Faculdade de Engenharia da Universidade do Porto

Algorithm design for the fleet sizing problem in grocery retail distribution

Tiago Beleza de Vasconcelos

Master in Electrical and Computers Engineering

Supervisor: Drª. Sara Martins

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Abstract

It is known that retail industry companies face an ever-increasing need for optimization of resources and cost-cutting strategies. Retailers strive to find efficient distribution networks that can both satisfy every customer need and reduce the overall costs of the operation. Grocery retailers, in particular, face even more complex challenges as products may have handling restrictions, special storing conditions and different fleet compositions.

Our study will take into account different temperature storage conditions (ambient, chilled and frozen), the existence of a mixed fleet with single and multiple compartment vehicles, as well as, a single depot. We propose a mixed integer linear programming formulation based on the single-commodity flow formulation. Multiple compartment vehicles will serve only direct deliveries and single compartment ones for multiple deliveries of a single commodity.

The efficiency of our novel proposal is shown in comparison to a MILP formulation, initially proposed by Ostermeier-Hübner, that we have adapted to our problem. The desired properties of the optimal routes are illustrated through a set of numerical experiments.

Resumo

É do conhecimento geral que as empresas do setor de retalho se deparam com a necessidade crescente de melhor otimizar recursos e de implementar estratégias de redução de custos. O setor dedica enorme esforço para determinar redes de distribuição eficientes que possam satisfazer as necessidades de cada cliente e reduzir os custos gerais da operação. O retalhista alimentar, em particular, debate-se com desafios ainda mais complexos na medida em que os produtos podem ter restrições acrescidas de manuseamento, condições especiais de armazenamento, para além de dispor de composições de frota variadas.

Este trabalho terá em linha de conta restrições do produto, a existência de uma frota mista com veículos de compartimento único e múltiplo, bem como a existência de um único armazém. Será proposto um modelo de formulação matemática que utiliza os veículos com vários compartimentos apenas para entregas diretas e os de compartimento único para entregas múltiplas de uma única mercadoria.

A eficiência da nossa nova proposta é apresentada em comparação com uma modificação da formulação de Ostermeier-Hübner. As propriedades desejadas das rotas ótimas são ilustradas através de um conjunto de experiências numéricas.

Résumé

On observe à l'heure actuelle que les societés industrielles de vente au détails ont des besoins croissants dans le domaine de l'optimisation de ressources et de coût. Les détaillants s'efforcent de trouver des réseaux de distribution efficaces capables de satisfaire tous les besoins des clients et de réduire les coûts globaux de l'opération. Les détaillants en alimentation, en particulier, font face à des défis encore plus complexes, car les produits peuvent être soumis à des restrictions de manutention et des conditions de stockage particulières. Des problèmes de logistique de transport concernant leur parc de camions deviennent de plus en plus importants et critiques.

Nous étudions un système avec un dépôt et plusieurs clients ayant des demandes à satisfaire pour 3 types de produits : ambients, surgelés et frais. Deux types de véhicule assurent le transport du dépôt vers les clients: véhicule à un seul compartiment et véhicule à multi-compartiments. Le premier type simple peut visiter plusieurs clients dans une tournée en servant un seul type de produit, tandis que le deuxième véhicule à plusieurs compartiments peut uniquement servir un client à la fois, en lui servant toute sa demande de trois types de produits. On doit noter que cette formulation a l'avantage d'être plus compacte que d'autres modélisations qui doivent être adaptées à des contraintes similaires.

Cette formulation est comparée à celle proposée par Ostermeier-Hübner adaptée à notre problème. Les propriétés souhaitées des chemins optimaux sont illustrées par un ensemble d'expériences numériques.

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Tiago Beleza de Vasconcelos

"*It is not the answer that enlightens, but the question.*"

— Eugène Ionesco

playwright (26 November 1909 – 28 March 1994)

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Abbreviations and Symbols

CVRP	Capacitated Vehicle Routing Problem
HVRP	Heterogeneous Vehicle Routing Problem
FSM	Fleet Size and Mix Vehicle Routing Problem
FSMTW	Fleet Size and Mix Vehicle Routing Problem with Time Windows
HF	Heterogeneous Fixed Fleet Vehicle Routing Problem
HFTW	Heterogeneous Fixed Fleet Vehicle Routing Problem with Time Windows
MCV	Multiple Compartment Vehicle
MILP	Mixed Integer Linear Programming
MTVRP	Multiple Trip Vehicle Routing Problem
RGTS	Reactive Guided Tabu Search
SCV	Single compartment vehicle
SDMTPVRP	Site-dependent Multi-Trip Periodic Vehicle Routing Problem
VRP	Vehicle Routing Problem
VRPTW	Vehicle Routing Problem with Time Windows

Chapter 1

Introduction

In today's global competing markets, companies face an ever-increasing need for constant optimization of resources and cost-cutting strategies if they intend to survive. The case of the retail industry is no different. Currently, retailers strive to find an efficient distribution network that can both satisfy every customer need and reduce the overall costs of the operation.

Grocery retail, in particular, poses even more complex challenges, as products may have handling restrictions, special storing conditions, and a multitude of other variables and constraints. On top of this, current fleet composition, work-force restrictions and even the possibility of outsourcing part of the routes can be taken into consideration when optimizing the distribution network.

The aforementioned issues can be considered as extensions for the more restrict Vehicle Routing Problems, where the goal is to determine an optimal routing plan for a fleet of vehicles to serve a group of customers. This study aims to tackle these and other variables that comprise day-to-day challenges in the real world, optimizing fleet usage in order to reduce costs and eliminate unnecessary expenses. Our study will take into account product limitations, the existence of a mixed fleet with single and multiple compartment vehicles (SCVs & MCVs), as well as allowing for multiple parallel trips.

This thesis is organized as follows. The remaining of this chapter describes the motivation and aim of this project. Chapter 2, Literature Review, presents the studied literature regarding Vehicle Routing Problem and its extensions for fleet sizing in grocery retail. Chapter 3, Problem Description, characterizes the problem under investigation and includes a proposed solution, as well as, the Ostermeier-Hübner formulation and a variation of it. Chapter 4, Computational Results, will explain our tests and show simulation results. To conclude, Chapter 5, Conclusions and Future Developments, will summarize the content of this dissertation as well as give some insights about future developments.

1.1 Motivation

One key strategic problem in grocery retail distribution is the definition of the fleet size, that is, to determine the quantity and the type of vehicles to be used. In real life, different situations will require a strategic and/or tactical capacity adjustment. Therefore, fleet composition, resizing and allocation are important tasks that fleet owners and managers are faced with. Moreover, service time, vehicle capacities and customer demand can influence a fleet's composition, since a variety of constraints can alter different vehicle type needs. Although there is already an extensive literature on fleet sizing, none of it portrays the many existing challenges found in grocery retail distribution. Some works do not explore different types of products or the specifications of the retail commodities, and so compartment based vehicles are not considered. Moreover, the distinction between vehicles only relies on capacity, which is too simplistic, when heterogeneous vehicles are considered.

This study can bring novelty and valuable results since it explores and considers different types of goods, compartment based vehicles, multiple trips and vehicle limitations, in a grocery retail distribution. These constraints have been discussed in the literature, yet not studied in the scope of a retail network, making this dissertation a pioneer.

1.2 Objectives

This project aims to analyze the fleet sizing problem in grocery retail distribution together with its constraints. Therefore, a mathematical formulation will be proposed for this problem, in order to optimize the fleet sizing problem bearing in mind the following practical requirements:

- Different variety of products for each order;

- Different types of vehicles used:
  - single compartment vehicles, or SCV for short;
  - multiple compartment vehicles, or MCV for short;
- Customers must be visited only once;
- Not splitting deliveries (i.e. a customer order for each commodity must be fulfilled in a single delivery).

The main goal is thus to optimize one fleet, i.e, to minimize the operational cost associated with vehicle selection. This may not be straight forward since, in some particular cases, using more vehicles may reduce costs, as not all vehicles are identical (mixed fleet). In order words, assuming that the optimal usage of a fleet is by employing the least amount of vehicles is not a universal truth. Additionally, different models and analysis will be performed in order to understand how these requirements will affect the costs and the planning of the fleet.

