the mathematics used in physics. The researchers investigating instructors and students' perceptions of mathematics use in physics, however, have theorized from responses to their studies a possibility of translation issues between mathematics and physics (Angell, Guttersrud, Henriksen, & Isnes, 2004; Frykholm, & Glasson, 2005; Guttersrud, & Angell, 2010; Kapucu, Ocal, & Simsek, 2016; Turşucu, Spandaw, Flipse, & de Vries, 2017).

Instructors Perceptions of Mathematics in Physics. When examining instructors' perceptions of mathematics used in physics, Angell, Guttersrud, Henriksen, & Isnes, (2004) found that instructors perceive mathematics use in problem-solving physics phenomenon to be

the most problematic strand of physics for students. While at the same time, the instructors indicate that being able to calculate problems using basic laws is the second most important aspect of physics next to understanding everyday phenomena. Frykholm, & Glasson, (2005), examining instructors' understanding of the knowledge transfer between mathematics and science, find a connection between mathematics and science to be important and intuitive for pre-service instructors. However, the pre-service instructors indicate that mathematics has always been taught as an independent subject and "content was typically fragmented, often in isolation from other topics that may have provided various contexts and/or connections" (p. 137). This isolation has led to a feeling of incompetence that creates a barrier for the instructors in creating a connection between the two subjects.

Mulhall and Gunstone (2007) find during their interviews with secondary level physics teachers that the teachers had never really considered mathematics place in physics only that it was a necessary tool for explaining physical phenomenon. All of the teachers believed that physics was mathematical. Ornek, Robinson, and Haugan (2008) research indicated that teachers believe that students lack higher level mathematics is one possible cause of students struggle in physics. Although the teachers believe that the lack of motivation to study is the highest reason for struggle and that good mathematics skills and background are necessary for success in physics. The teachers also believe that more real-life applications are needed to aid the students.

Also interested in the perception of instructors' beliefs of the transfer of algebra to physics, Turşucu, Spandaw, Flipse, & de Vries (2017), surveyed and interviewed pre-university instructors to find that mathematics instructors realize there is a translation problem between mathematics and physics with the physics instructors calling for more emphases between mathematics and physics. Both groups of instructors believe that students do not connect mathematics to physics. The interviews indicated that the instructors believe that the transfer of knowledge between the subjects happens automatically and can be improved only through intensive instruction in the mathematics course. Only a small group of instructors (mainly

physics instructors) viewed the need for collaboration between mathematics and physics in order to aid transfer knowledge. Overall, the mathematics instructors felt no need to collaborate to improve the transfer of mathematics.

It should be noted at this point that as early as 1968 a symposium of teachers made the following recommendations. The participants in the teaching of mathematics and physics Lausanne Symposium (1968) suggested "mathematics and physics have their own language and notations. To ensure that they are understood, teachers of both disciplines must explain how these languages connect" (p. 246). The participants also recommended that mathematics and physics curriculum should be a collaborative process and further add, "It is necessary to develop both the aptitude of pupils for identifying mathematical structures presented in situations encountered in physics (transfer of knowledge) and their skill in the use of key mathematical tools, particularly algebraic calculations" (p. 245).

Students' Perceptions of Mathematics in Physics. Although the beliefs and attitudes of the instructors drive the curriculum presented to the students, it is just as important to examine students' beliefs. Angell, Guttersrud, Henriksen, and Isnes, (2004) found that students felt mathematics was needed to describe physics systems and was hard. The students, however, did not feel that mathematics skill was a stumbling block to physics whereas their instructors disagree. The students did indicate issues with using formulas and interchanging symbols for numbers indicating that "it was hard to keep the various expressions and formulas apart especially since some of the same symbols appear in different contexts (such as W for work and W for watt)" (p. 693). According to the researchers, "It seems that it is the "translation" from the physical situation to a mathematical expression that causes trouble" (p. 692).

Kapucu, Ocal, and Simsek (2016) examining students' beliefs about the relationship between mathematics and physics found that students responded that success in physics was directly related to mathematics. The students felt that physics was more challenging than mathematics. Kapucu, Ocal, and Simsek indicated that students "could not easily construct the

relationships among what they learned about physics when comparing to that of mathematics" (p. 270). The researchers suggested further research into students' conceptions of the dependence of physics on mathematics and if that belief hinders the students' attitudes and performance in mathematics.

Contrary to Kapucu, Ocal, and Simsek (2016) research, additional research on students views on the difficulty of physics found that students believed the lack of higher-level mathematics was not a reason for struggle and mathematics was not required for success (Ornek, Robinson, & Haugan, 2008; Prosser, Walker, & Millar, 1996). This lack of belief in the need for mathematics agreed with Guttersrud and Angell (2010)research that investigated students' ability to describe physical phenomena through graphing. The researchers found high school students perceived graphing was not an issue in their struggles to describe physical phenomena. However, the students struggled with finding a mathematical expression that fit the data that they were graphing. The researchers concluded that students possibly do not realize the skills needed to be successful in physics or do not realize their skill level in applying the mathematics to physics.

Algebra Skills Needed for Physics. As the previous literary research has indicated, mathematics skills are not only necessary for success in mathematics but can cause struggles for students. The question then arises as to what skills instructors perceive as being necessary to be successful in physics. With limited researcher on instructors' perceptions of mathematics skill necessary for success in physics, it can be determined that several strands are commonly required. First, Gill (1999) found that the primary standard skills needed for physics included algebraic manipulation and graphing along with the ability to apply it to context. Next, Champagne, Klopfer, and Anderson (1980) examining factors that influence students' success in physics found that the following skills were necessary for physics: conversion from scientific notation, congruent triangles, conversion from one unit to another, proportional analysis, writing equations, and analysis of graphs. In addition, Delialioğlu, and AŞKAR (1999) examining the mathematical skill need to be successful in physics created a test on mathematical skills that contained content

the researchers found necessary for success in physics. The test evaluated the students' mathematical skills in the following areas: algebraic expression, ratios, geometric properties, formulas, equations, graphs, functions, and trigonometry. Finally, Hudson and McIntire (1977) examined the success in physics based on the students' mathematics skills including linear equations, quadratic equations, graphing and trigonometric functions.

With a common skill set needed for physics of graphing, it is important to examine the skill set in graphing that teachers find important for success in physics. Graphing, "a powerful tool for depicting data and as an effective communicating tool," is a skill necessary for physics success that is widely researched in both mathematics and physics (Kekule, 2008, p.1). Brasell and Rowe (1993) when investigating graphing found that students along with not enjoying working with graphs, "do not understand the fundamental properties and functions of graphs in representing relationships among variables" (p. 69). Kekule (2008) also interested in graphing skills of students in physics found that students struggled with the interpretations of graphs qualitatively. In addition, Planinic, Ivanjek, Susac, and Milin-Sipus (2013) found that students have difficulty in graphing concepts such as slope and area under the curve that increases with the application to other contexts such as physics or business.

It is important for researchers when examining the issues students have in physics to investigate the mathematical skills these students need in order to be successful in physics. Understanding the skills needed such as graphing can help researchers further investigate how these skills tie in with students and instructors' perceptions and students' student success in physics as demonstrated by Guttersrud and Angell (2010) research where students did not see graphing as a block to success to physics.

Mathematics Language and Symbolic Interpretations

Researchers have proposed that some of the student struggles in the transfer of mathematics knowledge to physics could possibly be struggling to translate the symbolism used in mathematics to the symbolism used in physics leading to a language translation issue. These

two topics, symbolism and language translation, though related have not thoroughly been investigated together. The discussion that follows will show the research done to date on the two concepts of symbolism and mathematics versus physics language.

Symbolism. The use of symbols in mathematics and science is extensive. Symbols are used not only to describe the operation of the mathematical equation but also are used to denote variables. Students' success in understanding the symbols used in both mathematics and physics is vital to success in both areas. This understanding requires that students are able to understand the different meanings of the symbols depending on the context in which the symbols are used. The students in Angell, Guttersrud, Henriksen, and Isnes, (2004) study indicated exactly this issue, "it was hard to keep the various expressions and formulas apart especially since some of the same symbols appear in different contexts (such as W for work and W for watt)" (p. 693). Realizing that the interpretation of the symbols meaning depending on the context can cause students to struggle, researchers have begun to examine such issues although limited (Clement, Lochhead, & Monk, 1981; Pietrocola, 2008; Torigoe, 2015; Torigoe & Gladding, 2007; Torigoe & Gladding, 2011).

To begin, Clement, Lochhead, and Monk (1981) finding that university students enrolled in engineering were experiencing difficulties in applying mathematics to their engineering courses, developed a series of short problems that they requested the students write an equation. The researchers found that the students are unable to solve a simple algebra problem that consisted of mainly variables indicating an issue of "translating into and out of algebraic notation" (p. 287). The students' issues consisted of reversing the variables in the following problems.

> Write an equation using the variables C and S to represent the following statement: At Mindy's restaurant, for every four people who ordered cheesecake, there were five who ordered strudel. Let C represent the number of cheesecakes ordered and let S represent the number of strudels ordered. (p. 287)

Out of the 497 engineering students who were assessed, only 39 passed. The unsuccessful students were unable to translate the words into mathematical equations showing possible areas for error when students are trying to translate between mathematics and physics.

Exploring the difference in students' solutions to problems that offer numerical values or symbolic variables, Torigoe and Gladding (2007) analyzed students' results on a mechanics exam. Students were given one of two problems. The first problem offers numeric values for the variables whereas the second problem offers only variables and requests the answer in terms of symbols. The researchers found no significance between the problems. However, students seem to have difficulty using the correct symbols in the symbolic problems. The researchers concluded that the students did not see the problems as identical due to the difference between numbers and symbols.

Torigoe (2015) and Torigoe and Gladding (2011) building on their previous research argued that the symbols used in mathematic equations for physics, which represent mathematic expression for certain physics systems confused students. For example, the students during their interviews found the following equation to be troublesome, $v_f = v_o + at$. First, students encountered difficulties in determining which system the equation represents. Second, the students were unable to determine which of the symbols used in the equation are experimental. Torigoe (2015) argued that the skills the students used in mathematics to solve numerical equations were different from the skills needed to solve symbolic equations. Torigoe (2015) suggested that the general symbolic equation was an important tool to help students understand since it allowed for understanding of the physical model and not just one instant.

Language. Students struggles in physics include not only the understanding of the symbols used but also the translation between the two languages of mathematics and physics (Pietrocola, 2008; Redish, 2005; Redish & Gupta; 2009; Redish & Kuo, 2015). In researching students competency in describing physical phenomena, Guttersrud, and Angell (2010) concluded

their research with a general statement on language issues between physics and mathematics, "We believe that emphasizing the connections between mathematics and physics by focusing on acquaintances between the general mathematical expressions $y = ax + b$ and $y = x^2 + bx + c$ on one hand and physics equations like $F = ma$ and $s = v_o x^2 + \frac{1}{2}x + c$ on the other is crucial" (p. 5). Guttersrud, & Angell also suggested the vocabulary of mathematics and physics, which is the equations, is difficult for students and more time in the translation of mathematics into physics could help improve students understanding of physics.

Redish (2005) opened his paper with the argument, "It almost seems that the "language" of mathematics we use in physics is not the same as the one taught by mathematician" (p. 1). Redish argued that physicists use the symbolic version of an equation until the end of a calculation where numerical values are then entered. This process was not the same as with students who work with numerical values from the start. Redish argued that students do not understand the symbolic mathematics as it describes the physical system; they only understand the procedure methods they acquired through mathematics courses. Redish also explained that in physics the goal of mathematics is not just to solve equations but to describe a physical system.

The success of students in physics class according to Redish went beyond the understanding of the mathematical language of physics but also included the student's attitude toward mathematics in physics. Redish explained, "Student expectations also play a powerful role in how they think they are supposed to use math in the physics (or science) classes" (p. 8). The students entered physics with the attitude that mathematics procedures will resemble problems seen in mathematics courses that are equivalent to plug-and-chug problems, which again did not meet the goals of mathematics in physics. Redish recommended that students need more than algorithmic approaches to mathematics to adequately apply the mathematics language to physics.

Redish and Gupta (2009) suggested that approaching the translation of mathematics into physics may require a cognitive semantic approach. A cognitive semantic approach can be examined at four levels: encyclopedic knowledge, conceptualization, embodied cognition and conceptual grounding. Here the researchers suggested that students' understanding of equations may be confusing based on their level of translation listed above. For example, F=ma and a=F/m should have two different meanings for students indicating an embodied cognition level. The first equation is a definition of force whereas the second equation explains acceleration in terms of force. This creates a different meaning for the equal sign. When observing two students solving physics equations, the researchers noted that the student with a higher level of cognitive semantic showed greater ability to access mathematics skills and relate the mathematics to physics and solve the problem.

Redish and Kuo (2015) continued with the same arguments by explaining the difference between the mathematics used in physics and the mathematics learned by the students. The authors explained "…while related, the languages of 'math in math' and 'math in physics' may need to be considered as separate languages" (p. 563). One of the main differences in the two languages was how meanings were attached to symbols. For example, physicists add units to symbols whereas mathematicians look at numerical values. Therefore, the authors found that "mathematicians teaching math classes focus on the mathematical grammar of an equation, ignoring possible physical meaning" (p. 566). The authors also spent time explaining cognitive semantics as covered in the previous article by Redish and Gupta (2009). The researchers argued that it is imperative that the students learn to interpret the physical meaning of the mathematical symbols and equations and not just the manipulation of the equation. The researchers concluded their argument by stating,

> Learning math in math class and that in physic class should be treated as learning two related but distinct languages: Although there is significant overlap, there are also important differences, and expertise in one does not guarantee expertise in another.

> Although both physics and math make meaning in the same way that any language does--by building on physical experience, by drawing on broad (encyclopedia) knowledge, and by contextualizing--they do so in different ways. (pp. 587-588)

The authors presented their argument for mathematics and physics as different languages based on symbolic meanings and cognitive semantics.

Summary

Reviewing the research literature related to the transferability of mathematics skills and knowledge to the study of physics, revealed gaps in the literature that this study will partially address. Researchers Redish and Kuo (2015) along with Pietrocola (2008) suggest further investigation of mathematics transfer to physics. The researchers call for offering students a translation between two very different languages of mathematics and physics. Also, Planinic, Ivanjek, and Susac (2013) investigating the transfer of mathematics to physics using isomorphic problems found that student issues with graphing "was not their lack of mathematical knowledge, but rather their lack of ability to interpret the meaning of the line graph slope in physics context" (p. 2). This line of possible research although not new has not been thoroughly investigated and by studying students' performance on parallel isomorphic college algebra and physics problems will provide possible data into students' transferability of mathematics language to physics. This study can then provide instructors with information to aid the students on make transfer between the two contexts and add to the literature on mathematics transferability and mathematics language used in college algebra compared to algebra-based physics.

Additionally, there is little research that investigates instructors of mathematics and physics perceptions of students' transferability and skill of mathematics knowledge to physics. Researchers investigating instructors and students' perceptions of mathematics use in physics, however, have theorized from responses to their studies a possibility of translation issues between mathematics and physics (Angell, Guttersrud, Henriksen, & Isnes, 2004; Frykholm, & Glasson, 2005; Guttersrud, & Angell, 2010; Kapucu, Ocal, & Simsek, 2016; Turşucu, Spandaw, Flipse, &

de Vries, 2017). This study can provide the instructors of both mathematics and physics insight into the similarities and differences between their perceptions along with adding to the literature on instructors' perceptions of mathematics transferability to physics studies.

Finally, researchers have examined students' issues in physics determining that one major factor influencing student success lies in the students' mathematics skills. Data indicates that the probability of a student's success in physics is highly correlated to the student's mathematical skill set, and students who do not possess the mathematics skill set necessary for mathematics application are less likely to be successful in physics than those students who demonstrate proficiency (Champagne, Klopfer, & Anderson, 1980; Hudson & McIntire, 1977), but not all data indicates mathematical skill set is the entire issue and some students may have proficient mathematics skills but struggle to apply that skill set to physics (Hudson & Liberman, 1982; Hudson & McIntire, 1977; Rebello, Cui, Bennet, Zollman, & Ozimek, 2007). The research on the students' mathematics skills needed to be successful in algebra-based physics is limited. This study can provide instructors of both physics and mathematics information on a strand of skills on which to focus student learning for success in studying physics in addition to adding to the literature on the mathematical skill needed for success in physics.

This review begins by offering the reader background into knowledge transfer and the how knowledge transfer can be used to examine students transfer of mathematical knowledge into all areas including physics. Further, by examining the problems solving process of students, researchers can further explore students' struggles with knowledge transfer. It is important to note that both the researchers in physics and mathematics model problems solving in similar manners allowing for a common link between the two. The paper then examines how both instructors and students perceive the use of mathematics in physics in order to observe any differences that may lead to mathematical issues in physics. Finally, by examining symbolic and possible language issues between mathematics and physics, a possible issue with translation

between mathematics and physics can be examined as a possible area of struggle for students in physics.

CHAPTER III

COLLEGE INSTRUCTORS' PERCEPTIONS OF TRANSFERABILITY OF ALGEBRA LANGUAGE AND SKILLS TO STUDIES IN PHYSICS

Target Journals: A. Science and Education

B. School Science and Math

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Abstract:

This mixed methods study explored college instructors perceptions of students' algebra skills, isomorphic problems and algebra language and the transferability of each to algebra-based physics. Thirty-one college instructors of college algebra and algebra-based physics participated in a survey with eight of the instructors participating in a semi-structured interview. A Kruskal-Wallis test of the closed-ended questions reveled a non-significant difference between the instructors perceptions. Individual instructor responses to the questions indicated that students need an understanding of linear, quadratic, and graphing concepts along the ability to manipulate variable equations to be successful in algebra-based physics. Instructors' responses also indicated that students may potential struggle with differences in use of vocabulary and variables between algebra and physics. Both mathematics and physics instructors can aid students in making connections between algebra and physics through application problems, discussions of variables, and connections between standard mathematics equations to physics equations.

Keywords: algebra skills, college instructors, instructor perceptions

Introduction

According to the National Academy of Sciences, National Academy of Engineering and Institute of Medicine (2010), "The United States ranks 27th among developed nations in proportion of college students receiving undergraduate degrees in science or engineering" (p. 8). Further, the President's Council of Advisors on Science and Technology (2011) reported that graduation rates in science, technology, engineering, and mathematics (STEM) areas needed to increase by 34% to meet the demand of the work force. Additionally, these studies pointed to the need to increase the enrollment and retention of students in STEM majors. Exploring enrollment and retention of students in STEM majors, researchers have indicated several areas that potentially affect retention: students' previous enrollment in high school STEM classes, success in college level gatekeeper STEM courses, and attitudes towards STEM (Maltese & Tai, 2011; Shaw & Barbuti, 2010). With physics considered one of the gatekeeper courses for students majoring in STEM fields, success in physics courses can potentially influence a student's decision to continue studying in a STEM field (Redmond-Sanogo, Angle, & Davis, 2016; Robinson, 2003).

Investigating potential hurdles to students' success in physics, researchers have found one common variable that remains constant, students' struggles with mathematics (Hansson, Hansson, Juter, & Redfors, 2015; Nguyen & Rebello, 2011; Pepper, Chasteen, Pollock, & Perkins, 2012; Winegardner, 1939). Interestingly research investigating students' struggles with mathematics in physics suggested that students' struggles are not always based in the students' mathematics skills but the application of those skills (Hudson & Liberman, 1982; Hudson & McIntire, 1977; Rebello, Cui, Bennet, Zollman, & Ozimek, 2007). Additionally, researchers have suggested that students in physics may also struggle with the translation of mathematics language from mathematics to physics (Pietrocola, 2008; Redish, 2005; Redish & Kuo, 2015).

Finally, when investigating students' struggles with the application of mathematics in the physics classroom, researchers should also examine instructors' perceptions. When specifically

investigating instructors' perceptions on mathematic use in physics, researcher have observed that instructors perceive a possible struggle with students' transferability of mathematics to physics (Angell, Guttersrud, Henriksen, & Isnes, 2004; Frykholm & Glasson, 2005; Ornek, Robinson, & Haugan, 2008; Turşucu, Spandaw, Flipse, & de Vries, 2017). With research on instructors' perceptions of the transferability of mathematics to physics limited to mainly pre-service and secondary instructors, expanding research on this topic to include perceptions of college instructors may offer researchers more insight into possible struggles of students with the transfer of mathematics to physics. This concept of translation struggles with mathematics language has led to the object of this study, to explore college instructors' perceptions of the transferability of mathematics language and skills to the study of physics.

