GRAVITY INTERPRETATION AND INFORMATION THEORY

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SUMMARY

Methods generally used to transform a gravity map into another one (smoothing, second and higher derivatives, analytic continuations upwards and downwards etc.) mean linear transmission. From the point of view of information theory these methods are not quite consequential, having unnecessary digressions and not utilizing all the advantages of linear transmission. Therefore we attempt to develop a new general method of transformation using throughout the concepts and relations of information theory.

Since our data are only samples taken at discrete points from a continuous function describing the original gravity field, a practical analysis involves digital rather than analog computation. In any case the formulas (1) and (2) are to be applied and their coefficients can only be changed, to obtain an optimum transmission. In this introductory paper, some details of formula (1) and (2) will be investigated.

Introduction

The purpose of prospection by gravity methods is to discover geological structures. It is necessary in this work to clear away all effects which have no bearing upon the image of the structure in question. To do so, the gravity map representing the measured values must be transformed into a different kind of map. During the last 20 years or so, numerous methods have been developed for the performing of this transformation. According to their theoretical approach, these could be classified into several groups: smoothing, calculation of first and second derivatives, of analytic continuations and others. The exact analytic form of the gravity field is unknown; the values of the function $f(x, y)$ are known only at the discrete points of measurement. As a result, formulae of approximation have to be employed throughout, in which the measured values figure explicitly. Two types of approximative formula are usually employed. The one is

$$g^*(x, y) = \sum_{k=0}^{m} a_k \overline{g(r_k)}, \tag{1}$$

where x and y are the coordinates of the point of reference,

$g(r_k)$ is the average over the circle of radius r_k drawn around the point of reference,

a_k is the coefficient attributed to this average,

m is the number of circles considered.

The other type is

$$g^*(x, y) = \sum_{k=1}^{n} c_k g(x + x_k, y + y_k), \tag{2}$$

where $x + x_k$; $y + y_k$ are the coordinates of the moving point figuring in the calculations,

c_k is the coefficient attributed to the moving point,

n the number of points considered, and finally,

$g^*(x,y)$ is the transformed value in both cases.

Formula (1) may be employed also in the case of an irregular network of measurements while the other is used in the case of regular, mostly square, grids.

The large number of proposals concerning the derivation of the coefficients a_k and c_k is well known. For the approximation of the second derivative alone, more than 50 different sets of coefficients have been proposed. Numerous publications deal with the comparison of various sets of coefficients.

This state of facts, however, is not a sign of definite success. It is common knowledge that even the sets of coefficients designed to meet one and the same requirement – e.g. the calculation of the second derviative – yield markedly different transformed maps (see Hergerdt, 1957, Grosse, 1957). In the course of experiments performed on artificially composed maps it frequently turns out that every shape of the assumed disturbing body is best approximated by a different set of coefficients. Finally – and this is most important for practical work – each area of prospection requires a different set of coefficients in order to assure optimum transformation.

From the above-said, some highly important conclusions emerge:

1. As the final result of the operation depends partially on factors other than the coefficient set, it is misleading to perform the comparison of the sets by applying them to maps, and a method of comparison independent of maps has to be devised.

2. It is a purpose doomed to defeat to begin with to find a "best" series of coefficients which can be used to the best advantage ever hereafter, as there is no such set of coefficients. There have to be developed appropriate principles which permit to design or to calculate the most appropriate set for any given region, in the knowledge of the geological and geophysical features of that region.

3. As a corollary of the above idea, it is superfluous to attempt the best possible approximation of a given theoretical, mathematical transformation. (For instance, to try to approximate as closely as possible the second derivative.) An excellent approximation of the theoretical operation may yield quite a false or at least an unfavorable result in practice. If the average over the circles is calculated from a sufficiently large number of points and if formula

(1) is applied, the Henderson Zietz formulae represent an excellent approximation of the second derivative, and the Elkins – Peters set of coefficients a fairly poor one (Meskó, 1965): nevertheless, the application of the second approximation to the measured field is found to furnish better results. This is explained by the fact that the second derivatives as well as the analytical continuations downward enhance unduly the abrupt changes of the field (Swartz 1954, Dean 1958). However, errors of measurement or an inappropriate grid spacing (too great distances between points measured) tend to cause the most important distortion just in the case of abrupt changes. There arises consequently the danger of introducing the largest amount of ,,noise" just into the most accentuated parts of the picture.

It follows from the above-said that the accuracy of the approximation of a theoretical concept cannot possibly be a criterion of goodness suited to serve as a basis of choice among sets of coefficients. It is best to consider it irrelevant and to leave it completely aside, by adhering closely to the aims set and the possibilities given in every step from the formulation of the problem onward.

The problem – restated so as to emphasize the essential points – is as follows.