Chapter 2

Literature Review

In this chapter, the literature review is discussed based on the most relevant constraints to achieve an optimal result. The main topic to consider is the fleet sizing problem. Moreover, as part of the investigation, Multiple Trip Vehicle Routing Problem is discussed, as well as Multiple Compartments vehicles, since both have a big influence in the final result. We address the problem of routing a fleet of vehicles from a central depot to customers with known demand. Routes originate and terminate at the central depot and obey vehicle capacity restrictions.

The classical Vehicle Routing Problem (VRP) considers an optimal routing plan for a fleet of homogeneous vehicles serving a set of customers where each vehicle route starts and ends at a depot and each customer is visited only once. This problem has been widely studied from the pioneering work of Dantzig and Ramser [1959]. In Toth and Vigo [2002], and on the references therein, a review of the VRP problem is offered.

The VRP has been extended to take into account more complex variants, often named by "rich VRP", such as time windows, single and multiple commodities, heterogeneous vehicles, single and multiple compartments, single and multiple trips, among others. In Baldacci et al. [2008], Hoff et al. [2010], Koç et al. [2016] an extensive revision of routing heterogeneous fleet is provided. Most of the solution techniques to tackle these rich VRP problems rely on heuristics. However, some exact methods can be deployed under more restrictive scenarios, regarding computational time and parameters size.

2.1 Fleet Sizing Problem

The Fleet Sizing Problem represents the optimization of a fleet composition. In the classical VRP a set of identical (or homogeneous) vehicles, with capacity limitations, is to be optimally routed to serve customers with known demands. Variants of this problem have been studied such as the Capacitated VRP (CVRP), that takes vehicle capacity as a constraint, and VRP with Time Windows (VRPTW), where customers have to be served within a specific time interval.

Vehicle Routing Problem was first introduced by Dantzig and Ramser [1959] with the term Truck Dispatching Problem, where a set of identical vehicles had to respond to petrol demand of several gas stations scattered across the map. Assuming a single depot the objective was to minimize the distance travelled.

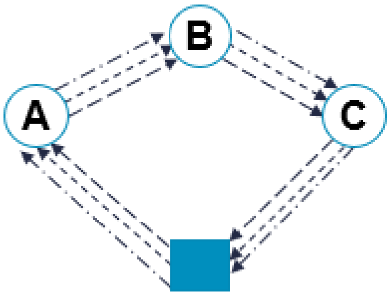
The complexity of the problem increases when a fleet is no longer a set of identical vehicles but is characterized by different capacities and costs. The extension of the VRP in which one must additionally decide on the fleet composition is known as the Heterogeneous Vehicle Routing problem (HVRP). Heterogeneous VRP consists on designing feasible routes with minimum total costs, ensuring that each customer is visited by exactly one route and the number of routes per vehicle type does not exceed the number of vehicles available for that type. Two major variants of the HVRP‘s are Fleet Size and Mix Vehicle Routing Problem (FSM), which work with an unlimited heterogeneous fleet, and the Heterogeneous Fixed Fleet Vehicle Routing Problem (HF), in which the fleet is limited to a predefined set of vehicles.

The first structured work on heterogeneous fleet VRP was due to the seminal paper of Golden et al. [1984] and a deep literature revision on the topic was first provided in Baldacci et al. [2008]. Problem variants such as Heterogeneous VRP with Fixed Costs and Vehicle Dependent Routing Costs (VVRPFD), Heterogeneous VRP with Vehicle Dependent Routing Costs (HVRPD), Fleet Size and Mix VRO with Fixed Costs and Vehicle Dependent Routing Costs (FSMFD), Fleet Size and Mix VRP with Vehicle Depedent Routing Costs (FSMD) and Fleet Size and Mix VRO with Fixed Costs (FSMF) were all discussed in this paper. It is also relevant to refer to Koç et al. [2016] where the authors made a review of three decades of research on heterogeneous vehicle routing. According to Hoff et al. [2010] “a heterogeneous fleet of vehicles is generally more flexible and cost-effective”. Thus, having a heterogeneous fleet is to be preferred.

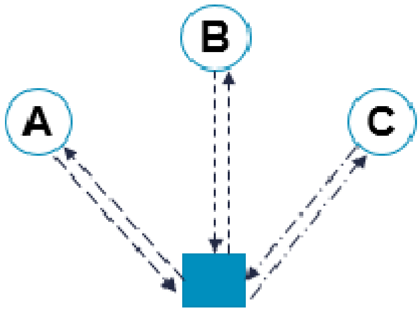
In order to better understand some of the possibilities of routing selection, and following closely Archetti et al. [2015], four different problems can be identified, each one with a different solution. Figure 2.1 illustrates all the variants of the problem. The blue square shape represents the depot, which is the beginning and the end of routes, the circles represent the customers and the arrows the routes. In Figure 2.1a a separate routing problem is shown. In this case, a specific set of vehicles is dedicated to each commodity and any commodity is delivered to any customer with a single visit. A customer will receive as many visits as the number of commodities requested. This is the case of the classical Vehicle Routing Problem. In Figure 2.1b the mixed routing problem is displayed. In this case, any vehicle can deliver any set of commodities. All requests, regardless of consisting in one or more commodities, will be carried out in one visit so no customer can be visited more than once. This is the case of a single classical VRP. In Figure 2.1c, the split delivery mixed routing is illustrated. Any vehicle can deliver any set of commodities and both split deliveries and commodities are allowed. A customer may be visited several times if beneficial, even if only one commodity is requested. Finally, Figure 2.1d exhibits the split commodities mixed routing problem. It considers the problem where vehicles are flexible and can deliver any set of commodities. Multiple visits of a customer are allowed whenever the customer requests multiple commodities. When a commodity is delivered to a customer, the entire amount requested by the customer is delivered. If customers are visited more than once, the different vehicles will carry different commodities. This latter case is the one closest to our problem. However, for our fleet composition, two types of vehicles, SCV and MCV, are taken into account.

Split Delivery, as mentioned on Archetti et al. [2015] examples, represents the separation of products requested by a single customer. This may create additional savings in terms of delivery cost since it can open opportunities to explore different types of routes. Even though this can be especially interesting if the demand is higher than the vehicle capacity, it may be also beneficial even when the demand is lower than the capacity. However, this has a major flaw. Having into consideration customer satisfaction usually is not well accepted splitting the delivery, especially in retail distribution, since customers are expecting the full delivery of a single commodity in a single trip and, in this case, customers will receive their orders on several visits. Therefore, even though this may increase the optimization of the operational cost, we do not take it into consideration in our formulation.

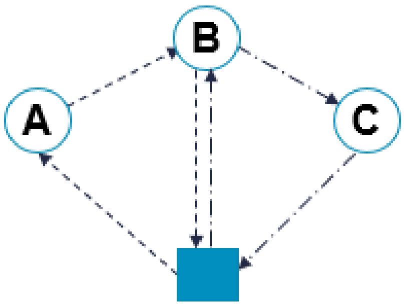
On the other hand, the split commodity hypothesis is considered in our problem. It represents the delivery of a single type of product to more than one customer, never letting the demand being higher than the vehicle capacity. Since there is a



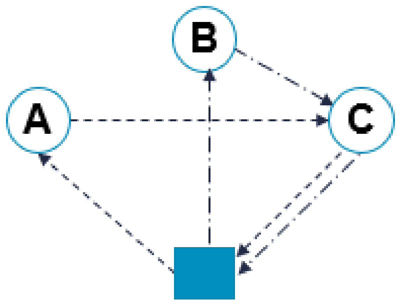
(a) Separate routings
(no split delivery / split commodity)



(b) Mixed routing
(no split delivery / no split commodity)



(c) Split delivery mixed routing
(split delivery / split commodity)



(d) Split commodities mixed routing
(no split delivery / split commodity)

Figure 2.1: Example of solutions according to [Archetti et al., 2015]

distinction between what product to carry, only single compartment vehicles can be used in this kind of optimization, bearing in mind multiple compartment vehicles will deliver all commodities at once. This is more suitable to the customer, since it is an internal operation, meaning that the customer is not affected by it. In each visit customers will receive all commodities of a certain type, and, therefore, customer satisfaction is not afflicted.

