ON THE ASTRONOMICAL TESTS OF GENERAL RELATIVITY

With 5 figures and 2 tables

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SUMMARY

The present state of observational verification of the three classical effects + 1. Perielion advance of planets, 2. Gravitational shift of spectral lines, 3. Light-deflection in graviational field – is outlined.

For forty years after the foundation of the general theory of relativity the astronomical observations of those three effects constituted merely the emprirical tests of the theory which Einstein himself has already mentioned: 1. the precession of the perihelion of Mercury, 2. the deflection of light rays passing near the Sun, 3. the gravitational shift of spectral lines. During the last years new methods have been suggested, particularly with the use of artificial satellites. For the empirical verification of the theory a new possibility of laboratory experiment opened by the Mössbauer effect, which enables us to measure extremely small changes of the frequency of gamma rays and, consequently, laboratory measurement of the gravitational red-shift has become possible [1, 2]. This is the more fortunate since astronomical observations concerning this effect are contradictory. This effect follows from the principle of equivalence* but it can be considered also as the energy-change of photons performing work in the gravitational field. Thus it happens that exactly this phenomenon proved in the laboratory does not present a crucial test of General Relativity.

The great elegance in principle and construction of the general theory of relativity has without doubt contributed to the fact that text – books of

* Gy. Marx had the kindness to call my attention to the fact that L. J. Schiff considers also the light-deflection as a consequence of the principle of equivalence [3]. A. Schild, however, is opposing Schiff, the more so, as Nordström's theory of gravitation, for instance, which also satisfies the principle of equivalence, gives a different value for the light-deflection [4]. See also another paper by R. U. Sexl [5].

the theory, particularly those published in the twenties, judged far too optimistically the first astronomical observations made to test the theory. We can read in several text-books that the gravitational shift of spectral lines has been successfully proved in the solar spectrum and for the white dwarf companion of Sirius, whilst light-deflection had already been demonstrated on the occasion of the total solar eclipse in 1919. This is not true at all, and we shall see that the astronomical verification of these two effects has remained uncertain even nowadays. Recently alternative theories of gravitation are frequently discussed, which either do not accept the principle of equivalence or that of covariance, such as, particularly, the theories of Whitehead [6] and Birkhoff [7]. These theories give the same values for the three above mentioned effects, which therefore do not afford decisive empirical criterions between the different theories.

The three theories give different numerical values for the rotation effect, i.e. the influence of the rotation of a body on the motion of its planet or satellite. The astronomical observation of this effect is nowadays impossible, since the Sun is rotating very slowly, and even the distance between the Sun and its nearest planet is far too great in this respect. In the solar system the effect is significant only for the fifth satellite of the quickly rotating Jupiter, this could be measured, however, with too much difficulty.

It should be mentioned that formulae given on this effect regard all weak fields, and the accordance with experiment would not yet guarantee the strict accuracy of Einstein's field equations. The basic assumptions of the theory of relativity, on the other hand, are not sufficient for the unambiguous determination of the field equations, and if some experimental or observational results would contradict the computations based on the field equations, this would not mean that the basic ideas of the general theory of relativity are not valid.

Relativistic effect in celestial mechanics.

Even according to classical mechanics the perihelion of a planet moves within its orbit plane in the direction of the motion of the planet, as a consequence of perturbations by other planets. In the theory of General Relativity such an effect occurs already in the one-centre problem, and the predicted advance in the longitude of the perihelion, D, expressed in seconds of arc per century amounts to

$$D = 3.34 \times 10^{33} \cdot a^{-5/2} \quad (1-e^2)^{-1} \tag{1}$$

wherin a is the semi major axis of the planetary orbit expressed in cm, and e is the excentricity of the orbit. D is greatest for Mercury; 43″, 03, whereas it is 8″, 63 for Venus, 3″, 84 for the Earth, 1″,35 for Mars, and 0″, 06 for Jupiter. In case of Venus the determination of D from observations is difficult, because of the small excentricity the location of the line of the apsides in the nearly circular orbit is rather uncertain.

