APPLICATION OF INVERSE FILTERING IN THE INTERPRETATION OF GRAVITY AND MAGNETIC ANOMALIES

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РЕЗЮМЕ

Аномалия моделей, применяемых в гравитационной и магнитной интерпретации может быть представлена в виде конволюции двух функций. Одна из них описывает плотность или намагниченность, а вторая – геометрию. Описание аномалий в конволюционной форме дает возможность применять аппарат линейной оптимальной фильтрации. Планирование обратных фильтров может быть доведено до конца только в случае определенной модели. Параметры обратного фильтра получаются как решения уравнения Вейнера с использованием в качестве автокорреляционной функции той функции, которая описывает геометрию. Параметры интерпретации получаются как результат конволюции измеренных данных и параметров обратного фильтра. Работа знакомит с планированием обратных фильтров и на синтетических примерах показывает определение параметров возмущающих тел.

SUMMARY

The anomalies of the models applied in gravity and magnetic interpretation can be obtained as a convolution of two functions. One of these describes the geometry (depth, shape) the second is connected to density distribution of magnetization. The description of anomalies by convolution makes possible the application of linear optimum filters. Inverse filters can be designed if the model is given. The coefficients of the inverse filters are obtained from the Wiener equation which contains the samples of the autocorrelation of the function describing the geometry of the models.

The parameters are supplied by the convolution of the inverse filter with measured data. Design procedure of the inverse filter is given and its application for determination of the parameters of the magnetic bodies is demonstrated by a synthetic example.

Introduction

The anomalies belonging to a class of models which are often applied in the interpretation of gravity and magnetic anomalies can be written as a convolution of two functions. One of these, denoted by $h(x)$, describes the density or magnetization (depending on the nature of the anomaly) the other one, denoted by $k(x)$, describes the geometry of the applied model. The anomaly in a point then becomes

$$a(x) = \int_{-\infty}^{+\infty} h(x)\, k\big((x'-x), Z_1, Z_2, \alpha, \beta\big)\, dx', \tag{1}$$

where the actual values of the parameters Z_1, Z_2, α and β can be determined by choosing an appropriate model; Z_1 and Z_2 denote the depths of the upper and lower boundaries of the causative body, α and β denote the direction of the Earth's magnetic field, and that of the magnetization of the body. (The two latter parameters obviously do not appear in the case of gravity anomalies). Formula (1) refers to two-dimensional causative bodies, but could be easily generalized to the three-dimensional case.

In a class of interpretational problems the function $h(x)$ is to be determined when $a(x)$ and $k(x)$ are known. B o t t, M. H. P. (1967 and 1973) suggested two procedures for the solution of such problems but solutions can also be obtained by inverse filtering (R o b i n s o n and T r e i t e l, 1967).

The inverse filter should output the $h(x)$, when the anomaly $a(x)$ is the input to the filter. The impulse response $w(x)$ can be determined by minimalization of the mean square deviation between actual and desired outputs, i. e. between $a(x) * w(x)$ and $h(x)$. In digital form the criterion can be written as

$$\mathcal{E}\left\{\left(h_i - \sum_j w_j a_{i-j}\right)^2\right\} = \min , \qquad (2)$$

when E denotes the computation of the expected value.

Computing the deviatives of (2) with respect to the w_j filter coefficients the well-known Wiener equations, a set of linear simultaneous equations

$$\sum_j w_j \varphi_{aa}(k-j) = \varphi_{ha}(k) \qquad k = 0, 1, \ldots, K \qquad (3)$$

is obtained and the solution of the equations supply the w_j coefficients. (In the equation φ_{aa} denotes the autocorrelation of the input, φ_{ha} denotes the cross-correlation of the desired output and the input).

The inverse filtering is but a special case of linear optimum filtering and the interpretation is based on the estimation of the φ_{aa} and φ_{ha} correlation functions.

The inverse filtering will be illustrated in the followings by syntetic examples in the interpretation of total magnetic fields due to two-dimensional prisms.

