# EULERIAN ANGLES AND THE PSEUDOSYMMETRY OF THE PLAGIOCLASE

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SUMMARY

Theoretical evidence for the solution of a practical problem has been given in the paper. Basing on measurements made by means of a Fedorov table new plagioclase twin laws were described in 1974, but the authors did not deal with the elimination of error caused by pseudosymmetry. Normal twin laws according to (110) and $(1\bar{1}0)$ as well as according to (130) and $(1\bar{3}0)$ could not be separated when basing only on external symmetry, since the angle deviation falls between the limits of measuring errors. But the relative position of crystallographic symmetry and of the optical indicatrix unequivocally determines whether in a given case we have to deal with a right or left form. For a correct determination of the plane index we have to state the relative position of the rectangular coordinate axes of irrational index taken of the basis of triclinic cristallographic directions.

When two new plagioclase twins were described *(L. Bondor – H. Szeberényi*, 1974) on the basis of data obtained from optical measurements by universal-stage, in all cases the Eulerian angles helped us to distinguish the right forms from the left ones. There was no problem to determine in the way of drawing on a stereographic net that the twin plane is the (110) or $(1\bar{1}0)$ face and in the other case it is the (130) or $(1\bar{3}0)$ face. It did not seem to be necessary to give the theoretical evidence of the applied method.

Because several mineralogists have doubted the possibility of the separation of the right and left forms, it seems to be necessary to return to this question.

The limit of error of the optical measurements is generally greater than it is in the case of measurements by goniometer. This fact may lead to an erroreous determination of the indices indeed in case of pseudosymmetry. But if the optical orientation is compared with the crystallographic one, the correct indices can be determined.

In the case of plagioclase crystals the triclinic crystallographic system of co-ordinates compared with the indicatrix would be a difficult thing. If we choose a Cartesian but irrational set after C. Burri *(C. Burri – R. L. Parker – E. Wenk*, 1967) it will be possible to use Eulerian angles, because the Eulerian angles relate to Cartesian set with the same origo.

The axes of the chosen crystallographic system of co-ordinates are indicated as $XYZ$

$$X = \frac{\perp [001]}{(010)} \qquad Y = \perp (010) \qquad Z = [001]$$

The optical axes are indicated as ABC

A = the highest optical elasticity direction, or optical main vibration direction

B = the medium optical elasticity direction

C = the lowest optical elasticity direction

In an other paper will be described the calculation of the indices on the basis of the optical and crystallographical data obtained from measurements by universal-stage.

Here we are dealing with the problem how the correct signs of the vector components and of the indices in the case of pseudosymmetry may be defined.

The first figure shows a plagioclase intergrowth which consists of six twin members, but the sixth is not measurable. In the first complex the intergrowth plane is the (010) face. Between the first and second individuals an Albite twin law is valid. The data are as follows:

| | | |
|---|---|---|
| Individual 1. | $n_a = 262.5°$ | $h_a = 37°$ |
| | $n_b = 18.5°$ | $h_b = 30°$ |
| | $n_c = 136°$ | $h_c = 39°$ |
| Individual 2. | $n_a = 190.5°$ | $h_a = 36.5°$ |
| | $n_b = 75°$ | $h_b = 30°$ |
| | $n_c = 317.5°$ | $h_c = 39°$ |
| Individual 3. | $n_a = 224.5°$ | $h_a = 29°$ |
| | $n_b = 318.5°$ | $h_b = 8°$ |
| | $n_c = 62{,}5°$ | $h_c = 60°$ |
| Individual 4. | $n_a = 267°$ | $h_a = 38.5°$ |
| | $n_b = 9.5°$ | $h_b = 15°$ |
| | $n_c = 122°$ | $h_c = 48°$ |
| Individual 5. | $n_a = 36°$ | $h_a = 80.5°$ |
| | $n_b = 144.5°$ | $h_b = 2.5°$ |
| | $n_c = 235°$ | $h_c = 10°$ |

The data of the measurement of the intergrowth planes:

| | | |
|---|---|---|
| | $n_I = 317°$ | $h_I = 0°$ |
| | $n_{II} = 79.5°$ | $h_{II} = 22°$ |
| The lines of growth: | $n_g = 317°$ | $h_g = 1°$ |
| Cleavage: | $n_{cl} = 80.5°$ | $h_{cl} = 21°$ |

The figure 2. shows the data of the measurements.

