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A Graph-Based Analysis of Anton Chekhov's *Uncle Vanya*

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Synopsis

We analyze the famous Anton Chekhov play *Uncle Vanya* by means of graph theory. Moreover, we make the 'brave' suggestion that Chekhov might have used graphs to represent the plot of the play and the relationships between characters. Our analysis also includes the analysis of a specific performance of the play held in Bielsko-Biała which differs slightly from the original script. The differences between the two versions are traced via graph-based analyses. When a first round of clique assignments did not give much insight we transformed them via a sequence of operations on consecutive graphs. The final graphs obtained this way from the two versions of the play differ only by one loop, which disturbs the initially intended symmetry. We explore some of the unexpected consequences.

Keywords: *community detection, graph transformations, interdisciplinary knowledge transfer, drama analysis.*

In the play *Humanka* of Jarosław Murawski, performed since 2017 at Theatre Polski in Bielsko-Biała (Poland), a graph and its transformations were used as virtual representations of a futuristic video-phone gadget used by the characters in the play for mutual connections. *Humanka* takes place in the 22nd

century; therefore, the scenographer exposes modernity via graphs! A successful connection by means of this futuristic gadget is graphically presented on the stage rear wall by means of dynamic transformations of the graph. The transformations consist of changes of the vertex positions randomly for several seconds, and finally after stabilization of the graph, the video-phone connection is performed.

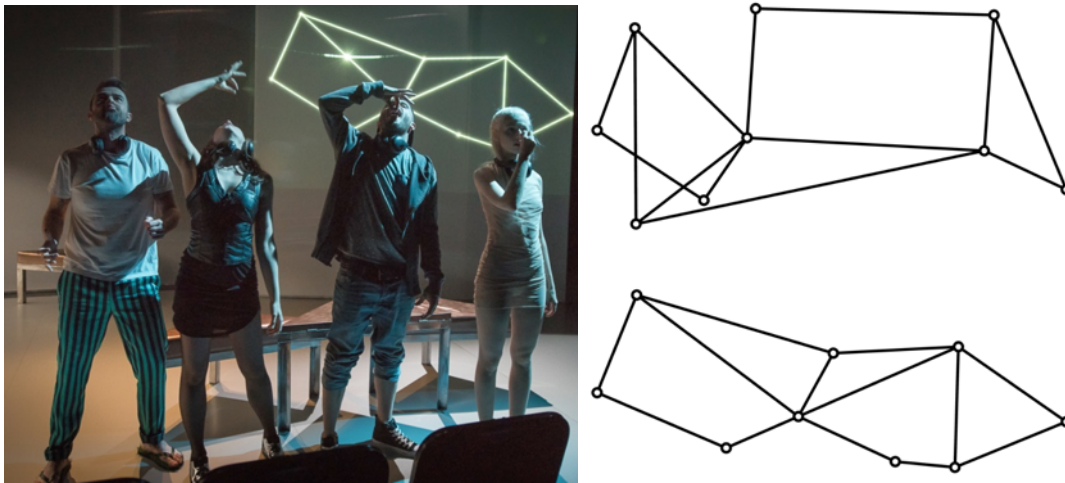


Figure 1: A scene of the play *Humanka* (a), and two versions of the graph presented in the stage rear wall (b).

As our main research area is graph-based modelling in engineering [49, 50], this neat graphical representation led us to several questions:

Could the graphs be not only for stage decoration?

How else could they be utilized?

What are the most fruitful applications of graphs in modelling a theatrical reality?

We wrote about some preliminary graph-based explorations of *Humanka* and some other plays performed in Bielsko-Biała in [51]. In this paper we present a similar analysis of the Chekhov masterpiece *Uncle Vanya*.

1. Background

1.1. Remarks on Anton Chekhov and *Uncle Vanya*

Anton Chekhov was an outstanding Russian playwright whose achievements are commonly recognized as evergreen and classic [55]. He was born in Taganrog in Russian Empire in 1860 and died in 1904 in Badenweiler near Freiburg im Breisgau (Germany). His most famous plays are *The Cherry Orchard*, *Three Sisters*, and *Uncle Vanya*. Taganrog is an harbour on the Azov Sea, connected via a narrow strait to the Black Sea. During Chekhov's lifetime, the place was full of immigrants and international merchants, so untypical for contemporary Russia. Chekhov was a physician and practiced his learned profession for his entire life. He lived for many years in Moscow and his plays were performed in the famous Stanislavski's Moscow Art Theatre as well as the rest of Europe.

Chekhov was extremely talented, but in our opinion one of the reasons why his works are still alive is that he was a practicing physician as well as a writer. In those days interviewing a patient was a crucial basis for a doctor's diagnosis. Also people told their doctors stories of their entire lives and not only the facts connected with their pains and diseases. Therefore he had a deep insight into life in different social strata.

Uncle Vanya was first published in 1898 and premiered in Moscow a year later. The text of the play in English is available online [54]. In the 20th century, there were several attempts to make a movie based upon *Uncle Vanya*: in USA (1957), by directors Franchot Tone and John Goetz, U.K. (1963), by director Stuart Burge, and finally in the Soviet Union (1970), by director Andrei Konchalovsky. Wikipedia tells us that this play is unique among Chekhov's major works because it is the result of the author reworking an older play of his that had originally been published a decade earlier [58]. The changes by Chekhov and by his adaptors are still investigated by scholars; see for instance a recent doctoral thesis [28] submitted to the University of Toronto in 2010.