Total magnetic field due to two-dimensional prisms

The total magnetic field due to a two-dimensional magnetized prisms can be easily evaluated by the logarithmic potential (G r a n t F. S., W e s t G. F., 1965). In the two-dimensional coordinate system (x,z), shown in Fig. 1., the total-field in an arbitrary point with coordinates x and z can be expressed as follows

$$t(x, z) = -2 \frac{\partial^2}{\partial \alpha \, \partial \beta} \int_{z_1}^{z_2} \int_{x_1}^{x_2} m(x', z') \ln\left((x-x')^2 + (z-z')^2\right)^{\frac{1}{2}} dx' \, dz' , \qquad (4)$$

where α denotes the direction of the Earth's magnetic field and β that of the magnetization, $m(x,z')$ is the magnetization of the prism. After some elementary manipulations the expression for the total-field in the point $P(x,o)$, shown in Fig. 1. becomes

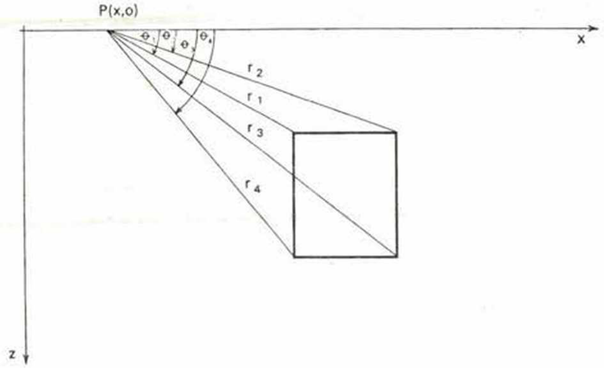

Fig. 1. The two-dimensional prism in the xz coordinate system

$$t(x,0) = -2m\Big[(\Theta_1-\Theta_2-\Theta_4+\Theta_3)\cos\tau\cos I+\ln\frac{r_1 r_3}{r_2 r_4}\cos\tau\sin I+ \\ +\ln\frac{r_1 r_3}{r_2 r_4}\sin\tau\cos I+(\Theta_2-\Theta_1-\Theta_3+\Theta_4)\sin\tau\sin I\Big] \quad (5)$$

assuming homogeneous magnetization within the prism (m is the magnetization with direction τ, while I denotes the inclination of the Earth's magnetic field).

For the sake of simplicity let us assume that the magnetic anomaly is computed at the level $z = 0$, the inclination of the Earth's magnetic field is 90°, and the direction of the magnetization of the prism is either 90° or 270°. (It can be shown, that the assumptions simplify the computations but do not modify the general validity of the suggested computational procedure).

By using the assumptions again in equ. (4) after elementary computations we obtain for the total field the relation

$$t(x,0) = 2\sin\tau\int_{x_1}^{x_2} m(x')\left(\frac{z_1}{(x-x')^2+z_1^2}-\frac{z_2}{(x-x')^2+z_2^2}\right)dx' \quad (6)$$

if the magnetization m does not depend on z. Let us introduce the function

$$g(x) = \frac{z_1}{x^2+x_1^2}-\frac{z_2}{x^2+z_2^2}\,. \quad (7)$$

The function defined by (7) depends on the geometry of the prism, only and it is independent of the magnetization. Therefore the anomaly can be written as

$$t(x, 0) = 2 \sin \tau \int_{-\infty}^{+\infty} m(x')\, g(x-x')\, dx' \qquad (8)$$

i. e. the total-field anomaly is indeed obtained as a convolution of a function $m(x)$ describing the magnetization and a function $g(x)$ describing the geometrical configuration. Equ. (8) is equivalent to equ. (6) only in those cases when the magnetization outside the prism is zero.

Determination of the inverse filter

In the previous part it was shown that the total-field anomaly can be written as the convolution

$$t(x) = m(x) * g(x)\,. \qquad (9)$$

Let us assume that the $t(x)$ anomaly is due to a random distribution of the magnetization $m(x)$. It can also be assumed that the expected value of the magnetization is zero and the magnetizations of the prisms are uncorrelated, i. e.

$$\mathcal{E}\{m_k\} = 0 \qquad (10)$$

and

$$\mathcal{E}\{m_k\, m_{k+l}\} = 0\,. \qquad (11)$$

A further assumption is that the anomaly $t(x)$ is a composite field due to prisms with different magnetization but identical shape.

The total-field in the i-th point of the magnetic profile within a stationary interval can be given by the general expression

$$t_i = m_i * g_i + n_i\,, \qquad (12)$$

where n_i denote the random noise component.

The inverse filter tries to reconstitute the series m_i from the data t_i describing the profile. The length of the deconvolution operator (i. e. the length of the inverse filter) be L. Then the desired output

$$r_i = m_{i+p}\,, \qquad (13)$$

where

$$p = \frac{L-1}{2}\,. \qquad (14)$$

The actual output of the deconvolution becomes

$$e_i = \sum_{k=0}^{L} w_k\, t_{i-k}\,. \qquad (15)$$

The operator $\{w_k\}$ minimalizes the deviation between the desired and actual outputs in the mean square sense, i. t.

$$\mathcal{E}\left\{(r_i - \sum_{k=0}^{L} w_k t_{i-k})^2\right\} = \min . \tag{16}$$

By the computation of the derivatives with respect to the filter coefficients w_k the Wiener equations are obtained:

$$\sum_{k=0}^{L} w_k \varphi_{tt}(l-k) = \varphi_{rt}(l) \qquad l = 0, 1, \ldots, L , \tag{17}$$

where φ_{tt} is the autocorrelation of the input, φ_{ri} the crosscorrelation of the desired output and the input.