In the second complex there are three measurable twin members. Between the third and fourth individuals there is an Albite-Carlsbad complex twin law, the twin axis is equal to the $X$ axis in the system of

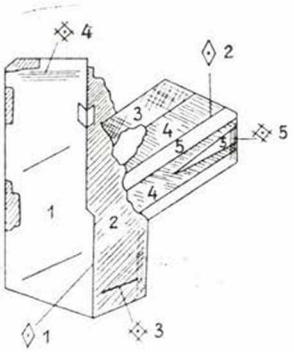



*Fig. 1.*

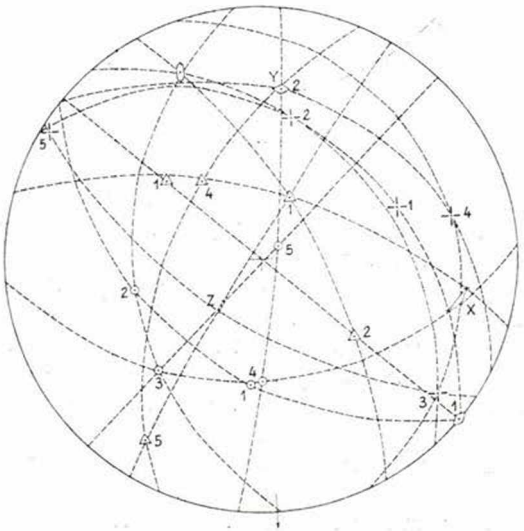



*Fig. 2.*

co-ordinates. Between the third and fifth members there is a Carlsbad twin law, and the twin axis is equal to the $Z$ coordinate axis. The fourth and the fifth are Albite twins, the normal of the (010) plane is the $Y$ axis.

If we choose such a stereographic projection plane which is the same as on the table 2. *(C. Burri – R. L. Parker – E. Wenk,* 1967) the $Z$ axis will be in the centre, and the $X$ and $Y$ axes will be on the outline of the circle. (Figure 3.)

Between the second individual and the third one there is a twin law and we want to define the twin axis. The second stereogram may be fitted on the table 2. in four different way (Figures 4a, 4b, 4c and 4d). So the requested twin axis may be near the (130) face and the ($\bar{1}30$) also. This uncertainty is caused by the fact that the positive and negative directions

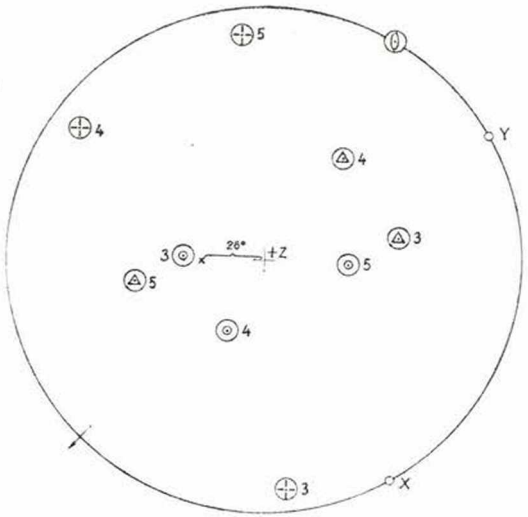



*Fig. 3.*

of the axes have not been taken into consideration. There are not sufficient crystallographic data to define the signs of the axes in many cases of investigation by universal-stage.

If this Cartesian set would be a rational crystallographic set (in the orthorhombic, and tetragonal system) the positive and negative directions of the axes would be invertable in the holohedral classes, because the symmetry of the right-angles system of co-ordinates requires all those points which are in the projection plane in the case of the four different fittings.

In the triclinic system the axes of the Cartesian set are not rational crystallographic directions, so only one fitting out of the four is possible. The inversion centre requires two points but either of them is on the lower half of the sphere.

The Eulerian angles proposed by C. Burri help us to choose the correct fitting.