On the other hand, the observation of Mercury belongs to the difficult tasks of positional astronomy, Mercury being observable but in daytime, and since Mercury always is near the Sun, the turbulence of the atmosphere heated by the Sun leads to serious errors in the determination of its position. The task becomes even more complicated by the variations of the shape of the planetary disk. Besides, the determination of the positions cannot be performed in a Newtonian coordinate-system, the observations refer to the moving equinox, and the exact determination of the precession of the equinox is no easy task at all. The perihelion advance due to the perturbations of the planets is considerably greater than the relativistic effect, hence a complete perturbation-theory of the planets should be worked out for the verification of the relativistic effect.

The US Naval Observatory in Washington has accomplished this immense work after World War II by means of electronic computing machines, thus bringing up to date Newcomb's computations performed around the turn of century. In Table 1 we show the results obtained by C. M. Clemence for Mercury [8] and by H. H. Morgan for the Earth [9]. We see which factors contribute to the perihelion movement according to Newtonian mechanics. The differences between observations and Newtonian theory is in good accordance with values predicted by formula (1), in case of the Earth, however, this is not of great weight, the error being 60 per cent of the result. It is the same with the results obtained for Mars and Venus. In all events, none of these results contradicts formula (1), and for Mercury the accordance is excellent.

Since the difference between Newtonian theory and observations has already been remarked by Leverrier, an explanation for this difference was endeavoured long ago before the theory of relativity became known. Leverrier himself tried to explain the difference by a planet within the orbit of Mercury, and he called this hypothetical planet Vulcan. Nowadays, we know that there is no such planet, but it may be that several planetoids are revolving near Mercury. Seeliger tried to give an explanation for the perihelion advance of Mercury by considering the effect of the dust-cloud causing the zodiacal light, but he assumed for it a density exceeding by several orders of magnitude the value derived from modern observations. Harzer tried to explain the difference by the oblateness of the Sun, but this effect is too small. It may be, of course, that axial rotation in the interior of the Sun is much quicker than at the surface, and the equipotential surfaces are there strongly flattened. As a matter of curiosity it should be mentioned that Grossman in the twenties computed the internal structure of the Sun on the basis of Jean's theory of radiative retardation. According to this theory the angular velocity is strongly increasing inwards. By taking into consideration the oblateness of the interior equipontential surfaces, Grossman obtained for the perihelion movement of Mercury exactly the value resulting from formula (1).

The orbits of several minor planets have a very great excentricity, and within a certain period it will be possible to study the movement of their perihelia resulting from (1). A particularly high value is expected for the minor planet Icarus 1566. In this case $a = 1{,}6 \times 10^{13}$ cm, $e = 0{,}8265$. From (1) we get for the perihelion advance during a century the value $10'',05$ [10]. For the planet Hermes the effect is $2'',62$, for Apollo $2'',10$, for Adonis $1'',80$. But all these planets can be seldom observed, and decades are required for the accu-

Table 1

Contributions to the perihelion movement of Mercury and the Earth according to G. M. Clemence and H. R. Morgan

	Mercury		Earth	
Mercury	0",03 ±	0,00	− 13",75 ±	2,3
Venus	277,85	0,68	345,49	0,8
Earth, Moon	90,04	0,08	7,68	0,0
Mars	2,54	0,00	97,69	0,1
Jupiter	153,58	0,00	696,85	0,0
Saturn	7,30	0,01	18,74	0,0
Uranus, Neptune	0,18	0,00	0,75	0,0
Solar oblateness	0,01	0,02	0,00	0,0
Precession	5025,65	0,50	5025,65	0,5
Sum	5557,19 ±	0,85	6179,1 ±	2,5
Observed motion	5599,74	0,41	6183,7	1,1
Difference	42,55 ±	0,94	4,6 ±	2,7
Relativistic effect	43,03 ±	0,03	3,8 ±	0,0

rate determination of the effect. By and by it will be possible to test relativistic movement of perihelion for close double stars with great excentricity. A very suitable case is DI Herculis having an excentricity 0,453 [11].

