APPENDIX

CONTRIBUTION TO THE LAW OF PROPORTIONALITY OF INERTIA AND GRAVITATION*

by

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Bibliography:
Annalen der Physik, (4) 68, 1922, 11 – 66.

„ARS LONGA, VITA BREVIS"

The warning of this ancient maxim induces the authors of this paper to edit the results of their investigations and submit it to the judgement of a higher scientific Areopagus.

Methods of observations are naturally improving and refining in the course of observations and thus no mortal being could be through with his task, would he follow without reserve his meritorius urge to steadily replace the utilizable by something better.

The authors submit to man's fate of finiteness and cede the task of improving these observations to future times and future workers who believe to be able to refine them by mature experience.

This treatise is the paper presented for the competition and rewarded with the first prize of the Benecke-fund for 1909 by the Philosophical Faculty of the Göttingen University[1]. Its publication was postponed so far, for the reason,

* This is a translation of the original paper, first published in Annalen der Physik (in German) as given in the Bibliography, then for the second time in „Roland Eötvös Gesammelte Arbeiten", edited by P. Selényi, Budapest, 1953. – Some of his notes were completed by the translator, marked by [*]

1. S. the literal text of the theme of competition in VIII. 78, p. 336. [*This citation of the „Gesammelte Arbeiten" refers to the paper of R. Eötvös: Über Geodätische Arbeiten in Ungarn, besonders über Beobachtungen mit der Drehwage. Bericht an die XVI. Allgemeine Konferenz der Internationalen Erdmessung", Budapest, 1909. The citation is, as follows:

„A very sensitive method was given by Eötvös to make a comparison between the inertia and gravitation of matter. Considering this and the new developments of electrodynamics, as well as the discovery of radioactive stuffs, Newton's law concerning the proportionality of inertia and gravitation is to be proved as extensively as possible."

The first prize, Mark 4500, was awarded to the abovenamed authors for their collective work in March, 1909.]

that similar new investigations with accomplished Eötvös' torsion balances were promising even a greater accuracy[2]. In recent times, however, Eötvös' torsion balance was applied to practical mining prospections, which in ever expanding frames, hindered the above-mentioned investigations. But with regard to the great interest — expecially for the postulate of general relativity given by Einstein — the authors of this treatise think not to retain it anymore from publicity. By so doing they believe to comply with the intention of the Baron *Roland* v. *Eötvös* who himself had already prepared the publication, but the completion was hindered by his death on the 8th April, 1919. The original of this competition essay had an extent of about 10 sheets whereby a considerable abridging of the paper became necessary, but without getting lost the originality of the work. So the long tables containing the readings and those parts which did not touch the essence of the whole were principally omitted[3].

1. The task as it was conceived and treated Newton's law may be expressed as follows: Every smallest part of a body attracts every other one with a force whose direction coincides with the connecting line of both parts, its magnitude being directly proportional to the product of the masses and inversely proportional to the square of their distance from one to the other. So. if M, m are these masses and r their distance, the mutual attraction has the value

$$P = f\frac{Mm}{r^2}$$

according to the principles of *Galilei's* and *Newton's* mechanics, the acceleration of the part of mass m toward M is

$$\gamma = f\frac{M}{r^2}.$$

Consequently, the proportionality of inertia and gravitation is equivalent to the constance of f (the constant of gravitation).

Now it should be examined by observations with Eötvös' torsion balance, how far do the gravitational phenomena agree with this postulate.

In this work the investigation will be conducted in two directions, at first about the question, whether the gravitational attraction is depending on the nature of the body, secondly as regards the question, whether an influence on the attraction of a body by the presence of other bodies would be perceptible, similarly to effects of different kind, like the phenomena of magnetic and electric inductions, and especially those of absorption of heat and light.

Experiments made with radioactive stuffs will be treated in a special chapter of this paper.

2. cf. the note at the end of this treatise.

[3] According to a discussion with Mr. D. Pekár, who kindly placed the original manuscript at our disposal, we inserted here again those parts omitted at the first time. For their indication they were put in brackets.

2. On observations with the object to decide the question whether gravitation is dependent on the nature of bodies.

First of all we have to consider the principles of argumentation presented by *Newton* himself in his Principia for the proportionality of inertia and gravitation of different bodies. Those are of two kinds: *astronomical*, referring especially to the motion of the satellites of Jupiter, and *terrestrial*, resting on observations on the free fall, and the oscillating motion of materially different bodies. Both kinds of demonstration gave the result, that the gravitational attraction seemed to be independent of the material nature of the bodies, though through those observations merely a difference of 1/1000 in the gravitational attraction of different bodies with identical masses and positions was recognized.

After Newton's times the continuous advance in the art of observation of terrestrial and celestial motions rendered it possible to carry out more precise investigations resting upon his law. So, above all, we want here to point to the classical pendulum observations of Bessel, by those the limit of a still possible difference in the attraction of different bodies was shifted from 1/1000 to 1/60 000. And this limit was recently still more significantly reduced to 1/20 000 000 by *Eötvös'* investigations, who had taken our most sensitive instrument, the torsion balance, for that purpose in service. As the methods and results of that research were published only in a short note in vol. 8. of Naturwissenschaftlichen Berichte aus Ungarn, 1890[1], we thought it for necessary to explain here somewhat more thoroughly the theme what was there only outlined.

We are regarding the force of gravity as the resultant of two forces, generally of different directions, one of which originates in the attraction of masses, the other one in the inertia of the bodies. For this reason observations directed to the direction of the gravity of different bodies may be used for the investigation of the relation between inertia and gravity.

The first force, the component of the gravity force of a unit mass at point P is expressed, according to the attraction of masses, by the integral

$$G = f \int \frac{dm}{\varrho^2},$$

and the *second one* is the component originated in the inertia, i. e. the centrifugal force expressed by

$$C = l\omega^2.$$

The notations used are, as follows: dm is an attracting unit mass, ϱ its distance from P, f the gravitational constant, b is the distance of P from the axis of rotation of the Earth, and a the angular velocity of the rotation. Fig. 1. shows a picture of these two components PG, together with their resultant Pg, i. e. the total force of gravity, shown by length and direction. One sees on the

[1] S. II. (43), pp. 17 – 20. [*Über die Anziehung der Erde auf verschiedene Substanzen, Akadémiai Értesítő 1, 1890, pp. 108 – 110, in Hungarian; Math. u. Naturw. Ber. aus Ungarn, 8, 1890, pp. 65 – 68.]

picture that the direction of the attractive force G deflects northwards from the direction of the force of gravity g by the angle ε on the northern hemisphere. Its value depending on the geographical latitude φ is calculated, as follows:

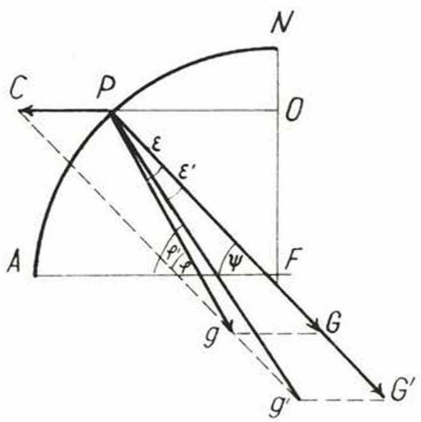

Fig. 1.

The total force at point P is in the vertical:

$$g = G\cos\varepsilon - C\cos\varphi, \tag{1}$$

but in the tangent plane the components of C and G are in equilibrium at point P, so, that

$$C\sin\varphi = G\sin\varepsilon, \tag{2}$$

and hence

$$\operatorname{tg}\varepsilon = \frac{C\sin\varphi}{g + C\cos\varphi} \tag{3}$$

[For the sake of better orientation we calculated and assembled in the annexed table the values g, G, C and ε, according to the *Bessel* ellipsoid and *Helmert's* formula, for every 5° of geographical latitude in a quadrant of the Earth. The values used here are, as follows: for the major semi-axis of the ellipsoid of the Earth

$$a = 637\ 739\ 700 \text{ cm},$$

for the minor semi-axis

$$b = 635\ 607\ 900 \text{ cm}.$$

Further is

$$g = 978{,}00(1 + 0{,}005\ 31\ \sin^2\varphi).$$

The centrifugal force was calculated from the formula:

$$C = l\omega^2 = \frac{a\cos\varphi}{\sqrt{1 - \frac{a^2 - b^2}{a^2}\sin^2\varphi}}\,\omega^2,$$

where

$$\omega^2 = 5{,}31751 \cdot 10^{-9}.$$

If we admit in the course of this research that the attraction of bodies with equal mass but of different nature could be different, so are the quantities G and f, consequently also g and ε to be considered as depending on that nature. Then we cannot speak shortly about gravity, or a level plane through a point, but distinction must be made between different gravities and different level planes according to the sorts of heavy bodies.