The anortite content of the individuals may be read off on the plate IX. *(C. Burri—R. L. Parker—E. Wenk,* 1967). The anortite content varies between 75—78%. We can see the I., II., and III. kinds of Eulerain angles in catalogue 41. of the above authors. That fitting will be correct, in which the correct Eulerian angles can be read off. When the third individual shows the corresponding Eulerian angles the twin axis coincides with the $(\bar{1}30)$ face (Figure 4a). The second individual shows the twin axis to coincide with the $(1\bar{3}0)$ face at the corresponding Eulerian angles.

The result of the drawing there is a twin law according to $(1\bar{3}0)$.

That has been an example how the problem should be solved by drawing. Now the theoretical evidence of the solution must be given.

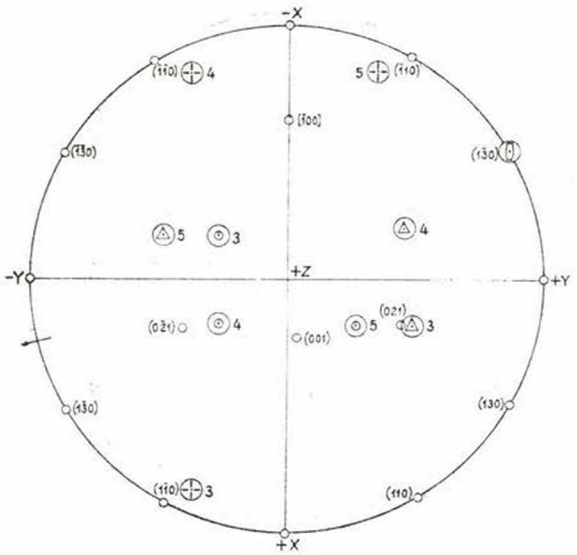
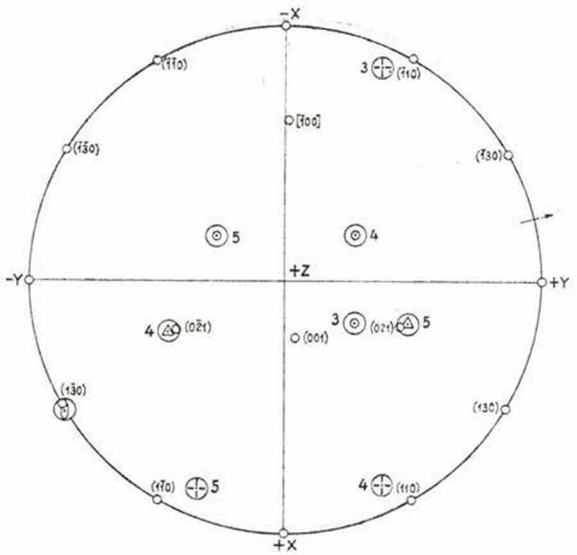

-X
(1̄10) 4
5 (11̄0)
[1̄00]
(1̄30)
(13̄0)
5
3
4
-Y
+Z
+Y
(0 2̄1)
4
(001)
5
(021)
3
(1̄30)
(130)
(1̄10) 3
(110)
+X

*Fig. 4a.*


-X
(1̄10)
3 (11̄0)
[1̄00]
(1̄30)
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+Z
+Y
4 (0 2̄1)
3
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5
(13̄0)
(130)
(1̄10) 5
4 (110)
+X

*Fig. 4b.*

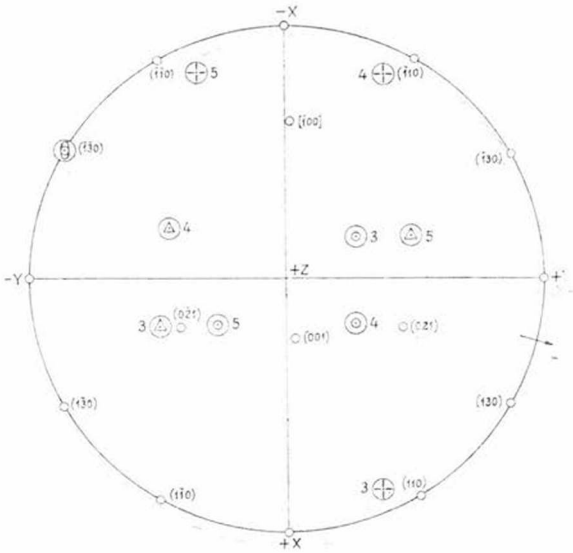
-X
(1̄10) 5
4 (110)
[100]
(1̄30)
(130)
4
3 5
-Y
+Z
+
3 (02̄1) 5
(001) 4 (02̄1)
(13̄0)
(130)
(11̄0)
3 (110)
+X