2.2 VRP with Multiple compartment vehicles

In areas like food delivery, petroleum distribution and waste recycling, due to the characteristics of the products, vehicles with multiple compartments are indispensable. Customers require the delivery of more than one type of product that may require different transportation conditions.

On grocery distribution, multiple compartment vehicles allow for different storage temperature products (ambient, chilled and frozen) to be separately loaded in one vehicle. All customer requirements may be fulfilled in a single trip, taking into account vehicle capacity limitations. Single compartment vehicles may be less costly but are restricted to deliver a single type of product at a time. Therefore, one must balance the cost of several trips to the same customer, which can lead to additional costs and time inefficiency, or do a single trip on a multiple compartment vehicle, which implies higher fleet costs. With the use of SCVs each trip is less costly and more vehicles can be available with MCVs. Moreover, multiple-compartment vehicles can be further optimized by adjusting compartments capacity, whenever possible. Usually, associated with multiple-compartment vehicles is a fleet of heterogeneous vehicles.

Derigs et al. [2011] considers the Vehicle Routing Problem with Compartments (VRPC) as an extension of the classical VRP, by introducing constraints based on compartment capacity and on different products. They assumed that all vehicles are homogeneous, i.e. with the same capacity, and all compartments are either flexible or fixed. The study was tuned for two types of industries that rely on heterogeneous products: petrol and food industries. The implemented solution is based on an integer program formulation, only for a homogeneous fleet, solved via a heuristic approach.

Ostermeier and Hübner [2018] considers MCVs as an attractive alternative to SCVs. MCVs allow the joint delivery of several product types for a customer at the same time. This is possible because compartments can be set to serve different types of products, independent of each other on the same vehicle. The number of compartments and their size can be adjusted flexibly without any loss in capacity. However, MCVs have slightly higher acquisition costs that impact the costs of transportation. This may be compensated by higher flexibility in routing. With the use of MCVs the number of stops reduces as orders are combined on the same vehicle. An important factor that the authors mention is the loading and unloading time/cost. When SCVs are used a single visit to a shipping gate is done. On the contrary, with MCVs the number of visits to a shipping gate depends on the number of different products that need to be carried. Therefore, when MCVs are used, loading and unloading time/costs are higher. In this work, the authors state that a mixed delivery fleet can yield significant savings when compared to a fleet of only one type of vehicles since it becomes possible to combine the advantages of both.

The case with heterogeneous fleet Multiple-Compartment Vehicle Routing Problem (MCVRP) is explored in Wang et al. [2014]. They consider that a mixed heterogeneous fleet brings additional advantages when compared with a homogeneous fleet since it can better meet the customers need. A metaheuristic solution procedure is proposed.

Over the recent years the literature in multiple compartment vehicles has been increasing. The most used applications occurs on petrol or waste distribution, which have their own specifications. In this work, we analyse cases similar to our problem, that is, cases where grocery retail distribution is being studied.

2.3 Our position into the literature

On literature the two papers that come closest to our problem were Ostermeier and Hübner [2018] and Archetti et al. [2015] however, they consider in their problem different variations.

In both Ostermeier and Hübner [2018] and Archetti et al. [2015] MCVs have flexible compartments that may be modified depending on the orders requested. In an extreme case, MCVs can become SCVs if only one commodity is used. However, even though our compartments are also flexible we do not consider the possibility of removing a compartment. That is, MCVs have always three different compartments, for the different commodities. Additionally, their MCVs may visit several customers if all customers order that types of commodities. We force that MCVs can only be used to serve a single customer and return to the depot.

For Ostermeier and Hübner [2018] the distribution cost can be divided into three phases: (i) loading, (ii) transportation and (iii) unloading, each one with a different cost. During the loading phase, a loading time/cost is considered. For SCVs this cost is lower since only one type of product has to the placed inside the vehicle. On the other side, when MCVs are used this cost becomes higher because each different commodity, that need to be transported, represents an additional loading time/cost. The same analysis can be made for phase (iii) but instead of loading the vehicles we are now unloading. During transportation phase two costs are considered a fixed cost, when a vehicle is used and a variable cost, depending on the distance travelled. Additionally, vehicles must go to the depot before they can begin their routes. In our case, we only focus on phase (ii), transportation, and so only fixed costs and variable costs are considered. Since we only approach the transportation phase, vehicles are assumed to be already full in the depot to begin their routes.

In Archetti et al. [2015] they illustrate, as explained in section 2.1 and in Figure 2.1, different distributions concepts. We can say that our case is a special case of Figure 2.1d. We take into consideration only split commodity and use both SCVs and MCVs. However, we take into account that MCVs can only visit one customer. Due to this consideration split commodity can only be performed with the use of SCVs.

Chapter 3

Problem Description

In this chapter, an extensive explanation of our problem is described, as well as, all formulations used during the thesis are exposed.

3.1 Problem definition

This work aims at designing a fleet of heterogeneous vehicles with multiple or single compartments for grocery retails distribution.

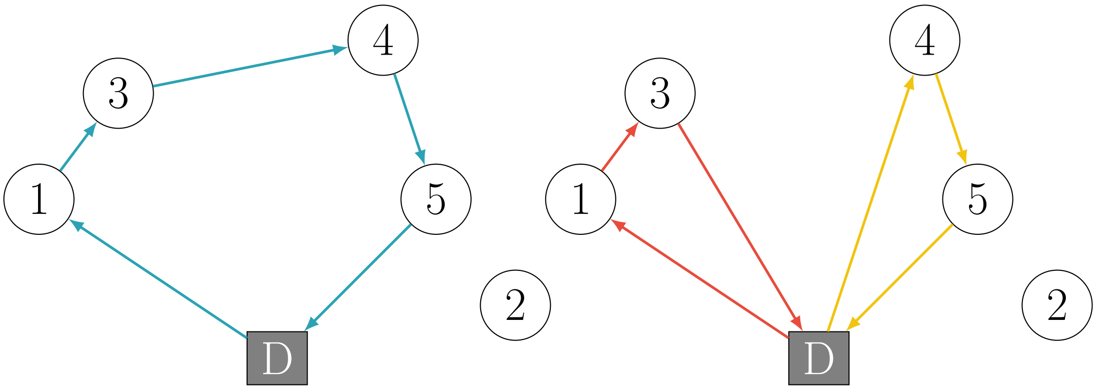
Our problem considers one depot, in which vehicles must depart from and arrive at the end of each route, multiple customers, able to request multiple volumes of three different commodities: ambient, chilled and frozen, and two types of vehicles: single compartment vehicles (SCV) and multiple compartment vehicles (MCV).

Associated with SCV there are two types of costs: a fixed cost, that represents the cost associated with the use of a vehicle, and a variable cost, that has a direct link with the route distance. Furthermore, SCVs have a limited capacity.

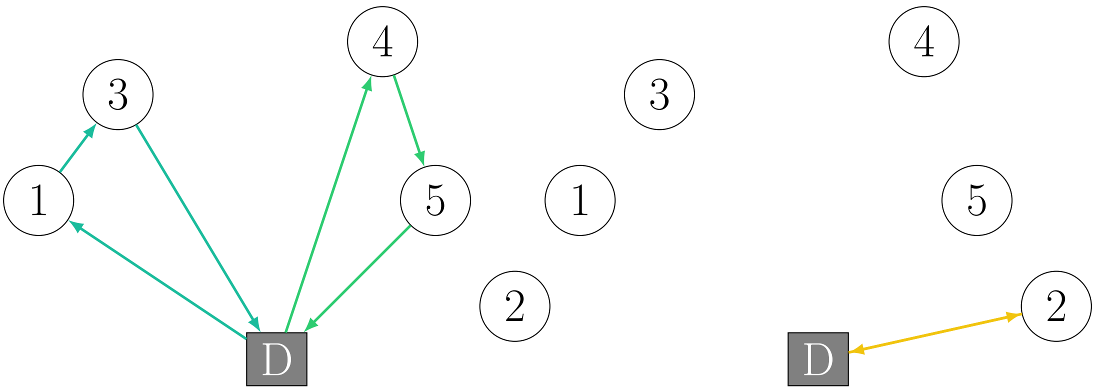
For MCV, only one cost is used, namely direct delivery cost. Although it is just one type of cost, it includes both the fixed and variable cost with the addition of a penalty variable, named *alpha*, that represents the additional cost for using MCVs. Unlike SCVs, with MCVs it is assumed that their capacity is always enough to carry all commodities of one customer.