Related Literature

Knowledge transfer can be used as a framework for investigating how students apply knowledge from one experience to another similar or new experience, such as applying mathematics knowledge to physics (Beach, 1999; McKeough, Lupart, & Marini, 2013; Perkins & Salomon, 1992; Singley, & Anderson, 1989). Knowledge transfer is a topic of interest for researchers of STEM since students' ability to transfer mathematics knowledge is important for the study of STEM courses such as chemistry, physics, and biology (Bassok, 1990; Hoban, Finlayson, & Nolan, 2013; Menis, 1987; Ngu & Yeung, 2012; Rebello, Cui, Bennett, Zollman, & Ozimek, 2007).

Knowledge Transfer

In the early 1900s, wanting to improve the learning process and expand existing knowledge on the science of learning, Thorndike, a psychologist, began researching the transfer of knowledge between subjects with dissimilar content. Examining the ability of individuals to transfer learned functions to a new situation, Thorndike along with his colleague Woodworth (1901) found that there was too large a difference between types of situations to observe any substantial transfer of knowledge. Thorndike theorized that transfer is applicable only between

activities that share common elements; these shared common elements are rarely identical making transfer of knowledge limited between activities (Singley & Anderson, 1989).

Continuing the research on knowledge transfer, researchers have identified three general types of potential knowledge transfer: classical knowledge transfer, process-based knowledge transfer, and sociocultural-based knowledge transfer. Classical knowledge transfer examines students' success in transferring knowledge from one problem to another similar problem (Bassok, 1990; Hoban, Finlayson & Nolan, 2013; Reed, Dempster, & Ettinger, 1985; Sloutsky, Kaminski, & Heckler, 2005). Process-based knowledge transfer examines the process, not necessarily the level of knowledge transferred, in hopes to find the mechanism of knowledge transfer (Bransford & Schwartz, 1999; Carraher & Schliemann, 2002; Rebello, Cui, Bennett, Zollman, & Ozimek, 2007). Lastly, sociocultural-based knowledge transfer examines the sociocultural influence on learning and knowledge transfer (Beach, 1999; Evans, 1999; Pea, 1987; Terwel, van Oers, van Dijk, & van den Eeden, 2009).

Specifically, when examining the knowledge transfer of mathematics to physics, researchers primarily utilized classical knowledge transfer. Using problems of similar content but in the context of mathematics versus physics (isomorphic problems), researchers documented students' transferability of mathematics to physics. These researchers found that graphing, linear equations, and quadratic equations are a few of the mathematical content areas that students find challenging when attempting to transfer their mathematics knowledge to physics (Bollen, De Cock, Zuza, Guisasola, & van Kampen, 2016; Guttersrud, & Angell, 2010; Planinic, Ivanjek, & Susac, 2013; Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012).

Further investigating students' transferability of mathematics knowledge to physics, Redish and Kuo (2015) along with Pietrocola (2008) suggested examining the transferability of mathematics language from mathematics to physics. Additional researchers found that students' struggles in physics include not only the transfer of mathematics language between mathematics and physics but also difficulties in the understanding the symbols (Pietrocola, 2008; Redish,

2005; Redish & Gupta; 2009; Redish & Kuo, 2015). Supporting research in the transferability of mathematics language, Guttersrud, and Angell (2010) stated, "We believe that emphasizing the connections between mathematics and physics by focusing on acquaintances between the general mathematical expressions $y = ax + b$ and $y = x^2 + bx + c$ on one hand and physics equations like $F = ma$ and $s = v_o x^2 + \frac{1}{2}x + c$ on the other is crucial" (p. 5). Guttersrud and Angell suggested that the vocabulary of mathematics and physics (the equations) is difficult for students. Further, they suggested more time needs to be spent supporting the students in the translation of mathematics into physics in order to improve students' understanding of physics.

When discussing the use of mathematics in physics, Redish (2005) argued, "it almost seems that the 'language' of mathematics we use in physics is not the same as the one taught by mathematicians" (p. 1). Redish pointed out that physicists use the symbolic version of an equation until the end of a calculation where numerical values are then entered. This process is not the same as with students who work with numerical values from the start. Additionally, Redish argued that students do not understand symbolic mathematics as it describes the physical system; they only understand the procedure they acquired through mathematics courses.

Redish and Kuo (2015) continued research into the transferability of mathematics language by examining the difference between the mathematics used in physics and the mathematics learned by students. They explained, "…while related, the languages of 'math in math' and 'math in physics' may need to be considered as separate languages" (p. 563). Redish and Kuo explained that one of the main differences in the two languages is how meanings are attached to symbols. For example, physicists add units to symbols whereas mathematicians look at numerical values. Therefore, the authors found that "mathematicians teaching math classes focus on the mathematical grammar of an equation, ignoring possible physical meaning" (p. 566). Redish and Kuo argued that it is imperative that the students learn to interpret the physical

meaning of the mathematical symbols and equations and not just the manipulation of the equation. They concluded their argument by stating,

> Learning math in math class and that in physics class should be treated as learning two related but distinct languages: Although there is significant overlap, there are also important differences, and expertise in one does not guarantee expertise in another. Although both physics and math make meaning in the same way that any language does--by building on physical experience, by drawing on broad (encyclopedia) knowledge, and by contextualizing--they do so in different ways. (pp. 587-588)

Further, Redish and Kuo suggested that an understanding of mathematics in both the context of mathematics and physics is necessary for a students' success in physics.

Perceptions and Skills of Mathematics Use in Physics

Based on Redish and Kuo's (2015) premise that there are separate languages to be learned in teaching mathematics and physics, it is also important to examine educators' perceptions of teaching and use of mathematics in the physics classroom including the required mathematical skills. Understanding these perceptions is one tool for identifying areas of concern in students' application of mathematics in the physics classroom (Archambault, Janosz, & Chouinard, 2012; Kim, Damewood, & Hodge, 2000).

When examining instructors' perceptions of mathematics used in physics, Angell, Guttersrud, Henriksen, & Isnes (2004) found that secondary physics instructors perceive mathematical application in physics to be the most problematic strand of physics for students. Secondary physics instructors also indicated that the second most important aspect of physics was being able to calculate problems using basic physics formulas. Additional research of physics instructors' perceptions found secondary physics instructors believed more real-life applications were needed to aid students in learning physics (Ornek, Robinson, & Haugan, 2008). Further, examining secondary pre-service teachers' understanding of the knowledge transfer between mathematics and science, Frykholm and Glasson (2005) found a connection between mathematics

and science to be important and intuitive for pre-service instructors. However, these pre-service instructors indicated that mathematics had always been taught as an independent subject and content was "typically fragmented, often in isolation from other topics that may have provided various contexts and/or connections" (p. 137). This isolation led to a feeling of incompetence that creates a barrier for these secondary pre-service instructors in creating a connection between the two subjects.

Turşucu, Spandaw, Flipse, and de Vries (2017) also surveyed and interviewed both mathematics and physics secondary instructors to examine issues related to knowledge transfer. The researchers found that mathematics instructors believed that there was a translation problem between mathematics and physics. The physics instructors in the study believing that students could be struggling with the transferability of mathematics to physics suggested activities that included more connections between the two subjects. Both groups of instructors perceived that students do not connect mathematics to physics. Interviews indicated that instructors' perception that the transfer of knowledge between the subjects happened automatically and can be improved only through intensive instruction in the mathematics course. Only a small group of instructors (mainly physics instructors) viewed the need for collaboration between mathematics and physics in order to aid transfer of knowledge. Overall, the mathematics instructors did not acknowledge a need collaborate to improve the transfer of mathematics.

As early as 1968 at a mathematics and physics Lausanne Symposium, teachers acknowledged the fact that "mathematics and physics have their own language and notations. To ensure that they are understood, teachers of both disciplines must explain how these languages connect" (p. 246). Further, they recommended that mathematics and physics curriculum should be a collaborative process and suggested that, "it is necessary to develop both the aptitude of pupils for identifying mathematical structures presented in situations encountered in physics (transfer of knowledge) and their skill in the use of key mathematical tools, particularly algebraic

calculations" (p. 245). Yet, decades later, researchers are still examining possible struggles with students' transferability of mathematics language to physics.

Minimal research has been conducted concerning instructors' perceptions of mathematics used in physics. The researchers investigating instructors' perceptions of mathematics use in physics, however, theorized from responses to their studies a possibility of translation struggles between mathematics and physics symbols and vocabulary. This gap in the research may offer possible insight into the struggles of students to transfer not only mathematics skill but also mathematical language to physics. To further the research into the transferability of mathematics skill and language to physics, this study was designed to investigate college algebra and algebra-based physics college instructors' perceptions of the transferability of mathematics language and skills to physics. The research questions guiding this study are: (a) How do instructors of college algebra and algebra-based physics perceive the transferability of algebra language and skills to the study of algebra-based physics courses? (b) Is there a significant difference between instructors of college algebra and instructors of algebra-based physics perceptions of transferability of algebra language and skills to physics?

Methodology

This study employed a mixed method design to analyze instructors' perceptions of transferability of mathematics skills and language to physics. Although the use of quantitative data offers data on instructors' perceptions, the use of qualitative data can further explain the instructors reasons behind their answers. The use of both quantitative and qualitative data allowed the researchers to gain a deeper insight into instructors' perceptions (Creswell, 2014).

Participants

Participants were solicited from instructors of college algebra and algebra-based physics courses in the 24 state colleges and universities in a Midwestern state. Thus, a census sampling was used to obtain participants. Full-time faculty members (n = 153) teaching college-algebra and algebra-based physics in the state during a spring semester were sent an email requesting

their participation in this study. Thirty-one instructors responded to the survey resulting in a response rate of approximately 20%. Of the respondents, 20 taught mathematics, seven physics, and four both mathematics and physics.

Eleven of the 20 mathematics instructors indicated that they were currently teaching at a two-year institution, eight at a regional university, and one instructor did not respond. The mean age of the mathematics instructors was 49 years old with a gender distribution of 65% female and 35% male. Of the seven physics instructors, one indicated that they were currently teaching at a two-year institution while five indicated teaching at a regional university and one at a research institution. The mean age for the physics instructors was 52 years old with a gender distribution of 43% female to 57% male. Four of the instructors surveyed indicated that they taught both mathematics and physics (combination instructors) with three indicating that they were currently teaching at a two-year institution and the fourth at a regional university. The mean age of the mathematics/physics instructors was 47 years old with a gender distribution of 75% female to 25% male.

Of the 31 instructors responding to the questionnaire, eight were contacted for semi-structured interviews based on their indication to participate and contact information offered in the questionnaire. Four of the eight interviews were with instructors of mathematics, two from a two-year college and two from a regional university. Two of the interviews were with instructors of physics, one from a regional university and one from a research university. Two of the instructors were combination instructors teaching both college algebra and algebra-based physics, one from a two-year college and the second from a regional university.

Measures

Data were collected from multiple sources including an instructor background survey, instructor perception survey, and semi-structured interview of instructors.

Instructor background questionnaire. In order to gain insight into the instructors' background, an instructor background questionnaire (See Appendix A) was used. The

questionnaire included questions about the instructors' age, gender, and race as well as their education attainment, years of experience, and whether they taught mathematics, physics or both.

Instructor perception questionnaire. The instructor perception questionnaire (See Appendix B) gathered instructors' (a) perceptions of students' mathematics skills, (b) perceptions of isomorphic parallel mathematics and physics problems, (c) and transferability of mathematics language to physics. The questionnaire contained both closed-ended and open-ended questions and concluded with a question asking the instructors about their willingness to volunteer for an interview.

The section of the survey dealing with instructors' perception of students included eight questions related to students' mathematics skills and 16 questions were related to transferability of mathematics to physics, both open-ended and closed-ended questions. The closed-ended questions used a forced-choice style on statements such as "Would you say college algebra courses do not prepare students for the mathematics needed in physics, college algebra courses moderately prepare students for the mathematics needed in physics, or college algebra courses do prepare students for the mathematics needed in physics." The forced-choice style was chosen to encourage respondents to more thoughtfully respond to the questions since the participants had to choose a response and not just agree or disagree (Lohr, 2010).

The final 23 questions related to instructors' perceptions of isomorphic parallel mathematics and physics problems. The mathematics and physics problems were isomorphic in that they were written to demonstrate similar content but different context. The section offered instructors a selection of isomorphic parallel mathematics and physics problems based on recommendations from Guttersrud and Angell's (2010) research along with other research on isomorphic problems (Bollen, De Cock, Zuza, Guisasola, & van Kampen, 2016; Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012). The problems for this study have been chosen from previous research (Bollen et al., 2016; Planinic et al., 2012) and students' textbooks (Aufmann, Barker, & Nation, 2008; Serway & Vuille, 2015). For each problem, instructors were asked

about the familiarity of the problems and whether or not they would utilize the problems in their own classrooms, if they considered the problems important for what they taught and to explain why the would/would not use the problem in their class.

Semi-structured interviews. Semi-structured interviews (see Appendix C) were conducted with eight instructors who volunteered. The semi-structured interviews provided additional data related to instructors' perceptions of the transfer of mathematics language to physics. The instructors were asked not only to further explain the skills needed for physics but also to reflect on possible situations in which students may encounter differences and similarities of mathematics languages between mathematics and physics courses. The interviews were approximately 50 minutes in length.

Data Analysis

Quantitative data were analyzed using descriptive and inferential statistics based on the respondent groups of mathematics, physics, and combination instructors. The data were further analyzed for significant differences among these groups. Due to the small and unequal sample size, the non-parametric Kruskal-Wallis test was utilized. This statistical analysis examined the difference in instructors' perceptions of skills, transferability of language, and isomorphic problems between instructors of mathematics, physics, and combination instructors.

Qualitative data from the instructor perception questionnaire were collected, organized, and analyzed. Responses to the open-ended questionnaire questions were transferred to an Excel sheet and descriptive coding was used to develop an initial set of codes (Creswell, 2007; Patton, 2015; Saldana, 2016). Interviews were transcribed, and an analytical memo was used in conjunction with descriptive coding for an initial set of codes. Finally, all data were combined for a second cycle of coding and examined using pattern coding for emergent themes covering all the data collection from open-ended questionnaires and interviews (Saldana, 2016).

Results

This study explored instructors of college algebra and algebra-based physics perceptions of students' skills, isomorphic problems, and transferability of algebra language. Data were analyzed to further study whether there was a significant difference between the instructors' perceptions.

Perception of Students' Algebra Skills

In order to examine the instructors overall perceptions of students' algebra skills, a total score was obtained across the six questions (see Table 3.1) and a Kruskal-Wallis analysis was conducted between mathematics, physics, and combination instructors indicated. Results revealed a non-significant difference between instructors and their perceptions of algebra skills ($\chi^2(2) = 1.06$, $p = .59$). These results suggest that mathematics, physics, and combination instructors in general have similar perception of the algebra skills needed for algebra-based physics.

Further, instructors were asked to list algebra skills they believed to be important for studies in physics, and the algebra weaknesses they perceived in their students. All instructors responses included: graphing, understanding linear and quadratic equations, and manipulation of equations. However, all four of the combination instructors included manipulation of algebra equations in their list indicating manipulation as an important skill set. When asked about possible student weaknesses in algebra, instructors indicated two major areas of concern: the ability of students to manipulate/solve variable equations and graphing.

Table 3.1

Descriptive Statistics for Instructors' Perceptions of Students' Algebra Skills

Questions	Math (n = 20) M (SD)	Physics (n = 7) M (SD)	Combo (n = 4) M (SD)
College algebra is (not important, moderately important, important, very important) for success in algebra-based physics.[a]	3.45 (0.60)	3.42 (1.13)	3.75 (0.50)
Application problems that involve physics concepts are (not important, moderately important, important, very important) for success in college algebra.[a]	2.45 (0.94)	2.57 (0.98)	1.75 (1.50)
Would you say college algebra (does not prepare, moderately prepares, prepares, highly prepares) students for algebra-based physics?[b]	2.80 (0.62)	3.00 (0.82)	3.00 (0.82)
College algebra (does not provide, moderately provides, provides, highly provides) the math skill students need for physics.[c]	2.80 (0.62)	2.71 (0.49)	3.25 (0.50)
My students' math skills for studying physics are (not adequate, moderately adequate, adequate, highly adequate).[d]	2.22 (0.65)	2.14 (0.69)	3.00 (0.82)
Importance (not important, somewhat important, important, very important)[a]			
Algebraic manipulation	3.65 (0.49)	4.00 (0.00)	4.00 (0.00)
Scientific notation conversions	3.35 (0.67)	3.43 (0.79)	3.50 (0.58)
Unit conversions	3.70 (0.57)	3.14 (1.07)	4.00 (0.00)
Writing equations	3.65 (0.49)	3.86 (0.38)	3.00 (1.41)
Linear equations	3.25 (0.55)	3.86 (0.38)	3.00 (0.82)
Linear functions	3.30 (0.57)	2.57 (0.79)	2.57 (0.79)
Quadratic equations	3.45 (0.51)	3.29 (0.76)	3.00 (0.82)
Quadratic functions	3.55 (0.51)	2.57 (0.98)	2.75 (0.96)
Graphing linear equations	3.35 (0.75)	3.29 (0.76)	3.00 (0.82)
Graphing quadratic equations	3.40 (0.58)	2.86 (1.07)	2.50 (1.29)
Interpreting graphs	3.89 (0.32)	3.71 (0.49)	3.50 (1.00)

Note. Combo refers to instructors that have taught both college algebra and algebra-based physics. Scale of the Likert questions is 1-4 with [a]not important = 1, [b]does not prepare = 1,[c]does not provide = 1, [d]not adequate = 1.

Perceptions of Isomorphic Problems

Instructors' responses to the isomorphic problems were not uncommon; mathematics instructors indicated higher means for problems presented in mathematics context compared to

those in the physics while physics instructors responded with higher means to problems in the physics context (see Table 3.2). It is interesting to note that combination instructors found all the questions to be familiar unlike the instructors of either mathematics or physics. Instructors' responses to closed-ended questions concerning the perception of isomorphic problems were analyzed using descriptive statistics and a Kruskal-Wallis test was used to determine any significant differences among the instructors. The isomorphic problems in both the context of mathematics and physics examined three areas of content: linear equation, quadratic equations, and graphing. An analysis was run on the responses to all the questions on the isomorphic problems and no significant difference was found between the mathematics, physics, and combination instructors' perceptions of isomorphic problems ($\chi^2(2) = .68$, $p = .71$). The analysis indicated that the three groups of instructors have similar perceptions of the isomorphic problems. Even though the overall analysis did not reveal a significant difference, the various content areas were indicated as critical algebra skills by the instructors, thus, analysis on individual content areas were examined as well.

Linear problems. First, a statistical analysis was run on instructors' responses to each set (i.e. physics and mathematics context) of the isomorphic linear problems using a Kruskal-Wallis test. Results indicated a non-significant difference for linear equations in the mathematics context while there was a statistically significant difference for the linear equations in a physics context among the between the mathematics, physics, and combination instructors ($\chi^2(2) = 6.34$, $p = .04$). Thus, a Mann-Whitney post hoc analysis was run revealing statistical significance with a small difference between mathematics and physics instructors ($U = 31.00$, $p = .03$, $r = .16$). Results to open-ended questions revealed that the mathematics instructors indicated that they perceived the linear equation to have too much physics context to be used in the mathematics class.

Quadratic problems. Next, analysis of instructors' responses about the quadratic questions in a mathematics context was not statistically significant. However, a statistical

difference was found among instructors responses to the quadratic equations in a physics context ($\chi^2(2) = 6.18$, $p = .03$). Further, post hoc analysis indicated a statistical significance, although a small difference, between mathematics and physics instructors ($U = 31.00$, $p = .03$, $r = .16$). An examination of instructors' open-ended responses further explained the difference with mathematics instructors commenting that the problem contained too much physics for a mathematics class. One instructor stated, "While it could be included as an application of a quadratic equation, in college algebra we are teaching the mechanics of solving quadratics. This problem has a lot more they would need to understand." A similar thought was echoed by another mathematics instructor, who responded

"the students would have to create the function themselves, and I find that creates issues."