*Fig. 4c.*

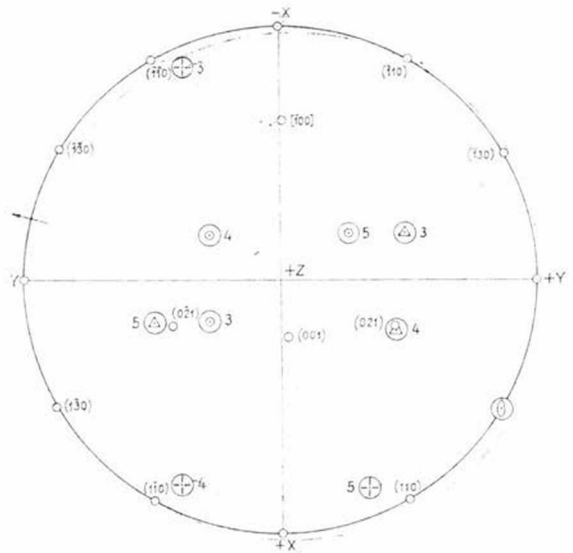
-X
(1̄10) 3
(3̄10)
[100]
(13̄0)
(130)
4
5 3
-Y
+Z
+Y
5 (02̄1) 3
(001) (02̄1) 4
(13̄0)
(11̄0) 4
5 (110)
+X

*Fig. 4d.*

What is unknown during the calculation or the drawing are those axial directions of the XYZ system of co-ordinates having a positive sign. If one of the three axes is taken with the opposite sign the set will be one of left-handed rotation. This mistake would not be committed if the rotation of the axes would be taken into consideration. All the axes can be inverted withouth an error, the result are the same, because the plagioclase crystals have an inversion centre.

Only one case should be examined i.e. when the sign of two axes are changed.

Point $P$ is the position of the pole of a particular face. The coordinates of point $P$ in the XYZ system of co-ordinates are indicated as $P(x, y, z)$. The vector of the normal of the face is:

$$p = xi + yj + zk$$

The same point P in the ABC set is indicated as P(a. b, c), the normal of the face:

$$p = am + bn + co \quad \text{if} \quad m, n \quad \text{and} \quad o$$

are the unit vectors in the directions of the corresponding axes.

Transformating the XYZ system of co-ordinates into the ABC system, the components of the point P are as follows:

$$a = x \cos (\widehat{AX}) + y \cos (\widehat{AY}) + z \cos (\widehat{AZ})$$

$$b = x \cos (\widehat{BX}) + y \cos (\widehat{BY}) + z \cos (\widehat{BZ})$$

$$c = x \cos (\widehat{CX}) + y \cos (\widehat{CY}) + z \cos (\widehat{CZ})$$

$$p = [x \cos (\widehat{AX}) + y \cos (\widehat{AY}) + z \cos (\widehat{AZ})]m + [x \cos (\widehat{BX}) + y \cos (\widehat{BY}) +$$

$$+ z \cos (\widehat{BZ})]n + [x \cos (\widehat{CX}) + y \cos (\widehat{CY}) + z \cos (\widehat{CZ})]o$$

