CYCLOGENESIS AND ENTROPY

by

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ZUSAMMENFASSUNG

Zwischen den in Wechselwirkung stehenden Körpern besteht fortwährend eine Tendenz des Ausgleichs. Ein allgemein brauchbares Mass dieses Ausgleichprozesses ist die Entropie. Die Artikel setz es zum Ziel, eine nähere Klärung des energetischen Hintergrundes der Entwicklung der synoptischen „Störungen" unter Betonung des recht engen Zusammenhangs zwischen den atmosphärischen Transformationsprozessen und der Entropieänderungen zu geben.

Mit Hilfe von die Werte der Entropie zeigenden Karten und mit besonderer Betonung der Rolle der orographischen Hindernisse wird auf die Bedeutung der durch die Reibung verursachten Entroipieänderung hingewiesen.

The mechanism of weather processes is determined by the relative position of acting forces and by the variety of interactions. If the system is under simultaneous action of several forces, these tend to reintegrate the system into its equilibrium state. The necessary and sufficient condition of the equilibrium is that we should have uniform temperatur, pressure etc. values within the system. In the course of approaching the equilibrium state various movements appear within the system and the energy needed for these movements will be covered by the respective reserve of potential and internal energy of the system. The amount of energy usable for transformation into kinetic one is given by the amount of the available potential energy.

For the atmosphere the equilibrium state is represented by the barotropic stratification and hydrostatic equilibrium, when the value of available potential energy is zero, while the entropy function set up for the characterisation of the equilibrium state reaches its maximum. The process control function of the entropy function is due to the fact that within a closed system only such processes can be generated, which do not lead to a decrease of the entropy of the system investigated, i.e. the entropy changes during the processes taking place within the body only if we are dealing with an adjustment trend: the smoothing of inhomogeneities is always accompanied by entropy production.

The individual change of the mechanical energy of a volume τ bounded by surface σ is given by the expression

$$\frac{d(K+\Phi)}{dt} = -\int_\sigma p V_n d\sigma + \int_\sigma V_n F d\sigma + \int_\tau \left(p\frac{d\alpha}{dt} - \delta\right) \varrho d\tau, \qquad (1)$$

where K is the kinetical, Φ the potential energy of the system, p is the pressure, $\alpha = 1/\varrho$ the specific volume, V_n the outward normal wind component, F the friction force, $\delta = F$ grad V the so called Stokes-dissipation function, $p\frac{d\alpha}{dt}$ is the work done by the fluid when it is extended (compressed) on to the specific volume α.

The individual change of the internal energy is represented by the formula

$$\frac{dI}{dt} = -\int_\tau \left(p\frac{d\alpha}{dt} - \delta\right) \varrho d\tau + \int_\tau \frac{dq}{dt} \varrho d\tau, \qquad (2)$$

where $\frac{dq}{dt}$ represents the heat-income (-expenditure) without the friction heat —, the individual forms of which are: the various radiation components, evaporation, precipitation etc. From (2) it follows that the change of internal energy is depending also on processes within the system this connection being implemented through the expansion work $p\frac{d\alpha}{dt}$ resulting in an increase in case of compression and in decrease during expansion, furthermore owing to the dissipation heat, this being always positive, thus causing an increase of the internal energy.

The change of specific entropy is given by the expression

$$T ds = dI + p d\alpha. \qquad (3.a)$$

In case of a dry adiabatic change we have

$$T ds = c_v dT + p d\alpha. \qquad (3.b)$$

Thus the individual change of entropy of the system is

$$\frac{dS}{dt} = \int_\tau \frac{\frac{dq}{dt} + \delta}{T} \varrho d\tau. \qquad (4)$$

Analysis of equations (1), (2) shows that the various forms of the entire energy are continuously transforming one into another. The transformation is accompanied by various weather processes, but as regards a longer time interval no unidirectional progress will be made, the entire energy of the atmosphere remains constant and we have the relation:

$$\frac{dK}{dt} = \frac{d\Phi}{dt} = \frac{dI}{dt} = \frac{dS}{dt} = 0. \qquad (5)$$

The relations (1), (2) and (4) demonstrate the existence of an energy cycle and if we add the identity (5), we can draw a few very important consequences regarding the characteristics of the circulation prccesses of the atmosphere.