1.2. Mathematics and the theatre

Much has been written on the connections of mathematics and the arts; see [1, 2, 4, 5, 14, 21, 26, 34, 42, 43] for an eclectic selection of related references.

The relationship of mathematics and theatre is not as commonly explored as the relationship of mathematics with music, fine arts, and architecture. However, there are some recent works in this direction, too. A brief historical review of the topic (mathematics and theatre) is given in [29], where the author mentions a course on mathematical teatrology taught in Bucharest in 1966.

There is also a plethora of mathematically inspired plays written and performed in the recent years. Tom Stoppard's *Arcadia* (1993) and David Auburn's *Proof* (2000) come to mind. Also see [15] for a more recent play written about the life story of Kepler in the context of a history of mathematics course.

Much of the relevant recent scholarship on mathematics and theatre explores how theatre can help teachers in learning different subject and skills — including mathematics [3, 11, 18, 19, 32, 35, 44]. Other papers on this topic are dedicated to the general relationship between theatre and mathematics [22, 37, 46] from the points of mathematicians and playwrights; analysis of the social network of characters in Homer's *Iliad* and their connections [23] as well as analysing texts of the plays via statistics methods [6, 9, 10].

1.3. Graph Theory and theatre

One of the pioneers of using graph theory in analysing works of literature and performance art was the famous graph theorist Frank Harary (1921-2005), who wrote on a famous Mozart's opera [17] and one of Iris Murdoch's novels [16]. Some theory and examples of modelling drama via graphs are described in [30]. In paper on Homer's *Iliad* mentioned earlier [23], the analysis of the network of characters is performed by means of graph theory, using concepts such as the diameter of a graph, link density, and average shortest path. In recent years, graphs are taught in the context of play analysis methodology at University of Nebraska in Omaha in USA, see [24, 25]. The versatility of these methods can be seen in the series of important papers by Amelia C. Sparavigna and collaborators [38]-[41]. Another new idea, that of interactive drama systems, is now used in the creation of plots for computer games, where graphs are essential [2, 34, 36, 42]. Graphs are also used in the detection of scene sequences [33], drama hierarchies [27], artificial storytelling, cinema analyses and extracting emotions from art [20], and robot-drama [31].

In general, schemes representing particular relations among play characters can be different depending on the plot of the original work. The graph in Figure 2 represents acquaintanceships between the characters in Marc Camoletti's *Boeing, Boeing*. We can see that the three stewardesses do not know each other. The character Bernard would like to avoid any meeting of these three in any case. The stewardesses' names are given twice due to the different versions of the play. The comedy leads up to the final meeting of the young women at least with Robert. So, the graph evolves throughout the course of the play. In other words, the graphs representing the network of connections can be stable or they can fluctuate during the course of the action. In case of *Uncle Vanya* the graph remains the same for the whole play time.

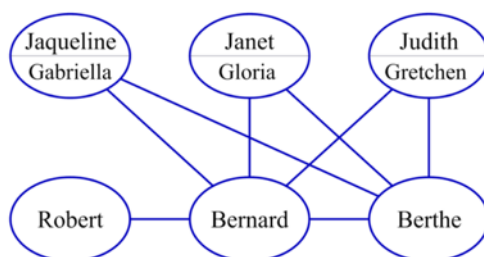


Figure 2: The initial connection graph for Marc Camoletti's *Boeing, Boeing*.

2. Chekhov and *Uncle Vanya* in Bielsko-Biała

Despite the fact that Anton Chekhov was a famous playwright, there are only a few items on his original plays in the libraries of Bielsko-Biała [7, 8]. The most interesting elaboration is René Śliwowski's book [45]. Śliwowski (1930-2015), born in France, was a professor of literature in Poznan, Poland [57]. His book covers not only a biography of Chekhov (illustrated by many photos) but also the analysis of Chekhov's plays as well as stories on their presentations in Moscow and other Russian theatres.

Plays of Russian authors are rarely seen in repertoires of Polish theatres these days. However *Uncle Vanya* has been performed in several venues: Gdansk,¹ Warsaw,² and Cracow.³ In 2012, journalist Łukasz Maciejewski wrote about the performance at the *Bagatela* Theatre that the interpretation of the director is pretty simple, traditional, and “like from the school book”, criticising the stage manager Waldemar Śmigasiewicz (see Footnote 1); the performance in Poznan was likewise criticised.⁴ Waldemar Śmigasiewicz is one of today's most outstanding Polish artists [53]. We learn from an interview with him that he decided to modernize the third adaptation due to the criticism he received [52]. And he did succeed in Bielsko-Biała! The reviews for this performance are fully positive, even enthusiastic.⁵ So what did he do in his third attempt in Bielsko-Biała to reach success?

One of the general changes made in Bielsko-Biała involved the stage: this performance used a one-scene decoration: a room joined with a greenhouse, where the latter stretched out deep inside the stage. Another change involved the four parts of the original play being aggregated into two drama acts. Furthermore, the actor playing Uncle Vanya was, at 27, much younger than the original character's 47-48 years. Moreover, one of Chekhov's characters, the watchman (or butler) was removed from the list of characters. But this does not seem enough to reach a success. And as one might suspect, there were a few other changes introduced, which we will explore in what follows. Unexpectedly, the discovery of these changes was possible just via our graph-based analysis!

¹A review in Polish is available at <http://www.dziennikteatralny.pl/artykuly/niewyrazny-wujaszek-wania.html>, last accessed on June 2, 2019.