We are confronted with a set of values of which a map of contours representing some parameter or other can be constructed. This set is not sufficiently easy to handle and to interpret. We may transform it by means of formula (1) or (2) depending on whether the grid of measurements in question is regular or not. Our purpose shall be to derive by an appropriate transformation of our set of values a map that represents as clearly as possible the object prospected. In what manner are we to choose the values of a_k and c_k figuring in the formulae ?

We are in the fortunate position of finding the concepts and methods appropriate to our task among the apparatus of information theory. It is these that are to be employed to the solution of the geophysical problem. In the present paper the author wishes to commence the development of a self-consistent method based on information theory. To this end, he first analyzes the formulae (1) and (2). It is appropriate, however, to review as an introduction the most important concepts of information theory to be applied below.

Basic concepts and formulae of information theory

According to information theory, our Universe and the phenomena taking place therein may be described by one- or multi-dimensional signals. These signals are in a general way functions of the three space coordinates and of the time coordinate. It is obviously sufficient to consider exclusively the coordinates along which the signal is not constant, as it is only these that carry real information. If the coordinates do not include time, we speak of a configuration. In this order of ideas, the gravity field as considered in its projection upon a horizontal plane of reference is a two-variable configuration of the form $f(x, y)$.

The field in its physical reality is a continuous, analog signal. The measurement results represent samples of this continuous function. The set of measurements is a digital representation of the continuous function. As the measured area is a finite domain of space, it possesses a finite content of information that can be fully and unequivocally represented by a finite number of samples. (It is consequently unnecessary to increase the number of measurements above a given point.) The minimum number of points needed to characterize the domain depends on the spectral composition of the continuous function to be sampled.

By the Fourier transform, we may attribute to $f(x, y)$ a function, likewise of two variables, the so-called complex spectrum:

$$F(\omega, \psi) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) e^{-i(\omega x+\psi y)} dx dy, \tag{3}$$

where the coordinates ω and ψ represent space frequencies. Their dimension is cm^{-1}. To revert from $F(\omega, \psi)$ to the original function we have to employ the inverse Fourier transform:

$$f(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(\omega, \psi) e^{+i(\omega x+\psi y)} d\omega d\psi \tag{4}$$

If in the complex spectrum the values of the function different from zero are restricted to the arguments less than ω_1 along the x axis and less than ψ_1 along the y axis, respectively, we obtain for the spacing of the sampling points, s_x and s_y:

$$s_x \leqq \frac{\pi}{\omega_1}; \qquad s_y \leqq \frac{\pi}{\omega_1} \tag{5}$$

ω_1 and ψ_1 are termed upper frequency limits. If (5) fails to be satisfied, the spectrum of the digital set of data does not agree with that of the original continuous function (aliasing). Distortion first sets in the high-frequency part of the spectrum.

If the point spacing is $s_x = s_y = s$ everywhere, we have a regular square grid and the connection between the analog and digital description becomes

$$f(x, y) = \sum_{k=1}^{m} \sum_{l=1}^{n} f(ks, ls) \frac{\sin \frac{\pi}{s}(x-ks)}{\frac{\pi}{s}(x-ks)} \cdot \frac{\sin \frac{\pi}{s}(y-ls)}{\frac{\pi}{s}(y-ls)}. \tag{6}$$

Let us note – although this does not form an organic part of our line of thought – that as (6) furnishes the description of the field in analytical form, one can derive from it very simply and – if the number of coefficients to be utilized is also fixed, unequivocally – the coefficient set best adapted to certain theoretical purposes (Tomoda, Aki 1955; Tsuboi, Tomoda 1958).

The application of formulae (1) and (2) is essentially a linear transmission. „Transmission" is understood here in a very general sense, as the assigning of given output signals to certain input signals. In gravity interpretation the input signal is the untransformed map, the output signal the map subsequent to transformation. The transmission is essentially the performing of the operations of transformation. Transmission is linear if the transformation engendered by the operations in question is given for the sum of two signals as follows:

$$T\{f_1+f_2\} = T\{f_1\}+T\{f_2\}.$$

For a linear transmission, the complex spectra of the input and output signal are related as follows:

$$F^*(\omega, \psi) = G(\omega, \psi) \cdot F(\omega, \psi),$$

where $F(\omega, \psi)$ is the complex spectrum of the input signal,
$F^*(\omega, \psi)$ is the same for the output signal, and
$G(\omega, \psi)$ is the transfer function of the system.

The transfer function consists in general of complex values. But if phase shift is zero, the imaginary part becomes zero as well. In such cases the transmission may be described by a single two-variable function, the amplitude distortion of the operation.

To (7) there corresponds in the spatial domain

$$f^*(x, y) = g(x, y) * f(x, y),$$

where $g(x, y)$ is the weighting function of the system, and
$*$ indicates a convolution operation.

