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Connecting Mathematics and Community: Challenges, Successes, and Different Perspectives

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In this article, we summarize our personal journey to establish a successful Math Circle in a community that is not very familiar with such mathematics enrichment programs. We share the story of how our Math Circle began three years ago, as well as the lessons we learned and our organizational challenges and successes. Additionally, we outline three primary perspectives: the founder perspective, the student volunteer perspective, and the faculty volunteer perspective.

Keywords: Math Circle, Peer Teaching, Exploding Dots

1 Introduction

Through math competitions, guest speakers, math festivals, summer math programs and internships, Orlando Math Circle[1] (OMC) promotes engagement and creative expression in mathematics. The variety of such math events in the circle provides a large selection of extracurricular activities for students in Orlando.

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[1]https://www.orlandomathcircle.org/

In developing lesson plans that are accessible for all learners, we routinely explore means of offering math enrichment to our students, regardless of their age or ability. One way to do this is to provide an innovative peer teaching and learning experience in a Math Circle environment. The OMC creates academic collaboration between elementary, middle school, high school, and college age participants across the greater Orlando area. Given the various levels of experience and the need to tailor the activities differently at each level, one tool OMC routinely uses to challenge its more advanced students is to task the older students with preparing lesson plans for the younger students that will be taught in local community centers and libraries. This peer teaching model is important to not only hone the older students' understanding of a topic, but also to give them the opportunity to develop their skills in training, mentoring, public speaking and project planning. In addition, this practice creates a collaborative learning environment for all Math Circle participants and increases student ownership in the circle. The method of peer teaching has proved to help students learn how to share knowledge and build a collaborative learning environment on their own [2]. We have developed and refined this model in the three years that the OMC has been operating.

Webb and Farivar [11] indicate that one of the major benefits of peer teaching is the ability of peers to communicate complex ideas more clearly to each other because they all share a similar language. We introduce students to mathematical ideas that they do not come across in the curriculum. For example, OMC explores topics like combinatorics, number theory, and graph theory – all of which are beyond normal school curriculum. Even advanced students who take calculus in high school are usually unfamiliar with these kinds of topics. Through lesson plans such as those found in [1], our circle challenges students to think outside the box and to engage in mathematical conversations beyond classroom mathematics.

There are also several challenges and concerns regarding peer teaching practices. For example, Sprinthall and Scott [8] note that in a setting where 11^{th} graders tutored fourth and fifth graders on algebra, the program benefited tutors' psychological development, but did not improve their algebra achievement. In school math tutoring programs, the age gap difference between the tutor and the tutees as well as gender mix of the group were studied extensively in the literature [7, 8]. Both Sprinthall [8] and Robinson [7] emphasize that peer and cross-age tutoring programs benefit students who participate as tutors and tutees. In our activities, we encountered age gaps between activity leaders and participants as large as six or seven years, but most of the time activity leaders teach students two or three grades below them. We did not observe that age difference in our practices has a negative impact on student

learning. Ideally, we prefer a girl and a boy to lead an activity, but we had situations where girls were leading a group boys or boys were leading a group of girls.

In several ways, our circle is similar to the Orange County Math Circle. However, in our program, college students work closely with high-school students and help them teach math activities to younger mathematicians. College students also take on administrative tasks. For example, Rollins College, Winter Park, FL helped bring the 2019 Julia Robinson Festival by recruiting their students as volunteers for half of the activities. Valencia College, Orlando, FL participated in our circle by bringing their high school scholarship students to the 2019 Julia Robinson Festival as part of volunteer programming in the Horizon Scholars Program.

1.1 The Challenge of Establishing Math Circles in our Community

From the beginning, our goal has been to make OMC a program that served the needs of all students who enjoy learning mathematics and inspire those who struggle with the subject to not give up. We do not specifically attempt to improve students' test scores. We want students to learn how to explore mathematical subjects, ask "why" questions, and improve their mathematical reasoning skills. While it was not difficult to convince STEM professionals and/or higher income families the value of our outreach, it was more difficult to convince people in underserved communities the value of our work, and thus to reach these communities.