²A review in Polish is available at <https://teatr6pietro.pl/spektakle/wujaszek-wania/>, last accessed on June 2, 2019.

³A review in Polish is available at <http://kultura.trojmiasto.pl/Wujaszek-Wania-s432.html>, last accessed on June 2, 2019.

⁴See for instance a review of the Poznan performance (in Polish) available at <http://www.encyklopediateatru.pl/artykuly/25607/nastroj-chwili-wizja-poetycka>, last accessed on June 2, 2019.

⁵See a review in Polish available at <http://www.bielsko.biala.pl/aktualnosci/33750/milosc-w-szklarni>, and another at <http://www.dziennikzachodni.pl/wiadomosci/bielsko-biala/a/wujaszek-wania-czyli-dlaczego-warto-szukac-u-czechowa-obawy-przed-marnowaniem-zycia,11544124/>, both last accessed on June 2, 2019.

3. Graphs representing *Uncle Vanya*

The theatre in Bielsko-Biała offers a variety of events to attract a wide audience. One of them is a special performance for students which involves a meeting and a discussion with the actors after the usual performance. The first author SZ attended the performance of *Uncle Vanya* twice, first at the ordinary performance and then during the student open night. During the latter event, participants asked questions about why Vanya is called an uncle by Sonya even though they are (at a first glimpse) of the same age and why Sonya and Helena are enemies at the beginning. Obviously the relations among the main characters were initially confusing to the spectators who were unfamiliar with the classic story. However we saw that via our graph-theoretical approach these problems could easily be cleared.





A graph in mathematics is equivalent to a relation, as a subset of the Cartesian product of two copies of a set V . Relation is a formal mathematical notion, but the word ‘relation’ has many synonyms, such as connection, tie, bond, nearness, relationship and many others. The collocation ‘interpersonal relationship’ even has its own entry in Wikipedia [56].





One could easily determine that in the Bielsko-Biała version of *Uncle Vanya*, eight characters took the stage, see Figure 3. Nonetheless, there is not any single main character. Also see Table 1.



Figure 3: The Bielsko-Biała cast of *Uncle Vanya* on the stage.

Table 1: Characters and the graph labels.

Vertex label	Character	Actor
P	Aleksandr Vladimirovich Serebryakov — Professor emeritus	 Jerzy Dziedzic
H	Helen Andreyevna Serebryakov (Yelena) — Professor Serebryakov's young and beautiful second wife	 Oriana Soika
S	Sofia Alexandrovna Serebryakov (Sonya) — Professor's daughter from first marriage	 Daria Polasik-Bułka
V	Ivan Petrovich Voynitsky ('Uncle Vanya') — in Bielsko-Biała — 27yo because played by a young actor	 Michał Czaderna

Vertex label	Character	Actor
A	Mikhail Lvovich Astrov — Medicine doctor.	 Piotr Gajos
M	Maria Vasilyevna Voynitsky — Widow and mother of Vanya	 Brygida Turowska (as Guest)
I	Ilya Ilych Telegin — Living in the house, divorced	 Grzegorz Sikora
N	Marina Timofeevna — An old Nanny	 Jadwiga Grygierczyk

We display the family ties at the beginning of the play in Figure 4.

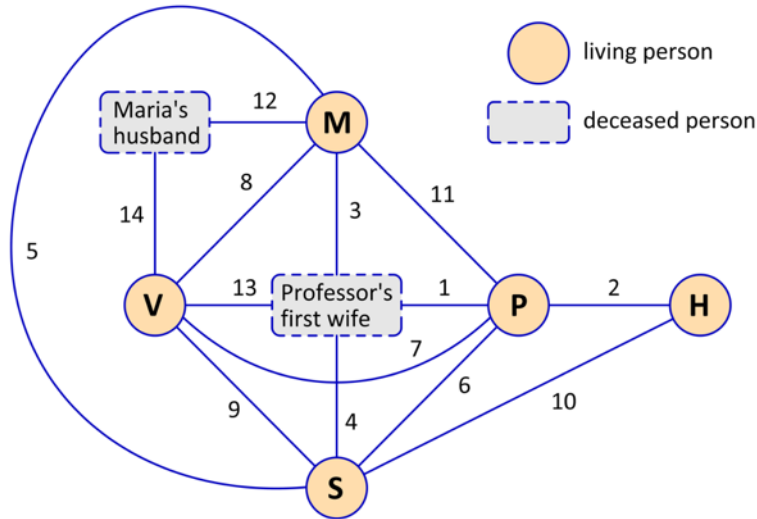


Figure 4: The graph of family ties of the main characters of Uncle Vanya.

It seems that Vanya is equally important as other personages like e.g. Professor Aleksandr Serebryakov, his wife Helen, doctor Mikhail Astrov and Sonya, daughter of Vanya's sister. Other heroes are as follows: Maria Voynitsky, Vanya's mother, as well as Ilia Telegin and Maria Timofeevna, Nanny, having cordial feelings to other members of the family. The rule for the edges in Figure 4 is as follows: the ties are written top-down or left-right. For example, the edge labeled 2 denotes a relationship between husband and wife, and this corresponds to Helen being the professor's (second) wife as well as Professor Aleksandr Serebryakov being Helen's husband. Similarly the edge labeled 6 denotes a relationship between a father and a daughter, and this corresponds to the professor being Sonya's father and Sonya being the professor's daughter. In every case, both persons are equally important so "husband \leftrightarrow wife" is in fact a pair which is a two-element set {husband, wife} where any order is not theoretically inserted. The full meaning of the family relations described by consecutive edges in Figure 4 is summarized in Table 2. We can consider this table as an element of interdisciplinary knowledge transfer. The graph vertices are shown as circles encircling the initials of the particular characters according to the rules in Table 1: Helen = H, Sonya = S, Mikhail Astrov = A etc., in such a manner that the descriptions are unique.