First we estimate the autocorrelation φ_{tt} and cross-correlation φ_{rt}, then the solution of the set of linear simultaneous equations (17) yields the coefficient w_k. The definition of the autocorrelation φ_{tt} is

$$\varphi_{tt}(l) = \mathcal{E}\{(m_i * g_i - n_i)(m_{i+l} * g_{i+l} + n_{i+l})\} \tag{18}$$

and after taking the expected value of the product it yields

$$\varphi_{tt}(l) = S\,\varphi_{gg}(l) + \varphi_{nn}(l) , \tag{19}$$

where φ_{gg} and φ_{nn} are the autocorrelation of the function describing the geometry and that of the random noise component, respectively; while S denotes the autocorrelation of the magnetization for the argument zero. In deriving (19) we made use of the assumption that the noise and the data of the noise-free profile is uncorrelated. The definition of the cross correlation $\varphi_{rt}(l)$ is

$$\varphi_{rt}(l) = \mathcal{E}\left\{m_{i+p}\left(\sum_{k=0}^{L} g_k m_{i-l-k} + n_{i-l}\right)\right\} \tag{20}$$

which gives

$$\varphi_{rt}(l) = S\,g_{-l+p} . \tag{21}$$

Putting the correlations (19) and (21) into equation (17) we obtain

$$\sum_{k=0}^{L} w_k\big(\varphi_{gg}(l-k) + N^* \delta_{kl}\big) = g_{-l+p} \qquad l = 0, 1, \ldots, L, \tag{22}$$

where $N^* = N/S$ and δ_{kl} is the Kronecker delta symbol, i. e.

$$\varphi_{nn}(k) = \begin{cases} N & k = 0 \\ 0 & k \neq 0 . \end{cases} \tag{23}$$

Application of the inverse filters to synthetic models

For the solution of the equation (17) one has to know the function $g(x)$ and the autocorrelation $\varphi_{gg}(l)$. The latter can be computed from its definition

$$\varphi_{gg}(l) = \mathcal{E}\{g(x)\, g(x+l)\} \tag{24}$$

therefore

$$\varphi_{gg}(l) = \pi\left(\frac{2\,z_1}{\Delta_1} + \frac{2\,z_2}{\Delta_2} - \left(\frac{1}{\Delta_3^2 + 4\,l\,2z_1^2}(z_2\,\Delta_3 + z_1\,\Delta_4)\right) - \right.$$

$$\left. - \left(\frac{1}{\Delta_4^2 + 4\,l^2\,z_2^2}(z_1\,\Delta_4 + z_2\,\Delta_3)\right)\right), \tag{25}$$

where

$$\Delta_1 = l^2 + 4\,z_1^2; \qquad \Delta_2 = l_2 + 4\,z_2^2;$$

$$\Delta_3 = l^2 + z_2^2 - z_1^2; \qquad \Delta_4 = l_2 + z_1^2 - z_2^2\,. \tag{26}$$

Fig. 2. shown the normalized autocorrelation $\varphi_{gg}(l)/\varphi_{gg}(0)$.

In the model computions the effects of the random noise will be neglected. An efficient algorithm was developed by N. Levinson for the solution of the set of equations (17). (See e.g. in Wiener, 1947)

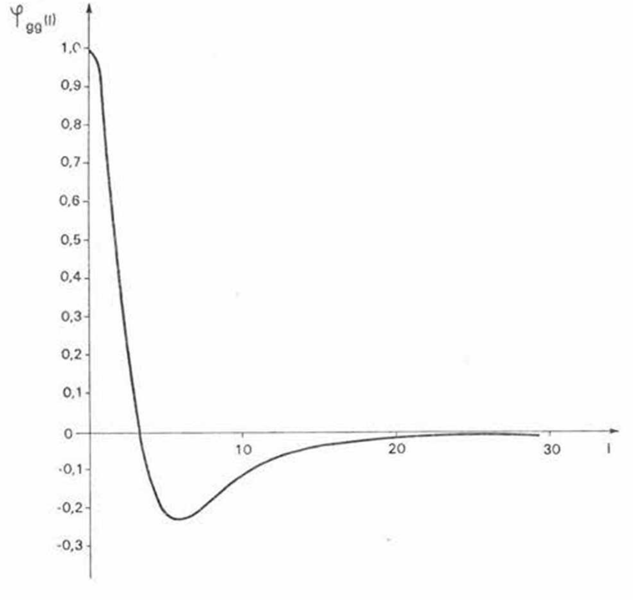

Fig. 2. The normalized autocorrelation of the function describing the geometry of the model