The transfer functions of the transformations employed in gravity interpretation have been calculated for certain special cases by several authors (Swartz, 1954, Dean, 1958, Byerly, 1965, Meskó, 1965). The transfer function of (2) is

$$G_2(\omega, \psi) = \sum_{k=1}^{n}{}' c_k e^{i(\omega x_k+\psi y_k)}, \tag{9}$$

The transfer function of (1) depends also on the number of points utilized in forming the average over the circles. If the number of the points tends towards infinity, we have

$$G_1(\omega, \psi) = \sum_{k=0}^{m} a_k J_0(\sqrt{\omega^2+\psi^2} \cdot r_k) \tag{10}$$

where J_0 is the zero-order Bessel function of the first kind.

It is obviously impossible to utilize an infinity of points, but (10) furnishes a fairly good approximation if a sufficient number of points is taken into consideration. One of the tasks to be solved hereunder is to give a more accurate description of the term „sufficient number".

A description independent of the maps is consequently feasible. In the general case, two separate two-variable functions are required as we must form both the real and the imaginary part of the transfer function. In the absence of a phase shift it is sufficient to form a single two-variable function. Let the upper frequency limit equal Ω along both the x and y axis. Let us suppose that the grid spacing was sufficiently close and that consequently the set of measured values contains indeed all the information included in the measured field. (Let us leave aside for the time being the influence of amplitude quantization.) We then obtain from (5)

$$s \leqq \frac{\pi}{\omega_1},$$

that is

$$\omega_1 \leqq \frac{\pi}{s}. \tag{11}$$

(For a regular grid, s is the grid interval; for irregular networks, we shall suppose that (11) is valid for the average grid point interval, too.)

We have assumed that the input signal contains no components having a frequency higher than π/s in any of the directions. (7) then reveals that the output signal will not contain any such components, either, and that this circumstance is entirely independent of the transfer function. It is then sufficient to define the transfer function for arguments less than π/s; that is, the value of $G(\omega, \psi)$ has to be calculated for the arguments falling into the quadrangle

$$|\omega| \leqq \frac{\pi}{s} \quad |\psi| \leqq \frac{\pi}{s}.$$

Let us introduce the relative frequencies ω' and ψ' by the equations

$$\omega' = \omega s; \ \psi' = \psi s. \tag{12}$$

Relative frequency is a dimensionless number. It is to be determined – independetly of grid spacing – always in the range

$$|\omega'| < \pi; \quad |\psi'| < \pi. \tag{13}$$

Investigation of the process of averaging on a circle

In this section the transfer functions of various averaging procedures will be computed and compared. It is important in practice to know how many points must be considered to receive a suitable transmission under given conditions. Namely – as it will be seen – the variation in the transfer functions grows more and more insignificant with increase of the number of values used in computation. This means that increasing that number beyond a point is unnecessary. The transfer function of the average computed from an infinite number of values (average by integration) is given by (10). The transfer functions of averages computed from some numbers of values (4, 6, 8 or 16) (average by summation) can be obtained from (9).

For simplicity it will be assumed that the points are symmetrically distri buted on the circle. Then, for $n = 4$:

$$G_{4,r}(\omega, \psi) = \frac{1}{2} \cos(\omega r + \cos \psi r), \tag{14}$$

for $n = 6$:

$$G_{6,r}(\omega, \psi) = \frac{1}{3} [\cos \omega r + 2 \cos(\omega r \cos 60^\circ) \cos(\psi r \sin 60^\circ)], \tag{15}$$

for $n = 8$:

$$G_{8,r}(\omega, \psi) = \frac{1}{4} [\cos \omega r + \cos \psi r + 2 \cos(\omega r \cos 45^\circ) \cos(\psi r \sin 45^\circ)]. \tag{16}$$

and lastly for $n = 16$:

$$G_{16\,r}(\omega, \psi) = \frac{1}{8} [\cos \omega r + \cos \psi r + 2 \cos(\omega r \cos 45^\circ) \cos(\psi r \sin 45^\circ) +$$

$$+ 2 \cos(\omega r \cos 22{,}5^\circ) \cos(\psi r \sin 22{,}5^\circ) + 2 \cos(\omega r \sin 22{,}5^\circ) \cos(\psi r \cos 22{,}5^\circ)]. \tag{17}$$

Introducing a new, dimensionless parameter μ by the definition

$$r = \mu s,$$

we can substitute $\mu\omega'$ and $\mu\psi'$ for $r\omega$ and $r\psi$ respectively, where ω' and ψ' are the relative space frequencies.