Nowadays the possibilities to determine the movement of perihelion and the relativistic effect of the Earth's rotation by artificial satellites are much talked over. The artificial satellites with their small masses, however, will hardly be suitable to demonstrate effects like that, since collisions with meteorites as well as air resistance may influence their orbit.

An interesting new suggestion came from L. I. Schiff [12] for the observation of the precession of the axis of a gyroscope in a gravitational field. The axis of a gyroscope on the Earth, if the centre of mass of the gyroscope rests with respect to the Earth, would precede 0",4 in a year. This precession could be increased, if the gyroscope would be placed on an artificial satellite, because then, beside the rotation-effect of the Earth there is a precession of the axis depending on the velocity of the satellite. This experiment is by no means easy, neither on the Earth where the gyroscope ought to be suspended against gravity, nor on an artificial satellite where suspension is not necessary, but other difficulties would arise.

Astronomical Measurement of the Gravitational Shift of Spectral Lines

According to the theory of relativity, the spectral lines in a gravitational field of potential Φ show — as compared with those in a gravitation-free field — a red-shift $\Delta\lambda$ according to the quation

$$\Delta\lambda/\lambda = \Phi/c^2 \tag{1}$$

The shift of the lines in the solar spectrum can be computed from this formula, since the gravitational field of the Earth can be neglected. A red-shift of 0,0063 A can be expected for the wave-length of 3000 A, and one of 0,0148 A for 7000 A. Accordingly, the red-shift is equivalent to an apparent

Doppler-effect of +0,636 km/sec. For a star of an arbitrary mass M and radius R, the relativistic shift corresponds to a Dopplershift of

$$v=0{,}636\ \mathrm{M/R}\ (\mathrm{km/sec}) \tag{3}$$

if M is given in solar mass and R in solar radii. Since in the case of stars of great mass also the radius is usually large, no greater values than 4 km/sec can be expected for v even for stars of the greatest mass. Most hopeful is the situation for the white dwarfs having radii smaller than that of the Sun by two orders of magnitude. As their mass is about $M=1$, some white dwarfs may be expected to show a red-shift equivalent to $v=+60$ km/sec. The radii of the recently discovered sub-white-dwarfs are still smaller, but their masses are small as well.

When studying the red-shift in the solar spectrum, we are compensated for the smallness of the effect by being able to use high dispersion spectrographs. Measurement of the effect is difficult even then, the effect being a small fraction of the widths of the spectral lines. In the first measurements a great number of lines were used, later on the measurements were rather concentrated on a few carefully chosen Fraunhofer-lines. The most careful measurements have been performed by Miss M. G. Adam [13]. According to the results:

1. the red-shift shows a systematic trend with the intensity of the lines.

2. the red-shift in the centre of the Sun's-disk is much smaller than required, near the limb of the Sun it suddenly becomes greater, and beyond the limb it reaches high values (limb-effect).

Schrőter very carefully studied all data of observations concerning the red-shift of solar spectral lines [14]. It turned out, that among the 1500 spectral lines measured by St. John, only four per cent were free of blends. The wave-length errors of the lines used for comparison contributed to the errors of observation. When comparing the solar Fraunhofer-lines with the emission lines of the comparison spectra, systematic errors depending on the width of the Fraunhofer-lines may occur as well. Much trouble is caused by the circulations in the solar atmosphere. According to Schrőter, the differences between theory and measurements arise, above all, from the up- and down movements of the solar granulae. In consequence of local Dopplershifts, each Fraunhofer-line is the superposition of lines originating in granulae *and* intergranulae. The small asymmetry of the resulting line-contours causes an apparent line-shift. Thus Schrőter succeeded in explaining the limb-effect as a function of line-strength.