φ	G	$C = l\omega^2$	ε	G	$G \sin \varepsilon = C \sin \varphi$
0°	978,0000	3,3912	0' 0"	981,3912	0,0000
5	978,0394	3,3784	1' 2"	981,4049	0,2944
10	978,1566	3,3400	2' 2"	981,4461	0,5800
15	978,3479	3,2764	2'58"	981,5130	0,8480
20	978,6075	3,1871	3'49"	981,6038	1,0903
25	978,9275	3,0753	4'33"	981,7156	1,2997
30	979,2983	2,9393	5' 9"	981,8449	1,4697
35	979,7085	2,7848	5'35"	981,9878	1,5951
40	980,1457	2,6014	5'51"	982,1399	1,6721
45	980,5966	2,4019	5'57"	982,2969	1,6984
50	981,0475	2,1841	5'51"	982,4528	1,6731
55	981,4847	1,9495	5'35"	982,6042	1,5969
60	981,8949	1,6999	5' 9"	982,7459	1,4721
65	982,2657	1,4371	4'33"	982,8740	1,3025
70	982,5857	1,1633	3'49"	982,9842	1,0931
75	982,8453	0,8805	2'58"	983,0736	0,8505
80	983,0366	0,5908	2' 2"	983,1394	0,5818
85	983,1537	0,2966	1' 2"	983,1795	0,2954
90	983,1932	0,0000	0' 0"	983,1932	0,0000

Accordingly, even in an approximative representation of the gravity conditions, in place of an only *Bessel* ellipsoid and an only *Helmert's* formula, it would be necessary to put a lot of such ellipsoids and formulas adequate to the different bodies.]

It seems to be most practicable to fix the gravity conditions of a normal substance and to characterize those of other ones by the departures from those. As a normal substance could serve, e. g. the water.

For our contemplations the difference in the directions of gravity of different bodies according to this idea is of first importance. Putting for a body

$$C \sin \varphi = G \sin \varepsilon,$$

and for an other one

$$C \sin \varphi' = G' \sin \varepsilon',$$

we can calculate the angle between the directions of their gravities:

$$\varepsilon' - \varepsilon = \varphi' - \varphi,$$

because the directions of the forces of attraction G and G' are the same, i. e., conforming to our figure

$$\varphi = \psi + \varepsilon \quad \text{and} \quad \varphi' = \psi + \varepsilon'.$$

Considering the smallness of these angles, we obtain

$$\varepsilon' - \varepsilon = \varphi' - \varphi = -\frac{G' - G}{G \cos \varepsilon - C \cos \varphi} \sin \varepsilon. \tag{4}$$

In view of equation (1), we have

$$\varepsilon' - \varepsilon = \varphi' - \varphi = -\frac{G' - G}{g} \sin \varepsilon.$$

If G is related now to the normal substance (water), and we write

$$G' = G(1 + \varkappa),$$

it follows:

$$\varepsilon' - \varepsilon = \varphi' - \varphi = -\frac{G}{g} \varkappa \sin \varepsilon. \tag{5}$$

Thus, the quantity $\varkappa$ takes on the signification of a specific constant of attraction, for

$$\frac{G'}{G} = \frac{f'}{f},$$

consequently

$$f' = f(1 - \varkappa).$$

Newton's pendulum experiments indicated just, that $\varkappa$ is less than 1/1000, those of *Bessel*, that $\varkappa$ is less than 1/60 0000, those of *Eötvös*, that $\varkappa < 1/20\,000\,000$.

To enlighten our next considerations we introduce in addition the angle of deflection η, what the direction of gravity of any substance makes with that of the normal substance (water) towards the poles, i. e., to the North on the Northern hemisphere. Being

$$\eta = \varepsilon - \varepsilon',$$

we can write

$$\eta = \frac{G}{g} \varkappa \sin \varepsilon. \tag{6}$$

Let us consider, in which way would be manifested a difference like this in the direction of the gravity of different bodies. First of all the demand arises that plumb lines determined by different stuffs and fluids would give different directions of the vertical, when they were in standstill. In general, the plumb line would also not be normal to the resting fluid level.

The differences of directions are at the 45° latitude:

for $\varkappa = 1/1\,000$	0,357 sec of arc
$\varkappa = 1/60\,000$	0,003 95 sec of arc
$\varkappa = 1/20\,000\,000$	0,000 018 sec of arc.

[No direct observations of such differences in the directions were carried out so far with the intention to solve the problem we are interested in, but we have to remember the experiments of *Guyot*, which aroused great interest in his days. *Guyot* observed in 1836 in the Paris Pantheon the mirror images of marks reflected by a resting mercury surface, the marks coming along a 57 m long pendulum and he found that its end deviates from the normal of the fluid surface by 4,5 mm to the South.[1]. The legality, to conclude from that to a deviation of the direction of the gravity was strongly disputed. The author had the opportunity to be convinced by suspending pendulums of different materials in a tower of 22 m height, using diverse wires for the suspension, that the ends showed really some deviations, which originated but in the pressure of the irregularly heated and moving air.

A further consequence of the direction of gravity depending on the material nature would be an inconformity of the gravitational level planes of different substances.

APN is a meridional quadrant of the equipotential surface belonging to the normal substance (water, s. fig. 2.), *A'P'N'* the same for an other substance having the coefficient of attraction $\varkappa$. The distance between both equipotential surfaces passing through the point of the equator is easily calculable. Do we move a unit mass of the second substance from A along the equipotential surface of the normal substance to P—then from P to P', and again along the second equipotential surface back to A, so is the whole performed work along this way equal to zero. Hence it is

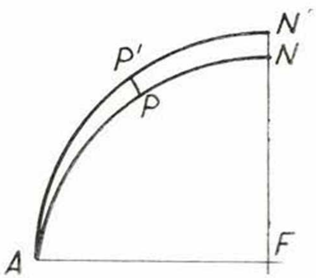

Fig. 2.

$$\int_0^{\varphi} g'\eta ds + zg' = 0,$$

where ds is an element of arc of the meridional quadrant, z the positive distance to the equipotential surface $AP'N'$ downward.

Using the relations (6) and (2), we have

$$\int_0^{\varphi} \frac{g'}{g} \varkappa C \sin \varphi \, ds = -g'z.$$

To avoid tiresome calculations, with sufficient approximations we put here g for g', and

$$ds = r d\varphi$$

$$C = r \cos \varphi \omega^2,$$

denoting the mean of the Earth's semi-diameter by r. We have thus:

$$z = -\frac{1}{2} \frac{\varkappa}{g} r^2 \omega^2 \sin^2 \varphi$$

[1] Guyot: La pendule n'est pas perpendiculaire à la surface des liquides tranquilles. C. R. XXXII, Fortschritte der Physik. VI.

and for $\varphi = 90°$, i. e. at a pole of the Earth:

$$z = -\frac{1}{2}\frac{\varkappa}{g} r^2 \omega^2.$$

Taking the values

$$r = 636\,740\,000 \text{ cm}, \; g = 983{,}19 \text{ cm sec}^{-2}, \; \omega^2 = 5{,}31751 \cdot 10^{-9}$$

we obtain for the greatest distance between the equipotential surface of any substance and that of the water at the poles

$$z = -1\,380\,250\,\varkappa \text{ cm},$$

accordingly

$$\begin{aligned} \text{for } \varkappa &= 1/1000 & z &= -1\,380 \text{ cm}, \\ \varkappa &= 1/60\,000 & z &= -\;\;23 \text{ cm}, \\ \varkappa &= 1/20\,000\,000 & z &= -\;\;0{,}069 \text{ cm}. \end{aligned}$$

To positive values of $\varkappa$ corresponds at the poles an elevation, to negative values a depression of the equipotential surface.

We could think on a separation of terrestrial substances so that those with positive $\varkappa$ should be piled up at the poles, on the other hand those with negative $\varkappa$ in the equatorial regions, but the eventual forces acting this way are certainly too small, and the resistances acting against them are too large to permit of separations of this kind].[1]

[1] We recognize by these never published reflections of Eötvös the ideas disclosed in the footnote of VIII (78), p. 272. But the phenomenon in question is considered also here rather from a practical experimental standpoint, than as a matter of principle. [*The quotation refers to the treatise of Eötvös: Bericht über Geodätische Arbeiten in Ungarn, besonders über Beobachtungen mit der Drehwaage, Verhandl. d. XVI. allg. Konferenze der Internat. Erdmessung in London – Cambridge, 1909. I. 319 – 350.

The treatise ends with the phrase: „Would ever the physicist detect by further refinement of his experimental methods even minor spoors of selective attraction of the Earth, the activity of the geodesist, should be confined just as before to measure out the dimensions of only one geoid, valid for every sort of substances."

The text of the quoted footnote is:

From these reflections of Eötvös, especially from the above last phrase immediately follows that a body floating on the resting surface of a fluid, e. g., on the surface of water, should move by itself to the North, or to the South, respectively, if its gravitational constant were greater or less than that of the water, or generally: the rotation of a fluid or gaseous celestial body would operate the segregation of its constituting substances having different gravitational constants. The editor was apparently the first to point to these simple inferences, many years ago. It was shown in a short preliminary notice (M. Rózsa and P. Selényi, Über eine experimentelle Methode zur Prüfung der Proportionalität der trägen und gravitierenden Masse, ZS. f. Phys, 71, 814, 1931), that observations aiming at that and executed with the most primitive technique give for $\varkappa$ a relatively small value ($\varkappa < 1/100\,000$). In two further notices (P. S., Inert and Heavy Mass, Term. tud. Közlöny, Supplement, Ápr. – Jún. 1940 (in Hungarian). and *P. S.* Inertia and Gravity of Matter, Hungarica Acta Physica, vol. 1. no. 5, 1949) this subject was considered from the side of principle and dared the assertion, that possibly the non-occurrence of the mentioned phenomena might be regarded as the most immediate and clearest evidence for the proportionality of inertia and gravity, what makes at the same time plainly represent the importance and far-reaching of this law of Newton in the constitution of the universe.]