We have to investigate the goodness of approximation in the following equations:

$$J_0 (\sqrt{\omega'^2 + \psi'^2} \cdot \mu) \approx \frac{1}{2} [\cos \mu\omega' + \cos \mu\varphi'], \tag{18}$$

$$\approx \frac{1}{3} [(\cos \mu\omega' + 2 \cos(\mu\omega' \cos 60^\circ) \cos(\mu\psi' \sin 60^\circ)]. \tag{19}$$

$$\approx \frac{1}{4} [\cos \mu\omega' + \cos \mu\psi' + 2 \cos(\mu\omega' \cos 45^\circ) \cos(\mu\psi' \sin 45^\circ)], \tag{20}$$

$$\approx \frac{1}{8} [\cos \mu\omega' + \cos \mu\psi' + 2 \cos(\mu\omega' \cos 45^\circ) \cos(\mu\psi' \sin 45^\circ) +$$

$$+ 2 \cos(\mu\omega' \cos 22{,}5^\circ) \cos(\mu\psi' \sin 22{,}5) +$$

$$+ 2 \cos(\mu\omega' \sin 22{,}5^\circ) \cos(\mu\psi' \cos 22{,}5)] \tag{21}$$

The $G(\omega', \psi')$ frequency responses can be illustrated by surfaces over the ω', ψ' plane. Because of their symmetry properties, it is sufficient to represent the surfaces for the following intervals of their independent variables:

$$0 < \omega' < \pi; \quad 0 < \psi' < \pi$$

which is one quarter of the range given by (13).

The functions on the (common) left sides of (18) – (21) give a surface of rotation. The transmission corresponding to the integral average is nondirectional.

The other surfaces given by the functions on the right sides of (18) – (21) differ more or less from this surface of rotation. The deviation depends:

on the ratio $m = \frac{\psi'}{\omega'}$ (direction);

on the ratio $\mu = \frac{r}{s}$; (radius of circle vs. distance to sampling station);

and – obviously – on the number of values used in averaging (n).

Let us consider a straight line through the origin of the ω', ψ' plane. Its slope fixes a direction. A plane containing this line and perpendicular to the ω', ψ' plane intersects the surfaces in certain curves which represent the frequency responses in the considered directions.

To estimate the deviation of a surface corresponding to a given number n of points from the surface of rotation, it is sufficient to examine only two suitable directions. The first one is always (i.e. for every n) $m = 0$; while the second one depends on n as follows:

$m = \mathrm{tg}\, 45°$	if	$n = 4$
$m = \mathrm{tg}\, 30°$	if	$n = 6$
$m = \mathrm{tg}\, 22{,}5°$	if	$n = 8$
$m = \mathrm{tg}\, 11{,}25°$	if	$n = 16$

Let us consider now a fixed n. It would be easy to show that any curve defined by any arbitrary direction lies between the curves defined by the above-mentioned pair of directions. ($m = 0$, $m = \mathrm{tg}\, 45°$ for $n = 4$, etc.) If these two curves are both close to the curve representing the nondirectional integral average, the whole surface gives a good approximation of the surface of rotation.

Introducing the directions in question into the formulas on the right sides of (18) – (21) we obtain the following functions to be investigated:

$n = 4$, $m = 0$

$$G(\omega') = \frac{1}{2}\,[\cos \mu\omega' + 1], \tag{23}$$

$n = 4$, $m = \mathrm{tg}\, 45$

$$G(\omega') = \cos \mu\omega', \tag{24}$$

$n = 6$, $m = 0$

$$G(\omega') = \frac{1}{3}\,[\cos \mu\omega' + 2\cos(\mu\omega' \cos 60°)], \tag{25}$$

$n = 6$, $m = \mathrm{tg}\, 30$

$$G(\omega') = \frac{1}{3}\,[\cos \mu\omega' + 2\cos(\mu\omega' \cos 60°)\cos(\mu\omega' \sin 60°\, \mathrm{tg}\, 30°)]. \tag{26}$$

$n = 8, \quad m = 0$

$$G(\omega') = \frac{1}{4}\left[\cos\mu\omega' + 1 + 2\cos(\mu\omega'\cos 45^\circ)\right]. \tag{27}$$

$n = 8, \; m = \mathrm{tg}\, 22{,}5^\circ$

$$G(\omega') = \frac{1}{4}\left[\cos\mu\omega' + \cos(\mu\omega'\,\mathrm{tg}\, 22{,}5^\circ) + \right.$$
$$\left. + 2\cos(\mu\omega'\cos 45^\circ)\cos(\mu\omega'\sin 45^\circ\,\mathrm{tg}\, 22{,}5^\circ), \right. \tag{28}$$

$n = 16, \; m = 0$

$$G(\omega') = \frac{1}{8}\left[\cos\mu\omega' + 1 + 2\cos(\mu\omega'\cos 45^\circ) + 2\cos(\mu\omega'\cos 22{,}5^\circ) + \right.$$
$$\left. + 2\cos(\mu\omega'\sin 22{,}5^\circ), \right. \tag{29}$$