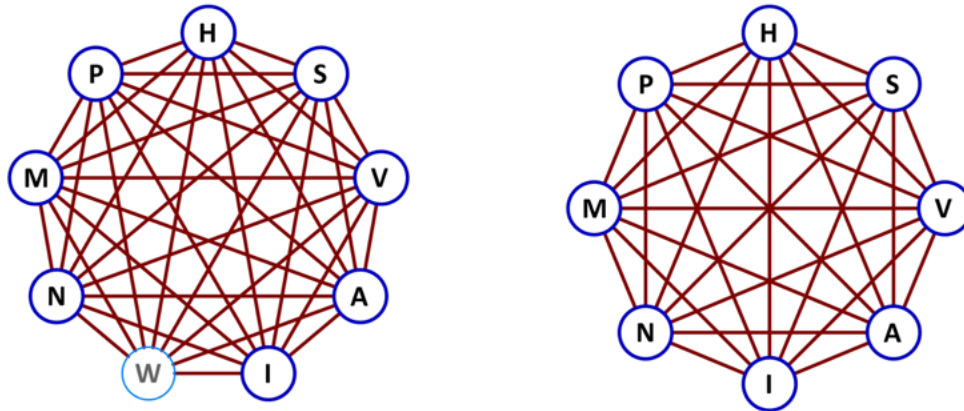
Edge label	Edge description
1	husband \leftrightarrow wife (professor and his first wife)
2	husband \leftrightarrow wife (professor and his second wife Helen)
3	mother \leftrightarrow daughter (Maria and professor's first wife)
4	mother \leftrightarrow daughter (professor's first wife and Sonya)
5	grandmother \leftrightarrow granddaughter (Maria and Sonya)
6	father \leftrightarrow daughter (professor and Sonya)
7	brother-in-law \leftrightarrow brother-in-law (professor and Vanya)
8	mother \leftrightarrow son (Maria and Vanya)
9	uncle \leftrightarrow niece (Vanya and Sonya)
10	stepmother \leftrightarrow stepdaughter (Helen and Sonya)
11	mother-in-law \leftrightarrow son-in-law (Maria and professor)
12	husband \leftrightarrow wife (Maria and her husband)
13	brother \leftrightarrow sister (Vanya and professor's first wife)
14	father \leftrightarrow son (Maria's husband and Vanya)

Table 2: Edges of the graph shown in Figure 4 describing the family ties - knowledge transfer: graph theory \leftrightarrow theatre and family.

The action of the play takes place at an estate in a rural area, near a bigger town (probably Moscow). Professor Serebryakov and his wife come to the big house of the estate. The normal, idyllic, slow life is disturbed by their arrival at the house — the unique venue of the play.

What is the play about? There are different opinions. The actors of Bielsko-Biała Theatre reported that it is about feelings, about impossibility in general or impossibility of reaching happiness. Another suggestion was that it is about boredom. But what makes Chekhov's plays classics? What makes them so widely popular around the world? It turns out that Chekhov's characters are the clue. Being more precise, their feelings, dreams, affections, passions, sympathies and love.

In the following we will refer to the Bielsko-Biała performance which was slightly changed from the original text. The relations between heroes can be drawn as cliques because all the heroes know each other; see Figure 5a. The original text of the play introduced nine characters. One of these, the Watchman, was omitted in the Bielsko-Biała version of the play, so the graph in Figure 5a turns into the one shown in Figure 5b.



a) the complete graph K_9 ; b) the complete graph K_8 , W removed.

Figure 5: Graphs representing the relationships between the characters in *Uncle Vanya*.

As we noted above the characters of *Uncle Vanya* know one another from the beginning of the play till the end. Therefore the graph of acquaintanceships remains stable. Hence the question arises: How can we extract more information from our graphs?

We present some relevant metrics in Figure 6 measuring how much speaking time each character gets throughout the play, according to the English translation [54] of the original text. The characters Mother, Ilya, and Nanny have the fewest turns (Figure 6a) as well as the fewest number of spoken words (Figure 6b).

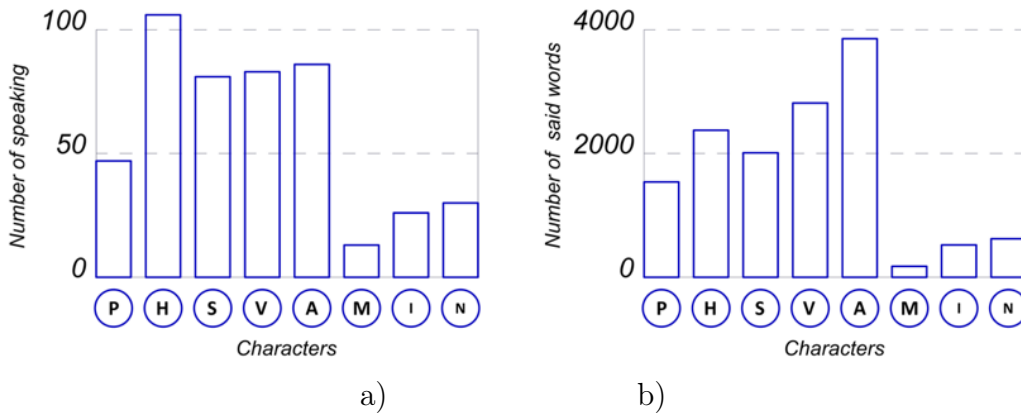


Figure 6: Measuring how much each character speaks based upon the English translation.