If we take the opposite direction of the X and Y axes as positive directions and the indication of these are as X' and Y', and we take a point P' which has the same coordinates as the point P, then P'(x', y', z'). The value of x', y' and z' in the X'Y'Z system of co-ordinates are the same as those of x, y and z are in the XYZ set. Transformating the X'Y'Z system of co-ordinates into the ABC set, the components are as follows:

$$a' = x \cos (\widehat{AX'}) + y \cos (\widehat{AY'}) + z \cos (\widehat{AZ})$$

$$b' = x \cos (\widehat{BX'}) + y \cos (\widehat{BY'}) + z \cos (\widehat{BZ})$$

$$c' = x \cos (\widehat{CX'}) + y \cos (\widehat{CY'}) + z \cos (\widehat{CZ})$$

If the three members of the sum are different from one another and from zero, the value of the sum cannot remain the same if two of the members are taken with opposite sign. If are there any cases when $a = a'$,

$b = b'$, $c = c'$ or $a = -a'$, $b = -b'$, $c = -c'$ errors are possible, and we cannot eliminate the uncertainty of the indices. The first three equalities cannot be realized if x and y differ from zero.

The above second three equalities may exist in some special cases. Two members of the sums are zero, if all the three axes of the ABC system of co-ordinates coincide with the axes of the XZY set. The variations are as follows:

| | | | |
|---|---|---|---|
| If | $A = X$ | $B = Y$ | $C = Z$ |
| then | $a = x$ | $b = y$ | $c = z$ |
| | $a' = -x$ | $b' = -y$ | $c' = -z$ |
| if | $A = Y$ | $B = Z$ | $C = X$ |
| then | $a = y$ | $b = z$ | $c = x$ |
| | $a' = -y$ | $b' = -z$ | $c' = -x$ |
| if | $A = Z$ | $B = X$ | $C = Y$ |
| | $a = z$ | $b = x$ | $c = y$ |
| | $a' = -z$ | $b' = -x$ | $c' = -y$ |

In the other three cases we take the negative directions of the axes of either of the two system of co-ordinates. The absolute values of the components are different independence on the axes of the ABC set coincide with which axis of the XYZ system. Consequently $a = -a'$, $b = -b'$, $c = -c'$. Because the absolute values of the co-ordinates are the same as all the three have an opposite sign, it is impossible to define the correct indices.

In another special case one axis of the XYZ system of co-ordinates coincide with one axis of the ABC set. The other four axes must be in one plane.

If we choose the opposite sign of those two axes of the XYZ set which lie in the common axial plane, then one member of two sums will be zero. Taking the other two members with an opposite sign will give the same absolute value. The third component has two members equalling zero.

Let us see that case, when $C = Z$. The XYZ system of co-ordinates transformating into the ABC system:

$$a = x \cos(\widehat{AX}) + y \cos(\widehat{AY})$$

$$b = x \cos(\widehat{BX}) + y \cos(\widehat{BY})$$

$$c = z$$

$$a' = -[x \cos(\widehat{AX})] - [y \cos(\widehat{AY})]$$

$$b' = -[x \cos(\widehat{BX})] - [y \cos(\widehat{BY})]$$

$$c' = -z$$

So $a = -a'$, $b = -b'$, $c = -c'$ equalities exist in this case also.

Any other symmetrical position is geometrically impossible between the two systems of co-ordinates.

Practically we cannot define the correct indices in two special cases: 1. The axes of the two systems of co-ordinates are near each other. 2. One axis of the ABC set is near one axis of the XYZ system of co-ordinates, and the other axes are near in one plane.

We may say it in an other sense that the transformation of a Cartesian set into another right-angles system of co-ordinates (with the same origo) is unambiguous excepting the above mentioned two special cases.

Working with Eulerian angles on the plane of the stereographic projection the solution of that problem is more simple. Taking the first kind Eulerian angles the drawing is as follows:

The axes of the first system of co-ordinates are $n_\alpha$, $n_\beta$, and $n_\gamma$ optical vibration directions which must be transformed into the XYZ set. First we make a right-handed rotation around Z as an axis of rotation, so the line of intersection between the XY and the $n_\beta$ $n_\gamma$ planes must fall into the X direction. The rotation angle is $-\Phi$, if the $n_\beta$ direction is in the negativ part of the Y axis after rotation. Making a right-handed rotation around the $+$X axis $n_\alpha$ falls into the $+$Z direction. The rotation angle is $\Theta$. Finally we make a right-handed rotation around Z, so $n_\beta$ falls into $+$X and $n_\gamma$ falls into $+$Y. The rotation angle is $\Psi$.