It is quite surprising that so tiny differences in the directions are sufficient to provoke mechanical impulses, which can be perceived and even measured with the torsion balance.

Is the swinging body of the torsion balance consisting of masses of different materials m_1, m_2, m_3 etc., so according to our considerations the axis of rotation represented by the measuring fibre should be deflected from the direction of the gravity of water to the North by an angle, that is easily calculable. Taken, namely, the conditions of equilibrium for such a body swinging about a horizontal axis 0, and oriented in West-East (fig. 3.) we have for the torque of gravity of a homogeneous part of mass m_1 the quantity:

$$-m_1 \varrho_1 g_1 \sin(\gamma_1 - \eta_1),$$

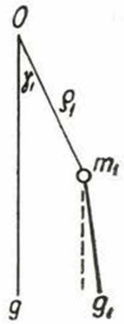

Fig. 3.

and for the condition of equilibrium:

$$\Sigma\, m_\varkappa \varrho_\varkappa g_\varkappa \sin(\gamma_\varkappa - \eta_\varkappa) = 0,$$

where $\varrho_\varkappa$ means the radius of rotation of the centre of gravity for the mass $m_\varkappa$, $\gamma_\varkappa$ the angle between $\varrho_\varkappa$ and the direction of gravity of the water; $g_\varkappa$ is the gravity of the unit mass $m_\varkappa$; and $\eta_\varkappa$ the deflection of its direction from the gravity of water.

Placing approximately:

$$\cos\eta_\varkappa = 1,$$

$$\sin\eta_\varkappa = \eta_\varkappa,$$

$$g_\varkappa = g(1+\varkappa),$$

we have then

$$\Sigma\, m_\varkappa \varrho_\varkappa g \sin\gamma_\varkappa + \Sigma\, m_\varkappa \varrho_\varkappa g(\varkappa \sin\gamma_\varkappa - \eta_\varkappa \cos\gamma_\varkappa) = 0,$$

and denoting the mass of the whole swinging body by M, and the radius of rotation of its centre of gravity by R, we obtain

$$MRg \sin E + \Sigma\, m_\varkappa \varrho_\varkappa g(\varkappa \sin\gamma_\varkappa - \eta_\varkappa \cos\gamma_\varkappa) = 0,$$

whence it is easy to see, that E remains always a small angle, not surpassing the order of $\varkappa$ and $\eta_\varkappa$.

In the plane of rotation is acting hence on every part of mass $m_\varkappa$ a component of gravity directed to the pole, which can be expressed by

$$m_\varkappa g_\varkappa(\eta_\varkappa - E).$$

We want now to refer our further calculations to a rectangular coordinate-system, where the z-axis coincides with the axis of rotation (i. e., with the measuring fibre), directed downwards, while the x-axis should be directed to the North, and the y-axis to the East.

The torque of gravity originating in the before considered differences in directions is then

$$-\Sigma\, m_\varkappa g_\varkappa y_\varkappa(\eta\,\varkappa - E) = -\Sigma\, m_\varkappa g_\varkappa \eta_\varkappa + E\,\Sigma\, m_\varkappa g_\varkappa \eta_\varkappa;$$

but as on account of equilibrium about the z-axis we have

$$\Sigma\, m_\varkappa g_\varkappa y_\varkappa = 0,$$

the torque will be limited to the first member:

$$-\Sigma\, m_\varkappa g_\varkappa y_\varkappa \eta_\varkappa.$$

On the torsion balances used here were placed the different masses at the end of a straight beam. One end of the beam should be denoted by a, the other one by b, and we write then m_a, l_a, g_a, and η_a for the masses which lie along the beam between the axis of rotation up to its end a. For the other side of the beam similar notations are due. Introducing the notation α for the azimuth of the beam, denoting by it the angle what the beam oriented from b to a makes clockwise with the x-axis pointing to the North, we obtain the former torque in the form:

$$(\Sigma\, m_b l_b g_b \eta_b - \Sigma\, m_a l_a g_a \eta_a)\sin\alpha,$$

and using equation (6), but neglecting the terms which are multiplied by $\varkappa^2$ we become for the torque

$$D = (\Sigma\, m_b l_b \varkappa_b - \Sigma\, m_a l_a \varkappa_a)\, G \sin\varepsilon \sin\alpha. \qquad (7)$$

The value of this eventual torque be illustrated by an example.

On both ends of a 40 cm long homogeneous beam should be suspended two masses of different stuff, 25 g each. At 45° latitude, where is

$$G \sin\varepsilon = 1{,}7,$$

in the case that the end a of the beam points to the North, we have for the torque

$$D = 25 \cdot 20 \cdot 1{,}7(\varkappa_b - \varkappa_a) = 850(\varkappa_b - \varkappa_a),$$

but when the end a points to the West:

$$D' = -850(\varkappa_b - \varkappa_a),$$

hence

$$D - D' = 1700(\varkappa_b - \varkappa_a).$$

Would be $\varkappa_b - \varkappa_a = 10^{-6}$, we had

$$D - D' = 0{,}0017,$$

and this torque would cause a torsion to a fibre, having a constant of torsion of 0,5 and the required weight-carrying capacity. Then the torsion, read at a distance of 1500 scale divisions, expressed in scale divisions, would be

$$n - n' = \frac{0{,}0017}{0{,}5} \cdot 3000 = 10{,}2 \text{ scale divisions.}$$

Yet, the matter is not so plain. The torsion of the measuring wire will not be effected by the just calculated torque D alone, but by the torque originated in the spatial changes of the force of gravity. In closed rooms of observation, namely in cellarlike spaces, it can be even quite considerable.[1]

A hanging system as shown in fig. 4 is consisting in a horizontal tube which is loaded at its end b by a weight M_b inserted to it, and by a suspended weight M_a at its end a, so, the centre of gravity of M_a being by h deeper than that of the weight at b. For this system *Eötvös* writes the torque effected by the increment of gravity, as follows:

$$F = \left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) K \frac{\sin 2\alpha}{2} + \frac{\partial^2 U}{\partial x\, \partial y} K \cos 2\alpha - $$
$$- \frac{\partial^2 U}{\partial x\, \partial z} Mhl \sin\alpha + \frac{\partial^2 U}{\partial y\, \partial z} Mhl \cos\alpha .$$

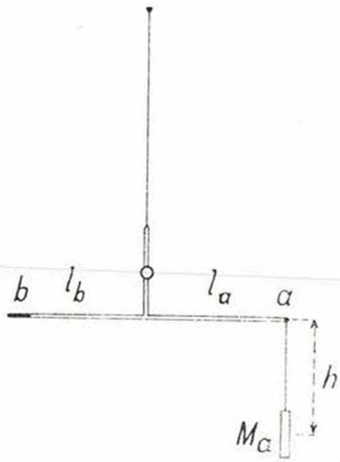

Fig. 4.

U means here the potential of gravity, K the moment of inertia of the suspended system. As the total torque of the forces of gravity acting at it is

$$D + F,$$

the angle of torsion ϑ according to the torsion of the fibre is in the position of equilibrium:

$$\vartheta = \frac{1}{2} \frac{K}{\tau} \left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) \sin 2\alpha + \frac{K}{\tau} \frac{\partial^2 U}{\partial x\, \partial y} \cos 2\alpha - \frac{M_a hl}{\tau} \frac{\partial^2 U}{\partial x\, \partial z} \sin\alpha +$$
$$+ \frac{M_a hl}{\tau} \frac{\partial^2 U}{\partial y\, \partial z} \cos\alpha + \frac{1}{\tau} (\Sigma\, m_b l_b \varkappa_b - \Sigma\, m_a l_a \varkappa_a) G \sin\varepsilon \sin\alpha, \tag{8}$$

where τ is the constant of torsion.

This equation was set up by taking into consideration that those small differences caused by the different attractions of different stuffs, which would only slightly alter the second derivatives of the potential, were negligible as compared with the last term of the equation. Attention must be paid to the fact, that the quantity

$$\Sigma\, m_b l_b - \Sigma\, m_a l_a$$

is here no more to be taken for zero, nevertheless its order of magnitude remains equal to that of the quantity

$$\Sigma\, m_b l_b \varkappa_b - \Sigma\, m_a l_a \varkappa_a,$$

[1] S. the treatise VI (76) in this volume. [*R. E ö t v ö s Bestimmung der Gradienten der Schwerkraft und ihrer Niveauflächen mit Hilfe der Drehwaage, Verhandl. d. XV. Allg. Konferenz der internat. Erdmessung in Budapest, 1906, vol. I, pp 337 – 395]

because for the equilibrium about a horizontal axis the relation

$$\Sigma\, m_b l_b g_b - \Sigma\, m_a l_a g_a = 0$$

must be valid and the ratio g_b/g_a is differing from the unit only by a fraction λ which is of the same order as $\varkappa$.

For the swinging-system of the torsion balance of the described type we may write in the case that its both ends are of equal length and homogeneous, having everywhere the same thickness, we have

$$\Sigma\, m_b l_b \varkappa_b - \Sigma\, m_a l_a \varkappa_a = M_b l_b \varkappa_b - M_a l_a \varkappa_a,$$

thus, neglecting the terms multiplied by $\lambda\varkappa$, we obtain

$$\Sigma\, m_b l_b \varkappa_b - \Sigma\, m_a l_a \varkappa_a = M_a l_a (\varkappa_b - \varkappa_a). \tag{8'}$$

Equations (8) and (8') will indicate later, how to determine by the aid of observations the quantities $\varkappa_b - \varkappa_a$, after eliminating all other unknowns, and thus, the problem can be solved, whether their value regains the limit of measurability.