As we can see from Figure 6b, the characters who utter the most words are Doctor Astrov and Uncle Vanya. The text used in Bielsko-Biała was obviously a rearranged Polish translation, but it was not available for the paper authors. Thus we decided to assume that the general proportion of spoken words would be approximately the same, though we also noted that the director changed a few things; for example, the Nanny says almost nothing, and Mrs. Voynitsky plays via pantomime as well as facial expression, both of which are difficult to measure. She plays a nervous tic which prevents her from speaking. Therefore we chose not to take into full account the data from Figure 6 in the proposed graphs.

In the end, we can present Ilya and Nanny as smaller circles and cut the relevant vertices from the graph; this yields the two graphs in Figure 7. The Mother vertex remains because she is a member of the family and plays via pantomime movements.

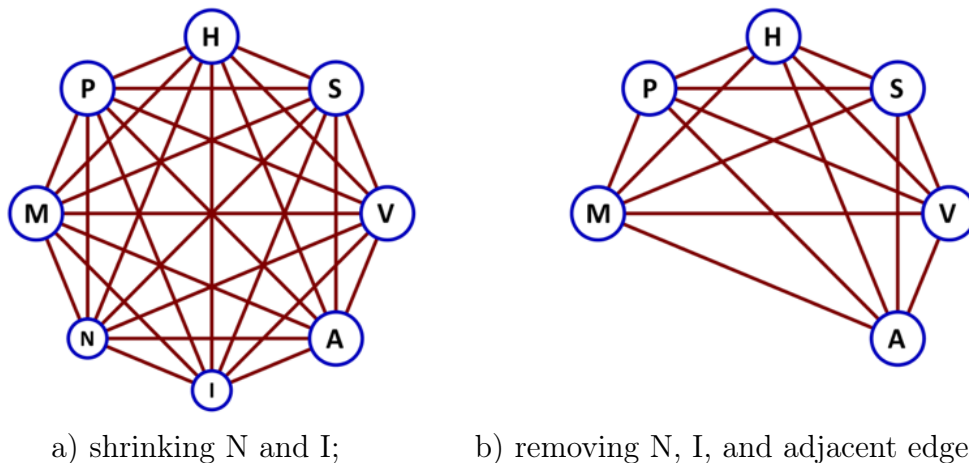


Figure 7: Graphs representing the relationships between the characters in *Uncle Vanya* with the less significant characters removed.

The final graph we obtain (Figure 7b) is much simpler than the original one (Figure 5a), but still it does not capture all the important points. Another possible approach is to use a weighted graph; this is shown in Figure 8.

Once more, we begin with the clique K_8 . Every person knows every other, but some connections are stronger than others. Recall that a graph, in which all the edges are assigned certain values, called *labels* or *weights*, is called a *weighted graph*. In our case, the edge weights depend on the intensity of the

links, feelings, emotions between the particular characters of the play. Here we assigned these values based on our own impressions of the relationships involved. We could have possibly determined these values by, for example, counting the number of spoken words or verbal encounters between the characters or the total number of pairwise common presences on the stage. We decided to keep things simple however, and ended up denoting the stronger links with continuous lines and the weaker links with dashed lines. Subsequently, we performed a graph transformation (that is, just an operation on a graph, as in [48]) removing the weak links (dashed connections). The resulting graph is presented in Figure 8b.

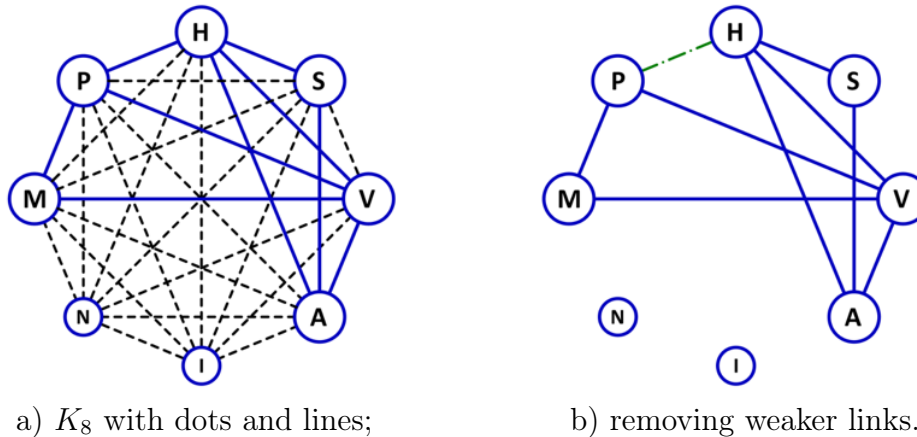


Figure 8: The complete graph K_8 represents the situation where every character knows everyone else. We represent the stronger relations with continuous lines and weaker relations with dashed lines (a); then we remove all the dashed edges (b).

The graph in Figure 8b is disconnected. In other words, the picture disintegrates into three separate parts, otherwise known as connected components. From this graph we could conclude that Ilya and Nanny are characters of secondary importance, which is in accordance with our earlier analyses visualized in Figure 7. This makes sense of course; almost nobody talks to Ilya, and even when Ilya says something almost nobody continues the dialog with him.