Recently the French astronomers, J. E. Blamont and F. Roddier determined the profile of the solar strontium line at 4607 angstroms with great precision [15]. They brought the light to be analyzed into the vapour of strontium, and they measured the intensity of the narrowband resonance radiation induced in the metal vapour, at right angles to the exciting beam, by photomultipliers. The intensity of the resonance radiation re-emitted by the vapour is proportional to the incident intensity. The resonance frequency was shifted by applying a magnetic field to the vapour. In this way the profile of the exciting light could be determined from the dependence of the intensity of the re-emitted light on the magnetic field. The profile of the strontium-line thus letermined resulted in a red-shift which agreed well with the relativity prediction.

As to the red-shift of the companion of Sirius, the first measurements by A d a m s and M o o r e gave a value of 30 km/sec apparently in full accordance with the theory. But it soon turned out that in formula (3) for the radius of Sirius B a too great value was used. The radius was computed from the spectral type and from the absolute luminosity of Sirius B. The luminosity is distorted by Sirius A which is 20000 times brighter than Sirius B and the separation of the two stars never exceeds 10″. The diffuse light of Sirius A influences in the same way the lineshift, because the same spectral lines occur in the spectra of both stars. Sirius B is, consequently, not suitable at all for testing the red-shift.

More suitable is for this purpose another white dwarf, 40 Eridani B. It is a close companion of an M-type star, 40 Eridani C. As the two stars differ strongly in spectral type, the spectral lines of the white dwarf companion are not influenced by its close neighbour. P o p p e r obtained from 37 spectrograms taken at the Mt. Wilson-reflectors a red-shift of 21 ± 4 km/sec [16]. The theoretical value based on $M = 0{,}43$ computed from the orbit of the BC-system, and on $R = 0{,}016$ obtained from the spectral type and the photometric data proved to be 17 ± 3 km/sec in good agreement with the observations.

Many suggestions have already been made for using artificial satellites to test the gravitational line-shift. Such an experiment has so far not yet been carried out, so I only refer here to a paper by G i n z b u r g [17]. In the light-source placed into the artificial satellite a violet-shift is, as a matter of fact, expected relative to light sources on the Earth.

The Light-Deflection

According to the general theory of relativity a light ray passing at a distance d from the centre of the Sun is subjected to a deflection

$$L = 1'', 75/d \tag{4}$$

directed radially outwards from the centre. d should be given in this formula in solar radii.

Even in the distance of several solar radii from the Sun's centre the deflection could easily be measured under ordinary conditions with the usual methods of positional astronomy. But measurements have so far been possible only during total solar eclipses, because only then are stars visible sufficiently near to the Sun's limb. A total solar eclipse is a rare opportunity and the longest possible duration of the totality is at most 7,5 minutes. The shadow of the Moon passes only a narrow belt of the Earth and within this belt it is mostly difficult to find a favourable place for the observations. Hence it is easy to understand that since 1916 there were only ten expeditions which succeeded in making photographs for the study of the light-deflection, on the occasion of six different solar eclipses, sometimes under rather bad weather conditions. At some eclipses there were rather few stars near the Sun, or the stars were situated asymmetrically around the Sun, and all these factors influenced unfavourable the accuracy of the results. The stars nearest to the Sun's disk, where the effect is the greatest, often cannot be used at all, since the light of the solar corona suppresses them. The greatest number of measurable stars are in a distance of $d = 4 - 10$ from the Sun's centre, where the hyperbola representing the deflection as a function of d has already a linear course.

