

ON THE KERNEL OF A SINGULAR INTEGRAL OPERATOR WITH NON-CARLEMAN SHIFT AND CONJUGATION

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Abstract. On the Hilbert space $\tilde{L}_2(\mathbb{T})$ the singular integral operator with non-Carleman shift and conjugation $K = P_+ + (aI + AC)P_-$ is considered, where P_{\pm} are the Cauchy projectors, $A = \sum_{j=0}^m a_j U^j$, a, a_j , $j = \overline{1, m}$, are continuous functions on the unit circle \mathbb{T} , U is the shift operator and C is the operator of complex conjugation. Some estimates for the dimension of the kernel of the operator K are obtained.

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