Let us take the equation (1) describing the change of the mechanical energy. If τ denotes the volume of the whole atmosphere and if we take relations (5) as granted, we get

$$\int_\tau p \frac{d\alpha}{dt} \varrho \, d\tau = \int_\tau \varrho \delta \, d\tau . \tag{6}$$

Since $\delta > 0$, it follows – with a view to a compensation of the dissipation of the kinetic energy – the necessity that the expansion should take place at a pressure higher than average, while the compression occurs at a pressure lower than the average value.

Let us express the value of $p \frac{d\alpha}{dt}$ from (3.b) and apply from (5) the relation $\frac{dI}{dt} = 0$, we get

$$\int_\tau T \frac{ds}{dt} \varrho \, d\tau = \int_\tau \varrho \delta \, d\tau . \tag{7}$$

From (7) it follows that the increase of entropy is taking place at a temperature higher than the average, while its decrease occurs at a lower temperature. If we substitute the potential temperature into the expression giving entropy, we get the expression

$$\int_\tau c_p \frac{T}{\Theta} \frac{d\Theta}{dt} \varrho \, d\tau = \int_\tau c_p \left(\frac{p}{p_{00}}\right)^{R/c_p} \frac{d\Theta}{dt} \varrho \, d\tau > 0 \tag{8}$$

meaning that the increase of potential temperature should take place at a pressure higher than the average, while its decrease occurs at a lower than the average pressure level. Referring again to (4) and (5) we obtain

$$\int_\tau \frac{1}{T} \frac{dq}{dt} \varrho \, d\tau = - \int_\tau \frac{\delta}{T} \varrho \, d\tau . \tag{9}$$

Owing to $\delta > 0$

$$- \int_\tau \frac{\delta}{T} \varrho \, d\tau < 0 ,$$

i.e. in view of a compensation of entropy increase setting in spontaneously as a consequence of friction, heat sources of the atmosphere occur at a temperature higher than average, while sinks are located at places with a temperature lower than the average.

Consequences drawn from (8) respective (9) are essentially the mathematical expressions of the Sandström – Wenger circulation theory, according to which: a stationary circulation can be maintained only in the case, if heat sources are on the average situated at higher pressure than heat sinks.

Further application of the Sandström circulation theory favours a closer knowledge of the atmospheric circulational systems (Bjerknes 1902) and what is more, we get an explanation of the circulation mechanisms originating from the interaction of the atmosphere and Earth. The Sandström – Höiland – Hansen circulation theorem states that: if we disregard friction, on the northern hemisphere the streamlines show a turning to the right at the windward side, thus creating an anticylonic circulation. In case of an overflowing of the crest we experience a turning to the left behind the mountain (lee-side), thus a cyclonic curvature appears.

Let us complete this theorem by the equation under (4). If we take friction into account, the friction heat owing to $\delta > 0$ causes an increase of the entropy, thus it seems also energetically justified that cyclones degenerate when approaching orographic obstacles. Although owing to (2) the positive value of the dissipation function results in an increase of the internal energy too, the irreversibility of the process makes its transformation into kinetic energy impossible. This rule involves essentially the idea of the "perpetuum mobile psychikum" the verification of which seems to be possible by the analysis of the equation describing the variation of the available potential energy.

Let τ denote the entire volume of the atmosphere defined by the surface pressure p from below and by the $p = 0$ isobaric surface from above and σ the entire surface of the Earth, then the variation of the available potential energy will be given by the expression

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} = \frac{1}{g} \int_{\sigma} \int_{0}^{p_0} \left[\left(\frac{p^k - p_r^k}{p^k} \right) \left(\frac{dq}{dt} + \delta \right) + \alpha\omega \right] dp d\sigma , \qquad (10)$$

where p_r is the so called reference-pressure – practically the pressure belonging to the equilibrium state –, $k = \frac{R}{c_p}$, $\omega = \frac{dp}{dt}$, the vertical velocity. The first term of the right side of (10) is a "generation term", while the second one represents the transformation into kinetic energy. The ascent of warm air and the descent of cold one fives a negativ sign to the term $\alpha\omega$, i.e. the available potential energy decreases, the process supports the transformation into kinetic energy.

Let us study the events in connection with the cold air coming up againts the mountains, where it is partly forced to rise, partly it flows round. In the rising process of the cold air the sign of the term $\alpha\omega$ is positive, thus an increase of the available potential energy is to be expected, while a decrease of the kinetic energy is probable.