The other two kinds of Eulerian angles are obtained by left-handed rotation.

The second kind Eulerian angles: Making a left-handed rotation around axis Z, the line of intersection between the XY and the $n_\alpha$ $n_\gamma$ planes falls into the X axis. The rotation angle is R, so $n_\beta$ falls in the positive part of the Y axis. We make a left-handed rotation around $+$X, the $n_\beta$ falls into $+$Z, the rotation angle is I. Then we make a left-handed rotation around $+$Z, the $n_\gamma$ falls into the $+$X, the rotation angle is L.

The third kind Eulerian angles are as follows: First we make a left-handed rotation around axis Z as an axis of rotation, so the line of intersection between the XY and $n_\alpha$ $n_\beta$ planes must fall into the X axis. The rotation angle is D, n is in the $+$Y axis. Then making a left-handed rotation around $+$X, so the $n_\gamma$ falls into the $+$Z axis. The rotation angle is N. Finally we make a left-handed rotation around $+$Z so the $n_\alpha$ must fall into the $+$X axis. The rotation angle is K.

If the opposite directions of two axes of the XYZ set will be taken as positive axes, and the rotation will be made as above, the result is as follows:

| | | |
|---|---|---|
| . $-\Phi' = 180° - \Phi$ | $\Theta' = \Theta$ | $\Psi' = \Psi$ |
| . $R' = 180° + R$ | $I' = I$ | $L' = L$ |
| . $D' = 180° + D$ | $N' = N$ | $K' = K$ |

One of the three Eulerian angles is certainly not the same as it is in the first case.

If the $-\Phi$ angle is near 90° and the $\Psi$ is also near 90°, the $n_\gamma$ vibration direction falls near the negative direction of the Y axis. At the law-temperature plagioclase crystals in the case of 20% anorthite content the $-$Y axis and the $n_\gamma$ vibration direction form an angle of 1.3°. Here a very

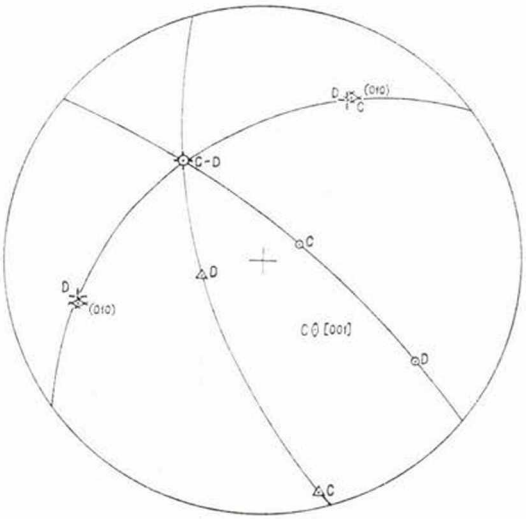


*Fig. 5.*

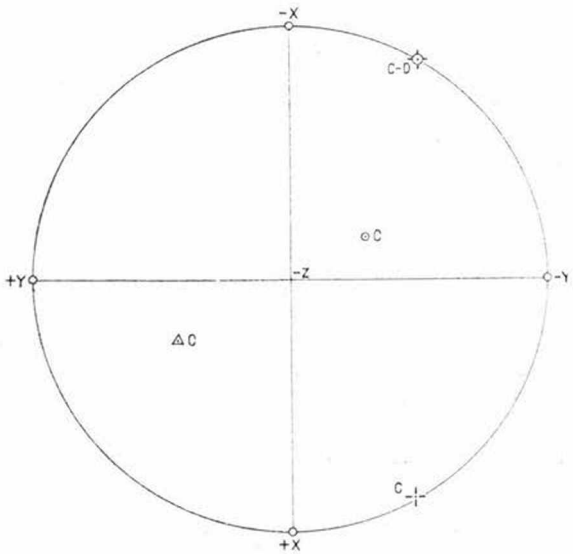


*Fig. 6.*

precise microscope and universal-stage is needed to eliminate the error. In the case of the other members of plagioclase series there is no problem like this.