As we watched our student population grow, we also observed that most of our participants who have parents who are STEM professionals and/or high or middle-income parents seek out a Math Circle for their children, while families with financial hardship or families with non-technical degrees did not send their kids to attend our Math Circle program. Most of the time, those parents did not know what a Math Circle is. Some parents cannot enroll their children in any program that requires transportation after school. For us, this sparked the challenging goal of finding ways to reach out to those parents and the students to introduce them to the idea of Math Circles. OMC first started doing outreach at a low-income public school where a personal connection helped our founding high school students run a math enrichment program in one school in 2017. When we presented a formative evaluation report to our school district in order to be formally recognized as a free school enrichment program, we were denied permission to establish a program at that school because our sample data did not show a significant positive impact on student standardized test performance.

Disappointed, but not deterred, the next places where we tried to introduce a Math Circle program were two different after-school programs. One of them was the Boys and Girls Club of Central Florida. The director of Boys and Girls Club was interested in starting a trial program, but the only way it could be done was if the site leaders would support it. We were asked to present our proposal on the value of a Math Circle program in front of 20 site leaders. Unfortunately, they were not interested in our proposal. One of the funders was presenting at the same meeting and during her presentation, we learned that attendance at the site, attendance at school, and juvenile detention were the major metrics to determine funding a trial program at the Boys and Girls Club. So again, we changed tack.

Our next attempt was a local community center with an after-school program for low income students who were bused to the center after school, while some of their peers stayed for after-school clubs and programs at the school. Students who came to the community center were the ones who did not have a ride home after school. The only ride available was the bus that took them to the community center until their parents could pick them up. The community center did not have any specialized programs for the children until this year. Our offer to come in to the community center and run a math enrichment program was not accepted until we established a contact with a community organizer, LaWanda Thompson of Equity Council, who promoted the benefits of a Math Circle program on our behalf. With the support of this organizer, we were able to run 60-75 minute weekly sessions at the community center for this past year.

Even though the benefits of our program are clear to us, we learned that it is important to communicate those benefits to community organizers, teachers, and administrators who are overworked and stressed about being judged by standardized test scores. We acknowledge that there is a need in schools for mathematics tutoring that could help to raise test scores on standardized tests, but we believe that there is also a need for programs that highlight the "fun and interesting" side of learning mathematics. One of our biggest challenge is to convince school administrators to see the value of a Math Circle that does not offer a traditional tutoring program. Our plan is to evaluate our program by collecting more data to show that Math Circle programs provide better learning opportunities [2, 3, 5].

2 Three Different Perspectives on Math Circles

Our Math Circle was established as a student-led program. It was launched by Ari Azbel in January of 2016. At the time Ari was a student at Lake High-

land Preparatory School and valued learning as a creative and collaborative process. He recruited other students to serve as mentors as well. In Spring 2017, we reached out to local universities and colleges to connect with interested mathematics faculty and recruit student volunteers from their campuses. Currently, our circle includes a board of directors who are community volunteers, an advisory board of mathematicians, and a group of college student volunteers. Perspectives from the founder, two student mentors and a faculty volunteer are included below.

2.1 Math Circle Founder Perspective

I started competition math in eighth grade. I loved that it focused on math beyond the typical school curriculum, but the competitive aspect was never my cup of tea. I began to realize that not only were math competitions focused on results, but so was math instruction. For instance, AP Calculus was completely indexed towards the exam, which meant that our learning was constrained by the parameters of the exam. Instead of primarily focusing on the concepts behind the equation, most instructors just teach how to get the right answer the fastest way.

The pressure of performing well and scoring highly slowly wore away at my passion for math, to the point where I began to question whether I even enjoyed math at all. I needed to find a way to foster my vision of math. Learning should be a creative and collaborative process. Students should learn for the sake of learning, and not for the sake of simply doing well on an exam. Orienting education towards a fixed path begins to limit what students can learn, missing creative possibilities.

When I realized the need for an environment that would focus on less traditional algorithms of math, I decided to take matters into my own hands. With this goal in mind came the Orlando Math Circle – an organization targeting students of all ages and backgrounds that promotes inclusivity and emphasizes creative applications of math. My work with the OMC has brought about many opportunities. For example, I regularly taught at a local middle school on Saturdays, where I was able to foster math education in a non-test-oriented setting. Through the growing community of its students, OMC has been able to foster creative ways to explore advanced mathematical concepts, such as real-world applications of fractals, binary representations and number theory. These concepts were able to be taught in a way that prioritizes the process over the solution.