In the perspective of the customer, splitting deliveries is not acceptable in our formulation, since they are not well seen in retail market. Usually, in grocery retail distribution, this kind of splitting is not considered since customers do not accept to have their orders delivered along several visits. Therefore, the only suitable splitting in our problem is the split commodity, i.e, each commodity can be delivered in separate visits by SCVs.

To summarize, the model considers that SCVs can visit multiple customers with one commodity with no split delivery. On the other hand, MCVs can transport all commodities to just one customer. In Figure 3.1 graphic representations of routes are drawn to illustrate the type of service considered.



(a) Routes for commodity one (b) Routes for commodity two



(c) Routes for commodity three (d) Routes for direct delivery

Figure 3.1: Representation of routes by commodities and direct delivery

Figures 3.1a-3.1c shows a route for commodities 1, 2 and 3, respectively, and Figure 3.1d for a direct delivery. As it can be assessed from these figures, routes start and end at the depot and customers are visited at most once per commodity. In the case shown, customer 2 is not included in the single commodity routes because it would exceed the vehicle capacity. Therefore, in Figure 3.1d a direct delivery route is depicted for this customer. Different routes for each commodity must be made since customers demands can differ per commodity.

Next, our proposal, the Flow formulation, is formalized. In order to check that our formulation brings better and improved results than some previous works in the literature, a mixed integer linear programming formulation given in Ostermeier and Hübner [2018] has been modified and adapted to explore the same restrictions for comparison purposes. Both the original and its modification are also detailed in the next sections.

To this extent, some tests will be performed to assess on the impact of different parameters, such as vehicle capacities, number of usable vehicles, additional cost using MCVs (i.e. *alpha*), fixed costs and commodities, among others.

3.2 Proposed mathematical problem

In this section, we propose a mixed integer linear programming formulation (MILP) based on single-commodity flow formulation (Baldacci et al. [2008]). An explanation of all variables and their bounds is presented together with the objective function and its restrictions is presented following.

Let $G = (N, A)$ be a directed graph where $N = Nc \cup \{0\}$ is the set of nodes, in which $Nc = \{1, ..., n\}$ represents the customers nodes and 0 the depot, and $A = \{(i, j) : i, j \in N\}$ is the arc set. Let d_{ij} be the minimal traveling distance from i to j, $\forall (i, j) \in A$. Let P be the set of commodities (products) the depot delivers to the customers, with each node $i \in N$ associated to a non-negative demand q_{ip} for each commodity $p \in P$ (for notation convenience, the depot is assigned a demand $q_{op} = 0, \forall p \in P$). The distribution can be performed using a fleet of SCVs defined by the set V. Each vehicle $v \in V$ used has associated a fixed cost f and a unitary travel cost u. Note that each vehicle can only transport one commodity to one or multiple customers. It is assumed that each vehicle $v \in V$ has a capacity Q and there is an infinite supply of vehicles. If a customer i does not receive at least one of the commodities ordered, a direct delivery cost dd_i is payed to fulfill the missing order. The vehicles associated with this cost are MCVs. If the cost function is lower using this type of vehicles, the formulation will choose to use them rather than the SCVs.

Some assumptions made are as follows. All orders are taken into account separately and independently from others. Without loss of generality one can assume the demand of each customer is less than or equal to the vehicle capacity. Distances between customers and/or depot can both be symmetrical and asymmetrical. It is considered that the fixed cost, f and the unitary travel cost, u, are cheaper (lower value) than the direct delivery cost, dd_i.

The objective of the problem is to determine the optimal fleet size with the minimum total distribution cost, which considers fixed and variable routing costs and direct delivery costs, while satisfying the customers demand. The formulation proposed uses the following variables:

- x^v_{ijp} equal 1 if vehicle $v \in V$ travels directly from node i to j $(i, j \in A)$ with commodity $p \in P$, 0 otherwise;
- w_{ijp} amount of commodity $p \in P$ transported on arc $(i, j \in A)$;
- y_i equal 1 if customer $i \in N_c$ requires a direct delivery, 0 otherwise.

Objective Function

$$min \sum_{v \in V} \sum_{j \in N_c} \sum_{p \in P} f \ . \ x^v_{0jp} + \sum_{v \in V} \sum_{(i,j) \in A} \sum_{p \in P} u \ . \ x^v_{ijp} \ . \ d_{ij} + \sum_{i \in N_c} dd_i \ . \ y_i \tag{3.1}$$

Constraints

$$\sum_{v \in V} \sum_{j \in N} x^v_{ijp} = 1 - y_i, \quad \forall i \in N_c, \, p \in P \tag{3.2}$$

$$\sum_{v \in V} \sum_{i \in N} x^v_{ijp} = 1 - y_j, \quad \forall j \in N_c, \, p \in P \tag{3.3}$$

$$\sum_{i \in N} x^v_{ihp} = \sum_{j \in N} x^v_{hjp}, \quad \forall h \in N_c, p \in P, v \in V \tag{3.4}$$

$$|N| \ . \sum_{j \in N_c} x^v_{0jp} \geq \sum_{i \in N_c} \sum_{j \in N} x^v_{ijp}, \quad \forall p \in P, v \in V \tag{3.5}$$

$$\sum_{j \in N_c} \sum_{p \in P} x^v_{0jp} \leq 1, \quad \forall v \in V \tag{3.6}$$

$$\sum_{j \in N} w_{ijp} = \sum_{j \in N} w_{jip} - q_{ip} \ . \ (1 - y_i), \quad \forall i \in N_c, p \in P \tag{3.7}$$

$$q_{jp} \ . \sum_{v \in V} x^v_{ijp} \leq w_{ijp} \leq (Q - q_{ip}) \ . \sum_{v \in V} x^v_{ijp}, \quad \forall (i,j) \in A : i \neq j, p \in P \tag{3.8}$$

$$w_{i0p} = 0, \quad \forall i \in N, p \in P \tag{3.9}$$

The objective function (3.1) minimizes the overall costs, both fixed and variable costs of routing either using SCV or with direct delivery using MCV. Constraints (3.2)-(3.3) guarantee that each customer is visited at most once for each commodity or for a direct delivery. Constraint (3.4) indicates that if a vehicle visits a customer, it has to leave it. Constraint (3.5) enforces that customers can only be visited by a vehicle if this one leaves the depot. Constraint (3.6) ensures that just one commodity is delivered in each vehicle that leaves the depot. Constraints (3.7)-(3.8) define the commodity flows and eliminate sub-tours. Constraint (3.9) makes sure that all commodities are delivered and no product arrives at the depot.

3.3 Modified Ostermeier-Hübner formulation

During the literature review, multiple formulations were taken into consideration. However, he problem formulated by a MILP in Ostermeier and Hübner [2018] was considered to be the closest one to our problem. Therefore, this formulation is implemented for comparison purposes and to assess the efficiency of our first flow based formulation.

This formulation is a compartment based formulation where each commodity is allocated in a different compartment. Orders are defined by customers, products segments and quantity. A product segment consists of items (products) that belong to one temperature zone. The orders have to be collected from the depot and transported to the customers. A customer may be visited several times (in different tours) in order to deliver different commodities. A split delivery of an order of one commodity of a single customer is not possible. Each customer places at least one order.

Vehicles are defined by the vehicle type, the number of compartments used on the vehicle and a given transportation capacity for each vehicle type. The set of vehicle types includes SCVs and MCVs. If an SCV is used, the number of compartments is limited to one compartment. If an MCV is used, a predefined number of compartments is available for each vehicle. Total vehicle capacity for MCVs can be divided into the maximum capacity of compartments for each vehicle. It involves a sufficient number for all vehicle types to meet customers demand. Three types of costs are considered: loading and unloading costs and transportation costs. The loading of a new commodity also requires the opening of a new compartment. The number of compartments determines the number of loading processes at the depot.

The original Ostermeier-Hübner MILP formulation can be seen in the Appendix A.