Finally, let us explain the green dotted line between P and H in Figure 8b. We know that the Professor and Helen are husband and wife. Nonetheless we think we have to cut the dashed edge between P and H due to their interactions through the play (see Table 3). The Professor tries to kiss his

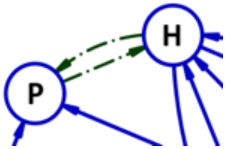
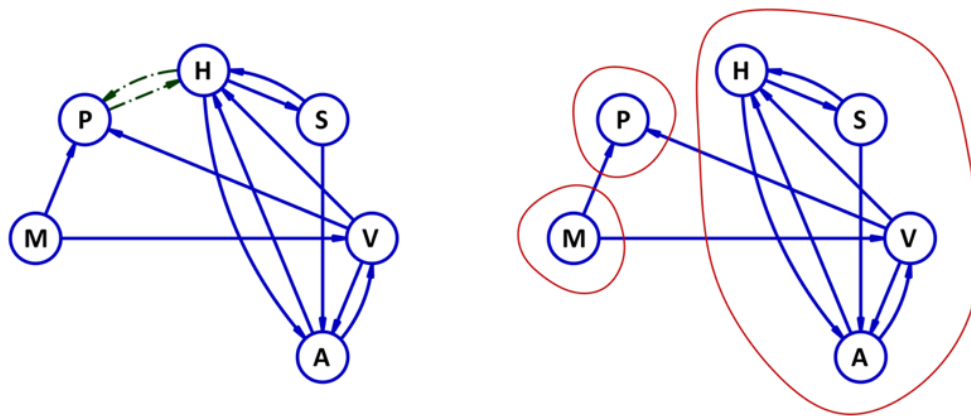
		Arguments	
		Based on play	General
	Helen does not help her suffering husband		The highest percentage of divorces is registered in group with significant age gap [13].
	Helen does not let P kiss her (shown)		
	Helen in monologue considers meeting with other man (original text — not shown)		

Table 3: Cutting the edge between P and H or the directed edges (P→H) and (H→P) - knowledge transfer: graph theory ↔ theatre and general knowledge

wife but she tries to avoid being close to him and she even pushes him back. Helen does this tenderly but emphatically. So even though she also promises to grow old together with him, we decided to remove this edge, and our decision was based upon context analysis, a somewhat different justification from our earlier reasoning.

Through further analysis we end up converting the edges into arcs and thus turning the graph into a digraph (Figure 9a); this leads us to eventually be able to explicitly point out the strong connected components (Figure 9b).



a) Figure 8b as a digraph;

b) strongly connected components.

Figure 9: We convert Figure 8b into a digraph (directed graph) (a) and obtain distinguished strongly connected components (b).

How do we convert the graph from Figure 8b into a digraph (directed graph)? For example, the arc $V \rightarrow P$ written as (V,P) means: Vanya has a positive attitude toward the Professor, appreciates him as a person and scientist; moreover he admires him for his scientific achievements — until the rapid change. On the contrary, the Professor does not reciprocate these feelings at all; therefore the arc $P \rightarrow V$ i.e. the pair (P,V) does not exist. The Professor never offers to raise Vanya’s salary, and when announcing his plans of selling the house, he does not include Vanya and his fate in his considerations. Specific interpretations of and justifications for the other directed arcs in Figure 9 are described in Table 4.






No	Graph arcs	Meaning of graph arcs
1		Mikhail Astrov and Helen: “love affair”; Mikhail’s proposal, some intimacy, Helen eventually pulls back.
2		Astrov and Vanya: good friends; they chat frequently, Vanya believes that the doctor can treat the Professor.
3		Helen and Sonya: the two women initially have strong negative feelings toward one another, though eventually are reconciliated.
4		Sonya loves Mikhail Astrov, but he does not pay any attention to her, so we denote this one-way relationship with a single arc.
5		Vanya declares his love to Helen but she rejects his proposal immediately and clearly. We denote this one-way relationship with a single arc.

Table 4: Meaning of arcs or pair of arcs in digraph Figure 9) — knowledge transfer: graph theory \leftrightarrow theatre.

The next step in our process involves the decomposition of our digraph into strongly connected components. (See [47] as well as [12]; the latter also describes several methods for the detection of communities in a graph.) The idea of strong connectivity describes the situation when “something is connected, but in a deformed, incomplete way”. That is, when we walk along the digraph arcs and we enter a strongly connected component, then it is impossible to get out of it. For other components there is no way to come back; if one goes out of it, then it is impossible to return.

It is really amazing how productive the idea of strong connectivity components is for analyzing the situation in Uncle Vanya. For example, the professor is thinking only about himself; he does not pay attention to anybody, with one exception (an attempt to come near his wife). He permanently goes to his room for work. But when he is ill, everyone (except his wife) tries to come to his aid, so they all seem to care for him. In consequence, the arcs are directed just to him (vertex P). On the other hand, nobody pays any attention to Maria. She is just a person who radiates feelings, especially towards the professor (she surrounds herself with his books and keeps reading them). The second destination of her love and attention is her son Vanya though she does rebuke him when the latter does not respect the professor's orders.

All in all, we have three strong connectivity components, as shown in Figure 9b. After removing the single-vertex components, we zero in on the greatest strong connectivity component for further consideration. More specifically we rotate this connected component 60° counterclockwise and then add loops to the vertices to denote monologues of the relevant characters. See Figure 10.