Graphing problems. The final two isomorphic questions dealt with linear graphing equations. Analysis of the instructor responses about linear graphing in a mathematics context revealed a statistically significant difference among instructors' responses ($\chi^2(2) = 10.94$, $p = .004$). A further Mann-Whitney post hoc analysis was run indicating statistical significance, although small difference, between mathematics and physics instructors ($U = 25.00$, $p = .01$, $r = .18$). While there was a significant difference between the physics and combination instructors ($U = 2.00$, $p = .02$, $r = .07$), analysis revealed no practical significance. Additionally, mathematics instructors indicated in the open-ended responses that they would use the linear graphing problem in the mathematics context in college algebra because it was a basic problem for the content required. However, physics instructors indicated that they would only use the problem if it had more physics context. Statistical analysis of the isomorphic linear graphing problem in the physics context revealed a non-significant difference between the three groups of instructors indicating that the three groups of instructors did not perceive the graphing in the physics context differently.

Table 3.2

Descriptive Statistics for Instructor's Perceptions on Isomorphic Problems

Question	Math (n = 20) M (SD)	Physics (n = 7) M (SD)	Combo (n = 4) M (SD)	χ^2	p
Linear (math context)	5.60 (0.88)	5.17 (1.21)	5.50 (1.00)	1.34	.51
Do you consider this problem important for your course?	1.85 (0.37)	1.71 (0.49)	1.75 (0.50)		
Is this a familiar problem?	1.90 (0.31)	1.86 (0.39)	2.00 (0.00)		
Would you use this problem in your class?	1.85 (0.37)	1.57 (0.54)	1.75 (0.50)		
Linear (physics context)	4.55 (1.19)	5.71 (0.48)	5.50 (1.00)	6.34	.04
Do you consider this problem important for your course?	1.45 (0.51)	1.86 (0.38)	1.75 (0.50)		
Is this a familiar problem?	1.70 (0.47)	1.86 (0.38)	2.00 (0.00)		
Would you use this problem in your class?	1.40 (0.50)	1.86 (0.38)	1.75 (0.50)		
Quadratic (math context)	5.90 (0.31)	5.29 (1.11)	5.75 (0.50)	3.89	.14
Do you consider this problem important for your course?	2.00 (0.00)	1.86 (0.38)	2.00 (0.00)		
Is this a familiar problem?	1.95 (0.22)	1.86 (0.38)	2.00 (0.00)		
Would you use this problem in your class?	1.95 (0.22)	1.57 (0.54)	1.50 (0.58)		
Quadratic (physics context)	4.20 (1.01)	5.29 (1.11)	5..25 (0.96)	6.18	.03
Do you consider this problem important for your course?	1.35 (0.49)	1.86 (0.38)	1.75 (0.50)		
Is this a familiar problem?	1.55 (0.51)	1.86 (0.38)	2.00 (0.00)		
Would you use this problem in your class?	1.30 (0.47)	1.57 (0.54)	1.50 (0.58)		
Graph (math context)	5.70 (0.57)	4.71 (0.95)	6.00 (0.00)	10.94	.004
Do you consider this problem important for your course?	1.85 (0.37)	1.86 (0.38)	2.00 (0.00)		
Is this a familiar problem?	2.00 (0.00)	1.74 (0.49)	2.00 (0.00)		
Would you use this problem in your class?	1.85 (0.37)	1.14 (0.38)	2.00 (0.00)		
Graph (physics context)	5.20 (1.05)	5.57 (1.13)	5.75(0.50)	1.91	.39
Do you consider this problem important for your course?	1.70 (0.47)	1.86 (0.38)	2.00 (0.00)		
Is this a familiar problem?	1.80 (0.41)	1.86 (0.38)	2.00 (0.00)		
Would you use this problem in your class?	1.70 (0.47)	1.57 (0.54)	1.50 (0.58)		

Note. Combo refers to instructors that have taught both college algebra and algebra-based physics. The scale for the questions was 1-2 with no = a and yes = 2.

Transferability of Algebra Language to Physics

In order to examine the instructors' overall perceptions of the transferability of algebra language to physics, a total score was obtained for the seven questions (see Table 3.3) and a Kruskal-Wallis analysis was conducted between mathematics, physics, and combination instructors. Results revealed a non-significant difference between instructors' perceptions of transferability of algebra language ($\chi^2(2) = .3.08$, $p = .21$). These results suggest that mathematics, physics, and combination instructors in general have similar perceptions of the transferability of algebra language to algebra-based physics.

Further exploring the transferability of mathematics language from mathematics class to physics, the instructors were asked to explain whether they perceived their students could recognize several equations used in physics compared to the basic equations used in algebra. Starting with the equation, $F = ma$, instructors were asked if they perceived their students would be able to recognize the equation as linear. Mathematics instructors noted that their concern with students' recognition of the equation being linear was because of the number of variables (i.e. *F, m, and a*) in the equation. For example, one mathematics instructor stated, "They would see this function as having three variables (letters) and linear equations as having two--x and y--so this cannot possibly be a linear function." Similar concern was indicted by a second mathematics instructor stating, "They are taught that linear equations have an *x* to the first power. Rarely do we emphasize that there could be more than one variable in a linear equation, and rarely do we use other than *x*. Teaching the basics of linear equations, we often do not do application problems." However, not all the mathematics instructors perceived students to have issues recognizing the equation as linear since the lack of exponents would signal to the students that the equation was linear. Physics instructors along with combinations instructors responded similarly explaining that the equation was simple and when graphed the student would recognize that it was linear.

Continuing with linear equations, the instructors were then asked if they perceived their students would be able to recognize $v_f = v_o + at$ as a linear equation. Several mathematics and combination instructors indicated in their written responses that the believed that the equation more closely followed the standard equation for a line, $y = mx + b$, helping students make the connection. However, not all of the mathematics instructors perceived students responses similarly and had concerns that the use of any variables other than *x, y,* and *z* would confuse the students along with the use of subscripts. The physics instructors' written responses also expressed concern for the students' lack of recognition stating that students would be confused with subscripts such as v_o .

Finally, in support of the data indicating no significant difference in instructors' perceptions, the instructors felt their students would be able to recognize $s_f = v_o t + \frac{1}{2}at^2 + s_o$ as a quadratic equation. They indicated that the square term would be an indicator to the students that the equation was quadratic.

In summary, the concerns the three groups of instructors had with students' recognition of physics equations compared to the standard mathematical form of the equation were the use of variables. Instructors were concerned that students would not be able to translate the physics equation into the standard mathematical forms due to the use of variables other than *x, y,* and *z* along with the use of subscripts on the variables.

Table 3.3

Descriptive Statistics for Instructors' Perceptions on Transferability of Algebra Language to Physics

Question	Math (n = 20) $M(SD)$	Physics (n = 7) $M(SD)$	Combo* (n = 4) $M(SD)$
The math language used in college algebra (does not resemble, slightly resembles, resembles, definitely resembles) the math language used in physics.[a]	2.55 (0.51)	2.43 (0.79)	3.00 (0.82)
The math vocabulary used in college algebra (does not resemble, slightly resembles, resembles, definitely resembles) the math vocabulary used in physics.[a]	2.58 (0.61)	2.43 (0.79)	2.75 (0.50)
The math symbols used in college algebra courses (does not resemble, slightly resembles, resembles, definitely resembles) the math symbols used in physics.[a]	2.40 (0.75)	2.71 (0.49)	2.75 (0.50)
Quadratic equations in college algebra (does not resemble, slightly resembles, resembles, definitely resembles) the quadratic equations used in physics.[a]	2.65 (0.75)	2.57 (0.79)	3.25 (0.96)
My students are able to recognize that $F = ma$ is a linear equation. (never, seldom, some of the time, most of the time).[b]	2.55 (0.83)	2.71 (0.76)	3.25 (0.50)
My students are able to recognize that $v_f = v_o + at$ is a linear equation. (never, seldom, some of the time, most of the time).[b]	2.68 (0.89)	2.71 (0.95)	3.00 (0.82)
My students are able to recognize that $s_f = v_o t + \frac{1}{2}at^2 + s_o$ is a quadratic equation? (never, seldom, some of the time, most of the time).[b]	3.00 (0.88)	2.86 (0.90)	3.25 (0.96)

Note. Combo refers to instructors that have taught both college algebra and algebra-based physics. Scale of the Likert questions is 1-4 with [a]does not resemble = 1 and [b]never = 1.

In a further exploration of the instructors' perceptions in the transferability of mathematics language to algebra-based physics, the instructors were asked to describe three conditions: connections, difficulties, and challenges that students may encounter between algebra language used in mathematics and the algebra language used in algebra-based physics. When

describing perceived connections students would make from algebra language to algebra-based physics, instructors expressed many similarities between the two topics such as terms of slope, rate of change, linear, quadratic, etc. One mathematics instructor's further shared surprise at the, "implication that the 'algebra language' is different between the two courses? That seems unlikely." However, not all instructors had similar perceptions about similarity and some of the mathematics and physics instructors expressed that the "forms of the equations used and the methods of solving them are the same. Physics just has different symbols and deals with real problems. All problems involving math in real life are word problems, whether in physics or any other area."

When reflecting on the difference students may encounter between algebra language in mathematics and the algebra language used in algebra-based physics, instructors mirrored some of the differences expressed in the responses for connections perceiving two major difficulties: variables and application . One mathematics instructor explained,

> The biggest difference is going to be in the notation. Subscripts are used frequently in physics and rarely in college algebra. It is also more common to use t (time) as the independent variable in physics whereas college algebra that may happen 1 out of 20 or more problems/examples. There are also differences in convention that show up between the two (for example which is the standard order for the terms of a quadratic).

Other instructors expressed concern over the "total amount of word problems" expressing that in college algebra the students were given "about two weeks of word problems" where in physics the students would "spend the entire semester working nothing but word problems."

Lastly, when the instructors described the challenges they perceived students encountered in the transferring of college algebra to algebra-based physics, the instructors continued to echo concerns over variable usage and lack of application. One of the mathematics instructors shared thoughts on challenges with variables and vocabulary:

> Most likely going from the basic format of a linear or quadratic equation to equations that represent each of those - but with very different letters and in different orders…Instead of determining the 'vertex' for example, they may be asked to find the maximum height of a ball. So, questions aren't quite as straight forward in what students are supposed to find.

While a combination instructor expressed concern that "topics presented in physics are not introduced in college algebra. Instructors often skip the word problems and focus on the techniques used to solve problems instead of the application of techniques."

To help summarize instructors perceptions on the transferability of algebra language to physics, the instructors were asked to explain whether they believed there was a difference in the language between algebra and algebra-based physics. The instructors continued to express that variables and the lack of application problem were the largest differences between the transfer of algebra language to physics. Continuing to share perceptions that students struggles with variables, a mathematics instructor shared in an interview,

> There is a lot of communication between the languages of math and physics that are barriers. The students will say, "Well that's not how we write that or that's not the variable that we use." It's usually the variables…or they will say it even in a different way. Maybe I would say 'v sub naught' and they might say 'v subzero.' So, there's even those little things that could be a problem when it comes to working the questions.

Many instructors continued to express a difference in vocabulary or variables between the two subjects with another mathematics instructor indicating on the survey, "I can see that once you get away from discussing *x*'s and *y*'s to discussing actual situations like speed, distance, time, height, etc., it would take some work to help students to see the connections." The instructors also continued to reflect that students "do not do enough application in college algebra." Further expressing concerns with the lack of application, a mathematics instructor stated that students, "see the slope of the line is 5/8," but they "probably don't do enough word problems, application problems, where the slope has meaning to it." Sharing similar thought, a physics instructor

commented; “I think that's helpful in being able to take those skills and apply them to something you would actually have to use. I don't think it's a just a college algebra issue, I think it's a disconnect from math to physics.” Contrarily, combination instructors expressed minimal difference in the languages between algebra and algebra-based physics. In general, the combination instructors felt that there was very little difference in the languages since “math is the language of physics.”

Although mathematics instructors perceived the possibility of differences between the algebra language used in mathematics class and the algebra language in algebra-based physics, a theme present in both the survey responses and interviews was a belief that the difference between the two could be overcome by the instructor offering a connection between the mathematics and physics for the students. A mathematics instructor explained in an interview,

> I can't imagine if I were teaching physics that I wouldn't take that opportunity to say now remember when you did $y = mx + b$ and this is a linear equation, and this was the rate of change in front of the x and this was the constant or the beginning value in the y and what not. I can't imagine that I wouldn't start there and then transition to the linear function with the physics variables in it.

This thought was also evident in combination instructors responses such as during an interview with an instructor of both mathematics and physics:

> If I were to give them $4x^2 - 2x + 7 = 3$ to solve, I don't have a worry that all of them can do that because it's the standard *x* and *y* that they see in their math class all the time. But if I were to use *t* and *h* or something, there would be a pause, ‘Okay how do we do this?’ That’s my job as a college instructor to say, ‘technically you seen this all before let’s make the connection.’ So, if the teacher is not willing to take the time to create the neural pathways to connect between what the students already know and the new information…

Both mathematics and combination instructors perceived that any possible differences in mathematics languages between college algebra and algebra-based physics could be reduced by instruction including connections.

Discussion and Conclusion

In terms of the transferability of algebra language and skills to the study of algebra-based physics, instructors of mathematics, physics, and combination courses all seemed to have similar perceptions in the algebra skills needed to be successful in algebra-based physics. First, all three groups of instructors indicated that the algebra skills needed for a student to be successful include an understanding of linear concepts but more importantly quadratic concepts. Second, the instructors indicated that the students need to understand graphing concepts and be able to interpret graphs. Thus, this study supports previous research on algebra skills needed for physics (Bollen, De Cock, Zuza, Guisasola, & van Kampen, 2016; Guttersrud, & Angell, 2010; Planinic, Ivanjek, & Susac, 2013; Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012) which indicated that graphing concepts were necessary for the study of physics. Third, regardless of what they taught, instructors indicated a need for students to be able to manipulate and solve variable equations without numbers which supports Redish's (2005) research that indicated that manipulation of variable equations was important for the study of physics. Instructors perceive mathematics as an important skill needed for success in physics, but mathematics and physics instructors perceive that their students may not have those skills necessary to be successful. Conversely, combination instructors did not perceive their students' algebra skills as a concern, supporting research (Hudson & Liberman, 1982; Hudson & McIntire, 1977; Rebello, Cui, Bennet, Zollman, & Ozimek, 2007) that indicated that students' struggles with physics was not due to algebra skills.

When examining students' struggles with mathematics language transfer, instructors of both mathematics and physics indicated students struggle with the differences between vocabulary and variables in mathematics and physics along with application, supporting Redish

and Kuo (2015) finding that one of the main differences in the two languages is how meanings are attached to symbols along with meaning constructed during application. However, combination instructors did not consider language transfer to be an issue. In contrast to physics instructors, the instructors who taught either mathematics or both mathematics and physics tended to believe that it was the responsibility of the instructor to help students make connections between the way mathematics is talked about in the two classes. For example, they believe that physics instructors should explicitly make connections to linear equations in a mathematics context with the physic context where they are using linear equations. These findings are in line with Guttersrud and Angell (2010) research findings that suggest that instructors should aide students in making translation. Further investigation into the combination instructors' perceptions that their students have minimal struggles with transfer between mathematics and physics language should be further explored. Additional investigation of mathematics and combinations instructors' perceptions of the responsibility of the instructors to aid the students in translation between the languages should be explored to examine whether that translation instruction aids students.

Examining instructors' perception of skills, isomorphic problems, and transferability of language, data analysis indicated no statistical significance between the three groups of instructors' perceptions. However, differences in the means for individual questions between the three groups of instructors' perceptions such as skill level of their students, language transferability, and application to increase transferability were indicated suggesting that minimal differences exist in perceptions. Also, the differences in combination instructors' responses as compared to mathematics and physics instructors suggest that the combination instructors may have a slightly different perception of students' transfer of mathematics knowledge to physics. Further research into this difference of perception may offer more insight into how combination instructors offer algebra translation to students between mathematics and physics.

Further, implications from this study suggest that students would benefit from additional application problems in mathematics course to aid in transferring algebra skills to physics. Providing students with opportunities in mathematics class to make connections between linear and quadratic equations in mathematics to applications in physics could aid the students in transferring the mathematical knowledge to physics. Students would also benefit from additional application problems that provide connections for graphing to physics such as possible meanings of slope. Instructional connections between mathematics concepts and physics concepts in the physics classroom can also aid students in making connects between general mathematics equations and physics equations. Overall, instructors of both mathematics and physics can provide students with connections between mathematics and physics by providing students with the translation of the symbols and vocabulary in mathematics to other subjects the student will be studying such as physics.

CHAPTER IV

TRANSFERABILITY OF ALGEBRA LANGUAGE AND SKILLS TO PHYSICS: STUDENTS' PERCEPTIONS OF MATHEMATICS ABILITY

Target Journals: A. School of Science and Mathematics Association

B. MathAMatyc

Authors: Kathleen Otto and Juliana Utley

Abstract:

This study examined college students' perceptions of the transferability of algebra skills and language to the study of algebra-based physics. The convergent parallel mixed-methods study examined the perceptions of seventeen students enrolled in college, algebra-based physics course at a Midwestern two-year college through the use of a questionnaire and a semi-structured interview of three students. Realizing that college algebra was important for success in studying algebra-based physics, students indicated that they struggled with the transferability of algebra skills and language to physics. The students indicated struggles with the formulas and variables used in physics. Further, written responses by the students indicated that they did not necessarily perceive struggles with their algebra skills but the application of those skill to physics. Instructors of mathematics and physics can aid students in transferring algebra knowledge to physics through application problems, discussions of variables, and connections of standard mathematics equations to physics equations.

Introduction

Success in physics, one of the gatekeeper courses for majors in STEM fields, can potentially influence students decisions to remain in a STEM major (Redmond-Sanogo, Angle, & Davis, 2016; Robinson, 2003; Shaw & Barbuti, 2010). Researching potential hurdles to students' success in physics, educators have concluded that mathematics is a common variable for physics success (Hansson, Hansson, Juter, & Redfors, 2015; Nguyen & Rebello, 2011; Pepper, Chasteen, Pollock, & Perkins, 2012; Winegardner, 1939). Further investigating struggles in mathematics, researchers have documented that students' perceptions (attitudes) affect the performance of students in mathematics (Lamb & Daniels, 1993; Rice, Barth, Guadagno, Smith, & McCallum, 2012; Siskandar, 2013; Wilkins and Ma, 2003). Additionally, researchers when observing students' perceptions of mathematics use in physics have observed translation struggles between mathematics and physics (Angell, Guttersrud, Henriksen, & Isnes, 2004; Guttersrud, & Angell, 2010; Kapucu, Ocal, & Simsek, 2016). The object of this study is to explore students' perceptions of the transferability of mathematics skills and language to physics.

Related Literature

Using a framework of knowledge transfer, which examines the application of knowledge from one experience to a similar or new experience, researchers can observe students' transfer of mathematical knowledge to physics (Beach, 1999; McKeough, Lupart, & Marini, 2013; Perkins & Salomon, 1992; Singley, & Anderson, 1989). While examining this knowledge transfer, it is also important to examine students' perceptions of mathematics use in physics to possibly gain further insight into students' struggles (Angell, Guttersrud, Henriksen, & Isnes, 2004; Guttersrud, & Angell, 2010; Kapucu, Ocal, & Simsek, 2016).

Knowledge transfer has interested researchers since the early 1900s, when a psychologist, Thorndike, began expanding the research of knowledge transfer. Thorndike and Woodworth (1901) began to examine the ability of individuals to transfer new skills from similar situation to new situations. Through their research, Thorndike and Woodworth (1901) found that when a

large difference existed between the two situations, transfer of knowledge was not observable. Thorndike suggested that only when activities share common elements is transfer applicable; since rarely are shared common elements similar, transfer of knowledge is limited (Singley & Anderson, 1989). Today researchers in STEM continue to examine knowledge transfer of mathematics to core STEM courses such as physics, chemistry, and biology since mathematics transfer is important for success in these courses (Beach, 1999; McKeough, Lupart, & Marini, 2013; Perkins & Salomon, 1992; Singley, & Anderson, 1989).