Even though the original Ostermeier-Hübner formulation is the closest one to our model, it still needed some tweaks in order to fully respect the retail distribution constraints considered in our problem. Hence, a modified Ostermeier-Hübner formulation was developed and used for comparison tests.

The modifications in the formulation occur in the indexes used, objective function and in the types of vehicles. In our problem, orders are required based on customers and commodities so an index based on customers (i) and commodities (p) replace index order (o). No distinction between compartments is ever made in our formulation, so index compartments (c) must be replaced. Also the number of compartments correspond to the number of commodities and, so, index commodity (p) replaces index compartment (c). Additionally, a distinction between vehicle types is not defined as in the original formulation. Instead of having an extra parameter we separate our set of vehicles V into two subsets SCV and MCV, where $V = SCV \cup MCV$. Finally, our formulation only uses fixed cost and variable cost so changes must be made in order to guarantee the same results in the Ostermeier-Hübner formulation. Therefore, loading and unloading costs have been removed from the objective function, leaving only the transportation cost tc_v.

The full mixed integer linear programming with the required adaptations is provided following.

Index:

- $L^\star = L \cup \{0\}$, where $L = \{1, .., n\}$ is the set of customers and vertex 0 is the depot
- P, set ofcommodities $p \in P$
- SCV, set of single compartment vehicles
- MCV, set of multiple compartment vehicles
- V, set of vehicles ($SCV \cup MCV = V$)

Parameters:

- d_{ij}, distance between locations i and j
- q_{ip}, qualtity of commodity p for customer i
- Q, vehicles capacity
- tc_v, Transportation cost dependent on the vehicle v

Decision variables:

- $x_{ipv} = \begin{cases} 1, & \text{if commodity } p \text{ of customer } i \text{ is assigned on vehicle } v \\ 0, & \text{otherwise} \end{cases}$
- $b_{ijv} = \begin{cases} 1, & \text{if vehicle } v \text{ is traveling from customer } i \text{ to } j \\ 0, & \text{otherwise} \end{cases}$
- $a_{vp} = \begin{cases} 1, & \text{if commodity } p \text{ is on vehicle } v \\ 0, & \text{otherwise} \end{cases}$
- $u_{iv} = t,\ t \in \{1, ..., |L^\star|\}$ representing the position t of customer i on tour/vehicle v

The Objective Function

$$\min \sum_{v \in V} tc_v \ . \sum_{i \in L^\star} \sum_{j \in L^\star} (d_{ij} \ . \ b_{ijv}) \tag{3.10}$$

Constraints

$$\sum_{j \in L} b_{0jv} \ \leq \ 1, \quad \forall v \in V \tag{3.11}$$

$$\sum_{i \in L} \sum_{j \in L} b_{ijv} \ = \ 0, \quad \forall v \in MCV \tag{3.12}$$

$$\sum_{i \in L} b_{ijv} \ = \ \sum_{i \in L} b_{jiv}, \quad \forall j \in L, v \in SCV \tag{3.13}$$

$$\sum_{j \in L} j.b_{0jv} \ = \ \sum_{j \in L} j.b_{j0v}, \quad \forall v \in MCV \tag{3.14}$$

$$u_{iv} \ - \ u_{jv} \ + \ |L^\star| \ . \ b_{ijv} \ \leq \ |L|, \quad \forall v \in SVN, i \in L^\star, j \in L \tag{3.15}$$

$$u_{0v} \ = \ 1, \quad \forall v \in SCV \tag{3.16}$$

$$\sum_{i \in L} b_{iiv} \ = \ 0, \quad \forall v \in V \tag{3.17}$$

$$\sum_{i \in L} \sum_{p \in P} q_{ip} \cdot x_{ipv} \leq Q_v, \quad \forall v \in SCV \tag{3.18}$$

$$\sum_{v \in V} x_{ipv} = 1, \quad \forall i \in L, p \in P \tag{3.19}$$

$$\sum_{p \in P} x_{ipv} \leq \sum_{j \in L^\star} b_{jiv}, \quad \forall i \in L, v \in SCV \tag{3.20}$$

$$\sum_{p \in P} x_{ipv} = |P| \cdot b_{0iv}, \quad \forall i \in L, v \in MCV \tag{3.21}$$

$$\sum_{i \in L} x_{ipv} = |L| \cdot a_{vp}, \quad \forall p \in P, v \in SCV \tag{3.22}$$

$$x_{ipv} \leq a_{vp}, \quad \forall i \in L, p \in P, v \in V \tag{3.23}$$

$$\sum_{p \in P} a_{v+1p} \cdot v \leq \sum_{p \in P} a_{vp} \cdot v, \quad \forall v \in SCV : v < |SCV| - 1 \tag{3.24}$$

$$\sum_{p \in P} a_{vp} \cdot (v-1) \leq \sum_{p \in P} a_{v-1p} \cdot (v-1), \quad \forall v \in MCV + 1 \tag{3.25}$$

$$\sum_{p \in P} a_{vp} \leq 1, \quad \forall v \in SCV \tag{3.26}$$

$$a_{vp} \in \{0,1\}, \quad \forall p \in P, v \in V \tag{3.27}$$

$$b_{ijv} \in \{0,1\}, \quad \forall i,j \in L, v \in V \tag{3.28}$$

$$u_{iv} \in \{1, ..., |L^\star|\}, \quad \forall i \in L^\star, v \in SCV \tag{3.29}$$

$$x_{ipv} \in \{0,1\}, \quad \forall i \in L^\star, p \in P, v \in V \tag{3.30}$$

The objective function (3.10) minimizes the overall costs, both the cost of using a vehicle and variable costs of routing. Constraints (3.11)-(3.13) ensures that only one vehicle departs from the depot to visit a customer and for every vehicle that arrives to a customer also must depart from it. Constraints (3.12)-(3.14) indicates that MCVs go directly to one customer without visiting additional customers. Constraints (3.15)-(3.16) are used for sub-touring eliminations by assigning to each customer its position on the route. Constraint (3.17) ensures that a visit to the same customer never happens. Constraint (3.18) guarantees that the demand is never higher than the vehicle capacity. Constraint (3.19) ensures that only one vehicle can be used for each route. Constraint (3.20) indicates that if a customer order a commodity it must be visited. Constraint (3.21) indicates that when MCVs are used a single vehicle can visit all commodities requested. Constraint (3.22) indicates that SCV can visit multiple customers. Constraint (3.23) ensures that a commodity must be assigned to a vehicle if a customer is visited. Constraints (3.24)-(3.25) are used to select the first available vehicles, in an ascending order, for both types of vehicles. Constraint (3.26) ensures that for SCVs only one commodity can be carried. Constraints (3.27)-(3.28)-(3.29)-(3.30) indicate the variables' domain.

Chapter 4

Computational Results

In order to fully understand the quality and the impact of the problem parameters, several computational experiments have been carried out. This chapter is thus exclusively dedicated to show all the experiences we have made by describing the instances and by analyzing the results achieved. In section 4.1, the comparison to modified Ostermeier-Hübner formulation will take place. Later, in sections 4.3, 4.4, 4.5 an analysis to understand the impact of each parameter in the final result is performed. In this latter case, only the Flow formulation was considered.

We tested the Flow and Modified Ostermeier-Hübner formulations on a computer with a 8-core Intel® Xeon® E5-2687W 0 processor at 3.10 GHz, with 16 Logical Processors and 128 GB installed physical memory (RAM).

The implementations were performed on IBM® ILOG® CPLEX® Optimization Studio, version 12.5, with a time limit of one hour.

4.1 Comparison between formulations

In order to guarantee the quality of our formulation, some tests have been carried out to compare the results achieved with the proposed formulation, based on single-commodity flow, and modified Ostermeier-Hübner formulation. To ensure the validity of the results, same instances were used, so that, in the end, a direct comparison between both performances can be done. Three different tests were made with three different values of *alpha*, which can be seen in Table 4.1. A value of 1.15 for instance 1, 1.5 for instance 2 and 2 for instance 3.