The first example shows the magnetic field of a two-dimensional prism with the parameters $h = 20$, $Z_1 = 2$, $Z_2 = 4$ (measured in the units of the sampling interval), $m = 0{,}02$ (in cgs units) $I = 90°$, $\tau = 90°$ and the result of the deconvolution. The prism and its magnetic field $t(x)$ is given in the upper part of Fig. 3, and the result obtained by a deconvolution filter with 101 coefficients is given in the lower part of Fig. 3. The output of the filter is a pulse, whose length is equal to the length of the prism and the amplitude is equal to magnetization of the prism. In the computation of the magnetization the constant multiplier (left out in the preceeding part) was also taken into consideration. The magnetization obtained by the inverse filter is denoted by $m_d(x)$. It is clearly seen that the relative error is less than 10%.

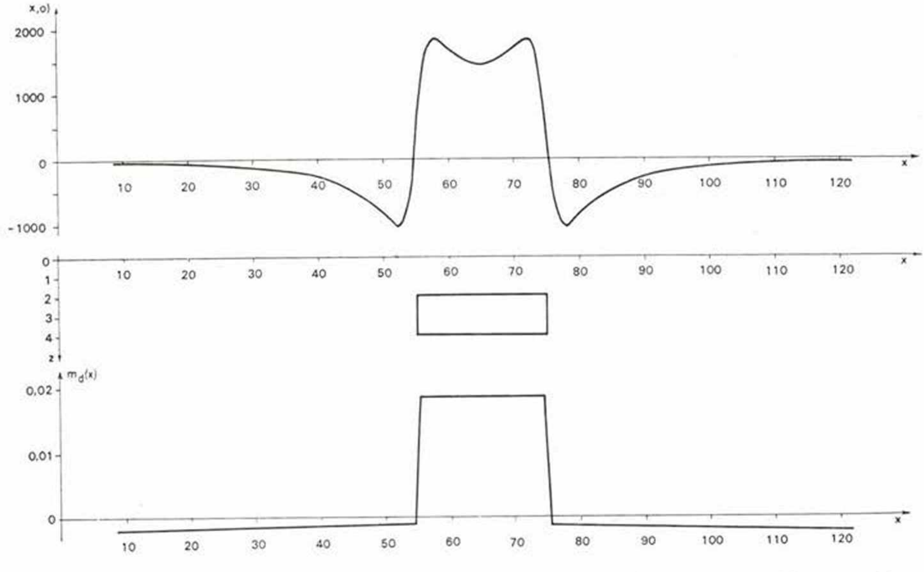

Fig. 3. Total magnetic field due to a vertically magnetized prism of length 20 × sampling distance (in $\gamma - s$),
the position of the prism,
output of the deconvolution filter (the magnetization m_d is measured in cgs units)

The filter containing 101 coefficients has also been applied to the magnetic field due to a prism with the parameters 4 = 2 (length), $Z_1 = 2$, $Z_2 = 4$ and $m = 0{,}02$ (in cgs units), $I = 90°$, $\tau = 90°$. The prism, its field and the result of the deconvolution are shown in Fig. 4. The relative error of m_d, obtained by the filter do not deviate more the 20% from the theoretical value 0.02.

The filter with 101 coefficients, which had been proved to be very efficient in the previous examples was applied to a magnetic profile