Through examination of students' perceptions of mathematics in physics, Angell, Guttersrud, Henriksen, and Isnes, (2004) found that students felt mathematics was needed to describe physics systems and felt physics was hard. The students, however, did not feel that mathematics skill was a stumbling block to physics whereas their instructors disagreed. The students indicated issues with using formulas and interchanging symbols for numbers "it was hard to keep the various expressions and formulas apart especially since some of the same symbols appear in different contexts (such as W for work and W for watt)" (Angell et al., 2004, p. 693). According to the researchers, "It seems that it is the "translation" from a physical situation to a mathematical expression that causes trouble" (Angell et al., 2004, p. 692).

Examining students' beliefs about the relationship between mathematics and physics, Kapucu, Ocal, and Simsek (2016) found that students believed that their success in physics was directly related to mathematics skills and felt that physics was more challenging than mathematics. Further, Kapucu, Ocal, and Simsek indicated that students "could not easily construct the relationships among what they learned about physics when comparing to that of mathematics" (p. 270). The researchers suggested further research into students' conceptions of the dependence of physics on mathematics and whether that belief hinders the students' attitudes and performance in mathematics.

Contrary to Kapucu, Ocal, and Simsek's (2016) research, additional research on students views on the difficulty of physics revealed that students believe the lack of higher-level mathematics was not a reason for struggle and that mathematics was not required for success in physics (Ornek, Robinson, & Haugan, 2008; Prosser, Walker, & Millar, 1996). This lack of belief in the need for mathematics was in line with Guttersrud and Angell (2010) findings that high school students perceived that graphing was not an issue in their struggles to describe physical phenomena. However, the students struggled with finding a mathematical expression that fit the data that they were graphing. The researchers concluded that students possibly do not realize the skills needed to be successful in physics or do not realize their skill level in applying the mathematics to physics.

With limited research on students' perceptions of knowledge transfer of mathematics to physics, a gap in research exists that could provide more insight into students' belief about their mathematics ability and possible struggles in transferring mathematics language to physics. The purpose of this study was to expand the research of algebra-based physics students' perceptions of the transferability of skills and language to algebra-based physics. The study was guided by the research question: How do students perceive the transferability of their algebra language and skills from mathematics courses to physics courses?

Methods

This study employed a convergent parallel mixed methods design to investigate students' enrolled in an algebra-based physics course, perception of the transferability of mathematics knowledge to physics. In order to provide a deeper examination, the use of both qualitative and quantitative data provided the researcher with deeper insight into students' perception of the transfer of mathematics knowledge to physics (Creswell, 2014).

Participants

Participants were selected using a single stage purposive sampling of students enrolled in a spring semester of algebra-based physics course in a rural Midwestern two-year college. The

sample consisted of seventeen students (35% male; 65% female) with a mean age of 22 years. The sample consisted of 6% Hispanic, 12% Native American, and 82% white. These students were required to have completed and passed college algebra with a grade of 'C' or better prior to enrolling in the course or obtained an ACT mathematics subset of 24 or higher. Three of the students (2 female and 1 male), Sam, Jorden, and Alec (names have been changed for anonymity), volunteered for a semi-structured interview. Two of the interviewed students were majoring in healthcare fields with one wanting to study occupational therapy and the other wanting to apply for medical school. The third student was studying in a technical field and was interested in working in refineries and gas plants. All three of the students had completed the mathematics requirements for the course with one of the students having previously taken calculus I.

Measures

The data were collected from multiple sources including a student demographic questionnaire, student perception questionnaire, and a semi-structured interview. The student demographic questionnaire (See Appendix A) collected responses on student demographics such as age, race, and major.

Student perception questionnaire. The student perception questionnaire (See Appendix B) gathered students' perceptions of mathematics skills and the transferability of mathematics to physics studies. The questionnaire contained both closed-ended and open-ended questions. During the laboratory period of the college algebra-based physics course, the survey was completed by the students on a voluntary basis. The questionnaire contained two sections of inquiry including a section on students' perceptions of their mathematic skills and a second section on the students' perceptions of the transferability of their algebra skills to physics.

The section of the questionnaire investigating the students' perception of their mathematics skills contained 11 questions, both open-ended and closed-ended. The closed-ended questions used a forced-choice style on statements such as "College algebra (a) did not provide

the math skills I needed for physics, (b) moderately provided the math skills I needed for physics, (c) provided the math skills I needed for physics, or (d) highly provided the math skills I needed for physics." The forced-choice style was chosen to encourage respondents to more thoughtfully respond to the questions since the participants had to choose a response and not just agree or disagree (Lohr, 2010).

The final 14 questions related to students' perceptions of the transferability of their mathematics skills to physics. The questions in this section examined students' perceptions on the transferability of language used in mathematics compared to the language used in physics such as, "The math vocabulary used in in college algebra (a) does not resemble the math vocabulary I used in physics, (b) slightly resembles the math vocabulary I used in physics, (c) resembles the math vocabulary I used in physics, or (d) definitely resembles the math vocabulary I used in physics." The questionnaire also examined the students' perception of the use of the following quadratic equations by asking the students if these questions were used in a math and/or physics setting and then to explain whether they were similar or different equations: $y = ax^2 + bx + c$ and $s_f = v_o t + \frac{1}{2}at^2 + s_o$.

Semi-structured interview. The semi-structured interviews (see Appendix C) were conducted with three students that volunteered. The semi-structured interviews provided additional data related to the students' perceptions of the transfer of mathematics language to physics. The students were asked not only to further explain their thoughts on their skills for physics but also to reflect on the similarities and differences between quadratic equations used in both mathematics and physics. The students were also asked to reflect on their perceptions of the mathematics language used in mathematics compared to physics courses.

Data Analysis

Quantitative data were analyzed using descriptive statistics to determine the frequency of student responses to the perception questions. Second, students' response to the open-ended

questions were analyzed by transferring their responses to Excel, and a descriptive coding technique was used to develop an initial set of codes (Creswell, 2007; Saldana, 2016). Third, the student interviews were transcribed, and an analytical memo was used in conjunction with descriptive coding. Finally, all data was combined for a second cycle of coding and examined using pattern coding for emergent themes covering all the data collection from open-ended questionnaires and interviews (Saldana, 2016).

Results

In order to explore how students perceive the transferability of algebra language and skills from their mathematics courses to physics courses, student responses to a survey and semi-structured interviews were examined. The following results are divided into students' perceptions of the importance of algebra skills and the transferability of algebra language to physics.

Importance of Algebra Skills

Descriptive analysis of students' responses to closed-ended questions concerning the importance of algebra skills (see Table 4.1) revealed that students felt that college algebra was important to the study of physics. However, students' responses to questions concerning algebra skill levels indicated they perceived that college algebra had not prepared them well for the study of physics. When asked to explain their mathematics level, students' written responses indicated the students perceived struggles not only with understanding the equations and the variables in the equation but also with the application of mathematics to physics. For example, one student commented, "I believe the confusion comes from not as many real world/word problems in my specific algebra courses. The hardest part for me is deciphering a question to find the variables I have and which variables I need to find." Similarly, another student commented, "My math skills for physics are slightly adequate because although I know the math, I do not know how to necessarily apply it to a specific problem." One student summed up their perceptions of their mathematics skills by stating that they did understand how to make connections to "the formulas and which to use."

Table 4.1

Descriptive Statistics for Students' Perceptions on Algebra Skills

Questions	*M*	*SD*
College algebra is (not important, moderately important, important, very important) for my success in algebra-based physics.[a]	3.59	0.51
Physics concepts are (not important, moderately important, important, very) important for my success in college algebra.[a]	2.24	0.83
Would you say college algebra (did not prepare, moderately prepared, prepared, highly prepared) me for physics.[b]	2.06	0.66
College algebra (did not provide, moderately provided, provided, highly provided) the math skills I needed for physics.[c]	2.29	0.69
I feel that my math skills for physics are (not adequate, moderately adequate, adequate, highly adequate).[d]	2.12	0.78

Note: Scale of the Likert questions is 1-4 with [a]not important = 1, [b]did not prepare = 1, [c]did not provide = 1, and [d]not adequate = 1.

To further explore students' perceptions of their mathematics skills, the students were asked to describe both their successes and struggles with mathematics in the physics classroom. Several of the students found that physics problems that used trigonometry offered them more success either because trigonometry was the most recent mathematics class taken or because they had a better understanding of the use of sine, cosine, and tangent. Additionally, several students indicated that once they understood what formula they needed, then they had no other frustrations with the mathematics. Overwhelmingly, nine of the seventeen students indicated that knowing what formula to use was their biggest mathematics struggle in physics. One student explained, "I do not understand how to use the formulas or what the symbols mean. I was not prepared for it at all." The students also suggested that application of algebra would have benefited them in the transfer of algebra skills to physics. For example, one student stated, "I believe not working on many word problems in algebra makes these problems in physics more confusing." These

students perceived being able to "take words and put them into equations" was the biggest mathematics struggle they had in physics.

Echoing similar thoughts, Jorden commented several times during an interview that the use of formulas and the variables caused struggles. Interestingly, Sam commented during an interview that "college algebra was really easy, though. It was really straight forward, not like physics….physics you have to manipulate things to find the answer, not like college algebra. College algebra you had the formula and you just plug them in there. Easy." Additionally, the interviewed students commented that knowledge of the quadratic formula helped them in solving projectile motion in physics.

Transferability of Algebra Language to Physics

To explore student perceptions of the transferability of algebra language to physics, descriptive statistics for the second part of the survey were analyzed (see Table 4.2). Students' responses to the questions covering mathematics language, vocabulary, and symbolism indicated that students perceived limited resemblance between mathematics and physics. When students were asked in which classes they had used the standard form of the quadratic equations, $y = ax^2 + bx + c$, 13 of the 17 students responded to having used the equation in mathematics class while four of the students indicated having used the equation in both mathematics and physics classrooms. When asked about the projectile motion equation, $s_f = v_o t + \frac{1}{2}at^2 + s_o$, 16 of the 17 students indicated they would have seen this equation in the physics classroom while only one indicated using it in both the mathematics and physics class.

When the students were asked whether the standard form of the quadratic equation and the projectile motion equation were similar or different and to explain, 12 of the students indicated that the equations were similar. Several of the students indicated that the equations were "generally just plug and chug equations" and that the two equations were "just different ways of showing a graphing equation." However, five of the students found no similarity

between the equations. The students stated that since the equations had different variables, then there was no similarity. When the same question was posed in interviews, only one student, Jorden, recognized the two quadratic equations as similar but the student was unable to explain why the equations were similar. Sam made the comment that only the standard form of a quadratic equation could be factored while the physics projectile motion equation could not.

Table 4.2

Descriptive Statistics for Students' Perceptions on Transferability of Mathematics Language

Questions	*M*	*SD*
Math language used in college algebra (does not resemble, moderately resembles, resembles, highly resembles) the math language I used in physics.[a]	1.88	0.60
The math vocabulary used in college algebra (does not resemble, moderately resembles, resembles, highly resembles) the math vocabulary we used in physics.[a]	1.82	0.53
Math symbols used in college algebra courses (does not resemble, moderately resembles, resembles, highly resembles) the math symbols we used in physics.[a]	2.18	0.64
The first equation $y = ax^2 + bx + c$ (we used in math class, we used in physics class, we used in both math and physics.[b]	1.47	0.53
The second equation $s_f = v_o t + \frac{1}{2}at^2 + s_o$ (we used in math class, we used in physics class, we used in both math and physics class).[b]	2.60	0.64
The two equations (are different types of equations, are the same type of equations).[c]	1.71	0.87

Note. First three questions are on a Likert scale of 1-4 with [a]does not resemble = 1. Remaining questions are 1-3 Likert scale with [b]we used in math class = 1 and [c]are different types of equations = 1.

During the interviews, the three students were further asked about the similarity between the general equation for a line, $y = mx + b$ and the linear motion equation, $v_f = v_o + at$. Sam did not recognize any similarities between the two equations but was able to talk about the slope and y-intercept in the general equation for a line. Alec had not previously made a connection between the two equations until the research presented both equations together and asked for

similarities and differences between the two. Similar to the quadratic equations, Jorden had already made a connection between the two equations indicating that correlation had been made in class. Jorden did relate the equation back to graphing, however; any connections between the two equations was never made. Alec and Jorden, even though they found the equations similar, were not able relate the slope in the linear velocity equation to acceleration or the starting velocity to the y-intercept.

The students' responses to the final question describing their thoughts on the transferability of mathematics language in college algebra to physics indicated that the students' struggled to find connections between algebra and algebra-based physics. Only two of the students indicated in their responses that they felt the skill they learned in college algebra could be transferred to physics. The remaining students indicated that they believed "there is very little transferability because it is like two different languages." When the three interviewed students were asked to give their perceptions of the transferability of mathematics language to physics, Sam explained that in algebra, "you're always solving for *x* or *y*, but in physics you're always solving for something. It's not always just *x*." Jorden further explained the struggle between algebra and physics by explaining, "if we were talking in high school math about speed or how many miles per hour or something, we didn't worry about velocity or the time that it actually took in seconds to get to that speed." Summarizing the two subjects of mathematics and physics, Alec explained,

> I've always seen physics as a science …then you get to it and it is almost more math than it is science…if instructors don't tie in that math than that's when the transition kind of gets funky. I think maybe if in math at the end (and I know time is always limited but…) even throw in little things like this is applied in physics …or you'll see this equation that's also this or they could show side by side formulas or vice versa."

In general, the students indicated frustrations over their ability to transfer mathematics language to physics.

Discussion and Conclusion

Realizing that college algebra was important for success in studying algebra-based physics, students struggled with the transferability of algebra skills and language to physics. The students indicated that they struggled with the formulas and variables used in physics. Further, students' written responses indicated that they did not necessarily perceive struggles with their algebra skills but the application of those skills to physics. This supported Angell, Guttersrud, Henriksen, and Isnes' (2004) research which indicated that students did not perceive their mathematics skills to be a stumbling block to physics.

When comparing the standard form of the quadratic equation and the physics projectile motion equation, the students indicated that they believed the two equations were similar, yet they were unable to verbalize similarities. Additionally, two of the three students interviewed had never realized the connections between the standard form of a linear equation and the linear motion equation used in physics. This observation raises the question to whether or not students are making connections between mathematics and the applications of mathematics, and further research in this area would benefit in understanding students' ability to transfer algebra knowledge to physics.

Additionally, the students repeatedly commented that the use of application problems in mathematics would have furthered their understanding of the use of mathematics in physics. Responding to students' request for more application of algebra, mathematics instructors can aid students in the transferring of algebra skills to physics by offering the students more opportunities to apply their algebra skills to physics. The instructors can additionally aid the students in transferring algebra language to physics by offering them opportunities to see equations using variables other than *x, y,* and *z* or solving for variables such as time or velocity. Additionally, physics instructors can assist students in making that transfer of algebra to physics by continuing

to make connections for the students, as Jorden indicated when explaining that the two quadratic equations were similar as explained by the physics instructor. As one student commented when reflecting on the transferability of algebra language to physics, “either side could have elaborated or made that connection and it would have been a little smoother going to physics.”

In summary, implications from this study suggest that students would benefit from additional opportunities to apply algebra knowledge to physics. Students would benefit from problems that aid the students in making translations between mathematics and physics such as problems that use variables with meaning such as time or velocity or explanations in the similarity between the standard form of the linear equation and equations used in physics. Further research into students transferability of algebra skills and language to physics could benefit instructors by offering insights into students’ struggles and whether or not students lack algebra skills or the ability to transfer those .

CHAPTER V

STUDENTS' TRANSFERABILITY OF THEIR ALGEBRA KNOWLEDGE TO PHYSICS

Target Journals: A. International Journal of Science and Mathematics Education

B. Investigation in Mathematics Learning

Authors: Kathleen Otto and Juliana Utley

Abstract:

The purpose of this mixed methods study was to examine students' transferability of mathematical knowledge of linear equations, quadratic equations, and graphing from a mathematics context to a physics context. Seventeen students from a midwestern two-year college algebra-based physics course participated in questionnaire containing isomorphic problems in the context of algebra and physics. Three of the students participated in a tasked-based interview using six of the questions from the questionnaire. A Wilcoxon signed rank test was used to analyze the students accuracy on the isomorphic questions indicating a significant difference between students' accuracy on the mathematics questions compared to physics questions ($z = -3.53$, $p < .001$) with a large effect size ($r = .86$). Results of the study suggested that students have difficulty interpreting graphs and solving quadratic equations regardless of the content. Students also struggled with interpreting variable meaning in physics and frustration in the use of formulas.

Introduction

Physics is considered one of the gatekeeper courses for students majoring in a STEM field, success in these courses can potentially influence a student's decision to continue studying in a STEM field (Redmond-Sanogo, Angle, & Davis, 2016; Robinson, 2003; Shaw & Barbuti, 2010). Over the past several decades, researchers examining students' retention in physics have concluded that success in mathematics is a common variable in the enrollment and retention of students in physics (Hansson, Hansson, Juter, & Redfors, 2015; Nguyen & Rebello, 2011; Pepper, Chasteen, Pollock, & Perkins, 2012; Winegardner, 1939). Research indicates that the probability of a student's success in physics is highly correlated to the student's mathematics skills set; students who do not possess the necessary mathematics skill set are less likely to be successful in physics (Champagne, Klopfer, & Anderson, 1980; Hudson & McIntire, 1977). Interestingly, some research suggests students may have proficient mathematics skills but struggle to apply that skill set to physics (Hudson & Liberman, 1982; Hudson & McIntire, 1977; Rebello, Cui, Bennet, Zollman, Ozimek &, 2007). With mathematics a variable to student success in physics, transferability of mathematics knowledge to physics is a possible avenue to investigate concerning the struggles of students in physics (Bollen, De Cock, Zuza, Guisasola, & van Kampen, 2016; Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012; Planinic, Ivanjek, Susac, & Milin-Sipus, 2013). Thus, the object of this study is to explore students, enrolled in an algebra-based college physics course, transferability of mathematics knowledge to the study of physics.

Related Literature

Knowledge transfer examines how knowledge gained from one experience is applied to another similar or new situation and can be used as a framework for examining how students translate mathematics to the study of physics (Beach, 1999; McKeough, Lupart, & Marini, 2013; Perkins & Salomon, 1992; Singley, & Anderson, 1989). Since courses such as chemistry,

physics, and biology require students to be able to transfer their mathematics skills, researchers of STEM fields are interested in the potential of knowledge transfer (Bassok, 1990; Hoban, Finlayson, & Nolan, 2013; Menis, 1987; Ngu & Yeung, 2012; Rebello, Cui, Bennett, Zollman, & Ozimek, 2007). However, before examining students' transfer of mathematics to physics, it is important to examine the algebra skills needed for students to be successful in algebra-based physics.

Research has indicated two major strands of mathematics content that are commonly required in physics. First, understanding linear and quadratic equations is required for algebra-based physics since students will be dealing with objects in motion both linearly and parabolically (Champagne, Klopfer, & Anderson, 1980; Delialioğlu & AŞKAR, 1999). Second, the ability to analyze graphs is imperative for success in physics when interpreting physical motion data (Champagne, Klopfer, & Anderson, 1980; Delialioğlu & AŞKAR, 1999; Gill, 1999; Hudson & McIntire, 1977).

With an understanding of the mathematics skills needed for physics, researchers can begin to examine students' transfer of mathematics knowledge to physics using classical knowledge transfer. Classical knowledge transfer occurs when the knowledge gained in one problem can be adapted and used in another similar problem (Bassok & Holyoak, 1989; Hoban, Finlayson & Nolan, 2013; Ngu & Yeung, 2012; Reed, Dempster, & Ettinger, 1985). By using classical knowledge transfer of isomorphic problems (problems of similar content but in the context of mathematics versus physics) researchers have identified a few areas students find particularly challenging: graphing, linear equations, and quadratic equations (Bollen, De Cock, Zuza, Guisasola, & van Kampen, 2016; Guttersrud, & Angell, 2010; Planinic, Ivanjek, & Susac, 2013; Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012).