However, experiments like these do us furnish only with an information on the attraction of one single body, i.e. the Earth. It is, certainly, of interest to investigate, whether the attraction of the sun and moon, which is really manifested in the tidal phenomena and in the variation of the plumb line, could contribute to the elucidation of our question? We want here to give answer in a short approximate discussion to this complicated phenomenon.

The so called tidal force can be composed of two components.

One of these components is the attraction exerted by the sun or moon on a particle of mass of the Earth; its value referred to the unit mass will be by assumption of a barycentric attracting body

$$f\frac{M}{\varrho^2},$$

where M means the mass of the sun or moon, ϱ the distance from its centre of attraction. We want to regard here this force, which has different length and different direction toward the different parts of mass of the Earth, as depending on the material nature, consequently, on $\varkappa$.

The second component of force acting here according to the inertia is the centrifugal force of the revolving motion described by the Earth round the center of inertia of Sun and Earth, respectively of Moon and Earth. For every part of the Earth free from rotation, this force is equal by size and direction; we want to denote it by C, referred to the unit mass.

Since the attraction exerted on the whole Earth and the centrifugal force of the Earth's whole mass has the same size, we write

$$C = f_0\frac{M}{D^2},$$

where D means the distance of the inertia-center of the Earth from the common inertia-center of Sun and Earth, or Moon and Earth, respectively. By f_0 is denoted here a mean value of the eventually different f values for the different substances of the Earth.

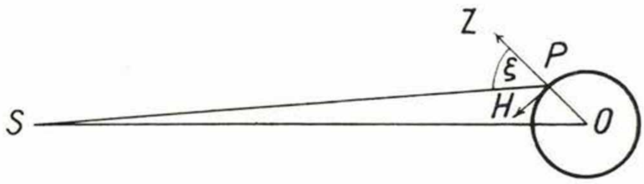

Fig. 5.

According to these considerations, and taking on a spheric form for the Earth, we obtain the components of the force as referred to a terrestrial co-ordinate-system: a vertical force directed upwards (fig. 5)

$$-Z = f\frac{M}{D^2}\cos\zeta - C\cos\zeta + fM\frac{a}{D^3}(2\cos^2\zeta - \sin^2\zeta),$$

and a horizontal force

$$H = f\frac{M}{D^2}\sin\zeta - C\sin\zeta + \frac{3}{2}M\frac{a}{D^3}\sin 2\zeta.$$

In these equations ζ is the zenith-distance of the Sun, or Moon, a the mean radius of the Earth, H is directed to that point of the horizon where the vertical plane of Sun, or Moon intersects the horizon, for which $\zeta = +\dfrac{\pi}{2}$.

A replacement of the approximate calculation brought up here by a more complete one would exceed the limits of this treatise.

Putting

$$f = f_0(1+\varkappa),$$

we obtain

$$-Z = \varkappa f_0\frac{M}{D^2}\cos\zeta + fM\frac{a}{D^3}(2\cos^2\zeta - \sin^2\zeta), \tag{9}$$

$$H = \varkappa f_0\frac{M}{D^2}\sin\zeta + \frac{3}{2}f\,M\frac{a}{D^3}\sin 2\zeta. \tag{10}$$

If $\varkappa = 0$, these expressions give us the usual components of the tidal forces (cf. e.g., *Thomson* and *Tait*, Natural Philosophy vol. 1, § 812).

But if $\varkappa$ be different of zero, an other term with a diurnal period according to the first terms would come in beside of the semi-diurnal tidal phenomena expressed by the seconds terms of these equations.

For $\zeta = 0$

$$-Z = \varkappa f_0 \frac{M}{D^2} + 2f \frac{M}{D^2} \frac{a}{D},$$

and for $\zeta = \pi$

$$-Z = -\varkappa f_0 \frac{M}{D^2} + 2f \frac{M}{D^2} \frac{a}{D}.$$

[The ratio of the first term to the second one is now $\varkappa$: 2a/D, so, if we take for the Earth and Sun a/D = 1/23600, and for the Earth and Moon a/D = 1/60.27 it would follow, that $\varkappa$ should have a value of 1/11800 to double the solar tide at the first time, then to annihilate it after half a day, and it ought not to be less then 1/30 to effect the same action against the semi-diurnal lunar tide.

Taking on that the force $-Z$ would be determinable from the tidal phenomena up to 1/100 of its size, so the observation of the solar tides would still lead to the recognition of such values of $\varkappa$, which are not greater then 1.10^{-6}, i.e., one million of the unit. But such an accurate observation of the 24 hourly tidal wave originating in the attraction of the Sun is hardly conceivable, for it would be difficult to isolate it from the radiation effects of the Sun, which repeat themselves in same periods.]

It is easier to make use of the equations (9) and (10) for observations with the torsion balance. Orienting namely, a torsion balance of the above described type so that the azimuth α of the beam be zero, i.e., the axis of the beam be in the meridian and its end a point to the North, then two external torques are acting on it. One is due to the gravity of the Earth and results in a torsion ϑ_0 of the fibre, independently of the time, the second torsion is according to the force H given by equation (10) and depending on the time.

If A means the azimuth of Sun or Moon, the component of H normally to the beam-axis is $-H \sin A$, and we obtain for the torsion of the measuring fibre

$$\begin{aligned}\vartheta = \vartheta_0 - \frac{1}{\tau} f_0 \frac{M}{D^2} \Sigma\, m_a l_a \varkappa_a - \Sigma\, m_b l_b \varkappa_b) \sin\zeta \sin A \\ - \frac{3}{2}\frac{1}{\tau} f_0 \frac{M}{D^2}\frac{a}{D} (\Sigma\, m_a l_a - \Sigma\, m_b l_b) \sin 2\zeta \sin A \qquad (11) \\ - \frac{3}{2}\frac{1}{\tau} f_0 \frac{M}{D^2}\frac{a}{D} (\Sigma\, m_a l_a \varkappa_a - \Sigma\, m_b l_b \varkappa_b) \sin 2\zeta \sin A.\end{aligned}$$

The last term on the right side of this equation can be neglected because of the smallness of the factor a/D, likewise the term preceding it, because

$$\Sigma\, m_a l_a - \Sigma\, m_b l_b$$

is of the same order as

$$\Sigma m_a l_a \varkappa_a - \Sigma m_b l_b \varkappa_b,$$

so that it is admissible to use the approximate formula:

$$\vartheta = \vartheta_0 - \frac{1}{\tau} f_0 \frac{M}{D^2} \{\Sigma m_a l_a \varkappa_a - \Sigma m_b l_b \varkappa_b\} \sin\zeta \sin A. \tag{12}$$

We can get some information on the size and measurability of that torsion by means of an example. We are using the above described instrument for which we take

$$\Sigma m_a l_a \varkappa_a - \Sigma m_b l_b \varkappa_b = M_a l_a (\varkappa_a - \varkappa_b)$$

with the dates

$$M_a = 25g, \quad l_a = 20 \text{ cm}, \quad \tau = 0{,}5.$$

We take further

for the Sun: $\quad f_0 \frac{M}{D^2} = 0{,}586$

for the Moon: $\quad f_0 \frac{M}{D^2} = 0{,}00332.$

We obtain then a torsion according to the attraction of the Sun

$$\vartheta = \vartheta_0 - 586(\varkappa_a - \varkappa_b) \sin\zeta \sin A,$$

and to the Moon

$$\vartheta = \vartheta - 3{,}32(\varkappa_a - \varkappa_b) \sin\zeta \sin A.$$

We want to deal mainly with the first one, since the second one has little significance because of its many times less value.

If $(\varkappa_a - \varkappa_b)$ be different from zero and positive, i.e., the mass unit of the mass M_a suspended on the North-end of the beam be stronger attracted by the Sun than the mass unit of M_b, the torsion-beam should show a daily oscillation so that its end a should be deflected from the middle position to the East at sunrise, and to the West at sunset.

Since at sunrise and sunset $\sin\zeta = 1$, the value of this deflection is

$$\vartheta - \vartheta' = 586(\varkappa_a - \varkappa_b)(\sin A' - \sin A),$$

and in the case, when

$$\sin A' - \sin A = 2$$

as it turns out to be approximately so at equinox, the elongation is

$$\vartheta - \vartheta' = 1172(\varkappa_a - \varkappa_b)$$

or in scale divisions at a distance of 1500 scale units

$$n - n' = 3\,516\,000(\varkappa_a - \varkappa_b).$$

For $\varkappa_a - \varkappa_b = 1.10^{-6}$ we should have therefore an elongation

$$n - n' = 3{,}5.$$

For the observational method based on these reasonings the required sensitivity is therefore only the third part of that given by *Eötvös*, as long as the same instrument will be used. Notwithstanding, this new method is promising some advantages as leaning on observations made by a stable instrument and in this way a greater sensitivity is utilizable. Eötvös' gravity compensator[1] permits of increasing up to an arbitrary limit the sensitivity of such stable torsion balances, when perturbing influences are eliminated.

Both methods are complementing each other so that the first one furnishes the required information on the attraction of the Earth, the second on that of the Sun.