The first term of the right side of (10) contains the product of the expression "$\frac{dq}{dt}+\delta$" and of the "efficiency factor" $\frac{p^k - p_r^k}{p^k} = N$. In case the absorbing of heat takes place at a pressure higher than the reference

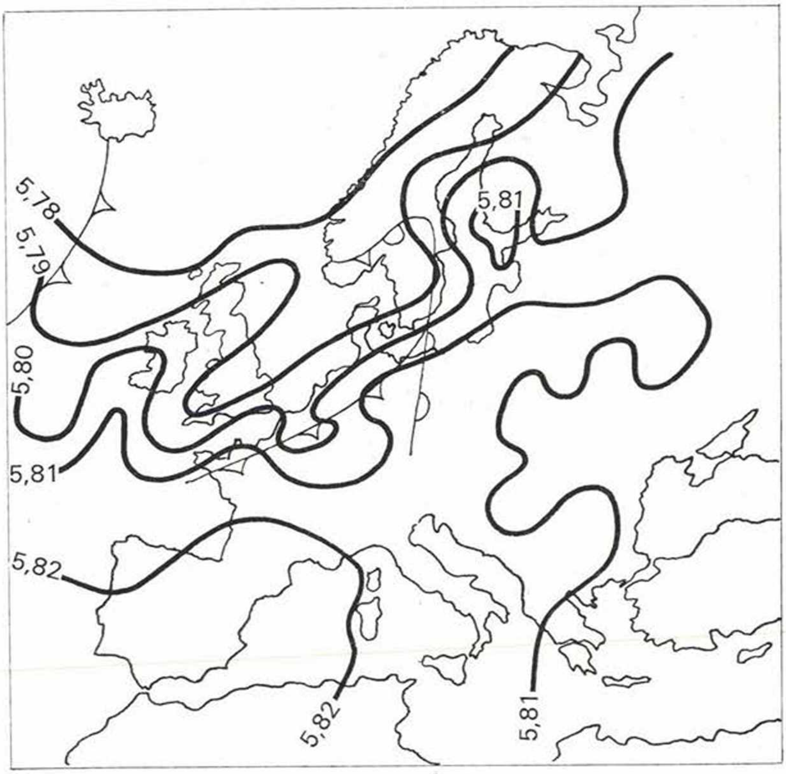

Fig. 1. Average entropy value of the layer between the 1000 and 200 mb surfaces in. units of 10^{10} erg/cm² at 12 hours on the 19th October 1970

pressure, and the loss of heat at a value lower than p_r, we can count on an increase of the usable potential energy. Regarding the dissipation heat δ we know that it is always positive, so that in case of $p<p_r$ a decrease of the available potential energy takes place, although the value of the internal energy increases. The actual sum of the internal and potential energy became higher, but the usable part is not defined by this sum alone. Taking into account (4) we see that there was an increase in the

value of the entropy and as a result of diminution of the original temperature differences the system came nearer to the equilibrium state.

The basic effects changing the entropy are coming to the air mass mainly from below. The dissipation-heat brought about by the friction is important obviously in the lower layer of the troposhere and particularly in the vicinity of orographic obstacles, but it is not to be neg-