When using isomorphic problems to examine how students of both algebra-based and calculus-based physics approaches kinematics graphing based on context, Bollen, De Cock, Zuza, Guisasola, and van Kampen (2016) found that students in algebra-based physics struggled more

in transferring knowledge from mathematics to physics than the calculus-based physics students. Additionally, the researchers indicated that students in algebra-based physics were better able to answer questions concerning graphs that were context-free as compared to those with context. Similarly, examining students' level of transferability of graphing knowledge using isomorphic problems, Planinic, Ivanjek, Susac, and Milin-Sipus (2013), found that students struggled with transferring knowledge of slope between several different contexts (mathematics, physics and other areas). Researchers explained that problems with slope in contexts other than math required "one more step in solving: interpretation and translation of context into mathematical language" (p. 7).

Planinic, Milin-Sipus, Katic, Susac, and Ivanjek (2012), examining students' transfer of mathematics knowledge to physics, developed two sets of isomorphic graphing problems for both mathematics and physics. The researchers found that the students were more likely to answer graphical problems in the mathematics context correctly as compared to the physics context. The researchers also suggested that the students found little similarity between problems in the same content when presented in different contexts, and the students used different skills to answer the questions based on the context. Bassok and Holyoak (1989) examined the transfer of isomorphic series and sequence problems from algebra to physics and found that knowledge transfer was evident between isomorphic problems in physics and algebra when similar variables were used. However, data did not show the same knowledge transfer on isomorphic equations that use different quantities suggesting that more time may be needed in connecting concepts.

With limited research on knowledge transfer of isomorphic mathematics and physics problems and existing research examining mainly graphing content, a gap in research exists around transfer of isomorphic problems in other mathematical content such as linear and quadratic equations. The purpose of this study was to expand the research of algebra-based physics students' transferability of mathematical knowledge of linear equations, quadratic equations, and graphing from a mathematics context to a physics context by examining students'

work on isomorphic mathematics and physics problems. The study was guided by the research question: Does a student's knowledge of linear equations, quadratic equations, and graphing differ between a mathematics context and a physics context?

Methodology

The study used a convergent parallel mixed methods design investigating how students enrolled in an algebra-based physics course transferred mathematics knowledge to physics. Implementing a mixed methods study allowed researchers to examine the "complexities of current educational issues" using a "multifaceted research design" (Hart, Smith, Swars, & Smith, 2009, p. 27). Additionally, the combination of qualitative results of isomorphic questions along with qualitative work on the problems and task-based interviews provided a deeper examination of students' ability to transfer mathematics knowledge to physics.

Participants

Participants were selected using a single stage purposive sampling of students enrolled in a spring semester algebra-based physics course in a rural Midwestern two-year college. The sample consisted of seventeen students (35% male; 65% female) with a mean age of 22 years. These students were required to have completed and passed college algebra with a grade of 'C' or better prior to enrolling in the course or to have obtained an ACT mathematics subset score of 24 or higher. Three of the students (2 females and 1 male), Sam, Jorden, and Alec (names have been changed for anonymity), volunteered for a task-based interview.

Measures

Data were collected from multiple sources including a set of questions using isomorphic mathematics and physics questions along with task-based interviews using selected questions from the problem set. Upon completion of course lectures on the topics in the study, students were requested to complete the question set. Four weeks later, three students were asked to complete six of the original problems for the task-based interviews.

Isomorphic problems. Isomorphic parallel mathematics and physics problems (see Appendix F and G) were given to students enrolled in the algebra-based physics course. The isomorphic problems were designed to cover content over linear equations, quadratic equations, and graphing based on previous research on mathematics skills needed for success in algebra-based physics. Graphing problems were designed based on recommendations of Guttersrud and Angell's (2010) research along with other research on graphing (Bollen, De Cock, Zuza, Guisasola, & van Kampen, 2016; Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012). Problems were used from previous research (Bollen et al., 2016; Planinic et al., 2012) and adapted from the students' mathematics and physics textbooks (Aufmann, Barker, & Nation, 2008; Serway & Vuille, 2015).

The isomorphic problems covered the content area of linear equations (eight questions), quadratic equations (six questions) and graphing (eight questions). Each question was multiple choice; however, the students were asked not only to circle the correct answer but to show their work providing a brief description of how they made their choices. The set of linear isomorphic questions included four questions in the form of $y = mx$ with another four in the form of $y = mx + b$. Questions for the quadratic problems asked the student to solve two of the questions for *y* while the other four questions required the student to solve for *x*. The isomorphic graphing questions include four questions asking the students to find slope while the final four questions asked the students to interpret linear graphs (see Figure 51).

Task-based interview. The task-based interview used six of the multiple-choice isomorphic problems (see Appendix H) given to the students earlier with two from each content area, including one with a mathematics and one with a physics context. The students' "think aloud" process while they solved the problems provided insight into understanding of these problems and whether students could apply their understanding of the mathematics in the physics context.

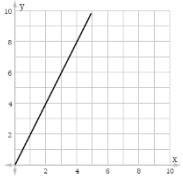
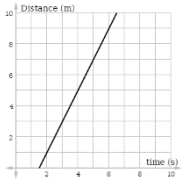
Mathematics	Physics
Linear $y = mx$	
Lisa is growing basil from a seed and is tracking the progress of her plant's growth. The plant grows 0.4 cm/day. How many days has it grown to get to reach 30 cm? (Aufmann, Barker, & Nation, 2008)	Calculate the acceleration of a 2000 kg, the single-engine airplane just before takeoff when the thrust of its engine is 500 N. (Serway & Vuille, 2015)
Linear$y = mx + b$	
The number of miles that remain to be flown by a commercial jet traveling from Boston to Los Angles can be approximated by the equations $miles\ remaining = 2650 - 475t$ Where t is the number of hours since leaving Boston. In how many hours will the plane be 1000 miles from Los Angles? (Aufmann, Barker, & Nation, 2008)	A boat moves slowly out of a marina with a speed of 1.50 m/s. As soon as it passes the breakwater, leaving the marina, it throttles up and accelerates at 2.40 m/s^2. How fast is the boat moving after accelerating for 5 seconds? (Serway & Vuille, 2015)
Quadratic $y = ax^2 + bx + c$, Solve for y	
An object is launched at 19.6 m/s from a 58.8 m tall platform. The equation for the object's height *s* at time *t* seconds after launch is $s(t) = -4.9t^2 + 19.6t + 58.8$, where s is in meters. What is the height of the object in 5.8 seconds? (Aufmann, Barker, & Nation, 2008)	A race car accelerates uniformly at 11.2 m/s from a velocity of 18.5 m/s in 2.47 seconds. Determine the distance traveled by car. Remember that $s_f = v_o t + \frac{1}{2}at^2 + s_o$ where $\frac{1}{2}a = 5.6\ m/s^2$ (Serway & Vuille, 2015)
Quadratic $y = ax^2 + bx + c$, Solve for x	
A company has determined that the profit, in dollars, it can expect from the manufacture and sales of x tennis racquets is given by $P = -0.01x^2 + 168x - 120{,}000$. How many racquets should the company manufacture and sell to earn a profit of $518,000? (Aufmann, Barker, & Nation, 2008)	A truck accelerates at a rate of 0.444 m/s^2 from rest to a velocity of 2.80 m/s. How long does it take for the truck to reach 40.0 m? Remember that $s_f = v_o t + \frac{1}{2}at^2 + s_o$ where $\frac{1}{2}a = 0.222\ m/s^2$ (Serway & Vuille, 2015)
Graphing	
Consider the following line in the coordinate system. Which statement is correct?  A. The slope of the line is constant and different from zero. B. The slope of the line is constant and equal to zero. C. The slope of the line is constantly increasing. D. The slope of the line is constantly decreasing. (Planinic, Milin-Sipus , Katic, Susac, & Ivanjek, 2012)	Distant-time graph of an object's motion is shown below. Which statement best describes this motion?  A. The object is not moving. B. The object is moving at a constant velocity. C. The object is moving with a uniformly decreasing velocity. D. The object is moving with a uniformly increasing velocity. (Planinic, Milin-Sipus , Katic, Susac, & Ivanjek, 2012)

Figure 5.1. Sample isomorphic problems examining students' transferability of mathematics knowledge of linear equations, quadratic equations, and graphing from mathematics context to physics context.

Data Analysis

Students' responses on the multiple-choice isomorphic problems were first scored for accuracy, correct or incorrect. Then both descriptive (i.e., frequencies, means and standard deviations) and inferential statistics were used to analyze data. With a sample size less than twenty a nonparametric test, Wilcoxon signed rank test, was used to analyze students' responses to determine whether there was a significant difference in how students answered the mathematics context questions compared to the questions in the physics context. Secondly, students' brief descriptions of how they solved each of the isomorphic question were analyzed by transferring their responses to Excel, and a descriptive coding technique was used to develop an initial set of codes and the data was then code charted and moved into themes. Third, the task-based interviews were transcribed by the researcher and an analytical memo was used in conjunction with descriptive coding for an initial set of codes and the data was then code charted and moved into themes. Finally, the data was compared using a pattern coding and was interpreted (Creswell, 2007; Saldana, 2016).

Results

Analysis of the isomorphic problems showed that the seventeen students completed with an overall accuracy of 46% on the questions. In an examination of the mathematics context questions only, the students completed the questions with an accuracy of 69%. However, the students' accuracy for the physics-only context was 30%. Problems that were left blank were considered an incorrect response. A Wilcoxon matched pairs signed rank test analysis of the data using SPSS® indicated a statistically significant difference between students' accuracy on isomorphic mathematics questions compared to physics questions ($z = -3.53$, $p < .001$). The effect size for this analysis ($r = .86$) was found to exceed Cohen's (1988) convention for a large effect ($r = .50$). These results indicated that students demonstrated an overall higher level of success solving the mathematics context questions ($M = 8.29$, $SD = 2.05$) in relation to solving the physics context questions ($M = 3.59$, $SD = 1.42$).

Linear Problems

Analysis of responses to the isomorphic linear problems showed that the seventeen students completed the questions with an overall accuracy of 65%. Students' accuracy (see Table 5.1) on the mathematics questions covering linear content was 87% with accuracy on the physics questions at 44%. A Wilcoxon signed rank test analysis of the data using SPSS indicated a significant difference between students' accuracy on isomorphic linear mathematics questions compared to linear physics questions (z = -3.48, p = .001). The effect size for this analysis (r = .84) was found to be large. These results indicated that students demonstrated an overall higher level of success solving the linear questions in the mathematics context (M = 3.47, SD = 0.80) in relation to solving the physics context questions (M = 1.65, SD = 0.78).

Table 5.1

Students' (n=17) Accuracy on the Isomorphic Mathematics and Physics Questions

Content	Math % Accuracy	Physics % Accuracy	Z	p	r
Linear					
Overall	87	41	-3.48	.001**	.84
$y = mx$	91	35	-3.58	.001***	.87
$y = mx + b$	82	47	-3.00	.003**	.73
Quadratic					
Overall	55	22	-2.84	.005**	.69
Solving for 'y'	76	18	-2.67	.008**	.65
Solving for 'x'	47	19	-1.82	.068	.44
Graphing					
Overall	62	28	-3.23	.001**	.78
Interpretations	59	41	-1.50	.130	.36
Slope	72	19	-2.60	.009**	.63

*p<.05, **p<.01, ***p<.001

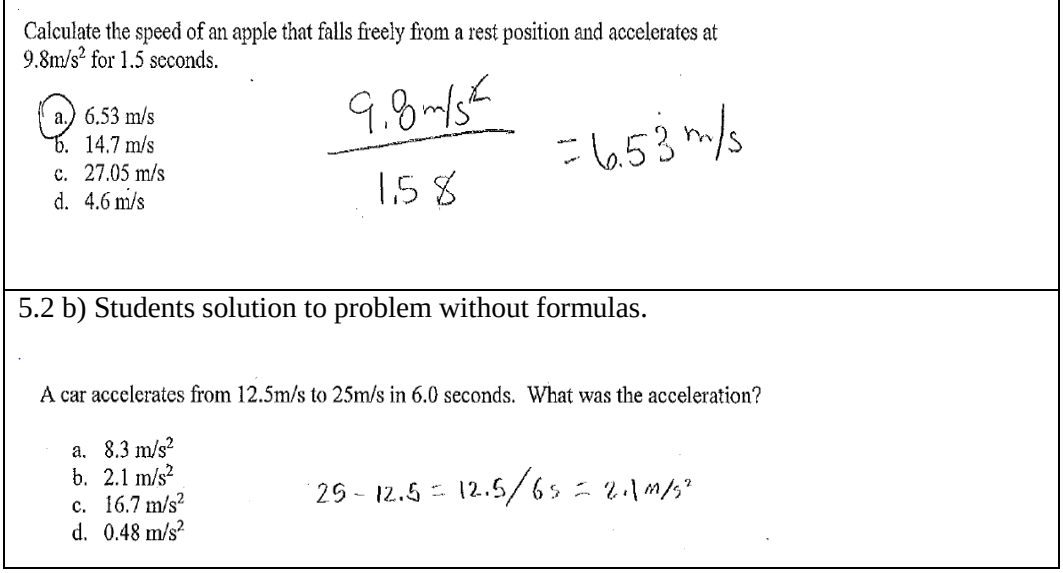
Specifically, when analyzing students' work on linear questions in the mathematic context of the form $y = mx$, a Wilcoxon signed rank test analysis indicated a significant

difference between students' accuracy on mathematics questions compared to linear physics questions (z = -3.58, $p < .001$). The effect size for this analysis (r = .87) was found to be large. These results indicated that students demonstrated an overall higher level of success solving the linear questions in the mathematics context (M = 1.82, SD = 0.53) in relation to solving the physics context questions (M = 0.71, SD = 0.59). A further examination of students' written work on linear questions in the mathematics context of the form $y = mx$ showed that the students who incorrectly answered the question typically made simple calculation errors.

In contrast, students' work on linear equations in the physics context revealed that most students tended to struggle more. In students' work with linear equations of the form $y = mx$ in the physics context, students repeatedly indicated that they were unsure of what formula to use. Additionally, one question gave force with units of Newton's which a student indicated an issue knowing the units. For the problems of the form $y = mx$ for which the students were asked to find speed, fewer of the students struggled then when they were asked to find acceleration. A common struggle on the questions was students using units incorrectly to solve the problems such as students dividing the acceleration by time instead of multiplying to find velocity (see Figure 5.2 a).

Analysis of student work on linear questions in the mathematics context of the form $y = mx + b$ using a Wilcoxon signed rank test analysis indicated a significant difference between students' accuracy on mathematics questions compared to linear physics questions (z = -3.00, p = .003). The effect size for this analysis, (r = .73) was found to be large. These results indicated that students demonstrated an overall higher level of success solving the linear questions in the mathematics context (M = 1.65, SD = 0.49) in relation to solving the physics context questions (M = 0.94, SD = 0.43). For the questions of the form $y = mx + b$, student work that was available typically showed simple calculation errors. Several students indicated with their answers on the problems, "simple equations" and "solve for x, simple math equation."

5.2 a) Student's misuse of units.



5.2 b) Students solution to problem without formulas.

Figure 5.2. Student work on linear equations in the physics context.

In the physics context for the linear equations of the form $y = mx + b$, students indicated a need for a formula similar to responses in the physics context of the linear equations of the form $y = mx$. For example, one student said, "Lost from beginning, what equation?" Another student indicated, "In college algebra we were taught different tricks to solve equations and in physics we actually have to know the math and make the equation." However, not all students relied on formulas to solve these questions. Those students who showed work and were accurate in their answers indicated a grasp of the physics concepts such that they did not rely on formulas nor had any issues with solving linear equations (see Figure 5.2 b). These students appeared to solve the equations in the same manner as the mathematical context without explicit formulas. It is important to note that the students were more accurate on the questions that provided only velocities and struggled more with the questions that provided acceleration.

During the task-based interview, two of the students worked the linear mathematics context problem without a reliance on formulas. Sam and Jorden were able to work through the problem without setting up an equation while Alec created an equation and then solved the

problems. When moving on to the physics context question, Alec and Sam were able to solve the problem utilizing a formula. For this problem, Alec and Sam both wrote the linear velocity equation, wrote down what was known, plugged the information into the equation and solved. Both indicated that they had just reviewed the physics equation a week before the interview in class. Sam did indicate that the problems would have been more complicated and frustrating had the class not reviewed the formula the week before. The third student, Jorden, struggled with working the problem. Jorden was able to get the velocity of the boat after acceleration but did not add it to the original velocity to find the final speed.

Examination of the students' solutions to the linear equations indicated that students were able to solve linear equations in mathematics context with ease. While the students' work on linear equations in the physics context suggested that they tended to rely on formulas to solve these questions. However, students who did not rely on formulas or were able to recall the correct formula easily solved the linear equation.

Quadratic Problems

Analysis of responses to the isomorphic quadratic problems showed that seventeen students completed all the questions with an overall accuracy of 38% on the problems. Student accuracy (see Table 1) on the mathematics questions covering quadratic content was 55% with accuracy on the physics questions at 22%. A Wilcoxon signed rank test analysis of the data using SPSS indicated a statistically significant difference between students' accuracy on isomorphic quadratic mathematics questions compared to linear physics questions ($z = -2.84$, $p = .005$). The effect size for this analysis ($r = .69$) was found to be large. These results indicated that students demonstrated an overall higher level of success solving the quadratic questions in the mathematics context ($M = 2.35$, $SD = 1.27$) in relation to solving the physics context questions ($M = 0.82$, $SD = 0.88$).

Specifically, in an analysis of data on quadratic questions when students were asked to solve for *y*, a Wilcoxon signed rank test analysis indicated that the isomorphic questions were

statistically significant (z = -2.67, p = .008) with a large effect size (r = .65) indicating that students' accuracy on mathematic context (M = 0.76, SD = 0.44) was higher than their accuracy in physics context(M = 0.18, SD = 0.39). An examination of students' work on the quadratic question in the mathematical context when asked to solve for *y* showed that the students who incorrectly answered the question typically had simple calculation errors. Examination of the quadratic problem in the physics context when students were asked to solve for *y*, revealed that ten of the students showed no work on the problem so no further analysis could be made. Those students showing work indicated no issue with solving the problem. One student did indicated confusion with the term s_o, in the physics equation $s_f = v_o t + \frac{1}{2}at^2 + s_o$.

Conversely, data analysis on quadratic questions when asking students to solving for *x*, a Wilcoxon signed rank test analysis indicated that the isomorphic questions were non-significant (z = -1.83, p = .068) indicating that students' accuracy on the mathematic context was similar to their accuracy in the physics context. When asked to solve for *x*, several students indicated frustration on how to approach this problem and several written comments on the problems revealed students' frustration such as, "Don't know how to start!" and "Wordy-making my brain shut off." These students were unable to set up the equations and ended up moving on to the next question with no work (see Figure 5.3 a). Additional student work and comments indicated that some of the students struggled with factoring a quadratic equation (see Figure 5.3 b).