3. Particulars on the execution of the observations according to the method given by Eötvös.

There were applied two instruments of the same kind as those used by *Eötvös* for his investigations about the local variations of gravity, and described by him in the first volume of „Abhandlungen der XV. Allgemeinen Konferenz der Erdmessung, 1906." These are torsion balances of great sensitivity, rotatory about a vertical axis, very suitable thus to the investigations treated here.

[Fig. 6. depicts one of the instruments, the *„single gravity variometer"*, so called by Eötvös. Its photo is shown on page 100[2].]

The housing is made of about 3 mm thick brass plates and pipes, which enclose the suspended system twofold, and even threefold at the hanging low part. This housing can be rotated about an adjustable vertical axis, and is resting on a solid base where a graduated circle is serving for indication of the angle of rotation. The graduation is by third degrees so that by the aid of a vernier one minute can be read.

The suspended system consists of a thin-shelled brass tube of about 40 cm length and 0,5 cm diameter; to its end *b* is inserted a platinum cylinder of about 30 g weight, while on the other end *a* were suspended by a thin cupper-bronze fibre the various bodies for examination. The weight of these bodies must be always so adjusted as to bring the other end loaded by a constant weight steadily in the same horizontal position. The suspension was done so that the inertia-center of the body came about 21 cm beneath the beam-axis. This length *h* had to be known more accurately especially for some parts of the experiments, to this end we used beside of the cathetometer a suitably shaped balance. By the aid of this we could determine the position of the gravity-center in the examined body, not always consisting of one single stuff; this position was determined by observing the change in the sensitivity of

[1] S. IV. (58), p. 63. [*The quoted treatise is: Untersuchungen über Gravitation und Erdmagnetismus, Math. és Term.-tud. Ért, 14, 1896, 221 – 266 (in Hung.); Math. u. Naturw. Beraus Ungarn, 13, 1896, 193 – 243; Ann d. Phys. u. Chem. Neue Folge, 59, 1896, 854 – 400.]

[2] [*The quotation refers to the above mentioned treatise.]

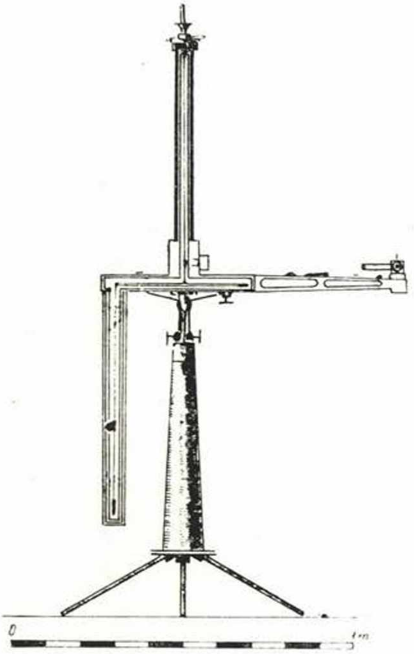

Fig. 6.

the balance occuring when the body was fixed to the beam of the balance. The attained accuracy for h was of about 0,1 mm, what was higher than needed.[1]

In order to read the position of the beam of the torsion balance, a mirror was fixed to it and a scale with half mm division at a distance of cca 62 cm from the axis of rotation. The readings were taken with a refracting telescope, with a view to set up and observe the instrument in a possibly small room.

We are using platinum-iridium fibres of 0,04 mm diameter and cca 60 cm length serving for the suspension of the swinging-system weighing cca 80 g and for measuring at the same time. The fibres loaded by 80 g weight were at first slowly heated over 100 C°, and then cooled down; after several repetitions of this procedure they attained in this way an almost perfect constancy of their equilibrium-positions after some months. Even the most violent shakings, accompanying the rotation of the instrument when the suspension-wires were in excentric positions, do not cause in general noticeable changes in the positions of equilibrium, but only seldom some small deviations.

[1] [*Determination of h was necessary, when the gravity-center of the body in question was not simply calculable from the form of the body. In this case a calibrating cylinder, then the body could be fixed to the beam of an analytical balance, near to the axis of rotation, specially designed for that purpose by E ö t v ö s. See the detailed description: J. R e n n e r, Experimental examination of the proportionality of gravitational attraction and inertia, Math. és Term. tud. Értesítő (in Hungarian), vol. 53, 1935, pp. 553 – 555.]

Experiments with quartz fibres did not give at all the same favourable results.

However, the position of equilibrium for loaded metal fibres is depending on temperature. It is the consequence of the torsion with vhich the fibres leave the eyelet, and so it is different for every piece of the fibre. This dependence is quite complicated namely the drift of the position of equilibrium is not depending on the variation of the temperature, but also on its course in time. Nevertheless, with so small and slow variations of temperature which occurred during the observations here treated, not exceeding some tenth of a grade in a day, this drift is satisfactorily represented by the individual temperature coefficient for each fibre. For the fibre in the simple variometer used by us this coefficient is $dn/dt = 0{,}4$, where n is the scale reading for the positions of equilibrium, and t is taken in centigrades. For the fibres in the second instrument used here it is still less.

That second instrument is *a double gravity variometer* (s. p. 101)[1], so called by *Eötvös*, for it consists in two parallelly suspended torsion balances which lying on a common base can be turned about the same axis. Both these single balances are of the type of the single gravity variometer; their beams are almost parallel, but so oriented as their suspended weights M_a lie on the opposite ends. Thus, when the suspended weight of one beam points to North, that of the other balance points to South.

In the beginning we followed *Eötvös'* instructions, but in the course of the observations we succeeded in a simple way to make the instruments more efficient. As according to our method of observation the position of equilibrium of the beam itself is read in the moment when it came just to rest, in this way the determination of a new position of equilibrium acted by a rotation requires a certain time depending on the resistance acting against the motion of the beam. Thus, the time-interval between two consecutive readings could not be fixed primarily for less then two, sometimes three hours. But simple calculations, the presentation of which would be here out of place, showed us, that this time-interval can be reliably reduced to one hour, when the resistance acting against the motion of the beam can be increased to the lowest limit required to make the motion aperiodic. The wanted increase was reached by inserting of brass plates of suitable size to the base and lid of the innermost housing. The innermost clearance was reduced so to 9 mm.

By applying those plates our performance could be increased by two-three-times higher than before.

Observations with so delicate instruments had to be done in shake-proof rooms protected at once against changes of temperature, and in consequence, possibly againts one-sided temperature radiations. Cellars without window would best fit to this condition. Unfortunately we did not dispose of such. Time was pressing, and so we had to be satisfield with a room for the observations, what lies at the first floor of the laboratory being at our disposal and has two windows opening to the South. Yet, higher buildings shadowed these windows for most part of the day, shutters did blanket them too, so the room was always held in dark. To complete this protection for each of the

[1] [*The quotation refers to Eötvös' treatise of 1906.]

instruments a celt was built, with strong double-lined walls between the frames, filled with sawdust, the linen stitched like counterpanes.

As the room used for observation lies out of the way of street traffic, we were not anxious about heavier shocks. Unfortunately, the conditions grew worse by a new building in progress, taken on in the immediate proximity, during the observations. Though the results of observations show no significant influence of these perturbances, we are aware of them, knowing that the observations disclosed here were not performed under the most favourable conditions and have not the perfection, as we thought to be able to reach. Well, "Ars longa, vita brevis", we must content ourselves with having proceeded a step forward.]

The considerations of the previous chapter serving for a theoretical basis of the experiments to be done suppose that the suspended parts of the torsion balance are not subject to other influences, but to those of the inertia and gravitational attraction of masses lying outside of them and the elastic force of the fibre acting againts the torsion.

Such a complete exclusion of all the effects which had to be preceeded by the knowledge of all natural forces, is beyond man's grasp, but at least those perturbing influences must be possibly avoided which are known to us to a certain extent.

We want to enumerate in order the most important influences and indicate also the way, how we invalidated them.

Magnetic forces, especially the geomagnetic force, must manifest themselves, if the swinging system contains some remanent magnetic parts. A fragment with a magnetic moment with only 1/1000 cgs magnetic moment, as about a fragment of a good steel magnet with 1/50 mg weight, could cause perturbing elongations of two scale divisions, after a rotation of the torsion balance. By careful selection of the parts composing the suspended system, it is attainable that it can be taken for non-magnetic, inspite of its great sensitivity; all the same, with our experiments attention had to be taken to this defect and prevent it in another way, while the suspended parts were repeatedly substituted by other ones. For this reason we compensated the horizontal component of the geomagnetic force so that in the space of the instruments H was reduced to zero by using permanent magnets and electromagnets.

[The compensating magnets had to be placed at a greater distance (about 1,4 m) so that they could not exert translatory forces on the temporary induced magnetism of the swinging system, effected mainly by the vertical component of the geomagnetic force. With extensive knowledge of the magnetic force it was easy to avoid its perturbance.]

The same can be said about the *electrostatic actions of outer bodies*, the influence of those on our torsion balance can be regarded as fully annulled by the threefold metal casing.

On the other hand, we have to regard for the *electrostatic forces between the suspended system and the enclosing housing* for they are not consisting of the same material. If the surfaces of the swinging system and the enclosing walls of the casing have different electric charges, electrostatic forces are produced, which might well be equal to null in a symmetrical mean position, but in the case of a deviation from it they can be sensible. Consequently, these forces must manifest themselves by that they influence the sensitivity of the instrument, i. e., the torque $\tau\vartheta$ acting againts the gravitational forces will be changed by them into $\tau'\vartheta$. In order to prevent the diverse electrical charges of the different parts of the surfaces we covered them with a uniform layer of soot. We also devised a method of observation, the results of which were not affected by small discrepancies in the quantity τ'.