Since the co-ordinates of the stars around the Sun are not known accurately enough, the light-deflection can be determined only by means of differential measurements. The photographs taken at the solar eclipse should be compared with night photographs of the same area of the sky. These must be taken some months after or before the eclipse, and even when they are made at the same place, with the same instrument in the same arrangement, it is still inevitable that there will be change in the focus of the telescope between the two exposures and thus the scale of the two plates will differ as well. The resulting error is increasing proportionally with d, and influences with full weight the results to be obtained for the light-deflection, since this latter is varying also linearly with d if $d > 3$. In case of a focal distance of 3,5 m, for instance, a focal change of 0,1 mm would cause at $d = 8$ a scale correction of the same order of magnitude as the Einstein-effect itself at the same distance. The scale difference, ΔS, between the two exposures causes an error

$$\Delta L = -\overline{d_i^2} \cdot \Delta S$$

in the value of the light-deflection at the Sun's limb, whereby $\overline{d_i^2}$ is the mean value of the squares of star distances from the Sun. $\overline{d_i^2}$ being rarely smaller than 20, every error in the scale causes an error in L which is twenty times greater.

The observations should be arranged therefore so that the scale of the plates could be determined independently from the stars around the Sun. This was done at the expedition of the Potsdam Observatory in 1929 when the scale-value was determined by an independent star-field during the eclipse itself, furthermore photographic reseaux were printed on all plates to determine scale changes between the exposures.

Table 2 shows the data characterizing the photographs at different expeditions and the values obtained for L. The results are judged, however, quite otherwise when the details are shown as well. For this purpose, I show you some figures taken from an elaborate paper by H. von Klüber [18]. Figure 1

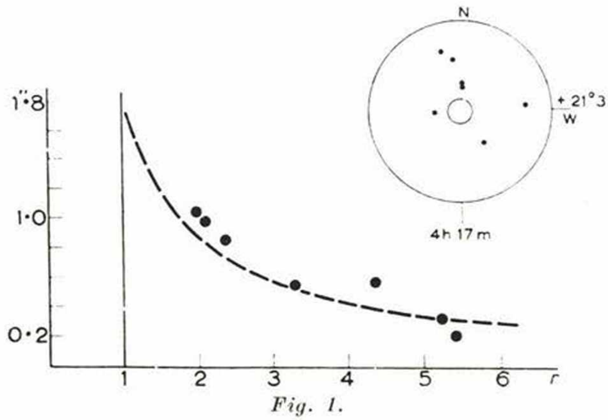

Fig. 1.

Table 2.

Observation of Light-Deflection at Different Solar Eclipses

Date	Station	f (cm)	No. of plates	Time of exposure (sec)	No. of stars	d	L	Observatory
1919 May 29	Sobral	570	7	26	7	2 – 6	1,78 ± 0,16	Greenwich
	,,	343	16	5 – 10	11	2 – 6	0,93	,,
	Principe	343	2	2 – 20	5	2 – 6	1,61 ± 0,40	,,
1922 Sep. 21	Cordillo	160	2	20 – 30	11 – 14	2 – 10	1,77 ± 0,40	Adelaide-Greenw.
	Dawns							
	Wallal	330	2	45	18	2 – 10	1,74 ± 0,30	Victoria
	,,	450	4	120 – 125	62 – 85	2,1 – 14,5	1,72 ± 0,15	Lick
	,,	150	6	60 – 102	145	2,1 – 42	1,82 ± 0,20	Lick
1929 May 9	Takengon	850	4	40 – 90	17 – 18	1,5 – 7,5	2,24 ± 0,10	Potsdam
1936 June 19	Kuybyshevka	600	2	25 – 35	16 – 29	2 – 7,2	2,73 ± 0,31	Sternberg
	Kosimizu	500	2	80	8	4 – 7	2,13 ± 1,15 *	Sendai
							1,28 ± 2,67 *	
1947 May 20	Bocajuva	609	1	185	51	3,3 – 10,2	2,01 ± 0,27	Yerkes
1952 Feb 25	Khartoum	609	2	60 – 90	9 – 11	2,1 – 8,6	1,70 ± 0,10	,,

f = focal length of telescope, d = distance of stars from centre in solar radii.