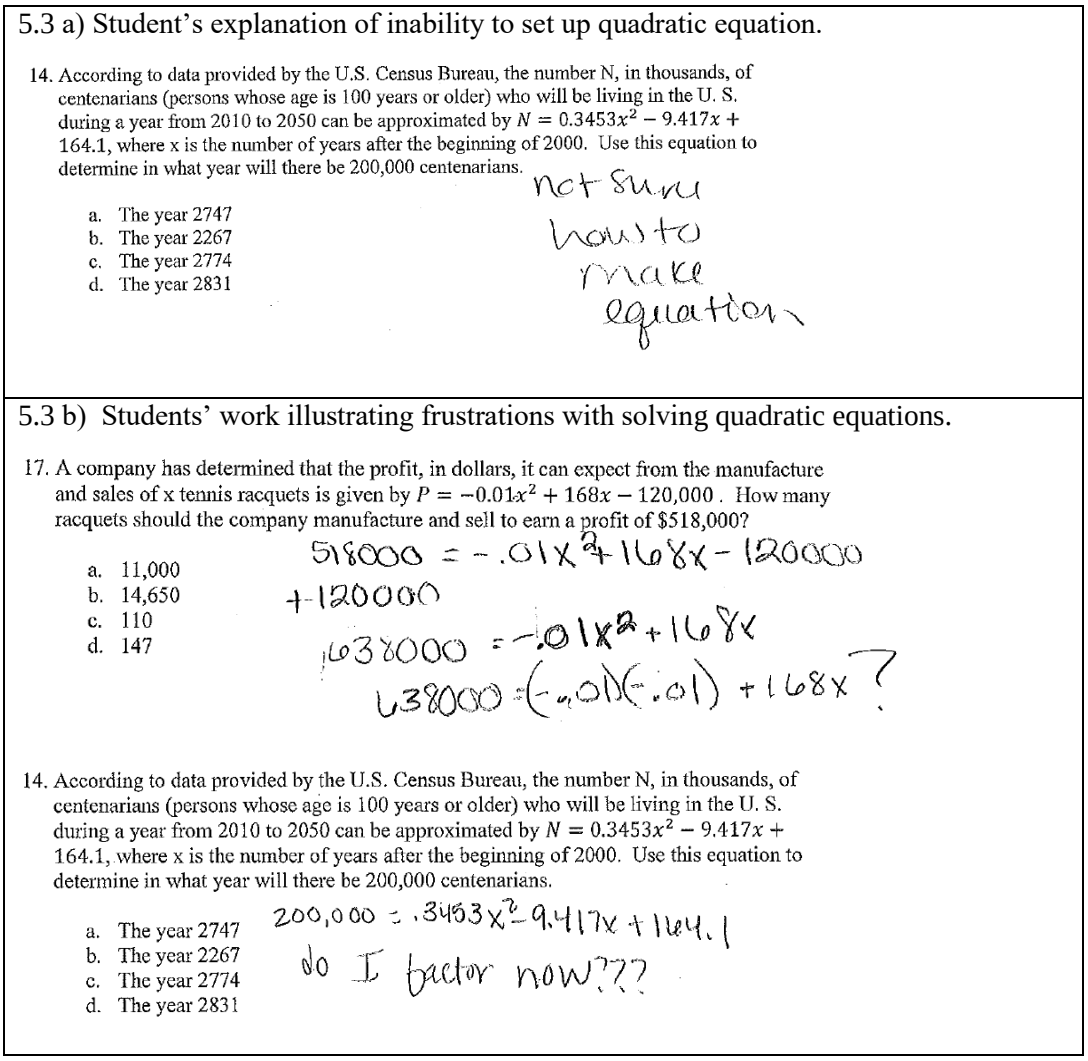


Figure 5.3: Student's solutions illustrating frustrations solving quadratic problems in the mathematics context.

Examination of the quadratic problems that asked the students to solve for *x* in the physics context, revealed at least five of the students' indicated confusion with the physics equation, $s_f = v_o t + \frac{1}{2}at^2 + s_o$, and term s_o in the equation. A student commented, "I don't know what s_o stands for because it looks like second, but time is *t.*" When time was the desired variable to solve for a student wrote, "Don't know where to start and how do you find time to plug in?" Students who did not struggle with the variables in the equation and were able to set up the problem, also showed frustrations with solving the quadratic equation, such as using factoring or the quadratic formula (see Figure 5.4). In summary, the students who struggled with the

quadratic questions in both the mathematics and physics context struggled either (a) with understanding the variables in the equation or (b) with finding the solution to a quadratic equation.

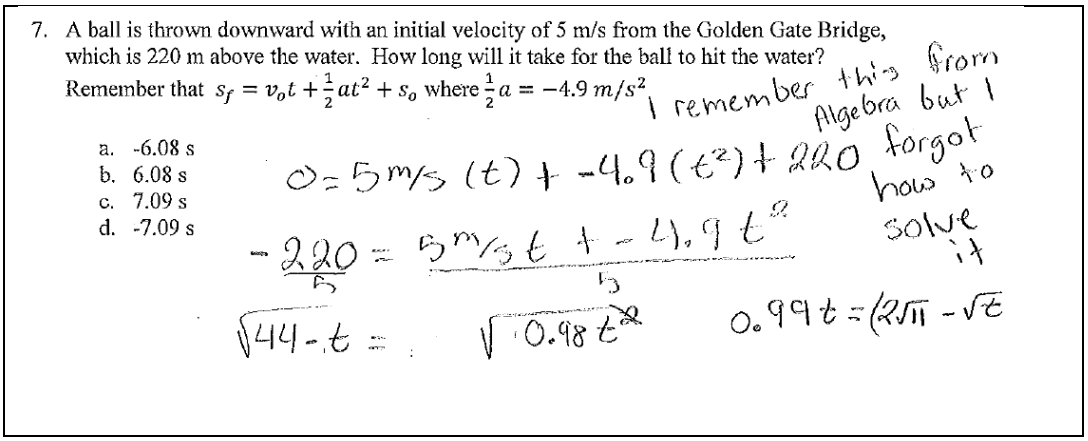


Figure 5.4. Student's solutions illustrating frustration on how to solve quadratic equations in the physics context.

During the task-based interview, all three of the students remembered and were able to utilize the quadratic formula when solving the quadratic problem in mathematics context for *x*. Sam and Alec had no troubles in solving the equation. Jorden, however, remembering the quadratic formula began plugging numbers into the equation before setting the equation to zero. For the quadratic problem in physics context, all three of the students struggled with the problems when interpreting the variables. Sam struggled the most with the variable *s* before deciding that it could be translated to delta *x*. However, Sam was unable to relate that $\Delta x = x_f - x_o$ to $\Delta s = s_f - s_o$ and left the s_o term in the original equation. Continuing to struggle with the variables, Sam finally, with a few leading questions from the researcher, was able to make the proper translation from s_o to x_o. From here, Sam set up the equation properly to solve for the squared term by taking the square root of both sides. Jorden also struggled with the *s* term before deciding that it was position. Although realizing *s* was position, Jorden also struggled with final position and was never able to set up the equation. Alec struggled the least with interpretation of the variables and after only a few seconds realized that *s* was position and translated to *d*. At this

point Alec commented on the confusion with variable meaning by explaining, “This is kind of a fuzzy area. Sometimes we use *s* and sometimes we use *d* and another thing, *d* can be density.” Alec finished the problem with little difficulty by taking the square root of each side.

Sam, Jorden, and Alec’s task-based interviews supported the translation struggle of variables that students indicated in the problem set. Although the students in the task-based interview did not struggle with solving quadratic equations, students’ responses on the questions indicated students struggled with solving quadratic equations in both the mathematics and physics context.

Graphing Problems

Analysis of responses to the isomorphic graphing problems revealed that seventeen students completed all the questions with an overall accuracy of 45% on the problems. Student accuracy (see Table 1) on graphing in the mathematics context was 62% with accuracy on the physics context of 28%. A Wilcoxon signed rank test analysis of the data using SPSS® indicated that students demonstrated an overall higher level of success solving the graphing questions in the mathematics context ($M = 2.47$, $SD = 0.87$) in relation to solving the physics context questions $M = 1.12$, $SD = 0.70$), $z = -3.23$, $p = .001$, $r = .78$.

Additionally, any differences in student success interpreting the slope of a graph or interpreting a linear graph given a mathematics and physics context were analyzed using a Wilcoxon Signed Rank test. Results indicated no difference between students’ accuracy on interpreting the slope of a graph in the mathematics context as compared to a physics context ($z = -1.50$, $p = .13$). When asked to interpret the slope of the line in the mathematical context problems, approximately half of the students related the line’s movement to slope and indicated that since the line was sloping up and to the right then the slope was constantly increasing. This idea continued over to the physics context with 94% of the students missing the acceleration question using the same thought pattern. Interestingly, the students did not have the same interpretation struggles with the velocity graph with only 29% missing the question.

Using the Wilcoxon Signed Rank test, analysis of the data indicated that students demonstrated an overall higher level of success calculating slope given a mathematics context (M = 1.35, SD = 0.70) in relation to a physics context (M = 0.35, SD = 0.70), z = -2.60, p = .009, r = .63. Further analysis of the student work on the graphing problems in mathematical context indicated that students had only small calculation errors when calculating the slope of a line at a given point. In the physics context, however, a large number of students (83% for both questions) did not find the slope but instead used the time given in the problems and read the distance off the graph interpreting that for the velocity or they calculated speed as distance divided by the time given instead of change in distance over time and many lacked confidence in their answers (see Figure 5.5).

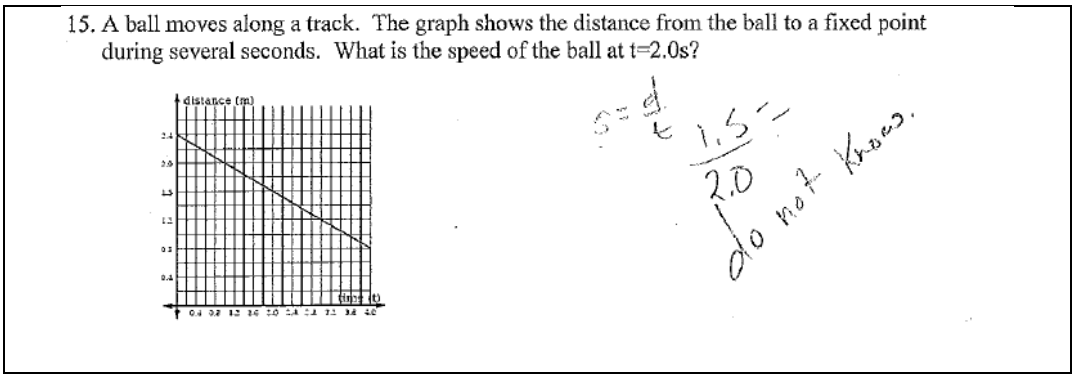


Figure 5.5. Students' solution demonstrating misuse of formula to find velocity.

During the task-based interviews, all three students were able to find the slope of the line given in the context of mathematics. Alec wrote both an equation for the slope and explained that slope was rise over run while Jorden used a velocity equation. Sam started off with an equation for a line and began to plug in the y-intercept and the calculated slope. After reading the problem again, Sam realized that the question only asked for slope so circled the work on slope and explained the process to find slope. However, when moving into the physics context, Jorden and Sam struggled with the problem. Jorden read the distance off the graph at the time given. Sam

started the question the same way by reading the point off the graph and after rereading the graph decided that was wrong. Sam struggled with equations and wrote down both the linear equation and the linear velocity equation and attempted to plug numbers in. Sam never made a connection of slope to acceleration in the equation. Sam finally gave up and moved on to the next question. Jorden used a formula for velocity to solve the problem. However, Jorden similar to the student in Figure 5 incorrectly used the formula as velocity equal to distance divided by time instead of change in distance over change in time never realizing that slope was equal to velocity.

In the isomorphic graphing problems, students had few issues finding the slope of a line in the mathematics context. In the physics context, students were able to use slope to find the velocity. However, the students struggled to use slope to find acceleration. In general, the students struggled interpreting graphs in both the mathematics and physics context.

Discussion and Conclusions

The purpose of the study was to examine differences of students' knowledge of linear equations, quadratic equations, and graphing in the context of mathematics and physics. The statistical analysis indicated that there was a significant difference in the students' knowledge of linear, quadratic, and graphing questions between the context of mathematics and physics. The students' results indicated that overall they were able to answer the mathematical questions with fewer issues than the physics questions indicating a possible struggle in transferring knowledge from mathematics to physics in all three content areas.

In terms of linear equations, students were able to solve linear equations in the mathematics context with minimal calculation errors and appeared to possess the mathematics skill necessary to transfer those skills to physics. However, some of the students struggled with translating the linear physics equations into mathematic equations. These students commented on their problems that they did not know which formula to use to solve the problem.

For quadratic equations in the mathematical context, student responses suggested that students, in general, were able to solve quadratic equations when asked to solve for the *y* term.

However, many of the students' demonstrated frustration when asked to solve for *x*. When examining the students' responses in the physics context, students demonstrated continuing frustration with solving quadratic equations along with understanding the variables' meaning in the equation. Since students' demonstrated frustration with solving quadratic equations in the mathematics context, it is hard to evaluate the transferability of mathematics knowledge to physics. Overall, this set of questions demonstrated students' struggles in solving quadratic equations for *x*, and a frustration in interpreting the meaning of the variables in the physics quadratic equations.

In terms of graphing students tended to possess knowledge of calculating slope but struggled with transferring that knowledge to physics' acceleration graphs. This supported Bollen et al. (2016) research that algebra-based physics students more accurately answered context free graphing questions. However, the students indicated limited frustrations in finding velocity from a time versus distance graph. When interpreting slope in both the mathematics and physics context, students struggled with the misconception that the characteristics of the graph related to the slope of the graph. Overall, the questions covering graphing content indicated that the students lacked an understanding of the concepts of graphing supporting previous research by Planinic et al. (2013).

In summary, findings from this study indicated several possible struggles for students: minimal understanding of the concept of slope, struggles in transferring knowledge of slope to acceleration, limited ability to solve quadratic equations when asked to solve for *x*, and reliance on formulas to solve physics equations. It is important for mathematics instructors to provide opportunities for algebra students to solve contextual problems involving slope, linear relationships, and quadratic relationships. When offering students more opportunities to observe slope in various contexts using application problems, mathematics instructors can prepare students to transfer concepts of slope to physics in the form of velocity or acceleration. Application problems that utilize linear and quadratic equations can also provide students

opportunities to set up and solve problems in both the context of mathematics and physics. Physics instruction can help students transfer mathematics knowledge to physics by offering the student translations of physics equations into the standard forms of mathematical equations. Finally, instructors of both mathematics and physics can aid students in translating the meaning of slope so that a connection between slope and velocity or acceleration becomes a logical connection.

Could the students' struggles in transferring the mathematical knowledge from mathematics to physics without a formula or with confusion over the meaning of the variables be a language issue? Students in this study indicated frustration at the symbols used in the physics quadratic equation along with Alec commenting on the variety of meanings a variable could have in physics. Further research into the struggles students have when moving from mathematics symbols to physics is a possible area of further research along with possible language differences. Additionally, investigating a larger set of students from a combination of colleges could further the research and provide more evidence on students' specific struggles with the transfer of algebra between mathematics and physics. Continued research in this area could help students and instructors close the knowledge gap between mathematics and physics and provide the students with the mathematics and translation skills necessary to focus on learning physics concepts and find success in physics.

CHAPTER VI

SUMMARY

The overarching goal of this study was to explore the transferability of mathematics knowledge to physics. More specifically, to meet this goal, the study examined both the instructors' and students' perspectives on the transferability of mathematics skills and language to physics by examining three different perspectives. The study was divided into three separate parts. This research proposed to answer the following questions in the three different studies:

- How do instructors of college algebra and algebra-based physics perceive the transferability of algebra language and skills to the study of algebra-based physics courses?
- Is there a significant difference between instructors' of college algebra and instructors' of algebra-based physics perceptions of transferability of algebra language and skills to physics?
- How do students perceive the transferability of their algebra language and skills from mathematics courses to physics courses?
- Does a student's knowledge of linear equations, quadratic equations, and graphing differ between a mathematics context and a physics context?

The overall research approach for this study was a convergent parallel mixed methods design using three parallel studies. For the first study, mathematics and physics instructors completed

an online questionnaire consisting of closed- and open-ended questions. The study further utilized semi-structured interviews. The second study used data from both questionnaires along with semi-structured interviews completed by students enrolled in an algebra-based physics course. The final study used a set of isomorphic problems along with a task-based interview to examine the ability of students enrolled in a college level algebra-based physics courses to transfer mathematics skills. Study results were organized into three manuscripts which are summarized below.

Summary of Findings

Chapter Three, titled, "College Instructors' Perceptions of Transferability of Algebra Language and Skills to Studies in Physic" focused on mathematics, physics, and combination instructors perceptions of the transferability of algebra language and skills to the study of algebra-based physics. The research questions answered by this study were: (a) How do instructors of college algebra and algebra-based physics perceive the transferability of algebra language and skills to the study of algebra-based physics courses? (b) Is there a significant difference between instructors' of college algebra and instructors' of algebra-based physics perceptions of transferability of algebra language and skills to physics?

The results suggested that the instructors in the study have similar perceptions in the algebra skills needed to be successful in algebra-based physics. All three groups of instructors indicated that the algebra skills needed for a student to be successful include: linear concepts, quadratic concepts, graphing concepts including interpretation of graphs, and ability to manipulate and solve variable equations without numbers. These results support previous research on algebra skills needed for physics (Bollen, De Cock, Zuza, Guisasola, & van Kampen, 2016; Guttersrud, & Angell, 2010; Planinic, Ivanjek, & Susac, 2013; Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012; Redish, 2005).

When examining instructors' perceptions of students' transferability of mathematics language to physics, instructors indicated students struggle with the differences between vocabulary and variables in mathematics and physics along with application. The instructors' perceptions supported Redish and Kuo's (2015) finding that one of the main differences in the language of mathematics compared to physics is how meanings are attached to symbols along with how meaning is constructed during application. Interestingly, combination instructors did not consider language transfer to be an issue. In contrast to physics instructors, the instructors who taught either mathematics or both mathematics and physics indicated a belief that it was the responsibility of the instructor to help students make connections between mathematics in both the areas. For example, these instructors believed that physics instructors should explicitly make connections to linear equations in a mathematics context with the physic context where they are using linear equations. These findings are in line with Guttersrud and Angell (2010) research findings that suggest that instructors should aide students in making translation.

Examining instructors' perception of skills, isomorphic problems, and transferability of language, data analysis indicated no statistical significance between the three groups of instructors' perceptions. However, differences in the means for individual questions between the three groups of instructors' perceptions such as skill level of their students, language transferability, and application to increase transferability suggested that minimal differences exist in perceptions. Also, the differences in combination instructors' responses as compared to mathematics and physics instructors suggest that the combination instructors may have a slightly different perception of students' transfer of mathematics knowledge to physics.

Chapter Four, titled, "Transferability of Algebra Language and Skills to Physics: Students' Perceptions of Mathematics Ability" focused on students' enrolled in an algebra-based physics class perception of the transferability of the algebra skills and language to physics. The research question answered by this study was: How do students perceive the transferability of their algebra language and skills from mathematics courses to physics courses? Results from the

study indicated that students realize that college algebra was important for success in studying algebra-based physics but struggled with the transferability of algebra skills and language to physics. The students indicated that they struggled with the formulas and variables used in physics. Further, students' written responses indicated that they did not necessarily perceive struggles with their algebra skills but the application of those skills to physics. This supported Angell, Guttersrud, Henriksen, and Isnes' (2004) research which indicated that students did not perceive their mathematics skills to be the stumbling blocks to physics.

When comparing the standard form of the quadratic equation and the physics projectile motion equation, the students indicated that they believed the two equations were similar, yet they were unable to verbalize similarities. Additionally, two of the three students interviewed had never realized the connections between the standard form of a linear equation and the linear motion equation used in physics. The students repeatedly commented that the use of application problems in mathematics would have furthered their understanding of the use of mathematics in physics.

Chapter Five, titled, "Students' Transferability of their Algebra Knowledge to Physics" focused on students enrolled in an algebra-based physics class transferability of the algebra knowledge to physics through examination of isomorphic algebra and physics questions. The research question answered by this study was: Does a student's knowledge of linear equations, quadratic equations, and graphing differ between a mathematics context and a physics context? A Wilcoxon signed rank test was used to analyze the students' accuracy on the isomorphic questions indicating a significant difference between students' accuracy on the mathematics questions compared to physics questions ($z = -3.53$, $p < .001$) with a large effect size ($r = .86$).

Students' responses from this study indicated that students struggle with interpreting graphs and solving quadratic equations regardless of the content. However, students did not indicate any difficulty in calculating slope and solving linear equations in the mathematics context. Yet when similar questions were presented in the physics context, the students indicated

some struggles. When examining transferability of algebra language to physics, students indicated in their responses a struggle with interpreting variable meaning in physics and frustration in the use of formulas.

Overall, the results of the study showed that both instructors and students have similar perceptions in the transferability of algebra skills and language to algebra-based physics. When examining both instructors' and students' perceptions of algebra skills, both the instructors and the students indicated concerns in the transferability of algebra skill to physics. The results to the isomorphic problems indicated that students were not able to demonstrate an understanding of graphing concepts in either the mathematics or physics context. These students also struggle with solving quadratic equations when asked to solve for the *x* variable, creating difficulty in examining the transferability of those skills from mathematics to physics. However, when the students did demonstrate the ability to solve the mathematics context problems, they struggled with translating those skills to the physics context.