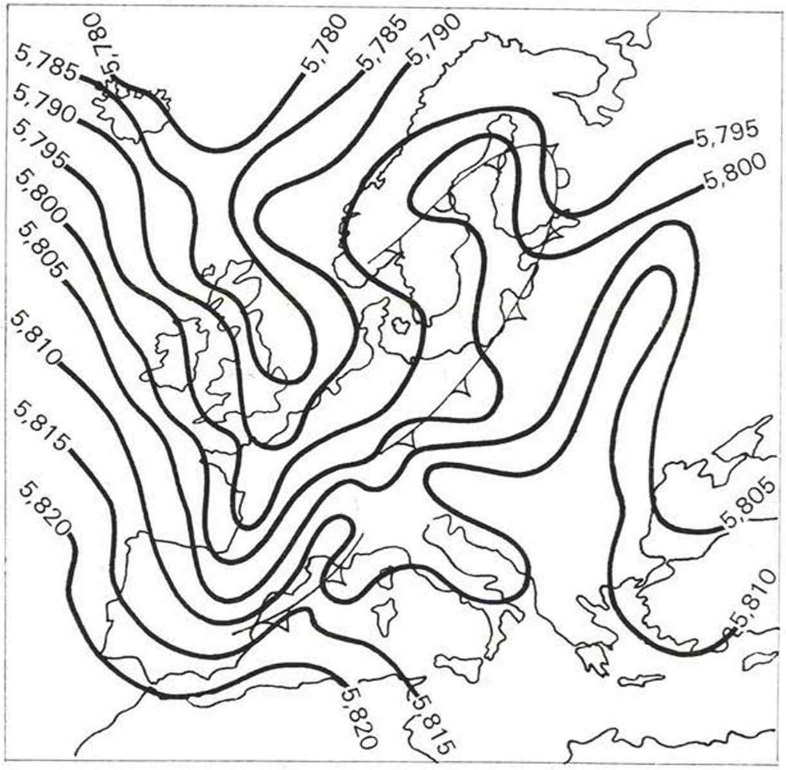

Fig. 2. The same as with Fig. 1. at 00 hour on the 20eth October 1970

lected even in the higher layers of the troposphere, where – owing to $\delta = \alpha p$ grad $\mathbf{V}$ – its significance increases because of the relatively high wind velocity and of a negative efficiency factor.

Our reasoning gives an energetical explanation for the weakening of cyclones or fronts coming accross the mountains. As regards the further development of the processes under special geographical conditions, we can sometimes witness – with a certain phase lag – the regeneration of cyclones registered earlier in the literature about cyclone-statistics in the form that "cyclones prefer inland seas". It is therefore not without interest studying the energetical background of this later process. Under such special geographical conditions, where on the lee-side of the mountains, respectively southwards of it an inflow of warm air is to be expected, in the warm air coming up to the mountains we have an increase of temperature owing to the dissipation heat and a steeper slope of the isosteric surfaces, thus an increasing inhomogeneity and so a strengthening of

Fig. 3. Entropy change between 12 hours on the 19th October 1970 and 00 hours on the 20eth October 1970 in units of 10^7 erg/cm^2

the baroclinic zones. Thus, in that process the value of the available potential energy will be increased, at the same time the total entropy of the system diminishes, while the term $\alpha\omega$ obtains a negative sign in the equation (10) describing the variation of the available potential energy, i.e. the transformation into kinetical energy will be favoured.

The quantitative interpretation of the equation describing the change of the usable potential energy becomes even more complicated if we restrict our considerations only to a small partial domain of the atmosphere, and we take into account also the energy-flow across the boundaries of the partial domain.

Between the interacting air masses a continuous adjusting tendency is in action and a commonly usable measure of this tendency is represented by the entropy. The maps representing the values of the entropy and its variations in magnitude and sign provide an orientation regard-

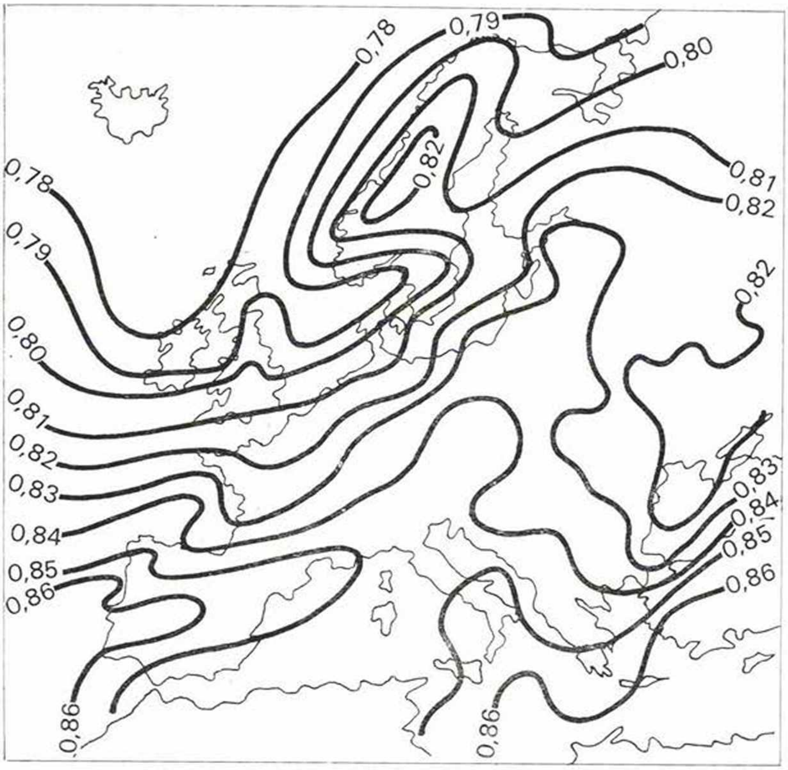

Fig. 4. Average entropy value of the layer between the 1000 and 900 mb surface in units of 10^{10} erg/cm^2 at 12 hours on the 19th October 1970

ing the possible development of the processes. Localities with low entropy values reflect the inhomogeneity and can be found obviously at places being asymmetrical from the point of view of temperature distribution, mainly at the frontal zones, where we have a great number of solenoids.