The direct effects of irradiation caused by external bodies are not sufficiently known to us. But the multiple metal cover of the housing serves for the reduction to minimum of this unknown influence. [Also the dimensions of the instruments used were accordingly chosen and application of smaller anf lighter swinging systems was avoided, as we had to consider that the force to be measured, which is proportional to the mass, should be great with respect to eventual forces, which are proportional to the surface. These forces are certainly very small with our instruments and concealed in the hazards adhering in form of errors to every series of observation.]

Effects originating in the differences of temperature between the diverse parts of the housing and the swinging system. The external variations in the temperature, in consequence of which heat will be transported to the instruments or taken away from them through radiation and conduction, produce discrepancies in the temperature of the parts of the balance and the enclosed air. The multiple metal cover of the housing serves for reducing this discrepancy to as small as possible; the soot cover of all inner parts mentioned above has the same purpose. [Supposed that a distribution can be reached by it, which is on both sides symmetrical to the vertical plane passing through the mean position of the beam, only the sensitivity of the instrument, i. e. the quantity τ' is to be substituted for the constant of torsion, in the same way, as it was mentioned concerning the internal electrostatic forces. Traces of asymmetrical warming up existing inspite of all protection, maintain still to-day their accidental character.]

Changes in the temperature of fibres, if small and of slow course, can be calculated by their individual coefficients, or even discarded by a suitably chosen method of observation.

Shocks are not absolutely ineffective, too. Namely the position of equilibrium of the end of a loaded fibre changes with the load as a result of the remanent torsion of the fibre, consequently, vertical shocks shall cause jumps of the beam. But these jumps are negligeably small if caused by usual street traffic. [Only in case of earthquakes do they reach perceptible values, and at this time they amount to several scale units. In the course of observations of several years we noticed so many earthquakes the occurence of which was stated later by the seismological reports. Exceptional cases, like those, are easily recognizable and have no significance for the totality of the observations.]

In the order of possible perturbances we have to think on changes, which take place owing to *variations in mass distribution in the environment* as acting to the second derivatives of the gravitational potential and specially to $\partial^2 U/\partial x\partial z$ and $\partial^2 U/\partial y\partial z$ and which may have measurable though not great values. Displacements of objects in the building are scarcely to be considered, but much more the accumulation of masses of water as it used to happen after cloud-bursts. [1 cm thick layer of water surrounding the building causes an effect on the position of equilibrium of our instrument, what amounts to about one hundredth of a scale division. Observations concerning changes of this kind had to be carried out systematically, but for that we found no time yet. Nevertheless, parts of our observational results were freed also from that possible influence.]

The execution and evaluation of our observations considering all those circumstances was developing and improving in the course of work. [Shortness of time did not allow us to carry out all, what was regarded as the best for our scheme, but it would involve a waste of time.] The results disclosed here were obtained by three different procedures, discriminated by us as the firts, second, and third procedure.

The first procedure supposes, that the quantities, $\partial^2 U/\partial x\partial z$ and $\partial^2 U/\partial y\partial z$ are constant and also the sensitivity of the instrument, i. e., τ remains steadily the same.

The second procedure rests like the first one on the constancy of $\partial^2 U/\partial x\partial z$ and $\partial^2 y\partial z$ but it admits the possibility, that τ be different during experiments with different suspended bodies and also it changes steadily in time.

The third procedure renders us at last independent from the supposition of the constancy of the quantities $\partial^2 U/\partial x\partial z$ and $\partial^2 U/\partial y\partial z$, as well as of τ.

All the three procedures repose on equations (8) and (8′), what we want to unite, and transform, by putting for ϑ the value

$$\vartheta = \frac{n_0 - n}{2L}$$

where n means the scale reading due to the position of equilibrium, n_0 a constant, and L the scale distance expressed in scale divisions. So we have

$$n_0 - n = \frac{L}{\tau}\varkappa\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)\sin 2\alpha + \frac{2L}{\tau}\varkappa\frac{\partial^2 U}{\partial x\,\partial y}\cos 2\alpha - \frac{2L}{\tau}M_a l_a h\frac{\partial^2 U}{\partial x\,\partial z}\sin\alpha +$$

$$+\frac{2L}{\tau}M_a l_a h\frac{\partial^2 U}{\partial y\,\partial z}\cos\alpha + \frac{2L}{\tau}M_a l_a \sin\varepsilon(\varkappa_b - \varkappa_a)\sin\alpha. \tag{13}$$

All observations were taken in four positions of the torsion balance, what we want to denote with respect to the end a of the beam as the northern, eastern, southern, and western positions by N, E, S, W and also the corresponding scale reading by n_N, n_E, n_S, n_W. Setting to N is easily done by the aid of a compass with knowledge of the magnetic declination; starting from that position the other ones are reached by successive rotations of the housing of the balance by 90°. However, in those positions the axis of the beam is not pointing precisely to the four quarters of the heaven. Be $\Delta\alpha$ the azimuth of the balance-axis in the initial N-position counted from North to the East, we obtain the following values for the four azimuths according to the four positions:

position N $\alpha_N = \Delta\alpha,$

,, E $\alpha_E = \Delta\alpha + \frac{n_N - n_E}{2L} + \frac{\pi}{2},$

,, S $\alpha_S = \Delta\alpha + \frac{n_N - n_S}{2L} + \pi,$

,, W $\alpha_W = \Delta\alpha + \frac{n_N - n_W}{2L} + \frac{3\pi}{2}.$

Considering, that $\Delta\alpha$ and also the quantities

$$\frac{n_N - n_E}{2L} \text{ etc.}$$

are small, we compute for the four positions from equation (13) the approximate values:

$$n_0 - n_N = \frac{L}{\tau}\varkappa\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)2\Delta\alpha + \frac{2L}{\tau}\varkappa\frac{\partial^2 U}{\partial x\,\partial y}$$

$$-\frac{2L}{\tau}M_a l_a h\frac{\partial^2 U}{\partial x\,\partial z}\Delta\alpha + \frac{2L}{\tau}M_a l_a h\frac{\partial^2 U}{\partial y\,\partial z},$$

$$n_0 - n_E = -\frac{L}{\tau}\varkappa\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)2\left(\Delta\alpha + \frac{n_N - n_E}{2L}\right) - \frac{2L}{\tau}\varkappa\frac{\partial^2 U}{\partial x\,\partial y}$$

$$-\frac{2L}{\tau}M_a l_a h\frac{\partial^2 U}{\partial x\,\partial z} - \frac{2L}{\tau}M_a l_a h\frac{\partial^2 U}{\partial y\,\partial z}\left(\Delta\alpha + \frac{n_N - n_E}{2L}\right)$$

$$+\frac{2L}{\tau}M_a l_a hG\sin\varepsilon\,(\varkappa_b - \varkappa_a),$$

$$n_0 - n_S = \frac{L}{\tau}\varkappa\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)2\left(\Delta\alpha + \frac{n_N - n_S}{2L}\right) + \frac{2L}{\tau}\varkappa\frac{\partial^2 U}{\partial x\,\partial y}$$

$$+\frac{2L}{\tau}M_a l_a h\frac{\partial^2 U}{\partial x\,\partial z}\left(\Delta\alpha + \frac{n_N - n_S}{2L}\right) - \frac{2L}{\tau}M_a l_a R\frac{\partial^2 U}{\partial x\,\partial y},$$

$$n_0 - n_W = -\frac{L}{\tau}\varkappa\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)2\left(\Delta\alpha + \frac{n_N - n_w}{2L}\right) - \frac{2L}{\tau}\varkappa\frac{\partial^2 U}{\partial x\,\partial y}$$

$$+\frac{2L}{\tau}M_a l_a h\frac{\partial^2 U}{\partial x\,\partial z} + \frac{2L}{\tau}M_a l_a h\frac{\partial^2 U}{\partial y\,\partial z}\left(\Delta\alpha + \frac{n_N - n_W}{2L}\right)$$

$$-\frac{2L}{\tau}M_a l_a G\sin\varepsilon\,(\varkappa_b - \varkappa_a).$$

We shall put in the following notations:

$$n_N - n_S = m \quad \text{and} \quad n_E - n_W = v,$$

and use the following equations as *basic equations:*

$$\left.\begin{aligned} m = &-\frac{4L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial y\, \partial z} + \frac{L}{\tau} K\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) \frac{n_N - n_S}{L} \\ &+ \frac{2L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial x\, \partial z} \left(2\Delta\alpha + \frac{n_N - n_S}{2L}\right), \end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned} v = &+\frac{4L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial y\, \partial z} - \frac{L}{\tau} K\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) \frac{n_E - n_W}{L} \\ &+ \frac{2L}{\tau} M_a l_a h \frac{\partial^2 U}{\partial y\, \partial z} \left(2\Delta\alpha + \frac{2n_N - n_E - n_W}{2L}\right) + \frac{4L}{\tau} M_a l_a G \sin \varepsilon (\varkappa_a - \varkappa_b). \end{aligned}\right\} \tag{15}$$

For the gravity variometers applied here the values occurring in these equations are collected in the following table, where M^* and h^* denote mean values, which were substituted by more accurate ones for the single observations.