I was amazed that people in mathematics like Douglas O'Roark (Chicago Math Circle), Dr. Loh (Carnegie Mellon University), and Dr. Tanton (Math-

ematical Association of America) were so responsive and helpful with this project. I joined the Global Math Project in 2016. I was intrigued that this math event would reach across different countries and cultures through the math activity of Exploding Dots [4]. I thought OMC would be a good vehicle to involve local students in this very same process.

2.2 Math Circle Student Mentor Perspectives

On February 26, 2018 Isabella Delbakhsh and Tami Heletz, two active Math Circle students, were offered the opportunity to develop a lesson plan to teach third grade students how to solve equations differently by using base two. Their lesson was based on and inspired by Exploding Dot activities [4] by Dr. Tanton and promoted by Global Math Project.

The Exploding Dots activity has beautifully connected students in math exploration regardless of their grade or level of math understanding. Middle school and high school students have successfully explored challenging Exploding Dots problems with college students, and these experiences have inspired the middle school and high school students to introduce the activity to elementary students. This wonderful activity has helped us fulfill our mission of creating an inclusive student community of mathematicians.

The intention of the lesson developed by Isabella and Tami was to not only give the third graders a chance to explore the Exploding Dots activity, but for these young middle school students to enhance their own comprehension by developing and implementing lessons for the younger students. Below, these young trainers share their experience of executing a lesson plan in a Math Circle setting.

Having been involved with Exploding Dots lesson [4] for a couple of years, we had a good understanding of how this model can be used to solve equations and thought it was an interesting way to convert from base ten to binary. When we were asked to prepare a lesson to share our knowledge with younger students, we were excited for the opportunity to instill a passion for math in kids like OMC did for us when we were first introduced to Exploding Dots. We wanted to develop a lesson that was not just about the learning, but that was engaging and would help break down the common fear around mathematics. We did not think this was going to be that hard and we were surprised to find it actually was much trickier than expected.

To get started, we met with our Math Circle leader to discuss what we wanted to cover in the lesson at a high level. We all agreed we wanted it to feel game-like for the students, so we collaborated to prepare a simple document with an outline of what topics to cover and questions we wanted to ask the kids

to get them thinking in the right way. Thirty minutes prior to giving our first lesson, we went over the outline and did a mock lesson with Margarita Azbel. Using her feedback, we adjusted a few things to help clarify the concepts, including adding an introduction with an abacus and an explanation of how to convert a base ten number to base two.

When the lesson began, we started to explain what an abacus was and how it could be used for count in base ten and then we gave a few examples showing place value using the abacus. The next topic we covered was how to convert from base two to base ten and then back from base ten to base two. We did not want to spend too long talking so the kids could learn through more hands-on activities, so we made sure to keep the lecture part under 10 minutes as we introduced the Exploding Dots. From there, the kids got to pick numbers to represent on the abacus and they took turns solving equations with it.

Next, we focused on teaching base two using Napier's checkerboard [9] which we found is a great tool for illustrating base two because it clearly shows how to add and subtract base two numbers by displaying them on the board with chips and then sliding the chips to the bottom to add them. After sliding, the next step is to "explode" each place where there are more than two chips and read the new base two number. We allocated 30 minutes to the activity, but it took a little more time to explain and run than anticipated, so we did not have time to do higher level problems.

Once all the students demonstrated an understanding of how to use Napier's checkerboard [9], we introduced the first equation: how to express the base ten number seven in base two. We attempted to explain it verbally at first, but the kids did not understand this so we needed to quickly change tack and tried explaining it a different way. They started to understand more when we used a visual demonstration with the chips that showed how two of them exploded and fused together to turn into one in the next box, but they still were not confident on the topic. Both of us had to think on the spot and find different ways to convert a number expressed in base ten to base two. To do this, we gave the kids M&Ms to represent a difficult number and walked them through the steps to express the number in base two. After this we were able to proceed to an exploration of how to add two numbers in the base two system.

When the lesson concluded, we both thought it was very successful, but there were definitely things that could have been improved, such as coming up with better ways to explain challenging ideas to the students. In the process, we learned to adapt to the kids and teach in a way that they would understand. Once we figured out how to be agile and flexible during the lesson, the rest of the session went much smoother.