Maps 1. and 2. represent average entropy values of the layer between the ground and the 200 mb-surface on the 19th October at 12^h and 20^{eth} October 00^h, given in units of 10^{10} erg/cm^2. On both maps thick lines denote the position of fronts near the ground, spikes on the lines denoting the direction of moving cold fronts, half-circles that of the warm fronts, as it is commonly used. An area characterized by high entropy values is running through Middle-Europe, Poland and the Baltic Sea, as well as over the southeast part of the Atlantic, over Spain and Asia Minor, while in the North-Atlantic area and over Western Europe, as well as across the Balkans and Eastern Europe up to the

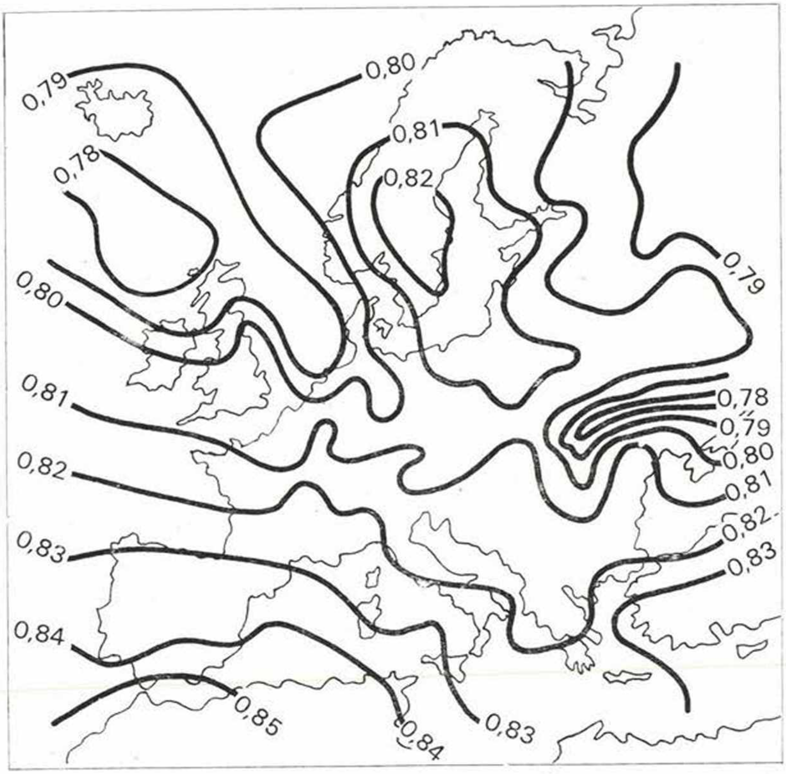

Fig. 5. The same as with Fig. 4. at 00 hours on the 20eth October 1970

70 degrees of Northern latitude we have got low values. After 12 hours, as it can be seen on Fig. 2. a significant change in the entropy-values has taken place: the total entropy of the area has increased. Owing to the occlusion of the cyclone of the North-Sea a significant adjustment occured over the northern territories, the average increase being 14.10^7 erg/cm^2. Because of the eastward shifting of fronts also the places with low entropy values can be found further eastwards and even in the area of the Mediterranean a significant decrease can be experienced, while

over Western Europe an increase has taken place. The amount of change is represented on Fig. 3, broken lines denoting zero value, continuous lines the increase, dotted ones decreasing entropy values. Since on Fig. 3. representing the change we find a more significant increase around the mountains, therefore we constructed maps of entropy of the layer between the 1000 and 900 mb-surfaces for both dates (Figs. 4, 5, and 6).