* Two comparison plates gave these alternative values when in different combinations with the eclipse plate.

shows the results of the Greenwich expedition in 1919. The figure shows the actually measured light-deflection for each star as function of d. Right above you see the corresponding starfield around the Sun. Coordinates of the Sun's centre for the equinox 1855 are indicated. The dotted curve represents the theoretical hyperbola. There are very few stars, as you see, and even these are rather asymmetrically distributed around the Sun. A representation of the observed values as accurate as through the theoretical hyperbola could be obtained also through a straight line, but this would lead to a value $L = 1,''05$. We must mention that the photographs have been taken with horizontal telescopes fed by coelostats. The mirror of the coelostat got deformed by the Sun's heat, and thus the photographs were deformed too.

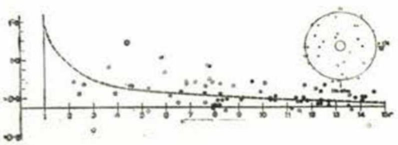

Fig. 2.

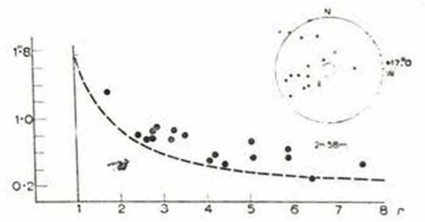

Fig. 3.

At the solar eclipse in 1922 (Fig. 2) the number of stars was sufficient, but I think I do not exaggerate when declaring that many different values can be derived for L depending on the method of reduction. The measurements of the Potsdam expedition in 1929 prepared with the greatest care led, using the theoretical hyperbola, to the value $L = 2,''24$. The accuracy looses much by the asymmetrical distribution of the stars (Fig. 3). It is difficult to understand how the observations in 1936 could result in $L = 2,''73$ (Fig. 4), or those in 1947 in $L = 2,''01$. (Fig. 5). Here all stars have been in a distance of $d > 3$ from the Sun's centre.

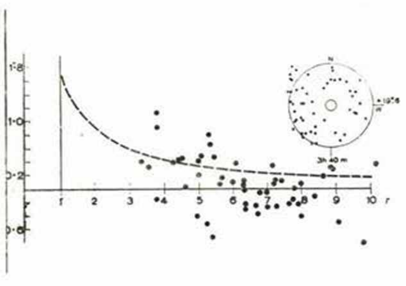

Fig. 4.

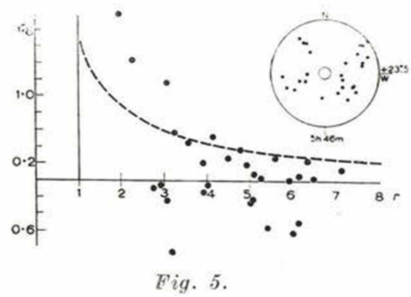

Fig. 5.

I think these figures could convince everybody of the fact that we cannot speak about the astronomical verification of the relativistic light-deflection. It is not impossible that the values obtained for L are influenced by systematic errors unknown so far. I am afraid there was some negligence when considering atmospheric refraction. The refraction-tables used were derived from night observations of stars and it is not impossible that in day time considerable differences may occur, even if the anomal refraction which might develop along the shadow-path is not considered as essential. A good improvement in the observation of the effect may be hoped for only by observations to be carried out from outside the atmosphere. Star light may be influenced also by the plasma clouds continuously flowing from the Sun, particularly at times of sun-spot maximum. It is, perhaps, worth-while to mention that the L-values in Table 2 are strongly correlating with the solar cycle: the highest L-values have been obtained at sun-spot maxima (in 1929 and 1947), and the lowest ones at sun-spot minima (in 1922 and 1952).

Summing up, it may be stated that there is no astronomical verification of the general theory of relativity, it is true, but on the other side there was no observation made which would contradict it. Apart from this, the theory of relativity did not solve the problem of gravitation, giving only the method for describing gravitational phenomena. From a satisfactory theory of gravitation it must be required that it should give an idea of the physical nature of gravitation. Consequently, the most important experiments made so far in connection with the nature of gravitation are even nowadays the fundamental experiments by Eötvös and their repetition with a more powerful apparatus by Dicke.

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