We were invited to teach this lesson to a much larger group at a local community center in partnership with two 9th graders who volunteer with OMC at the center. To prepare, we wrote a revised lesson plan, and we added more questions for each part of our lesson to help direct the learning.

Before the class, the two community center partners had no idea how to write numbers in the base two system, so we practiced with them as we would with the students. We came up with a plan for the four of us to work together cohesively, with us demonstrating the lesson details and the two boys engaging the kids and supporting them in the active learning process to draw out a solution to a question we would present.

Part way through the lesson, we realized some of the students appeared disinterested, so, again thinking on our feet, we decided to create a competition between the two tables of children. We gave the children numbers 1-20 to convert to base two and they raced to be the first team to solve these equations collaboratively, one team vs. the other team. Pretty quickly all the children were engaged in the process and were able to solve these problems without any help.

We did not realize it initially, but as the lessons progressed we started to guide our younger peers instead of teaching them. We would point them in the right direction and then they would carve their own path to solving the problem. Rather than saying, "No. That's not right, it has to be this way," we would say, "How did you get that?" and lead them in the right direction again. This was huge for the students because they weren't afraid of getting the wrong answer and they were able to use their own creative thinking to solve the problems, which is an experience that does not happen often with the math lessons that students experience in school.

For our own development, this experience challenged us to really think about different ways of modeling Exploding Dots and how to express these in a clear, digestible manner. Moreover, we learned that no lesson plan is ever "set" and that improvements need to be made after each iteration, especially in terms of time management and being able to come up with different examples if the planned examples do not work. We learned that it is helpful to have additional activities and questions. In the Appendix, we include Isabella and Tami's activity lists and plans inspired by Exploring Dots and Napier's checkerboard.

This adventure has shown us that kids (and adults) learn best when they are given the opportunity to find the way that works best for them. There are many ways to solve one simple problem, and everyone understands problems in their own way. If someone does not understand a certain way of looking

at a problem, a Math Circle facilitator can guide them toward a different way that they might understand better.

2.3 Math Circle Faculty Volunteer Perspective

I look at this opportunity as my chance to experience a Math Circle firsthand. A mathematics education faculty member and I planned to collaborate on establishing a theme-based Math Circle when I first moved to Florida in 2010. As a mathematics educator, his focus was to develop a Math Teachers' Circle. In my personal experience, most teachers struggle to find time to do anything, let alone a professional development program without any financial or career-based incentives. As we discussed who we should target for our Math Circle, we never actually got around to finalizing the project. Therefore, I was very excited to receive an e-mail from Margarita Azbel about an existing Math Circle for Students in the Orlando area.

The OMC organizers first contacted me about volunteering at the Julia Robinson Math Festival in February 2019. I offered this opportunity to my students in a 300-level mathematics elective course. They were very excited to be part of the festival. We practiced some of the online lesson plans [6] in class beforehand. Their favorite lesson plans were "Wolves and Sheep" and "The Game of Set." In the process, I realized that I do not have any experience in communicating mathematics to fourth through sixth graders. I had used board games and card games to teach some mathematics topics to my students at higher education institutions; however, I had never used games in my math activities with community partners.

My outreach efforts have concentrated on participating in "Women in STEM" and "Careers in Math" themed panels and providing quantitative data analysis support to community organizations. In Fall 2012, I also taught a first- year seminar course that has a significant service- learning component where my students collaborated with a community partner, Rollins College Child Care and Student Research Center (CDC), to raise awareness of childhood obesity [10]. In the process, students were involved in every aspect of a statistical study: preparing surveys about eating habits, going through the IRB process, collecting data on kids' eating behavior by regular on-site visits, analyzing the data, preparing reports by Excel, and finally presenting their findings to parents, faculty and the CDC staff. The statistical study on healthfulness of the CDC snack options helped CDC revise their menu options. During their first semester at Rollins, students saw that mathematics can influence social change. As a faculty member, I observed that such activities help make the class interactive and create an enjoyable learning environment

for students and participants. In some cases, I was worried about spending too much time with the community partner and not having enough time to cover the course material for the day. I believe that participating in OMC activities will help me learn how to easily and efficiently integrate community-engagement projects and hands-on activities into my own teaching.