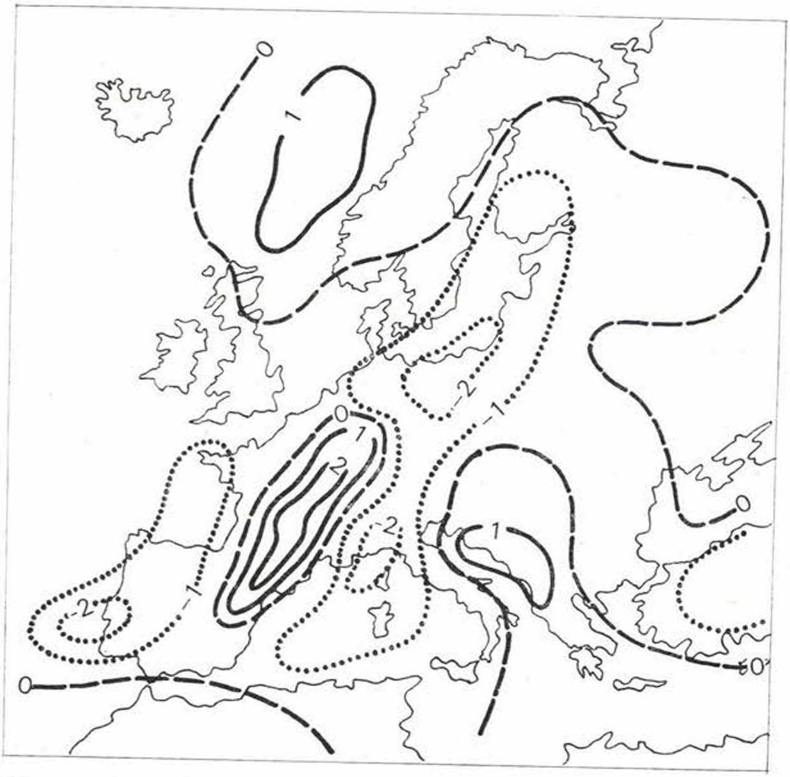

Fig. 6. Entropy change between 12 hours on the 19th of October 1970 and 00 hours on the 20eth of October 1970 concerning the layer between the 1000 mbar and 900/mb surfaces in units of 10^7 erg/cm^2

The distribution of entropy is like that of the layer up to the 200 mb-surface, although effects originating from the interaction of the ground surface and the moving air masses are decidedly predominant here. On the map of the entropy values of the lower layer and on that representing the changes we can clearly see the lagging behind of the lower section of the cold front which can be attributed to the effect of the Alps. This seems to be supported also by the significant entropy-increase seen on Fig. 6. A considerable entropy decrease can be found over the Mediterranean: this can be explained by the advection of warm air from south and by its meeting the cold air masses going round the Alps from southwest, thus producing a new frontal zone. The effect of friction caused by the mountains can be observed over the Balkans, Carpathians and even over the Scandinavian mountains too.

The areas of entropy decrease denote the places of departing from the equilibrium state, at the same time, however, the usable potential energy increases. An earlier study of us dealt with the clearing of the relation between the efficiency factor and non -adiabatic heating up concerning the same time interval from 19 to 21 October 1970. The results supported energetically the possibility of formation of a Genoa-cyclone as reflected by the data contained in the generation-term of the usable potential energy. The map of distribution of the efficiency factor also proved that especially in the lower layers of the troposphere, on the isentropic surface belonging to $\Theta = 290°$ K the diminution of the efficiency factor caused by the mountains presented itself in the configuration of the isolines in the foreground of the Alps (1973).

The entropy-analysis of the lower layer significantly indicates the action of local effects, although these cyclones do not attain the order of magnitude of the length of great atmospheric waves, i.e. 6000 – 8000 km, but they can even surpasse them as regards activity and effect. Furthermore, entropy analysis proves also that the Mediterranean cyclone can be classified as a synoptic disturbance of a smaller scale (their wave-length amounts to 2000 – 3000 km according to B j e r k-n e s and H o l m b o e 1944).

During our study we did not differentiate between coolings and heatings of different origin so that the released latent heat can also be of importance and its effect – according (10) – in the eastern basin of the Mediterranean together with positive efficiency factor can result in an increase of the usable potential energy, especially in the middle region of the troposphere. According to the calculation of R a d i n o-v i č (1960) in the development of the Mediterranean cyclone the released latent heat is of importance first of all in the later phase of development and it does not figure among the primary factors as regards the formation of the cyclone.

The maps shown for the period investigated represent a phase of cyclone development and if completed by the computations regarding the formation of usable potential energy as well as its transformation into kinetic energy, they will furnish in all probability detailed knowledge for clearing up of the energetical background of certain synoptic disturbances. Nevertheless we do not get by this a full explanation of the energy cycle, since besides solving our present problem we have still to clear up all data-to be taken into account- of the possible heat sources and sinks, which after all determine the specific distribution of the temperature.

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