The parameters are set in Table 4.2, Table 4.2 and Table 4.3.

parameters	value
Commodities	3
Customers	4
Vehicles	50
Fixed cost	0
Variable cost	5
Capacity	30
alpha	{1.15,1.5,2}

Table 4.1: Parameters values considered

Demand	Product 1	Product 2	Product 3
Depot	0	0	0
Customer 1	5	12	8
Customer 2	10	7	15
Customer 3	5	5	5
Customer 4	6	6	6

Table 4.2: Demand for products

Distance	Depot	Customer 1	Customer 2	Customer 3	Customer 4
Depot	0	10.82	9.22	11.66	12.65
Customer 1	10.82	0	18.44	5	14.32
Customer 2	9.22	18.44	0	17	11.18
Customer 3	11.66	5	17	0	10.20
Customer 4	12.65	14.32	11.18	10.20	0

Table 4.3: Distances between customers

The numerical results are synthesized in Table 4.4, where we report for each formulation, Flow and Modified Ostermeier-Hübner, the execution time, the Gap (%), the total fixed cost (%), the total variable cost (%), the number of SCV, the number of MCV and the cost for Instances 1, 2 and 3.

The formula for calculating the value of Gap (%) is:

$$Gap(\%) = \frac{Best\ integer\ value\ -\ Best\ lower\ bound}{Best\ integer\ value}\ .\ 100 \tag{4.1}$$

The criteria for selecting the best formulation will depend on the quality of the final result, in terms of the lowest possible cost, and the total execution time. By

Table 4.4: Results for Flow and Modified Ostermeier-Hübner formulations

				Flow formulation						
Instances	Fc	*alpha*	Q	Execution time (s)	Gap (%)	SCV fixed cost (%)	SCV variable cost (%)	Number of SCV	Number of MCV	Cost
1	0	1.15	30	0.08	0	0	0	0	4	510.025
2	0	1.5	30	0.16	0	0	0	0	4	665.25
3	0	2	30	0.56	0	0	100	4	0	749.75

				Modified Ostermeier-Hübner formulation						
Instances	Fc	*alpha*	Q	Execution time (s)	Gap (%)	Total fixed cost (%)	Total variable cost (%)	Number of SCV	Number of MCV	Cost
1	0	1.15	30	0.25	0	0	0	0	4	510.025
2	0	1.5	30	3.12	0	0	0	0	4	665.25
3	0	2	30	106.34	0	0	100	4	0	749.75

analyzing the table it is possible to detect that the final results, in terms of costs and routes, are the same, meaning that our changes in the original Ostermeier-Hübner formulation were valid. Therefore, the decision behind selecting the best formulation only relays on the execution time.

From a close inspection on the operation time column, it is clear that our proposed formulation is far better, especially when multiple compartment vehicles are more expensive, and/or more single compartment vehicles are required. From this comparative analysis it can be seen that the proposed formulation produces more efficiently the same optimal solutions provided by the modified Ostermeier-Hübner formulation. The advantages of our approach are two-fold: a formulation that embraces more information in fewer restrictions and lower computational time with regard to the best-known (adapted) formulation. It is considered that the equation used on Ostermeier-Hübner to eliminate sub-touring (3.13) is not so efficient as the single-commodity flow, influencing the computational effort. Instances using a higher number of customers were not examined since it would only aggravate the differences in the execution times.

4.2 Framework

The cost associated with direct delivery (dd_i) is not a criterion since its calculation depends on the values of fixed cost and variable cost, with the addition of a penalty cost, *alpha*.

Cost for direct delivery can be calculated by:

$$dd_i = (f \ + \ 2 \ . \ u \ . \ d_{0i}) \ . \ alpha \ . \ y_i \tag{4.2}$$

where d_{0i} represents the distance between depot (0) and customer i and y_i indicates if customer i has associated a direct delivery.

The capacity of vehicles was calculated according to Archetti et al. [2015]:

$$d_{max} \ = \ max_{i \in C} \sum_{j=1}^{m} \ . \ d_{ij} \tag{4.3}$$

$$Q \ = \ \alpha \ .d_{max} \tag{4.4}$$

where α takes two values: α=1.1 and α=1.5

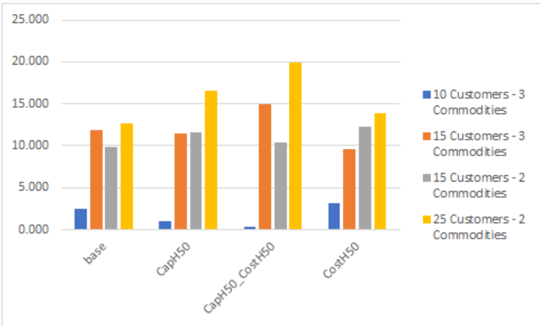
For our tests, we considered different degrees of flexibility towards demand, as presented in Archetti et al. [2015]. With the probability of 100% we ensure that all commodities are requested by all customers. Three different sets of instances are considered in this study:

- High variability demand (section 4.3): used instances with 10,15,25 random customers with an uniform demand between 1 to 100;
- Regular demand (section 4.4): the Solomon matrix was used to select 15 customers randomly out of 100 with fixed demands for each commodity (15 for commodity one, 10 for commodity two and 5 for commodity three);
- Homogeneous demand (section 4.5): a more real-life approach, which also uses the Solomon matrix to select 15 customers randomly among 100 with a demand between 40 and 60.

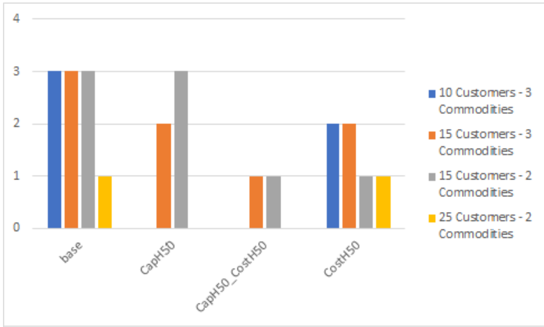
Figure 4.1 show the outputs of this first set of instances. Instances “CapH50”, “CostH50” and “CapH50_CostH50” are variations of the “Base” instance. Instance “CapH50” is the same instance as the “Base”, but with a higher vehicle capacity. “CostH50” has an increased cost for using direct delivery and “CapH50_CostH50” is a mix of both.

4.3 High variability demand

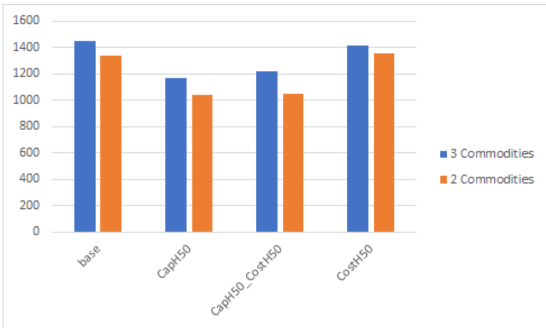
The first set of instances were generated based on Archetti et al. [2015]. We consider instances with 10, 15 and 25 customers, randomly located, with the possibility of having two or three commodities. Additionally, for each setting a group of five instances was considered.



(a) Average gap



(b) Number of direct deliveries



(c) Total cost

Figure 4.1: Average gap, number of direct deliveries and total cost for different commodities

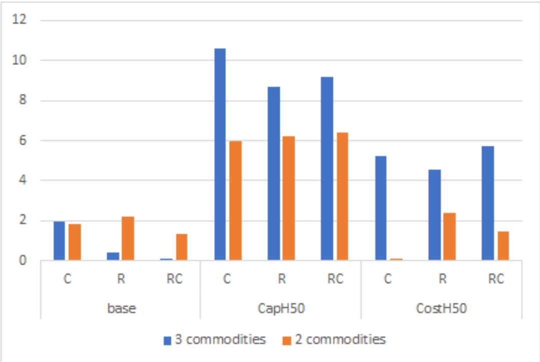
Looking at the results, in figure 4.1a it is not possible to detect a clear trend about the impact of each different setting. However, when the capacity of vehicles increases not only it is possible to reduce direct deliveries (Figure 4.1b) but it is also possible to reduce the overall operation cost (Figure 4.1c). The more expensive the direct delivery is the higher the total costs are.

Even though these results have shed light on some knowledgeable information, more instances are needed in order to fully understand the quality of the formulation, as well as to assess the impact of each parameter in the final result.