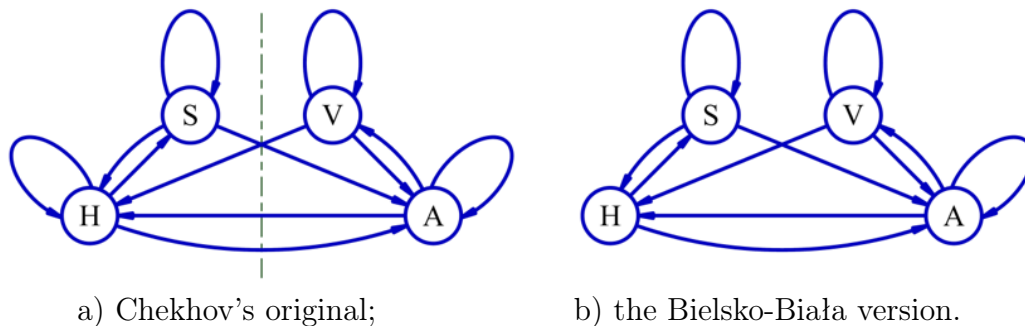


Figure 10: The digraph for the largest connected component in Figure 9b presented with loops depending on whether a character has a monologue or not: according to Chekhov's original text (a) and according to the Bielsko-Biała version (b).

In a typical graphical representation of a network, loops represent the connection of a particular person to themselves. In our case, this just means that the relevant character delivers a monolog about their feelings, plans, intentions, their life situation, etc. Such monologs by the main characters help set the stage, contextualize characters' behaviors and attitudes, and help the audience understand motivations and contexts. See Table 5 for an

overview of the monologs of several of the main characters, according to the Bielska-Biała version of the play.

Note that the graph representing Chekhov's original text is shown in Figure 10a and the Bielsko-Biała version is in Figure 10b. The slight difference between the two versions is intriguing. If, as in Figure 10a, there is a loop at the H vertex, then the graph has a vertical axis of symmetry. But without a monolog from Helen, as in the Bielska-Biała version, our graph is no longer symmetric, and the tension in the plot remains, making Helen's role more difficult and demanding.

Though we have no historical evidence for our convictions, we are quite convinced that Chekhov must have thought about the relationships between his characters in a similar way. We might even conjecture that he must have drawn these relationships; the symmetry in Figure 10a is that striking.

4. In Lieu of the Missing Loop at Vertex H: Helen's Inner Life

4.1. Introductory remarks

It was really interesting and exciting for us to realize that director Waldemar Śmigasiewicz modified Chekhov's text by dropping Helen's monolog. With this minor tweak, Helen is the only one of the four main characters in the play who does not have a monolog. Some journalists and critics in Bielsko-Biała responded negatively to this, arguing that the Helen character was too passive and her actions could not be understood. Indeed the change made the Helen role essentially different from all others. The actor playing Helen had to show without words what she thought. Oriana Soika, who played Helen in the Bielska-Biała performance, is indeed pretty and is able to display youthful optimism on her face. However, obviously, this would not suffice to perform Helen effectively. Luckily Soika has a wide range of dramatic expression skills. In the Bielska-Biała performance of *Humanka* for instance, she effectively played a robot without robot-like uniform and helmet, a really challenging assignment.

More specifically, without a monolog allowing the Helen character to express her inner life, the actor taking on the Helen role needs to communicate with her face and her body a range of wordless messages. And Soika does this most







No	Graph loop	Monolog	Remarks
1		Sonya speaks about her love for Mikhail, as well as about opinions of people about her.	We know why she tries to communicate with Mikhail.
2		Mikhail speaks of boredom, of how he thinks that life in the country is ordinary because the people are ordinary; he describes how a patient died during an operation (sign of his involvement in professional activities)	Possibly, we understand why he drinks vodka from time to time.
3		Vanya speaks about how his life has been wasted and how he used to believe that the professor was a great man, but it turns out that this was only an illusion. He proclaims dissatisfaction.	Possibly, we understand why he shoots at the professor.
4		NONE	Helen has no monolog in the Bielsko-Biała version (she does have a monolog in Chekhov's original).
5		The professor speaks about fame (this loop is not shown in the connected component we are looking at in Figure 10.)	The relevant vertex is not a part of the connected component we are looking at in Figure 10.
6		Ilia speaks about his marriage and children. He actually speaks in the presence of some other characters, but nobody converses with him. (this loop is not shown in the connected component we are looking at in Figure 10.)	The relevant vertex is not a part of the connected component we are looking at in Figure 10.

Table 5: Loops in the digraphs of Figure 10 and their meaning, according to the Bielsko-Biała version — knowledge transfer: graph theory \longleftrightarrow theatre.

effectively. At the beginning for example, she is a self-confident, freewheeling, carefree woman, the wife of a professor. For example, she sits on a sun bed (see Figure 11), takes off shoes, giving us the impression that they might be too tight. Her manner of walking also suggests serenity.



Figure 11: The Bielsko-Biała cast of *Uncle Vanya* on stage: Soika's Helen on a sun bed.

Though she is married, Helen becomes an object of interest for Vanya and Mikhail, two of the three main male characters of the play. However, both realize that she intends to remain faithful to her husband.

During the scene between Helen and Vanya, the latter unexpectedly declares his love. The dialog is short. She immediately rejects his affection. In the Chekhov original, Vanya is 48 years old, so one might assume that Helen, a young woman married to an older man, might not be interested in an older suitor. In the Bielska-Biała performance, Vanya is 27 years old, and so the whole situation seems more probable, though it still does not work out.