$n = 16, \; m = \mathrm{tg}\, 11{,}25$

$$G(\omega') = \frac{1}{8}\left[\cos\mu\omega' + \cos(\mu\omega'\,\mathrm{tg}\, 11{,}25^\circ) + \right.$$
$$+ 2\cos(\mu\omega'\cos 45^\circ)\cos(\mu\omega'\sin 45^\circ\,\mathrm{tg}\, 11{,}25^\circ) +$$
$$+ 2\cos(\mu\omega'\cos 22{,}5^\circ)\cos(\mu\omega'\sin 22{,}5^\circ\,\mathrm{tg}\, 11{,}25^\circ) +$$
$$\left. + 2\cos(\mu\omega'\sin 22{,}5^\circ)\cos(\mu\omega'\cos 22{,}5^\circ\,\mathrm{tg}\, 11{,}25^\circ)\right] \tag{30}$$

Figures 1—4. show the curves computed from (23)—(30).

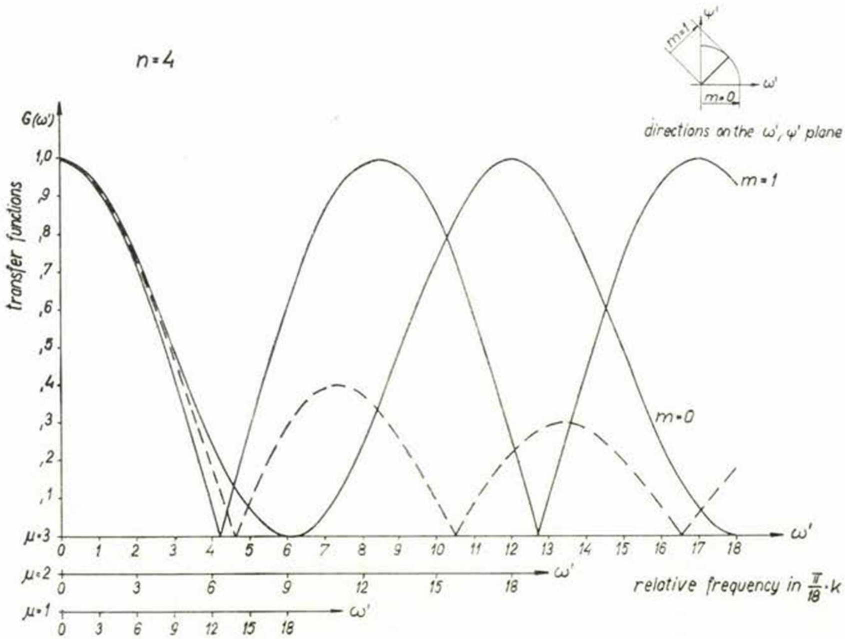
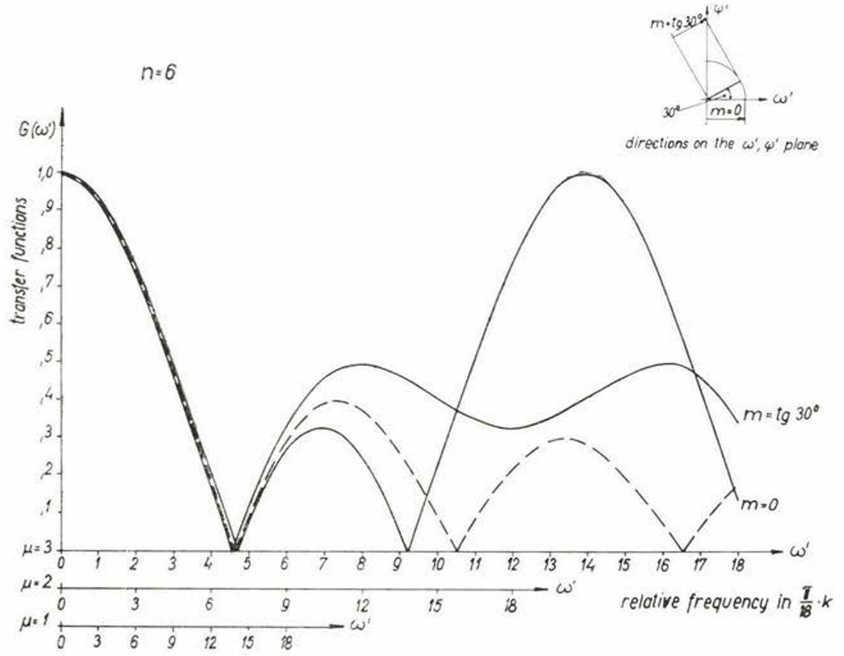
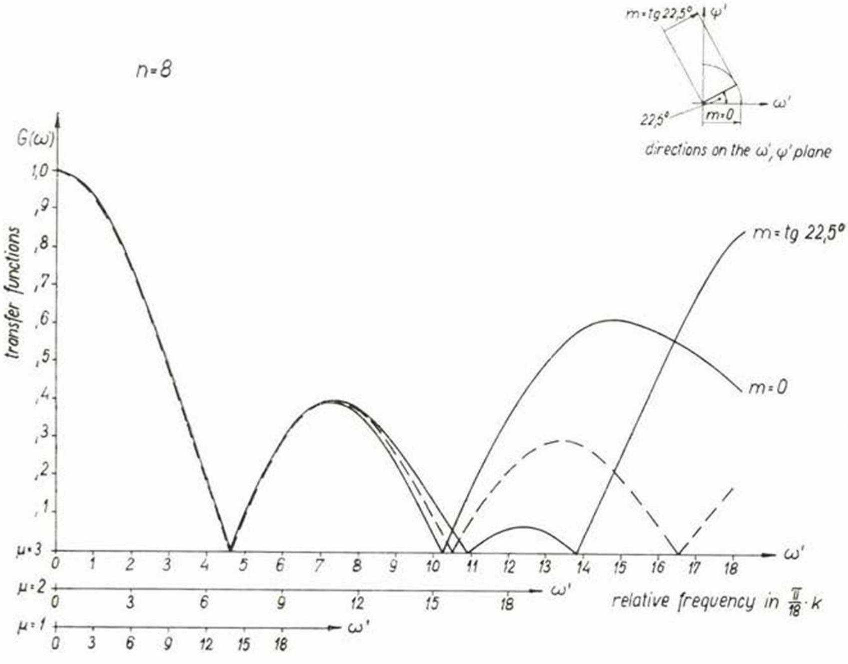