	τ	$\frac{K}{\tau}$	L	l_a	M_a^*	h^*	$G \sin \varepsilon$
Single Gravity variometer	0,5035	41 896	1232	20	25,4	21,2	1,6858
Double Gravity variometer							
Balance no. 1.	0,5073	43 081	1258	20	25,4	21,2	1,6858
Balance no. 2.	0,5116	43 849	1258	20	25,8	21,2	1,6858

4. Observations and their evaluation according to the first procedure

Only one swinging system of the torsion balance was used, i. e., that of the single variometer, or only one of the double variometer. End b remained steadily loaded by the same piece of platinum inserted to the tube.

End a was loaded so as before by the investigated body (e. g., by platinum), and the North-position of the instrument (with the end a to the North) as approximately defined on the graduated circle by the aid of compass. The admissible departure, i. e., the value of $\Delta\alpha$ may here reach some degrees.

Now, the instrument was in regular time intervals repeatedly set in two positions being distant from the approximately defined North-position by 90°, and 270°, respectively, i. e., in East- and West-position.

By reading the positions of equilibrium, we obtained then

$$v = n_E - n_W$$

so that this distance in each position was determined from the mean of the immediately preceding and subsequent opposite positions. Moreover the n values of the single variometer were reduced by a temperature correction of 0,4.

The quantity m will then be likewise determined for the same body (e. g., platinum), for which purpose even a few observations are sufficient, because at this procedure only the knowledge of an approximate value of this quantity is needed.

Having carried out these observations with a body at the place of it on the same end a, we suspended an other one (e. g., magnalium), of about the same weight and determined for it v' and m'. As the exchange of the body can only be done with arrested instrument, a small displacement of the first North-position is inevitable, we put therefore $\Delta\alpha'$ for $\Delta\alpha$. However, the quantity $\Delta\alpha' - \Delta\alpha$, which is measured in scale divisions, hardly reaches the value 1/1000.

Taken on that during this whole series of observation lasting for some weeks, the value of τ and the second derivatives of the gravitational potential remained constant, we obtain for the calculation of $\varkappa_a - \varkappa_a'$ i. e., the difference of the coefficients of attraction of both bodies (e. g., platinum and magnalium) according to equation (15)

$$v - v' = \frac{4L}{\tau} M_a l_a \frac{\partial^2 U}{\partial x\, \partial z}(h - h') + \frac{4L}{\tau} M_a l_a \frac{\partial^2 U}{\partial y\, \partial z}(h\Delta\alpha - h'\Delta\alpha')$$

$$+ \frac{4L}{\tau} M_a l_a G \sin\varepsilon(\varkappa_a - \varkappa_a'),$$

where we neglected those terms which were multipled by vanishingly small quantities like

$$\left(\frac{n_E - n_W}{L}\right) - \left(\frac{n_E - n_W}{L}\right).$$

This expression is still capable of a further simplification in that we disregard the small quantities of second order; so we have

$$v - v' = v\frac{h - h'}{h} - m(\Delta\alpha - \Delta\alpha') + \frac{4L}{\tau} M_a l_a G \sin\varepsilon(\varkappa_a - \varkappa_a'), \tag{16}$$

consequently

$$\varkappa_a - \varkappa_a' = \frac{\tau}{4LM_a l_a G \sin\varepsilon}(v - v') + \frac{m(\Delta\alpha - \Delta\alpha') - v\dfrac{h - h'}{h}}{4LM_a l_a G \sin\varepsilon}\tau. \tag{17}$$

a) *Observations concerning the difference* $\varkappa_{magnalium} - \varkappa_{platinum}$ performed with the first procedure using the single gravity variometer.

First series of observation

On the end a of the beam there was suspended by a 0,9 mm cupper-bronze wire a magnalium cylinder of 11,92 cm length and 1,01 cm diameter. We had

$$M_a = 25{,}402\ g,\ h = 21{,}20\ \text{cm}.$$

From the observed 114 v-values we received the mean value

$$v = +1{,}983 \pm 0{,}008,$$

and from the observed 64 m-values the mean value

$$m = +8{,}138 \pm 0{,}009.$$

Second series of observation

On the end a of the balance a platinum cylinder of 6,01 cm length and 0,50 cm diameter was suspended. We had

$$M'_a = 25{,}430\ \text{g and } h' = 21{,}24\ \text{cm}.$$

From the observed 48 m' and 56 v' values we obtained the mean values:

$$m' = +7{,}534 \pm 0{,}004 \text{ and } v' = +1{,}799 \pm 0{,}006.$$

For the calculation of $\varkappa_{magn} - \varkappa_{plat}$ *after the formula* (17) we took for M_a its mean value from the two series of observation, namely 25,416 g. With the values given previously for the instrument we obtained

$$\frac{\tau}{4LM_a l_a G \sin\varepsilon} = 0{,}1192 \cdot 10^{-6}.$$

As the readings in the North-positions were

$$n = 209{,}5 \text{ and } n' = 206{,}5,$$

we have

$$\varDelta\alpha - \varDelta\alpha' = \frac{n' - n}{2L} = -\frac{3}{2464} = -0{,}0012,$$

and with the mean value of $m = 7{,}84$,

$$m(\varDelta\alpha - \varDelta\alpha') = -0{,}009.$$

Further, as we have

$$\frac{h - h'}{h} = -0{,}002,$$

the term multiplied by this is to be neglected.

We obtain thus with the values found for m, m' and v, v'

$$\varkappa_{magn} - \varkappa_{pt} = +0{,}022 \cdot 10^{-6} - 0{,}001 \cdot 10^{-6} = +0{,}021 \cdot 10^{-6},$$

or with the mean error of this result:

$$\varkappa_{magn} - \varkappa_{pt} = +0{,}021 \cdot 10^{-6} \pm 0{,}001 \cdot 10^{-6}.$$

But this result indicating a value for the wanted difference surpassing the mean value should not mislead us. We mentioned already that with this first experimental disposition the constancy of τ was supposed; if we show together not only the values v, v' but also m, m', it is clearly seen that with the experiments with magnalium the value of τ was greater than with the platinum, namely

$$v = +1{,}983, \quad v' = +1{,}799, \quad m = +8{,}138, \quad m' = +7{,}534.$$

Pursuing the second procedure of observation, that will be described shortly, we shall be free from such effect of the dissimilarity and variability of τ, and we may apply the equations set up for the second procedure to evaluate the first experiments, inasmuch as we suppose that τ was constant during the experiments with magnalium, as well as with platinum, but τ and τ' had different values. [Reasons capable to cause such dissimilarities were already treated above.] Calculating the results of the preceding experiments by equation (20), wich will follow later, we obtain

$$\varkappa_{magn} - \varkappa_{pt} = +0{,}004 \cdot 10^{-6} \pm 0{,}001 \cdot 10^{-6}.$$

b) *Observations for the difference:* $\varkappa_{wood} - \varkappa_{platinum}$ performed after the first procedure, using beam 1 of the double gravity variometer.

First series of observation

On the end a of the beam was suspended a cylindric piece of snakewood, with 24,00 cm length and 1,01 cm diameter. There was

$$M_a = 24{,}925\,\mathrm{g} \text{ and } h = 21{,}03\,\mathrm{cm}.$$

From the observed 45 m values and 53 v values we obtained the means

$$m = +6{,}698 \pm 0{,}019 \text{ and } v = -1{,}797 \pm 0{,}008$$

Second series of observation

On the end a of the beam was suspended a platinum cylinder of 6.00 cm length and 0,50 cm diameter. We had

$$M'_a = 25{,}396\,\mathrm{g} \text{ and } h' = 21{,}18\,\mathrm{cm}.$$

We obtained from the observed 14 m' and 34 v' values the means

$$m' = +6{,}595 \pm 0{,}016 \text{ and } v' = -1{,}754 \pm 0{,}011.$$

For the evulation of $\varkappa_{wood} - \varkappa_{platinum}$ *after equation* (17) we take for M_a its mean value 25,160 g; we have then with the values given already for beam 1:

$$\frac{\tau}{4 L M_a l_a G \sin \varepsilon} = 0{,}1189 \cdot 10^{-6}.$$

In the North-position we had

$$n + 187{,}5 \text{ and } n' = 191{,}3,$$

consequently

$$\Delta\alpha - \Delta\alpha' = + \frac{3{,}8}{2516} = + 0{,}0015,$$

and with the mean value $m = +6{,}65$

$$m = (\Delta\alpha - \Delta\alpha') = + 0{,}010,$$

we had further

$$\frac{h - h'}{h} = - 0{,}007 \quad \text{and} \quad v \frac{h - h'}{h} = + 0{,}013,$$

then we got

$$\varkappa_{wood} - \varkappa_{Pt} = - 0{,}005 \cdot 10^{-6} - 0{,}000 \cdot 10^{-6} = - 0{,}005 \cdot 10^{-6},$$

or with the computed mean error

$$\varkappa_{wood} - \varkappa_{Pt} = - 0{,}005 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}.$$

But using equation (20) for the evaluation in the same way, as with the magnalium and platinum, we obtain

$$\varkappa_{wood} - \varkappa_{Pt} = - 0{,}001 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}.$$

5. *Observation and evaluation after the second procedure*

Only one swinging system was used, like with the first procedure. The comparative body was suspended on end a. The instrument was set always in equal time intervals, in order in the N-, E-, S-, W-position; this operation was sufficiently repeated.