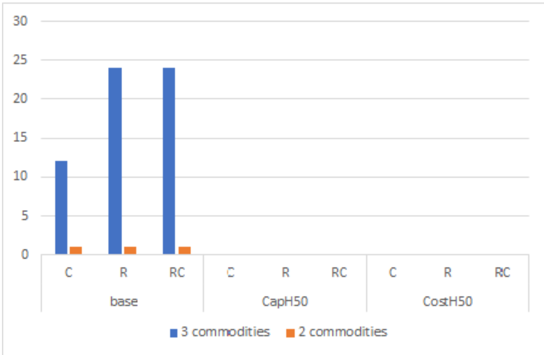
4.4 Regular demand

Demand can have a huge impact particularly on the number of direct deliveries. Therefore, in this experience a homogeneous demand is proposed, i.e., for each commodity the demand is the same for every customer. We consider that commodity one has a demand of 15, commodity two has a demand of 10 and commodity three has a demand of 5. This analysis was performed in order to understand if a more homogeneous demand can have an impact on the final result.

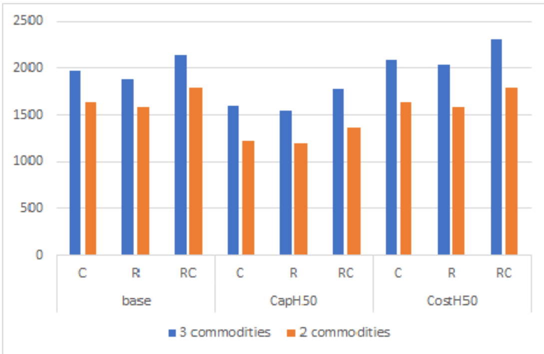
Figure 4.2 illustrates the results for this experiment.



(a) Average gap



(b) Number of direct deliveries



(c) Total cost

Figure 4.2: Comparisons between different instances in terms of (a) average gap, (b) number of direct deliveries and (c) total cost for regular demand instances

Unlike what was possible to see in the previous results, when a homogeneous demand is performed, differences regarding the average gap can be seen (Figure 4.2a). From a closer look in the "CapH50" instance, a peak of average gap emerges and the first thoughts may lead to assume that operational cost has also increased. But by looking at Figure 4.2c exactly the opposite happens. In "CapH50" the average cost is, in fact, lower, a common trend seen in prior results. This discrepancy may be due to the difficulty of updating the bounds when the program is running, making the gap larger, even.

(a) Routes for commodity one

(b) Routes for commodity two

(c) Routes for commodity three

(d) Routes for direct delivery

Figure 4.3: Representation of routes by commodities and direct delivery

After this experiment, it is possible to conclude that fixed cost, indeed, has an impact on the final result. When the value is lower better performances emerge. Additionally, average gap and number of direct deliveries also decrease when fixed cost is less expensive.

In Figure 4.3 the results of one instance are shown. For this test, 15 customers were considered. Vehicle capacity was limited to 30. Knowing the vehicle capacity and demand, we can conclude that for commodity one only two customers can be visited per route, for commodity two three customers and for commodity three six customers. As expected in Figure 4.3 we can observe this effect on routes. In Figure 4.3a only two customers are visited per route, in Figure 4.3b 3 customers and in Figure 4.3c 6 customers. Finally, since all customers must be visited, three additional routes must be performed (Figure 4.3d) to guarantee that customers 4, 5 and 6 receive their orders.

4.5 Homogeneous demand

In this section a more realistic approach to the problem was considered. With instances with 15 customers, we considered an uniform demand for each commodity between 40 and 60.

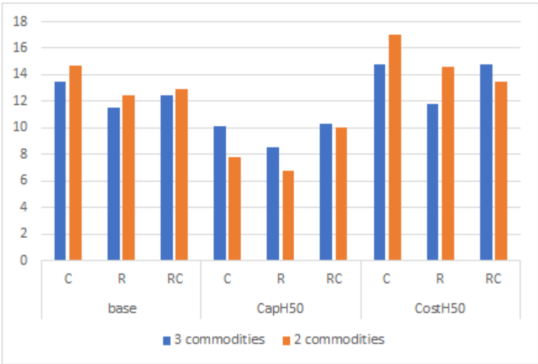
Unlike the previous test, only two different instances are considered for each set of parameters.

This experience can be separated into two parts. An analysis based on different number of commodities ordered is firstly performed and then on the value of the fixed cost for each vehicle.

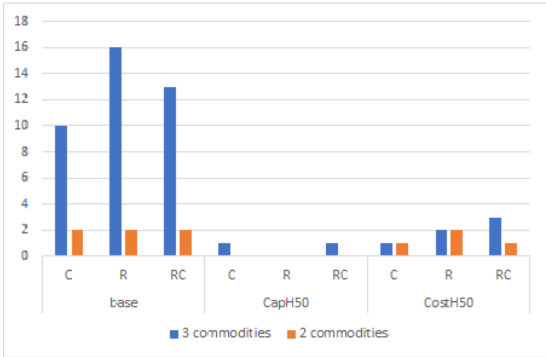
4.5.1 Different Commodities

The goal of this analysis is to understand the impact of the number of commodities in our problem. Therefore, instances are exactly the same with the exception of the number of commodities in order to be able to make a direct comparison looking at the final results. Figure 4.4 depicts the outputs for this instance.

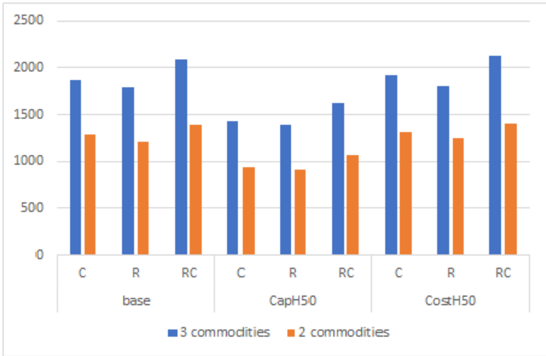
From the observation of Figure 4.4a it is not possible to detect any major differences in the average gap when different commodities are used and thus they do



(a) Average gap



(b) Number of direct deliveries



(c) Total cost

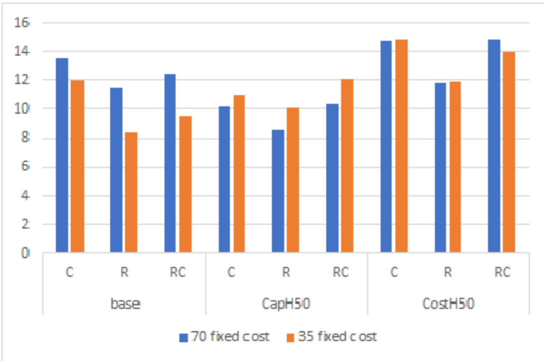
Figure 4.4: Average gap, number of direct deliveries and total cost for different commodities

not have an impact in terms of average gap. On the other hand, in Figures 4.4b and 4.4c it is clear that by reducing the number of commodities the number of direct deliveries are dramatically lower, especially in the "Base" instances, since on the others it is already normal not having a big amount of multiple compartment vehicles being used. Moreover, when looking at the total cost it is possible to see a reduction by 33% when two commodities are used, which is what it was expected since we are lowering the total expenses of the problem by around 33%.

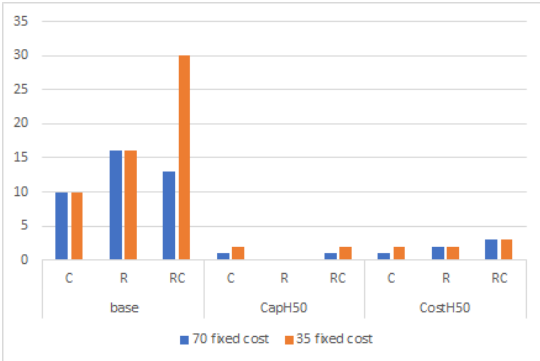
After analyzing the results it is possible to conclude that although in the average gap it is not possible to remark any differences, in terms of the number of direct deliveries and in total cost the number of commodities do have a big impact on the final result.