Fig. 1.



n=6
m=tg30°
φ'
30°
m=0
ω'
directions on the ω', φ' plane
G(ω')
transfer functions
1,0
,9
,8
,7
,6
,5
,4
,3
,2
,1
μ=3
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
μ=2
0 3 6 9 12 15 18
μ=1
0 3 6 9 12 15 18
m=tg 30°
m=0
relative frequency in $\frac{\pi}{18}\cdot k$

Fig. 2.



n=8
m=tg22,5°
φ'
22,5°
m=0
ω'
directions on the ω', φ' plane
G(ω)
transfer functions
1,0
,9
,8
,7
,6
,5
,4
,3
,2
,1
μ=3
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
μ=2
0 3 6 9 12 15 18
μ=1
0 3 6 9 12 15 18
m=tg 22,5°
m=0
relative frequency in $\frac{\pi}{18}\cdot k$

Fig. 3.

If it is necessary to compute averages on different circles during the transformation of a given map, the first thing to do is to determine the effective upper frequency limit ϱ_l of the radial frequency variable ϱ. To do so, we find on the map the direction in which the most rapid changes occur and then compute the upper frequency limit from the profile corresponding to this direction. The limit may be considerably lower than the folding frequency. We then compute the range of relative frequency in which the graphs are to

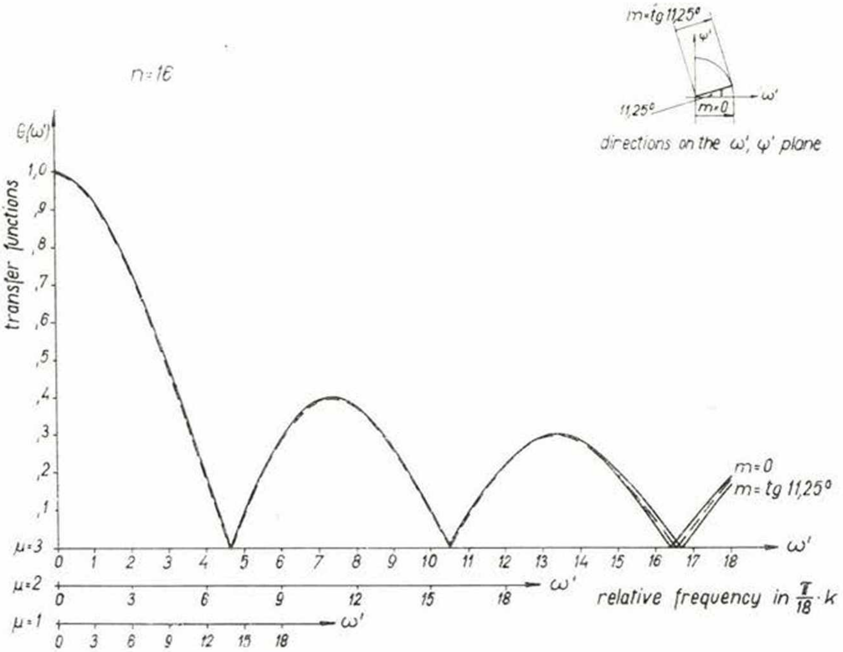


Fig. 4.

be compared. To facilitate this procedure, a simple parallel chart is given as Fig. 5. representing the connection between the true and relative frequencies for different station distances s as parameter. The given r and an average station distance (to be set in any case higher rather than lower) yields the value of parameter μ. Using the graphs in Figs. 1 – 4. there can be found the least number of points necessary to make the deviation from the curve of the non-directional frequency response negligible in the appropriate range of $\mu\varrho'$(or $\mu\omega'$).