When examining instructors' and students' perceptions of the transferability of algebra language to physics, both groups indicated similar concerns in the transferability of algebra language to physics. The instructors perceived the students would struggle with translating the vocabulary and variables used in the algebra to physics which was echoed in the students' responses. However, the students also indicated struggles with understanding the formulas used in physics. The students were not able to articulate specific similarities between the two forms of the quadratic equation used in both college algebra and physics. Interestingly, both instructors and students suggested that additional application problems would assist students in making connections between algebra and physics.

Implications

This research has supported previous research on students struggles in transferring mathematics to physics. However, the study has further research by examining both instructors and students perceptions of the transferability of algebra to physics. Instructor and student

responses from the study have indicated that both instructors and students believe that college algebra may not have provided students with the skills and language necessary to be successful in physics. Both instructors and students indicated that they believe that the differences in variables and symbols used in mathematics and physics may cause students to struggle in transferring mathematics to physics. Also, students responses on isomorphic problems suggest that students do not struggle in calculating slope or solving linear equations. However students do indicate struggles when applying those skills to physics. Examination of the isomorphic problems beyond graphing such as linear and quadratic content in this study, has provide further insight into students struggles with solving quadratic equations. This study has offered current research with more data on the transferability of algebra language and skills to physics.

Examining the responses from both instructors and students suggest the need for communication between instructors of mathematics and physics. Such conversations between instructors of mathematics and physics with the support of institutional administration could happen at the beginning of each academic year during instructor in-service time allowing instructors to share not only content specific skills but help develop methods to aid students in transferring mathematics to physics. Instructors would be able to see how mathematics applies in each area of students' studies not just their own teaching area. For example, such conversation between the instructors of both areas could provide physics instructors with mathematical teaching methods that would aid physics instructors in connecting mathematics content to physics for students. Further, conversations between the two areas of instruction would provide mathematics instructors insight into the application of mathematics to physics allowing them to see how students would be expected to use mathematics in physics. Time could also be spent on discussing the mathematical content needed for the students to be successful in physics. Conversations between the two groups of instructors would provide resources that would benefit the students in transferring mathematics to physics.

When moving into the instruction practice, instructors need to be aware of the use of mathematics in physic. This study suggest that students would benefit from additional application problems in mathematics course to aid in transferring both algebra skills and language to physics. First, providing students with opportunities in mathematics class to make connections between linear equations, quadratic equations, and graphing concepts in mathematics to applications in physics could aid the students in transferring the mathematical knowledge to physics. Second, mathematics instruction offering students more opportunities to see equations using variables other than *x*, *y*, and *z* or solving for variables with meaning such as velocity could help students with transferring algebra language to physics. Finally, physics instruction can help students transfer mathematics knowledge to physics by offering the student translations of physics equations into the standard forms of mathematical equations. A student summed up the need to make connections between algebra and physics when stating, "either side could have elaborated or made those connection and it would have been a little smoother going to physics."

Future Research

The differences in combination instructors' responses as compared to mathematics and physics instructors suggest that the combination instructors may have a slightly different perception of students' transfer of mathematics knowledge to physics. Further research into this difference of perception may offer more insight into possible instructional methods used by combination instructors in translation of mathematics knowledge to physics. Additionally, investigating a larger set of students from a combination of colleges could further the research and provide more evidence on students' specific struggles with the transfer of algebra between mathematics and physics. Also, examining isomorphic problems that were open-ended rather than multiple choice may offer more insight into students ability and though process. Continued research in this area could provide instructors with possible instructional knowledge to aid students in transferring knowledge between mathematics and physics so that students would be able to focus on learning physics concepts and not struggling with transferring mathematics.

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APPENDICES

Appendix A

Instructor Background Survey

1) Background Information

1. Gender (fill in)
2. Age (fill in)
3. Ethnicity (choose all that apply)
    a. American Indian or Alaska Native
    b. Asian
    c. African American/Black
    d. Native Hawaiian or other Pacific Islander
    e. Hispanic/Latino
    f. White
    g. Prefer not to report
4. What is your Bachelor's degree in? (fill in)
5. What is your Masters' degree in? (fill in)
6. If you have a doctorate, what is your doctorate in? (fill in)
7. What do you teach?
    a. Math
    b. Physics
    c. Both
8. How many graduate hours do you have in Math?
9. How many graduate hours do you have in Physics?
10. How many semesters have you taught college algebra?
11. How many semesters have you taught algebra-based physics?
12. Please give the title and the author for the college algebra and algebra-based physics text book you use for the classes you teach.

Appendix B

Instructor Perception Study

2) Math Skills

1. College algebra is
   a. Not important for success in algebra-based physics
   b. Moderately important for success in algebra-based physics
   c. Important for success in algebra-based physics
   d. Very important for success in algebra-based physics

2. Physics concepts are
   b. Not important for success in college algebra
   c. Moderately important for success in college algebra
   d. Important for success in college algebra
   e. Very important for success in college algebra

3. Would you say college algebra
   a. Does not prepare students for algebra-based physics
   b. Moderately prepares students for algebra-based physics
   c. Prepares students for algebra-based physics
   d. Higley prepares students for algebra-based physics

4. College algebra
   a. Does not provide the math skill students need for physics
   b. Moderately provides the math skill students need for physics
   c. Provides the math skill students need for physics
   d. Highly provides the math skill students need for physics

5. My students' math skill for studying physics are
   a. Not adequate
   b. Moderately adequate
   c. Adequate
   d. Highly adequate

6. Rate the following items for importance in studying physics using the scale 1- 4 with one being not important to 4 being highly important.

a. Algebraic manipulation	1	2	3	4
b. Scientific notation conversion	1	2	3	4
c. Unit conversions	1	2	3	4
d. Writing equations	1	2	3	4
e. Linear equations	1	2	3	4

f.	Linear functions	1	2	3	4
g.	Quadratic equations	1	2	3	4
h.	Quadratic functions	1	2	3	4
i.	Graphing linear equations	1	2	3	4
j.	Graphing quadratic equations	1	2	3	4
k.	Interpreting graphs	1	2	3	4

7. Please add any other college algebra skills not listed that you consider important in the studying of physics.

8. Please explain the level of algebra skills you believe students **need** to be successful in algebra-based physics.

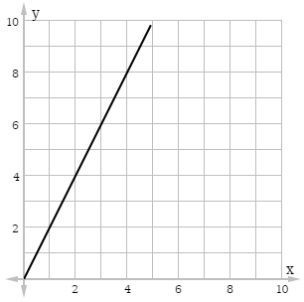
9. Please explain the level of algebra skills student **have** when they enter algebra-based physics.

10. List the algebra skills students are lacking when they enter algebra-based physics.

3) Problems

For questions 1-4, please refer to the problem below.

Consider the following line in the coordinate system. Which statement is correct?



E. The slope of the line is constant and different from zero.
F. The slope of the line is constant and equal to zero.
G. The slope of the line is constantly increasing.
H. The slope of the line is constantly decreasing.

1. Is this a familiar problem?
    a. No
    b. Yes

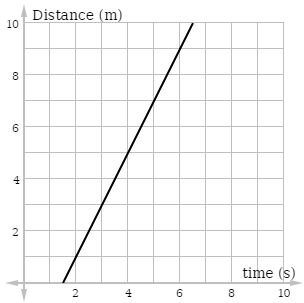
2. Would you use this problem in your class?
    a. No
    b. Yes

3. Do you consider this problem important for your course?
    a. No
    b. Yes

4. Explain why you would/would not use this problem in your class.

For questions 5-8, please refer to the problem below.

Distant-time graph of an object's motion is shown below. Which statement best describes this motion?



A. The object is not moving.
B. The object is moving at a constant velocity.

C. The object is moving with a uniformly decreasing velocity.
D. The object is moving with a uniformly increasing velocity.

5. Is this a familiar problem?
    a. No
    b. Yes

6. Would you use this problem in your class?
    a. No
    b. Yes

7. Do you consider this problem important for your course?
    a. No
    b. Yes

8. Explain why you would/would not use this problem in your class.

For questions 9- 12, please refer to the problem below.

The number of miles that remain to be flown by a commercial jet traveling from Boston to Los Angles can be approximated by the equations

$$miles\ remaining = 2650 - 475t$$

Where t is the number of hours since leaving Boston. In how many hours will the plane be 1000 miles from Los Angles?

9. Is this a familiar problem?
    a. No
    b. Yes

10. Would you use this problem in your class?
    a. No
    b. Yes

11. Do you consider this problem important for your course?
    a. No
    b. Yes

12. Explain why you would/would not use this problem in your class.

For questions 13- 16, please refer to the problem below.

A boat moves slowly out of a marina with a speed of 1.50 m/s. As soon as it passes the breakwater, leaving the marina, it throttles up and accelerates at 2.40 m/s^2. How fast is the boat moving after accelerating for 5 seconds?

13. Is this a familiar problem?
    a. No
    b. Yes

14. Would you use this problem in your class?
    a. No
    b. Yes

15. Do you consider this problem important for your course?
    a. No
    b. Yes

16. Explain why you would/would not use this problem in your class.

For questions 17- 20, please refer to the problem below.

An object is launched at 19.6 m/s from a 58.8 m tall platform. The equation for the object's height s at time t seconds after launch is $s(t) = -4.9t^2 + 19.6t + 58.8$, where s is in meters. When does the object strike the ground?

17. Is this a familiar problem?
    a. No
    b. Yes

18. Would you use this problem in your class?
    a. No
    b. Yes

19. Do you consider this problem important for your course?
    a. No
    b. Yes

20. Explain why you would/would not use this problem in your class.

For questions 21- 24, please refer to the problem below.

A truck accelerates at a rate of 0.444 m/s^2 from rest to a velocity of 2.80 m/s. How long does it take for the truck to reach 40.0 m? Remember that $s_f = v_o t + \frac{1}{2}at^2 + s_o$ where $\frac{1}{2}a = 0.222\ m/s^2$

21. Is this a familiar problem?
    a. No
    b. Yes

22. Would you use this problem in your class?
    a. No
    b. Yes

23. Do you consider this problem important for your course?
    a. No
    b. Yes

24. Explain why you would/would not use this problem in your class.

4) Transferability of Mathematics to Physics Math Skills

1. Math language used in college algebra
    a. Does not resemble the math language used in physics
    b. Moderately resembles the math language used in physics
    c. Resembles the math language used in physics
    d. Highly resemble the math language used in physics

2. The math vocabulary used in college algebra
    a. Does not resemble the math language used in physics
    b. Moderately resembles the math language used in physics
    c. Resembles the math language used in physics
    d. Highly resemble the math language used in physics

3. Math symbols used in college algebra courses
    a. Does not resemble the math language used in physics
    b. Moderately resembles the math language used in physics
    c. Resembles the math language used in physics
    d. Highly resemble the math language used in physics

4. Quadratic equations in college algebra
    a. Does not resemble the math language used in physics
    b. Moderately resembles the math language used in physics
    c. Resembles the math language used in physics
    d. Highly resemble the math language used in physics

5. My students are able to recognize that F=ma is a linear equation?
    a. No
    b. Yes

6. Explain your answer to 5.

7. My students are able to recognize that $v_f = v_o + at$ is a linear equation?
    a. No
    b. Yes

8. Explain your answer to 7.

9. My students are able to recognize $s_o = v_o t + 1/2at^2$ is a quadratic equation?
    a. No
    b. Yes

10. Explain your answer to 9.

11. How important is it for math and physics instructors to collaborate on topics and problems?

12. Explain your answer.

13. Explain your thoughts on the math language used in college algebra compared with the math language used in physics.

14. What are your thoughts about the differences students encounter between the math in college algebra and the math in algebra-based physics?

15. What thoughts do you have on the possibility of student challenges on the language transfer from math class to physics?

16. Do you believe there is a difference in the math language used in math class and the math language used in physics?
    a. No
    b. Yes

17. Explain your answer to 16.

18. What are your thoughts on the need for collaboration between math and physics instructors?

Appendix C

Semi-Structured Interview Questions

1. Tell me a little bit about what you teach and how long you have been teaching.
2. Describe the importance of college algebra in general physics?
3. What algebra skills do you believe students need to be successful in general physics?
4. What level of algebra skills do you believe students have when they enter general physics? Explain.
5. Do you find the math language used in college algebra to be similar to the math language used in physics? Explain
6. Think about a scenario in which the math language used in college algebra is similar to the math language used in general physics. Explain that scenario.
7. Think about a scenario in which the math language used in college algebra is different than the math language used in general physics. Explain that scenario.
8. What connections do you believe students make when moving from the equation for a line $y = mx + b$ in algebra to the linear motion equation $F = ma$ in physics?
9. What connections do you believe students make when graphing linear equations in college algebra compared to physics?
10. How easily do you believe students transfer math vocabulary from college algebra to physics?
11. Explain your thoughts on the role a collaboration between math and physics instructors would have on content in both courses.
12. Do you have anything you would like to add to our discussion or clarify?

Appendix D

Student Demographic Survey

1) Background Information

1. Gender (fill in)

2. Age (fill in)

3. Ethnicity (choose all that apply)

    a. American Indian or Alaska Native
    b. Asian
    c. African American/Black
    d. Native Hawaiian or other Pacific Islander
    e. Hispanic/Latino
    f. White
    g. Prefer not to report

4. What is your major?

Appendix E

Student Perceptions Survey

2) Math Skills

1. College algebra is
   a. Not important for my success in algebra-based physics
   b. Moderately important for my success in algebra-based physics
   c. Important for my success in algebra-based physics
   d. Very important for my success in algebra-based physics

2. Physics concepts are
   a. Not important for my success in college algebra
   b. Moderately important for my success in college algebra
   c. Important for my success in college algebra
   d. Very important for my success in college algebra

3. Would you say college algebra
   a. Did not prepare me for physics
   b. Moderately prepared me for physics
   c. Prepared me for physics
   d. Highly prepared me for physics

4. College algebra
   a. Did not provide the math skills I needed for physics
   b. Moderately provided the math skills I needed for physics
   c. Provided the math skills I needed for physics
   d. Highly provided the math skills I needed for physics

5. I feel that my math skills for physics are
   a. Not adequate
   b. Moderately adequate
   c. Adequate
   d. Highly adequate

6. Explain your answer to 5.

7. Explain how important you feel that math is for physics.

8. Explain any struggles you have with the math in physics.

9. Explain why you have had those struggles

10. Explain any success you have had with the math in physics

11. Explain why you have had those successes.

3) Transferability

1. Math language used in college algebra
   a. Does not resemble the math language I used in physics
   b. Moderately resembles the math language I used in physics
   c. Resembles the math language I used in physics
   d. Highly resembles the math language I used in physics

2. The math vocabulary used in college algebra
   a. Does not resemble the math vocabulary we used in physics
   b. Moderately resembles the math vocabulary we used in physics
   c. Resembles the math vocabulary we used in physics
   d. Highly resembles the math vocabulary we used in physics

3. Math symbols used in college algebra courses
   a. Does not resemble the math symbols we used in physics
   b. Moderately resembles the math symbols we used in physics
   c. Resembles the math symbols we used in physics
   d. Highly resembles the math symbols we used in physics

Examining the two equations $y = ax^2 + bx + c$ and $s_f = v_o t + \frac{1}{2}at^2 + s_o$ for the following questions 4 - 7

4. The first equation $y = ax^2 + bx + c$
   a. We used in math class
   b. We used in physics class
   c. We used in both math and physic

5. The second equation $s_f = v_o t + \frac{1}{2}at^2 + s_o$
   a. We used in math class
   b. We used in physics class
   c. We used in both math and physics class

6. The two equations
   a. Are different types of equations
   b. Are the same type of equations

7. Explain your answer to 6

8. How important is it for math and physics instructors to collaborate on topics and problems?
    a. Not important
    b. Moderately important
    c. Important
    d. Very important

9. Explain your thoughts on the math language used in college algebra compared with the math language used in physics.

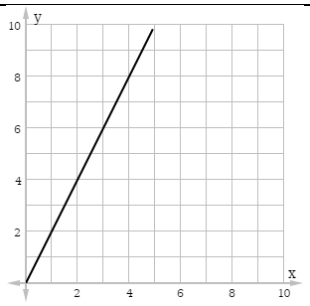
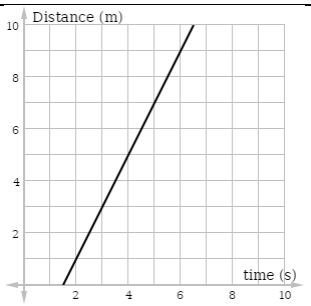
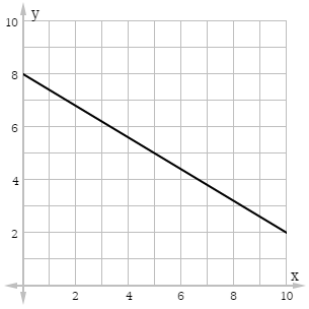
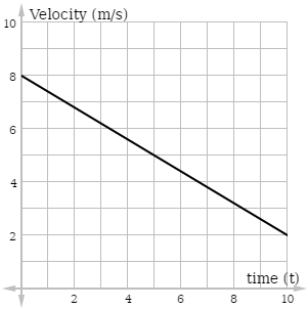
10. What are your thoughts on the need for collaboration between your college algebra and physics instructors for the creations of lectures and problems?