3 Conclusion

OMC aims to provide learning opportunities that enable students to work on their own understanding and improvement as they introduce mathematics activities and problems to others through peer teaching. Seeing OMC make a long-lasting impact in so little time is extremely exciting and rewarding for all of us, and we cannot wait to see how far its reach can span in the future.

We have indeed learned a great deal in these three years. For example, choosing a place that most students and parents are familiar with is important. For our Math Circle activities, we choose a meeting place, Winter Park Community Center, that is easily accessible to the students. Our prior experience showed that transportation could create problems for some students attending after-school enrichment programs. We also learned that it is important to clearly present the idea of our circle to the teachers and school administrators. Currently, many schools are under a lot of pressure to increase student scores on standardized tests. Therefore, they are mainly interested in activities that might improve student performance in testing situations. As we describe our Math Circle to teachers, school administrators, students, and their parents, we emphasize the importance of inspiring students to become lifelong learners in mathematics.

We still struggle with some aspects of running a Math Circle in Orlando. The culture of Math Circles is a relatively new phenomenon in Orlando. Most of our efforts go to recruitment and explaining the idea of a Math Circle to teachers and parents. As we promote engagement and creativity in mathematics, it is not easy to assess the program. We talk to students about their experiences and observe their involvement and the collaboration during the activities, but we do not have a rubric that helps us collect data to evaluate our program success. One of our future goals is to improve our assessment methods. In the future, we plan to continue to collect quantitative data to improve our program and widen its network.

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Appendix

M&M's with EXPLODING DOTS!!!

Directions: Base 2

*We are using a $2 \to 1$ machine which means if we have 2 dots (M&M's) they will "explode" since there cannot be two dots in one box

Step 1) Draw a 5 by 5 graph.

Step 2) As a group, pick a three digit number that is odd...to begin, let's chose $106 + 51 + 42$

Step 3) Write out 128, 64, 32, 16, 8, 4, 2, 1

- Why are we writing out those numbers?

Step 4) To make 106, we need what numbers from 128, 64, 32, 16, 8, 4, 2, 1? Take your M&M's and put one in the number boxes needed to create 106.

Step 5) We are going to do the same for 51 on the row and above our 106 now

Step 6) Do the same thing for the number 42...

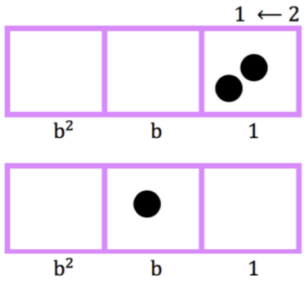
This is where it gets interesting...if you had the questions of why these are called EXPLODING DOTS, your mind is about to be blown...

Step 7) In each column, we are going to smush ALL of the M&M's together on the first row. MAKE SURE THE M&M's STAY IN THEIR COLUMN.

Step 8) If we are doing BASE 2, we are going to need to get rid of anything that is a two or a one...

Step 9) Add the numbers left, What do we get?

Step 10) EAT THE CANDY!!!



Notes from Trial Class:

- Figure out who says what so we can all say certain things.
- Know how to explain to kids. Think of different ways to explain so that if they do not understand one way we can use another.
- Do not spend so much time on the abacus.
- Get exact questions and path ready in this document.

REVISED PLAN:

- **ABACUS:**
  - This is called an abacus. Maybe you have played with it as a kid, but does anyone know what really it is used for?
  - You can actually add and subtract numbers on here.
  - Let's say the bottom row holds one place digits and the second holds the tens digits. What does the third on hold? And the 4th one?
  - Let's try to put a number on the abacus. Choose a number from 1-99. Map it out on the abacus.
  - Choose a different number from 1-99 and map it.
  - Now let's add the two numbers. First, map out the first one again. Here is a rule: If you add the numbers in the ones digit place and you get 10 or more, then you have to push them back or make them "explode", then add one to the tens digit row. Go over how it works and make sure they know why we do that.
  - Let them add two 2 digit number
- **BASE 2 WITH CHIPS:**
  - What we just did on the abacus was add numbers normally. But did you know that there are different languages of numbers? People usually use the language of base 10. We are going to learn how to translate it to base 2, or binary.