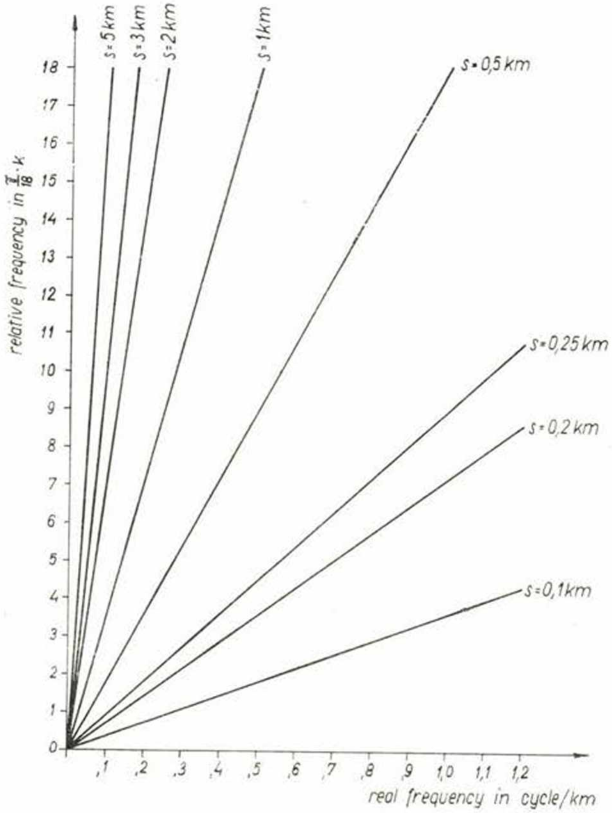


Fig. 5.

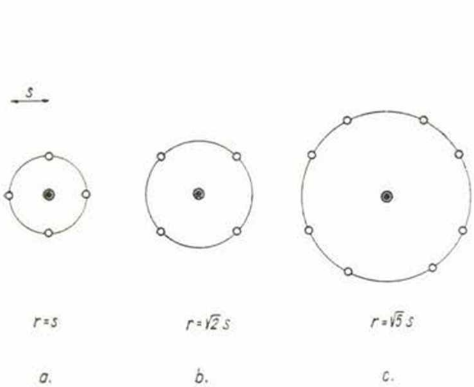


Fig. 6.

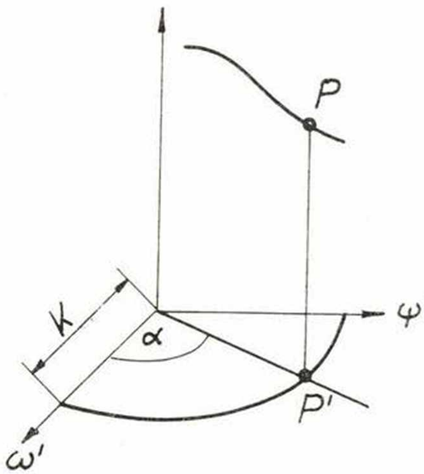


Fig. 7.

Some details of grid methods

In this section the transfer functions of some point arrangements used in grid computations will be computed. Figure 6. shows the simplest and most frequently employed arrangements. The transfer function for $\mu = 1$, $n = 4$ (6/a) given by (14) was already investigated. A simple computation yields

for $\mu = \sqrt{2}$, $n = 4$ (see Fig. 6/b)

and $\mu = \sqrt{5}$, $n = 8$ (see Fig. 6/c)

the following transfer functions:

$$G_{4,\mu=\sqrt{2}}(\omega', \psi') = \cos(\sqrt{2}\,\omega')\cos(\sqrt{2}\,\psi'), \tag{31}$$

and

$$G_{8,\mu=\sqrt{5}}(\omega', \psi') = \frac{1}{2}\left[\cos(2\sqrt{5}\,\omega')\cos(\sqrt{5}\,\psi') + \cos(2\sqrt{5}\,\psi')\cos(\sqrt{5}\,\omega')\right] \tag{32}$$