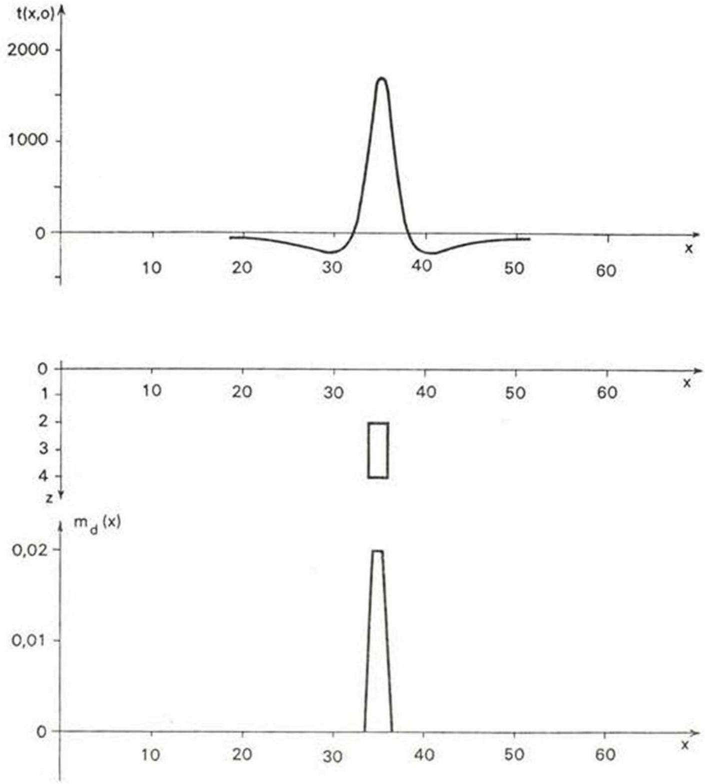

Fig. 4. Total magnetic field due to a vertically magnetized prism of length 2× sampling distance (in $\gamma - s$)
the position of the prism
output of the deconvolution filter (the magnetization m_d is measured in cgs units)

obtained as the composite field due to 13 prisms with different parametets. These parameters are summarized in Table I. (h denotes the lengths of the prisms, Z_1 and Z_2 denote the depths of the top and the bottom, respectively, m gives the magnetization in cgs units, I is the inclination of the Earth's magnetic field and τ is the direction of magnetization of the prisms, both measured in degrees).

Normal and inverse magnetizations regularly follow in the sequence. The synthetic profile is symmetric with respect to the center. The magnetic field due to the sequence of prisms is shown in Fig. 5., together with the output of the deconvolution filter (in the upper and lower parts, respectively).

The filtered profile $m_d(x)$ correctly restores the original lenghts of the blocks, while the relative error in the magnetization amounts to $10-20\%$.

Table I

No	Z_1	Z_2	h	m [cgs]	I [°]	τ [°]
1	2	4	20	0.012	90	90
2	2	4	10	0,012	90	270
3	2	4	20	0.012	90	90
4	2	4	4	0.012	90	270
5	2	4	2	0.012	90	90
6	2	4	6	0.012	90	270
7	2	4	20	0.030	90	90
8	2	4	6	0.012	90	270
9	2	4	2	0.012	90	90
10	2	4	4	0.012	90	270
11	2	4	20	0.012	90	90
12	2	4	10	0.012	90	270
13	2	4	20	0.012	90	90

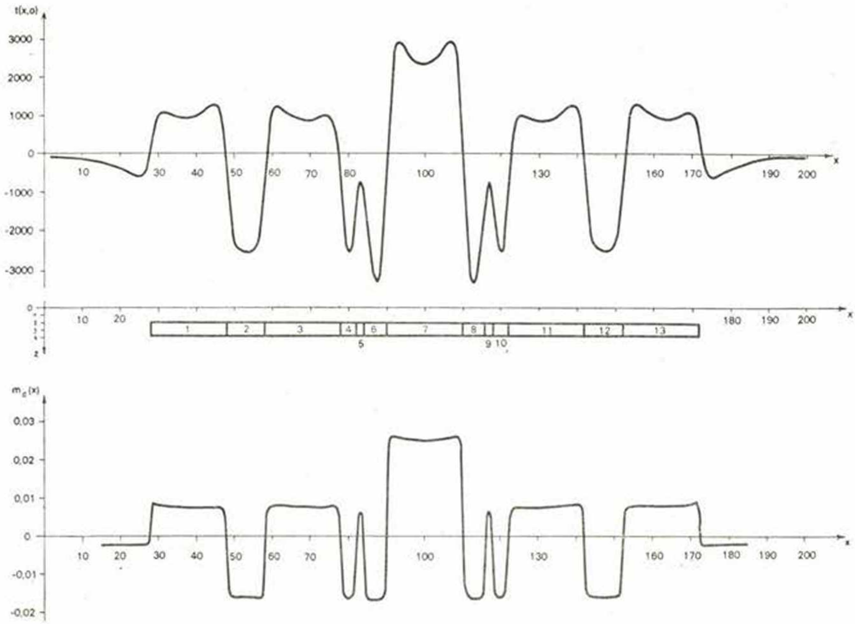

Fig. 5. Composite field of 13 prisms (in $\gamma-s$)
the position of the prisms,
the output of the deconvolution (m_d is measured in cgs units)

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