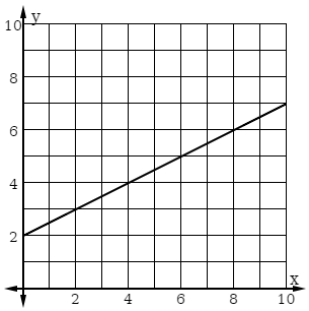
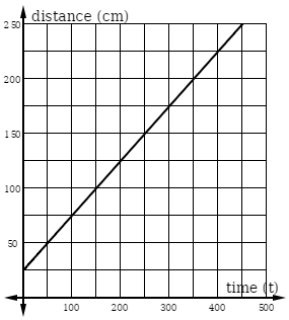
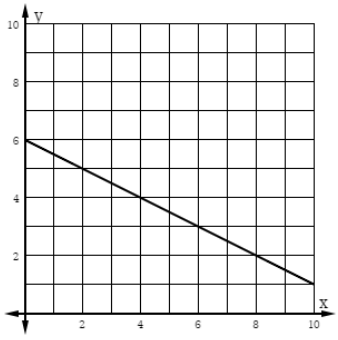
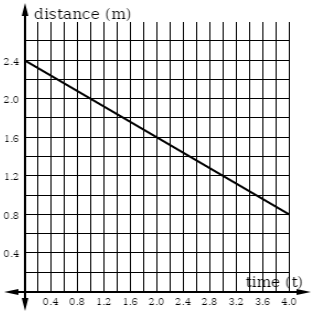
Appendix F

Isomorphic Mathematics and Physics Problems

Content	Mathematics	Physics
Linear $y = mx$ (problems 13 & 4)	Lisa is growing basil from a seed and is tracking the progress of her plant's growth. The plant grows 0.4 cm/day. How many days has it grown to get to reach 30 cm? (Aufmann, Barker, & Nation, 2008)	Calculate the acceleration of a 2000 kg, the single-engine airplane just before takeoff when the thrust of its engine is 500 N. (Serway & Vuille, 2015)
$y = mx$ (problems 10 & 20)	A machine salesperson earns a commission of $350 for every machine he sells. What would be the salesperson's income if he sold 75 machines in a month? (Aufmann, Barker, & Nation, 2008)	Calculate the speed of an apple that falls freely from a rest position and accelerates at 9.8m/s^2 for 1.5 seconds. (Serway & Vuille, 2015)
$y = mx + b$ (problems 6 & 16)	The number of miles that remain to be flown by a commercial jet traveling from Boston to Los Angles can be approximated by the equations $miles\ remaining = 2650 - 475t$ Where t is the number of hours since leaving Boston. In how many hours will the plane be 1000 miles from Los Angles? (Aufmann, Barker, & Nation, 2008)	A boat moves slowly out of a marina with a speed of 1.50 m/s. As soon as it passes the breakwater, leaving the marina, it throttles up and accelerates at 2.40 m/s^2. How fast is the boat moving after accelerating for 5 seconds? (Serway & Vuille, 2015)
$y = mx + b$ (problems 21 & 1)	Bennett and his friends decide to go bowling. The cost for the group is $15 for shoe rentals plus each game. If they played 5 games, what was the cost of each game if they spent $35? (Aufmann, Barker, & Nation, 2008)	A car accelerates from 12.5m/s to 25m/s in 6.0 seconds. What was the acceleration? (Serway & Vuille, 2015)
Quadratic $y = ax^2 + bx + c$ Solve for y (Problems 18 & 9)	A ball thrown off the Golden Gate Bridge can be approximated by $s = -16t^2 - 2t + 220$ with an initial velocity of 2 ft/s, which is 220 feet above the water. How far does the ball travel in 4 s?	A ball is thrown from the top of a 50 m building with an initial velocity of 20.0 m/s. Determine the time required for the ball to hit the street below. Remember that $s_f = v_o t + \frac{1}{2}at^2 + s_o$ where $\frac{1}{2}a = -4.9\ m/s^2$

	(Aufmann, Barker, & Nation, 2008)	(Serway & Vuille, 2015)
$y = ax^2 + bx + c$ Solve for y (Problems 3 & 23)	An object is launched at 19.6 m/s from a 58.8 m tall platform. The equation for the object's height *s* at time *t* seconds after launch is $s(t) = -4.9t^2 + 19.6t + 58.8$, where s is in meters. What is the height of the object in 5.8 seconds? (Aufmann, Barker, & Nation, 2008)	A race car accelerates uniformly at 11.2 m/s from a velocity of 18.5 m/s in 2.47 seconds. Determine the distance traveled by car. Remember that $s_f = v_o t + \frac{1}{2}at^2 + s_o$ where $\frac{1}{2}a = 5.6\ m/s^2$ (Serway & Vuille, 2015)
$y = ax^2 + bx + c$ Solve for x (Problems 17 & 12)	A company has determined that the profit, in dollars, it can expect from the manufacture and sales of x tennis racquets is given by $P = -0.01x^2 + 168x - 120{,}000$. How many racquets should the company manufacture and sell to earn a profit of $518,000? (Aufmann, Barker, & Nation, 2008)	A truck accelerates at a rate of 0.444 m/s^2 from rest to a velocity of 2.80 m/s. How long does it take for the truck to reach 40.0 m? Remember that $s_f = v_o t + \frac{1}{2}at^2 + s_o$ where $\frac{1}{2}a = 0.222\ m/s^2$ (Serway & Vuille, 2015)
$y = ax^2 + bx + c$ Solve for x (Problems 14 & 7)	According to data provided by the U.S. Census Bureau, the number N, in thousands, of centenarians (persons whose age is 100 years or older) who will be living in the U. S. during a year from 2010 to 2050 can be approximated by $N = 0.3453x^2 - 9.417x + 164.1$, where x is the number of years after the beginning of 2000. Use this equation to determine in what year will there be 200,000 centenarians. (Aufmann, Barker, & Nation, 2008)	A ball is thrown downward with an initial velocity of 5 m/s from the Golden Gate Bridge, which is 220 m above the water. How long will it take for the ball to hit the water? Remember that $s_f = v_o t + \frac{1}{2}at^2 + s_o$ where $\frac{1}{2}a = -4.9\ m/s^2$ (Serway & Vuille, 2015)
Graphing (Problems 2 & 19)	Consider the following line in the coordinate system. Which statement is correct?	Distant-time graph of an object's motion is shown below. Which statement best describes this motion?

	 I. The slope of the line is constant and different from zero. J. The slope of the line is constant and equal to zero. K. The slope of the line is constantly increasing. L. The slope of the line is constantly decreasing. (Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012)	 E. The object is not moving. F. The object is moving at a constant velocity. G. The object is moving with a uniformly decreasing velocity. H. The object is moving with a uniformly increasing velocity. (Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012)
(Problems 24 & 11)	Consider the following line in the coordinate system. Which statement is correct?  A. The slope of the line is constant and positive. B. The slope of the line is constant and negative. C. The slope of the line is constantly decreasing and is negative.	Velocity-time graph of an object's motion is shown below. Which statement best describes the motion?  A. The object is moving with a constant non-zero acceleration. B. The object is moving with zero acceleration. C. The object is moving with a uniformly increasing acceleration.

	D. The slope of the line is constantly decreasing and is positive. (Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012)	D. The object is moving with a uniformly decreasing acceleration. (Planinic, Milin-Sipus, Katic, Susac, & Ivanjek, 2012)
(Problems 22 & 5)	Using the graph, find the slope of the line at x=4.  (Bollen, De Cock, Zuza, Guisasola, & van Kampen, 2016)	The graph shows the water level in a flat-bottomed swimming pool at different times. How quickly does the water level change at t=200s?  (Bollen, De Cock, Zuza, Guisasola, & van Kampen, 2016)
(Problems 8 & 15)	Using the graph, find the slope of the line at x=6.  (Bollen, De Cock, Zuza, Guisasola, & van Kampen, 2016)	A ball moves along a track. The graph shows the distance from the ball to a fixed point during several seconds. What is the speed of the ball at t=2.0s?  (Bollen, De Cock, Zuza, Guisasola, & van Kampen, 2016)

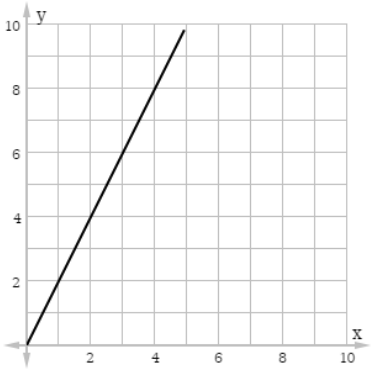
Appendix G

Isomorphic Parallel Mathematics and Physics Problems Student Evaluation Sheet

Show all your work. Circle the correct answer to the questions and then give a brief description of how you got the answer.

1. A car accelerates from 12.5m/s to 25m/s in 6.0 seconds. What was the acceleration?
   a. 8.3 m/s^2
   b. 2.1 m/s^2
   c. 16.7 m/s^2
   d. 0.48 m/s^2

2. Consider the following line in the coordinate system. Which statement is correct?



   a. The slope of the line is constant and different from zero.
   b. The slope of the line is constant and equal to zero.
   c. The slope of the line is constantly increasing.
   d. The slope of the line is constantly decreasing.

3. An object is launched at 19.6 m/s from a 58.8 m tall platform. The equation for the object's height s at time t seconds after launch is $s(t) = -4.9t^2 + 19.6t + 58.8$, where s is in meters. What is the height of the object in 5.8 seconds?

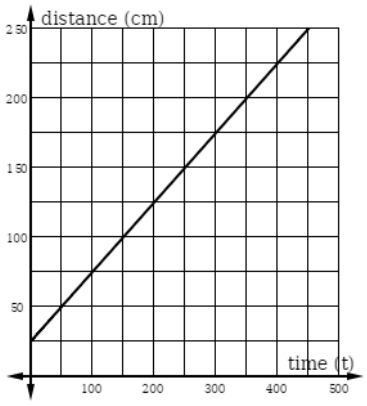
   a. 39.2 m
   b. 144.1 m
   c. 337.3 m
   d. 7.6 m

4. Calculate the acceleration of a 2000 kg, single-engine airplane just before takeoff when the thrust of its engine is 500 N.

   a. 2.5 m/s^2

b. 300 m/s^2
c. 0.4 m/s^2
d. 100,00 m/s^2

5. The graph shows the water level in a flat-bottomed swimming pool at different times. How quickly does the water level change at t=200s?



a. 0.63 cm/s
b. 125 cm/s
c. 2.67 cm/s
d. 0.50 cm/s

6. The number of miles that remain to be flown by a commercial jet traveling from Boston to Los Angles can be approximated by the equations

$$miles\ remaining = 2650 - 475t$$

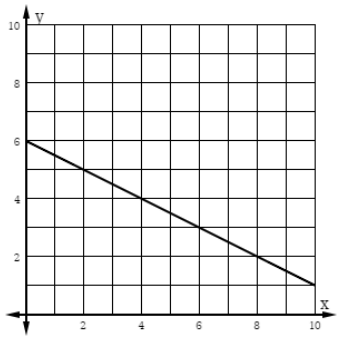
Where t is the number of hours since leaving Boston. In how many hours will the plane be 1000 miles from Los Angles?

a. 131 hours
b. 3.47 hours
c. 7.68 hours
d. 1175 hours

7. A ball is thrown downward with an initial velocity of 5 m/s from the Golden Gate Bridge, which is 220 m above the water. How long will it take for the ball to hit the water? Remember that $s_f = v_o t + \frac{1}{2}at^2 + s_o$ where $\frac{1}{2}a = -4.9\ m/s^2$

a. -6.08 s
b. 6.08 s
c. 7.09 s
d. -7.09 s

8. Using the graph, find the slope of the line at x=6.



a. 2
b. -2
c. 0.5
d. -0.5

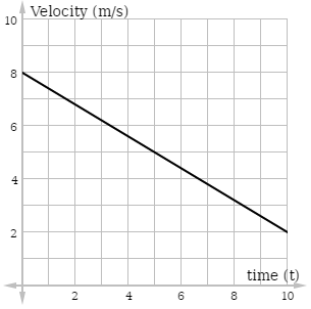
9. A ball is thrown from the top of a 50 m building with an initial velocity of 20.0 m/s. Determine the time required for the ball to hit the street below. Remember that $s_f = v_o t + \frac{1}{2}at^2 + s_o$ where $\frac{1}{2}a = -4.9\ m/s^2$

a. 1.72 s
b. -1.72 s
c. 5.72 s
d. -5.72 s

10. A machine salesperson earns a commission of $350 for every machine he sells. What would be the salesperson's income if he sold 75 machines in a month?

a. $4.67
b. $162.02
c. $131.25
d. $26,250.00

11. Velocity-time graph of an object's motion is shown below. Which statement best describes the motion?



a. The object is moving with a constant non-zero acceleration.
b. The object is moving with zero acceleration.
c. The object is moving with a uniformly increasing acceleration.
d. The object is moving with a uniformly decreasing acceleration.

12. A truck accelerates at a rate of 0.444 m/s^2 from rest to a velocity of 2.80 m/s. How long does it take for the truck to reach 40.0 m? Remember that $s_f = v_o t + \frac{1}{2}at^2 + s_o$ where $\frac{1}{2}a = 0.222\ m/s^2$

    a. 8.51 s
    b. 21.13 s
    c. 13.69 s
    d. 47.20 s

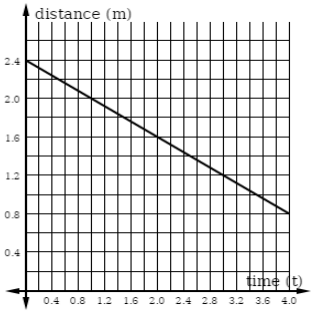
13. Lisa is growing basil from a seed and is tracking the progress of her plant's growth. The plant grows 0.4 cm/day. How many days has it grown to get to reach 30 cm?

    a. 0.013 days
    b. 12 days
    c. 4.8 days
    d. 75 days

14. According to data provided by the U.S. Census Bureau, the number N, in thousands, of centenarians (persons whose age is 100 years or older) who will be living in the U. S. during a year from 2010 to 2050 can be approximated by $N = 0.3453x^2 - 9.417x + 164.1$, where x is the number of years after the beginning of 2000. Use this equation to determine in what year will there be 200,000 centenarians.

    a. The year 2747
    b. The year 2267
    c. The year 2774
    d. The year 2831

15. A ball moves along a track. The graph shows the distance from the ball to a fixed point during several seconds. What is the speed of the ball at t=2.0s?



a. -0.4 m/s
b. 0.4 m/s
c. -1.6 m/s
d. 1.6 m/s

16. A boat moves slowly out of a marina with a speed of 1.50 m/s. As soon as it passes the breakwater, leaving the marina, it throttles up and accelerates at 2.40 m/s^2. How fast is the boat moving after accelerating for 5 seconds?

a. 1.20 m/s
b. 1.50 m/s
c. 3.15 m/s
d. 2.70 m/s

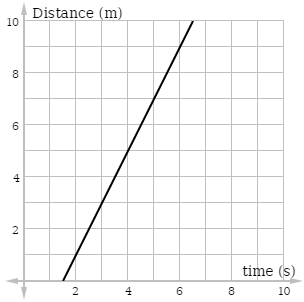
17. A company has determined that the profit, in dollars, it can expect from the manufacture and sales of x tennis racquets is given by $P = -0.01x^2 + 168x - 120{,}000$. How many racquets should the company manufacture and sell to earn a profit of $518,000?

a. 11,000
b. 14,650
c. 110
d. 147

18. A ball thrown off the Golden Gate Bridge can be approximated by $s = -16t^2 - 2t + 220$ with an initial velocity of 2 ft/s, which is 220 feet above the water. How far does the ball travel in 4 seconds?

a. 44 ft
b. -44 ft
c. -8 ft
d. 0.13 ft

19. Distant-time graph of an object's motion is shown below. Which statement best describes this motion?



a. The object is not moving.
b. The object is moving at a constant velocity.
c. The object is moving with a uniformly decreasing velocity.
d. The object is moving with a uniformly increasing velocity.

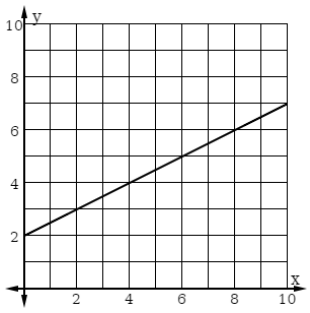
20. Calculate the speed of an apple that falls freely from a rest position and accelerates at $9.8 m/s^2$ for 1.5 seconds.

a. 6.53 m/s
b. 14.7 m/s
c. 27.05 m/s
d. 4.6 m/s

21. Bennett and his friends decide to go bowling. The cost for the group is $15 for shoe rentals plus each game. If they played 5 games, what was the cost of each game if they spent $35?

a. $4
b. $10
c. $22
d. $55

22. Using the graph, find the slope of the line at x=4.

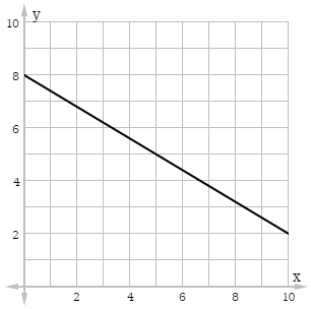


a. 2
b. -2
c. 0.5
d. 4

23. A race car accelerates uniformly at 11.2 m/s from a velocity of 18.5 m/s in 2.47 seconds. Determine the distance traveled by car. Remember that $s_f = v_o t + \frac{1}{2}at^2 + s_o$ where $\frac{1}{2}a = 5.6\ m/s^2$.

a. 428.34 m
b. 59.53 m
c. 114.03 m
d. 79.86 m

24. Consider the following line in the coordinate system. Which statement is correct?

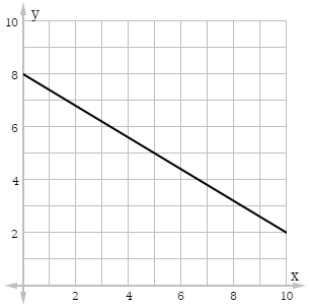


a. The slope of the line is constant and positive.
b. The slope of the line is constant and negative.
c. The slope of the line is constantly decreasing and is negative.
d. The slope of the line is constantly decreasing and is positive.

Appendix H

Task-Based Interview Problems

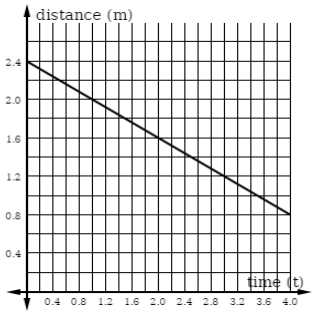
1. Using the graph, find the slope of the line at x=6.



2. A machine salesperson earns a commission of $350 for every machine he sells. What would be the salesperson's income if he sold 75 machines in a month?

3. A company has determined that the profit, in dollars, it can expect from the manufacture and sales of x tennis racquets is given by $P = -0.01x^2 + 168x - 120{,}000$. How many racquets should the company manufacture and sell to earn a profit of $518,000?

4. A ball moves along a track. The graph shows the distance from the ball to a fixed point during several seconds. What is the speed of the ball at t=2.0s?



5. A boat moves slowly out of a marina with a speed of 1.50 m/s. As soon as it passes the breakwater, leaving the marina, it throttles up and accelerates at 2.40 m/s^2. How fast is the boat moving after accelerating for 5 seconds?

6. A truck accelerates at a rate of 0.444 m/s^2 from rest to a velocity of 2.80 m/s. How long does it take for the truck to reach 40.0 m? Remember that $s_f = v_o t + \frac{1}{2}at^2 + s_o$ where $\frac{1}{2}a = 0.222\ m/s^2$.

Appendix I

IRB Approval Letter

Oklahoma State University Institutional Review Board

Date:	Wednesday, January 24, 2018
IRB Application No	ED17166
Proposal Title:	Transferability of Algebra Language and Skills to Algebra-based Physics: Students and Instructors Perceptions and Student Skills
Reviewed and Processed as:	Exempt

Status Recommended by Reviewer(s): Approved Protocol Expires: 1/23/2021

Principal Investigator(s):

Kathleen Otto
1220 E Grand
Tonkawa, OK

Juliana Utley
233 Willard
Stillwater, OK 74078

The IRB application referenced above has been approved. It is the judgment of the reviewers that the rights and welfare of individuals who may be asked to participate in this study will be respected, and that the research will be conducted in a manner consistent with the IRB requirements as outlined in section 45 CFR 46.

☐ The final versions of any printed recruitment, consent and assent documents bearing the IRB approval stamp are attached to this letter. These are the versions that must be used during the study.

As Principal Investigator, it is your responsibility to do the following:

1. Conduct this study exactly as it has been approved. Any modifications to the research protocol must be submitted with the appropriate signatures for IRB approval. Protocol modifications requiring approval may include changes to the title, PI advisor, funding status or sponsor, subject population composition or size, recruitment, inclusion/exclusion criteria, research site, research procedures and consent/assent process or forms.
2. Submit a request for continuation if the study extends beyond the approval period. This continuation must receive IRB review and approval before the research can continue.
3. Report any adverse events to the IRB Chair promptly. Adverse events are those which are unanticipated and impact the subjects during the course of the research; and
4. Notify the IRB office in writing when your research project is complete.

Please note that approved protocols are subject to monitoring by the IRB and that the IRB office has the authority to inspect research records associated with this protocol at any time. If you have questions about the IRB procedures or need any assistance from the Board, please contact Dawnett Watkins 219 Scott Hall (phone: 405-744-5700, dawnett.watkins@okstate.edu).

Sincerely,

Hugh Crethar, Chair
Institutional Review Board

VITA

Kathleen Elaine Blanton Otto

Candidate for the Degree of

Doctor of Philosophy

Thesis: TRANSFERABILITY OF ALGEBRA LANGUAGE AND SKILLS TO ALGEBRA-BASED PHYSICS: STUDENT AND INSTRUCTOR PERCEPTION AND STUDENT

Major Field: Mathematics Education

Biographical:

Education:

Completed the requirements for the Doctor of Philosophy in Mathematics Education at Oklahoma State University, Stillwater, Oklahoma in December, 2018.

Completed the requirements for the Master of Science in Secondary Education at Northwestern Oklahoma State University, Alva, Oklahoma, 2003.

Completed the requirements for the Bachelor of Science in Chemical Engineering at Oklahoma State University, Stillwater, Oklahoma, 1995.

Experience: Director of Institutional Research and Assessment, Northern Oklahoma College, Tonkawa, Oklahoma, 2013-present; Chair of Math and Engineering Division, Northern Oklahoma College, Tonkawa, Oklahoma, 2010-2013; Instructor of mathematics, chemistry, and physics, Northern Oklahoma College, Tonkawa, Oklahoma 2005-2010; Adjunct instructor of chemistry, Northern Oklahoma College, Tonkawa, Oklahoma, 2003-2005

Professional Memberships: School of Science and Mathematics Association, National Council of Teachers of Mathematics, Oklahoma Association for Institutional Research, Association for Intuitional Research