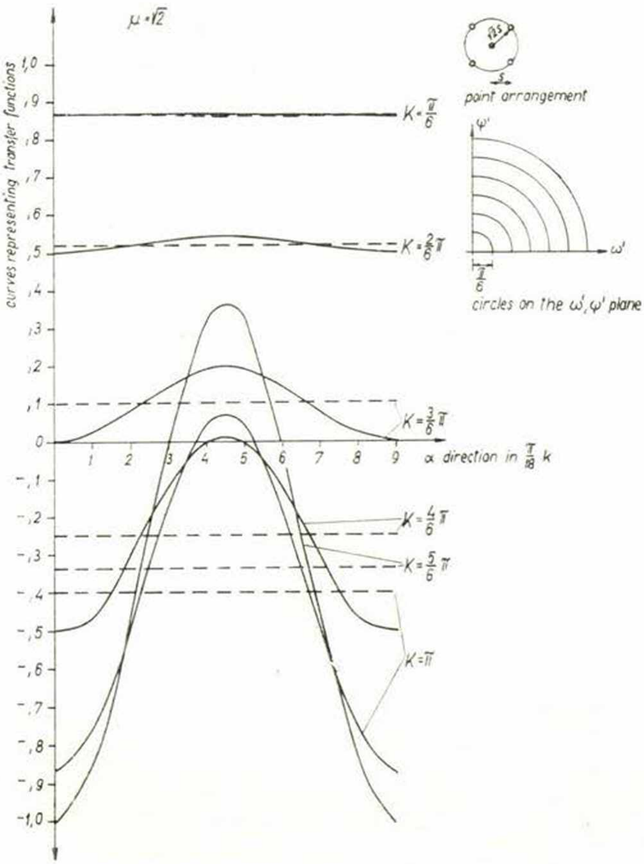


Fig. 8.

In the preceding section the surfaces representing the transfer functions were investigated along some fixed directions. Let us consider now a fixed circle on the ω', ψ' plane:

$$\omega'^2 + \psi'^2 = K_1, \quad \text{or:} \quad \varrho' = K(=\sqrt{K_1},)$$

A cylindric surface jacket containing this circle and perpendicular to the ω', ψ' plane intersects the surfaces in certain chcteraraistic curves (see Fig. 7.). The perpendicular distances of the points P on this curve from the ω', ψ' plane PP', depend on the direction α indicated by the point P' and on the radius K. Let us regard α as the argument and K as the parameter. Figs. 8. and 9. show the functions for the values of the parameter $K = \frac{\pi}{6} \cdot k$ (where $k = 0, 1, \ldots 6$). The curves represent the averages over 4, 6, 8 and 16 points, respectively, as well as the average by integration (dotted lines) on the circles $r = \sqrt{2}\, s$ and $r = \sqrt{5}\, s$.

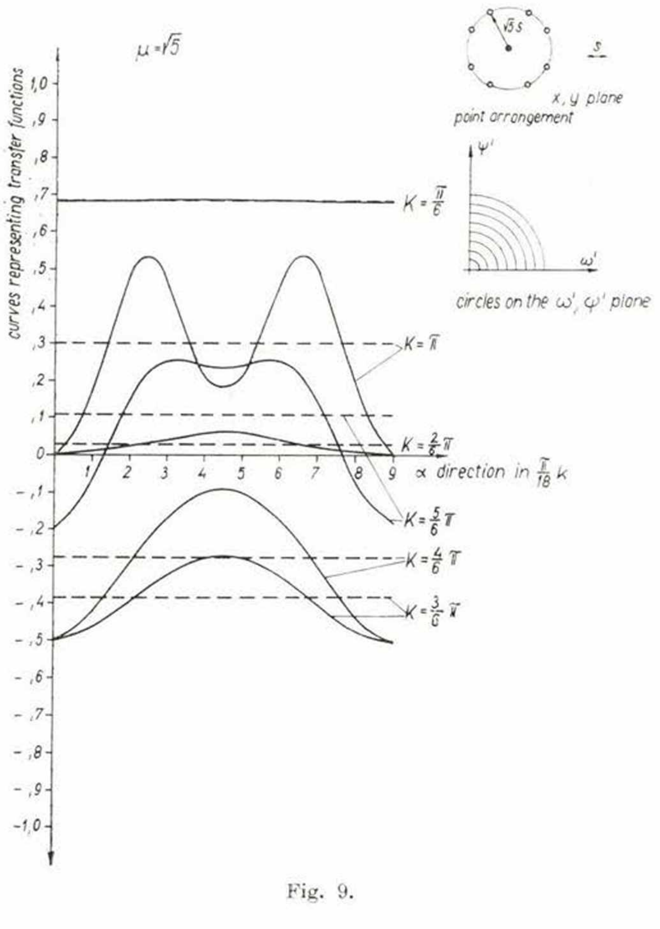


Fig. 9.

Using the equations (9) and (10) the transfer functions (frequency responses) of any given set of coefficients can be similarly computed and illustrated. These functions unambiguously characterize the transformations and permit to predict their effect on a map without any experimental computation.

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