On the other hand, Doctor Astrov proposes Helen mere sexual intercourse. The proposal comes across as rude and ugly, and absurd at the same time. As expected, therefore, Helen's first response is about the disrespect this proposal entails. Astrov knows that she is well educated, having received a diploma from the St. Petersburg conservatory, and thus deserving respect. However, as is common in classic literature, the persistence of the man seems to wear down the woman's defenses. At some point we know that Helen does temporarily give in to Doctor Astrov's persistent pursuit; during their second embrace both are equally enthusiastic. Still we really do not know what she

feels or what she is thinking about throughout. Helen never speaks to herself, and through such a monolog, to the spectators. She does not announce her feelings or thoughts, nor does she describe her state of mind.

Nevertheless, Oriana Soika does find an interesting and convincing way to display Helen's emotions. At the beginning her Helen is a relaxed, serene, and sunny woman. After the undesired and obtrusive wooings of two men, she starts to walk in a nervous way, makes movements stretching and flexing her body, and finally changes the position of her brassiere. Her costume in this part of the play consists of a long skirt and flower-like patterned, tight-fitting blouse. She touches her lips like dreaming or remembering a kiss.

What other ways do we have to learn about the inner life of Helen? Given that there is no monolog to tell us directly, how does Helen and the rest of the play inform the audience about what is going on in her mind and heart? In the next two sections we explore two other ways the audience gains insight into Helen's mental state and motivations.

4.2. *Circus in theatre?*

There is an interesting scene in the Bielsko-Biała performance where Helen goes along the barrier at the edge of the stage; see Figure 12. The barrier is covered by plush, and its surface is slightly uneven.

The walk could be really difficult! The actor goes barefoot and the spectators can see her feet — it really looks like she is walking on a circus tightrope. At some point she almost falls down, but fortunately she is caught by Vanya. Taking into account the scene arrangement and its dependence on the particularities of the stage, the scene must really have been invented in Bielsko-Biała and could be recognized as theatrical parable (from the Greek *parabolē*), symbolically describing Helen's path: hesitation, short fall, and recovery.

4.3. *Pencil versus map*

At some point in the play Doctor Astrov shows Helen the maps of the region registering changes of the natural environment (in terms of area of forests, occurrence of animal species, and so on) in the last twenty years. Helen declares that she is not interested in maps and their social and political interpretation. However, she asks Doctor for such a map. Why?



Figure 12: Soika's walk along the theatre barrier. Top two pictures: Helen's walk. Bottom left: moment of falling down. Bottom right: rescue by Uncle Vanya. The guard rail she is walking on is a remnant from the separation for the orchestra; the theatre building is more than 150 years old.

Helen's request is performed by Soika in a particular manner: Till this moment, Helen is a rather calm and graceful woman. In general, her walk and movements are elegant. During the scene of request, Helen walks and moves in a completely different way, like a vagabond. Moreover, she enunciates the words in the request in a particular way, reminiscent of slang use. However once again since Helen has no monolog in the play, the audience is not sure about her purpose in wanting a map. We are, however, tempted to think that she might want to keep a memento to remind her of Astrov. Indeed she carries it around with her for a while. See Figure 13. However, in one of the final scenes, presumably after some self-reflection (which we are unfortunately not privy to), she decides to break the relationship immediately and entirely. And as a symbol of this decision, she leaves the map on the stage.



Figure 13: Map lying on the floor near the sun bed on which Helen is sitting,

Here it might be worth it to note that Chekhov's original Helen does not show any interest in a map. Chekhov chose to use a pen in its stead. Moreover, in the original version of the play, Helen expresses openly why she is choosing this simple object: it is small and easy to hide. One could conceivably think about why the Bielsko-Biała folks did not choose to replace the pen with something more visible for spectators, like a big ball-point pen. However for a Polish audience a big ball-point pen might have other connotations, see Figure 14.

5. Final Remarks and Conclusions

The Bielsko-Biała performance of *Uncle Vanya* was recognized as a huge success. The cast performed brilliantly. The presentation was original and full of novel ideas. And we the authors enjoyed both the performances and our ensuing mathematical analyses of the play.

In particular we have seen that our graph-theoretical approach could be really fruitful in analyzing family connections and weighted relations among characters. Besides using standard graphs, we also used a couple graph transformations leading up to digraphs with loops. Comparing the graph obtained by our analysis of the Bielsko-Biała performance and the graph corresponding to the original text helped us identify and interpret the changes adopted for the Bielsko-Biała Theatre to make the play more attractive and



Figure 14: Left: Lech Wałęsa is signing the Gdansk Agreements with the Communist Government as the Head of the Trade Union of the Gdansk Shipyard in August 1980. Right: An ordinary pencil and a copy of the Lech Wałęsa's point-ball pen (bought by the first author at the International Solidarity Centre in Gdansk) on top of an A4-size paper.

more contemporary. Our graph-theoretic approach made the changes much more visible; without our graph-theoretic tools, we would expect that only drama experts or *Uncle Vanya* aficionados could recognize them. On the other hand it is easy for anyone to see, for example, the difference between the symmetrical and the unsymmetrical graphs in Figure 10.

We see this paper as a case study; even though the methods utilized here were mentioned in some of our prior publications (see, in particular, [51]), their joint use and application to *Uncle Vanya* is novel. We have also not highlighted elsewhere the central idea of multidisciplinary knowledge transfer (graph theory \longleftrightarrow drama) in other papers. Indeed we believe that the idea of inter-field or multidisciplinary knowledge transfer could be an additional tool for drama analysis.

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