4.5.2 Different Fixed Costs

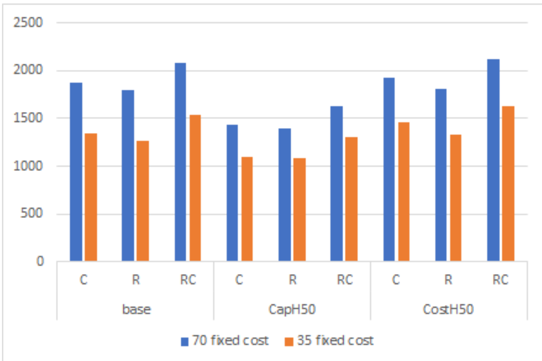
The purpose behind this experience is, likewise the previous one, to understand the impact of the parameters, in this case, the fixed cost. On the base case, a value of 70 is used for the fixed cost since it is the closest number for real-life cases. However, in this experiment, we will analyze the impact on the final results when the value of the fixed cost is lower than 35. The rest of the parameters used in the instances are the same, and so the results only change due to the change of fixed cost values.



(a) Average gap



(b) Number of direct deliveries



(c) Total cost

Figure 4.5: Average gap, number of direct deliveries and total cost for different fixed costs

Figure 4.5 shows the results for this trial. Looking at Figure 4.5a, where the average gap is analyzed, the differences between both are minimal and, therefore, it is impossible to conclude that fixed cost has an impact on the results. In figure 4.5b and especially on the "Base" instances, changing the fixed cost did alter the final results. By decreasing this value it is possible to see that the number of direct deliveries increases and this is due to the fact that multiple compartment vehicles are now cheaper since fixed cost enters in the calculation of direct delivery costs. Finally and focusing on Figure 4.5c it is clear that the higher the fixed cost the higher the overall operation cost is. This is expected because lowering fixed cost lowers vehicle usage cost and induces a lower number of direct deliveries.

After this experiment, it is possible to deduct that changing fixed cost can have a big influence on the number of direct deliveries, which could, later, influence the final result.

Chapter 5

Conclusions and Future Work

With this chapter, we conclude the exposition of these nine months of work. As the culmination of our activity, we summarize here the most relevant conclusions in this thesis.

Our work was focused on developing formulations for the Fleet Sizing and Composition problem on grocery retail distribution. Two Mixed Integer Linear Programming formulations were implemented on IBM CPLEX software[1] and each of them tested in a retail distribution environment to illustrate the applicability and to highlight the efficiency.

The main contribution of this thesis is the consideration of two different types of vehicles, single and multiple compartment, with different restrictions from the literature. In our approach, the multiple compartment vehicle is only used for direct deliveries, while the single compartment ones are operated for multiple deliveries of a single commodity. With this approach, the use of the latter is optimized and the former exploited only when cost-effective. A careful analysis of the impact of each parameter of the instances on the final result is presented and discussed.

Scientific Dissemination & Publications Together with the presented developments, the contents of this thesis have been presented in seminars and international meetings:

[1]CPLEX is a high-performance mathematical programming solver for linear programming, mixed-integer programming and quadratic programming

- S. Martins, T. Vasconcelos, A.Akbalik, C. Rapine. Fleet Sizing and Composition in Grocery Retailing, 7th Meeting of the EURO Working Group on Vehicle Routing and Logistics Optimization (VeRoLog), June 2-5, 2019, Seville, Spain
- DEGI Club, at Faculty of Engineering of the University of Porto, June 14, 2019, Porto, Portugal

and a paper is under preparation for submission to an international peer-review research journal.

Open Research Lines Many things have naturally remained to be done on the developments made. Some of them are just small ideas, and other, complex things that would entail a time that is far beyond the time available.

We here name some possible future courses of action that we think feasible and would improve this study:

- Inclusion of time windows;
- Addition of site-dependency, where some vehicles have access limitations to some customers;
- Multiple trip configuration, where a vehicle can complete several trips per day;
- Development of heuristics for larger instances (e.g. 50 customers)

Appendix A

Ostermeier-Hübner formulation

Index:

- $L^{\star} = L \cup \{0\}$, set of locations where $L = \{1, .., n\}$ is the set of customers, and vertex 0 is the depot
- P , set of product segments $p \in P$
- O , set of orders $o \in O$
- N_j , set of orders for customer j ($N_j \subseteq O$)
- S_p , set of orders for segment p ($S_p \subseteq O$)
- K , set of vehicle types $k \in K$
- V_k , set of vehicles of each type $k \in K, v \in V_k$
- C , set of compartments $m \in C$
- C_k , set of compartments for each vehicle type $k \in K, c \in C_k (C_k \subseteq O)$

Parameters:

- d_{ij} , distance between locations i and j
- q_o , quantity volume of order o
- Q_k , vehicle capacity of vehicle type k

Cost parameters:

- lc_m , loading cost depending on the number of compartments m, used on a vehicle
- tc_v , transportation cost dependent on the vehicle v

- ulc_v , unloading cost dependent on the vehicle v

Decision variables:

- $x_{ovc} = \begin{cases} 1, & \text{if order } o \text{ is assigned to the compartment } c \text{ on the vehicle } v \\ 0, & \text{otherwise} \end{cases}$
- $b_{ijv} = \begin{cases} 1, & \text{if the vehicle } v \text{ is traveling from customer } i \text{ to } j \\ 0, & \text{otherwise} \end{cases}$
- $a_{vc} = \begin{cases} 1, & \text{if the compartment } c \text{ is active on the vehicle } v \\ 0, & \text{otherwise} \end{cases}$
- $u_{iv} = t,\ t \in \{1, ..., |L^\star|\}$ representing the position t of customer i on tour/vehicle v

The Objective Function

$$\min \sum_{k \in K} \sum_{v \in V_k} [lc_{\sum_{c \in C_k}} a_{vc} + tc_v \left(\sum_{i \in L^\star} \sum_{j \in L^\star} d_{ij} \ . \ b_{ijv}\right) + ulc_v\left(\sum_{i \in L^\star} \sum_{j \in L} b_{ijv}\right)] \tag{A.1}$$

Constraints

$$\sum_{j \in L} b_{0jv} \leq 1, \quad \forall k \in K,\ v \in V_k \tag{A.2}$$

$$\sum_{i \in L^\star} b_{ihv} = \sum_{j \in L^\star} b_{hjv}, \quad \forall k \in K,\ v \in V_k,\ h \in L^\star \tag{A.3}$$

$$u_{iv} - u_{jv} + |L^\star| \ . \ b_{ijv} \leq |L|, \quad \forall k \in K, v \in V_k, i \in L^\star, j \in L \tag{A.4}$$

$$u_{0v} = 1, \quad \forall k \in K, v \in V_k \tag{A.5}$$

$$\sum_{o \in O} \sum_{c \in C_k} q_o \ . \ x_{ovc} \leq Q_k, \quad \forall k \in K, v \in V_k \tag{A.6}$$

$$\sum_{k \in K} \sum_{v \in V_k} \sum_{c \in C_k} x_{ovc} = 1, \quad \forall o \in O \tag{A.7}$$

$$\sum_{o \in N_j} \sum_{c \in C_k} x_{ovc} \leq |O| \ . \sum_{i \in L^\star} b_{ijv}, \quad \forall k \in K, v \in V_k, j \in L \tag{A.8}$$

$$\sum_{o \in O} x_{ovc} \leq |O| \ . \ a_{vc}, \quad \forall k \in K, v \in V_k, c \in C_k \tag{A.9}$$

$$\sum_{o \in S_p} x_{ovc} \leq |O| \ . \ (1 - x_{rvc}), \quad \forall k \in K, v \in V_k, c \in C_k, p, q \in P : p \neq q, r \in S_p \tag{A.10}$$

$$a_{vc} \in \{0,1\}, \quad \forall v \in V_k, c \in C, k \in K \tag{A.11}$$

$$b_{ijv} \in \{0,1\}, \quad \forall i,j \in L, v \in V_k, k \in K \tag{A.12}$$

$$u_{iv} \in \{1,...,|L^\star|\}, \quad \forall i \in L^\star, v \in V_k, k \in K \tag{A.13}$$

$$x_{ovc} \in \{0,1\}, \quad \forall o \in O, v \in V_k, c \in C_k, k \in K \tag{A.14}$$

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