The way in which the optical vibration directions relate to the crystallographic orientation is a characteristic feature of the minerals. In plagioclase crystals it is a lucky circumstances that the indicatrix and the crystal-

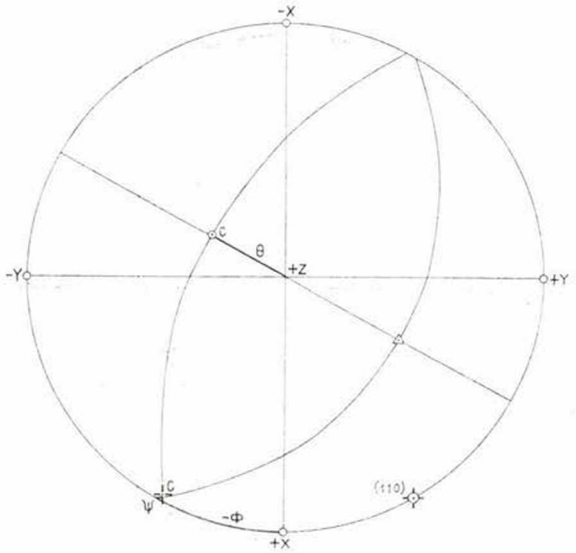


*Fig. 7.*

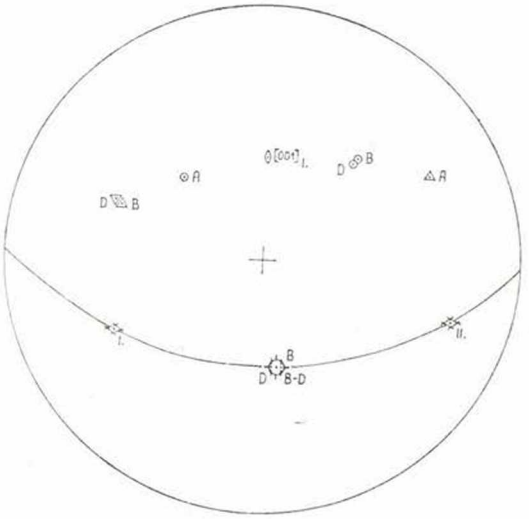


*Fig. 8.*

lographic orientation are in a general position and therefore the position of the ABC system of co-ordinates and of the XYZ set is also a general one.

The figures 5 – 9 show how the sign of the axes can be defined. The figure 5 shows the measured data of the C and D members of a plagioclase twin crystals.

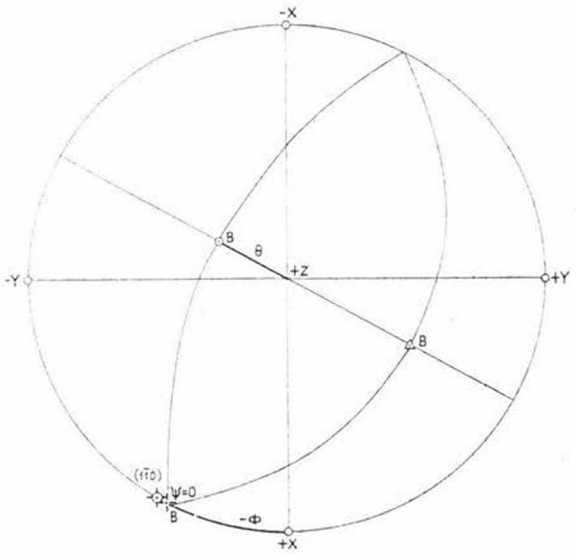


*Fig. 9.*

On the figure 6 the rotated member C can be seen, the projection plane being the XY plane. If this is rotated with 180° around the X axis, the figure 7 will show the Eulerian angles and the correct sign of the angles.

Figure 8 shows the measured data of the A, B and D members of an other plagioclase intergrowth. In the figure 9 the individual B is rotated into the XY plane. The Eulerian angles can be seen withouth further rotation.

Eulerian angles may also be used at monoclinic minerals in a similar way.

Eulerian angles and right-angle cristallographic co-ordinate systems in the literature are proposed only for plagioclase crystals. A Cartesian set can be established for any mineral whatever, provided that the most frequent faces and zone axes are taken into consideration.

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