We assume now, that τ and together with it m and v are varying with the time, but at least during the length of time necessary to six settings those variations can be taken as proportional to time. We obtain thus the values m according to the moment of readings, taken in the meridional position as the differences of these readings and the mean values of the reading taken in the preceding and the next following opposite meridional positions. The momentary values were similarly taken from the values read in the positions in the prime vertical. Whereas we compute the momentary values of v for the moment of a meridional reading as the mean of the preceding and next following reading of this quantity. And so viceverse.

We calculate now the ratio v/m, for which we obtain from (14) and (15) by neglecting small quantities of second order:

$$\frac{v}{m} = -\frac{\dfrac{\partial^2 U}{\partial x\,\partial z}}{\dfrac{\partial^2 U}{\partial y\,\partial z}} + \frac{2K}{\tau}\,\frac{\dfrac{\partial^2 U}{\partial x\,\partial z}}{\dfrac{\partial^2 U}{\partial y\,\partial z}}\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) - \left(1 + \frac{v^2}{m^2}\right)\Delta\alpha$$

$$-\frac{v^2}{m^2}\,\frac{m}{4L} - \frac{2n_N - n_E - n_W}{4L} + \frac{4L}{m\tau}\,M_a l_a G \sin\varepsilon(\varkappa_a - \varkappa_b), . \qquad (18)$$

After that the body at a will be replaced by an other one, and a new series of observations renders us the value for v'/m'.

For the computation of the $\varkappa_a - \varkappa_a'$ serves the approximate formula

$$\frac{v}{m} - \frac{v'}{m'} = +\frac{4LM_a l_a G \sin\varepsilon}{m\tau}\,(\varkappa_a - \varkappa_a') - \left(1 + \frac{v^2}{m^2}\right)(\Delta\alpha - \Delta\alpha'), \qquad (19)$$

whence

$$\varkappa_a - \varkappa_a' = \frac{m\tau}{4LM_a l_a G \sin\varepsilon}\left(\frac{v}{m} - \frac{v'}{m'}\right) + \frac{m\tau}{4LM_a l_a G \sin\varepsilon}\left(1 + \frac{v^2}{m^2}\right)(\Delta\alpha - \Delta\alpha'), \qquad (20)$$

where all quantities are neglected which contribute to the strict value of v/m by less then 1/1000.

a) *Observations concerning the difference* $\varkappa_{copper} - \varkappa_{platinum}$ carried out with beam 1 of the double gravity variometer, following the second procedure.

First series of observations

On end a of the beam was suspended a copper cylinder of 6,40 cm length and 0,77 cm diameter. We had

$$M_a = 25{,}441 \text{ g}, \quad h = 21{,}26 \text{ cm}.$$

From the 92 observed values we received the means:

$$m = +6{,}516 \pm 0{,}015 \text{ and } v = -1{,}923 \pm 0{,}005.$$

Second series of observations

On end a of the beam was suspended a platinum cylinder of 6,00 cm length and 0,50 cm diameter.
We had

$$M_a' = 25{,}437 \text{ g}, \quad h = 21{,}23 \text{ cm}.$$

From the observed 64 values we obtained the means:

$$m' = +6{,}536 \pm 0{,}013 \text{ and } v' = -1{,}982 \pm 0{,}011.$$

When computing $\varkappa_{Cu} - \varkappa_{Pt}$ *after equation* (20), we had to take exactly the mean values of the individually computed v/m and v'/m' values for the moments of readings. The laborious circumstantiality of computation of this nature moved us, however, instead of those to compute the mean values of v and m, as well as v' and m', and to form the ratios v/m and v'/m' from these. It is easy to prove that this mode of computation is permissible here within the limits of the accuracy found. By so calculating we obtained from the results of the two series of observations

$$\frac{v}{m} = -0{,}295 \pm 0{,}001, \quad \frac{v'}{m'} = -0{,}303 \pm 0{,}002,$$

we had further in the N-position the means

$$n = 214{,}5 \text{ and } n' = 208{,}0,$$

accordingly

$$\Delta\alpha - \Delta\alpha' = -0{,}002,$$

whence

$$\left(\frac{v^2}{m^2} + 1\right)(\Delta\alpha - \Delta\alpha') = -0{,}002.$$

Using the mean value $M_a = 25{,}439$ we obtain

$$\frac{m\tau}{4LM_a l_a G \sin\varepsilon} = 0{,}7687 \cdot 10^{-6}$$

and by this

$$\varkappa_{Cu} - \varkappa_{Pt} = +0{,}004 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}.$$

b) *Observations concerning an eventual change of $\varkappa$ with the reaction of silver-sulfate and ferrous sulfate.*

The great interest connected since the researches of *H. Landolt*[1] to the reaction

$$Ag_2SO_4 + 2\ F_0SO_4 = 2\ Ag + Fe_2(SO_4)_3,$$

who using an analytical balance proved recognizable changes in the weight, what made us investigate, whether that reaction had a result in the change of the coefficient $\varkappa$.

According to *Landolt's* dates we weighed first
1,56 g silver sulfate + 4,25 g water = 5,81 g,

then 4,05 g crystalline ferrous sulfate + 1,62 g water + 0,14 g dilute sulfuric acid = 5,81 g
and closed these two mixtures separately in two thin walled glass tubes. Then we have put these two mixtures commonly in a glass tube and then laid it aside, while after a week the perfect termination of the reaction was to be expected. In a series of observations the reacting mixtures being separate till now were introduced in a cylindric brass tube and suspended on the beam of the torsion balance. With a preceding series of observations the tube containing the products of reactions were examined in the same way.

[1] Zeitschr. f. physik. Chem, 12, 1893, p. 1.

First series of observations

Both glass tubes containing the two reacting mixtures placed one over the other were fixed in a cylindric brass tube of 12,91 cm length and 1,16 cm diameter, and suspended on the beam of the single gravity variometer. There was

$$M_a = 25{,}357 \text{ g}, \; h = 21{,}50 \text{ cm},$$

where 11,62 g falls to the share of the reacting mixture.

From observations of 132 values were derived the mean values

$$m = +7{,}590 \pm 0{,}011 \text{ and } v = -2{,}027 \pm 0{,}005.$$

Second series of observations

The glass tube containing the product of reaction was put into the brass tube used before and suspended together with the brass tube, we had

$$M'_a = 25{,}362 \text{ g}, \; h' = 21{,}34 \text{ cm},$$

where 11,62 g falls to the share of the products of reactions.

From the observed 132 values we obtained the mean values

$$m' = +7{,}622 \pm 0{,}08, \; v' = -2{,}032 \pm 0{,}005.$$

Using formula (20) *for the computation of* $\varkappa - \varkappa'$, we had

$$\frac{v}{m} = -0{,}267 \pm 0{,}001, \qquad \frac{v'}{m'} = -0{,}267 \pm 0{,}001,$$

further

$$n = 211{,}1 \text{ and } n' = 212{,}0,$$

consequently the term multiplied by $\Delta\alpha - \Delta\alpha'$ is to be neglected. With the mean value $M_a = 25{,}36$ we obtain

$$\frac{m\tau}{4LM_a l_a G \sin\varepsilon} = 0{,}9089 \cdot 10^{-6},$$

and so

$$\varkappa - \varkappa' = 0{,}000 \cdot 10^{-6} \pm 0{,}001 \cdot 10^{-6},$$

$\varkappa$ and $\varkappa'$ have the meaning of mean values for the inhomogeneous masses M_a and M'_a, which contained the reacting mixtures and products of reaction, respectively.

If we wanted to attribute an eventual, from zero different change of $\varkappa$ to the proceeded reaction, it should be

$$\varkappa_v - \varkappa_n = 0{,}000 \cdot 10^{-6} \pm 0{,}001 \cdot 10^{-6},$$

where $\varkappa_v$ relates to the same mass before the reaction and $\varkappa_n$ after it.

6. Observations and their evaluation after the third procedure

[This procedure yields values, which are independent not only from continuous changes of the sensitivity, but from changes of the local variations of the gravity at once.]

A double gravity variometer will be used to that purpose, but its beams should be only approximately parallel. The azimuth of the first beam in the N-position be $\Delta\alpha_{1I}$ with the first series of experiments, with the second series $\Delta\alpha_{1II}$, the azimuth of the second beam with the first series $\Delta\alpha_{2I}$, and with the second series $\Delta\alpha_{2II}$, this time the differences $\Delta\alpha_2 - \Delta\alpha_1$, should not surpass about two degrees, what is easy to achieve.

While the b ends of both beams are loaded by the inserted platinum pieces, one of the comparative bodies with $\varkappa_a$ was suspended on the a end of the swingling system no. 1., and the other one with $\varkappa'_a$ suspended on the a end of the swinging system no. 2.

The observations will be then in the consecutive N-, E-, S-, W-positions so arranged, as with the second procedure.

We obtain, thus, after equation (18)

$$\frac{v_1}{m_1} = -\frac{\dfrac{\partial^2 U}{\partial x\,\partial z}}{\dfrac{\partial^2 U}{\partial y\,\partial z}} + \frac{2K}{\tau}\,\frac{\dfrac{\partial^2 U}{\partial x\,\partial z}}{\dfrac{\partial^2 U}{\partial y\,\partial z}}\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) - \left(1 + \frac{v_1^2}{m_1^2}\right)\Delta\alpha_{1I}$$

$$-\frac{v_1^2}{m_1^2}\,\frac{m_1}{4L_1} - \left(\frac{2n_N - n_E - n_W}{4L}\right)_1 + \left(\frac{4LM_a l_a}{m\tau}\right)_1 G\sin\varepsilon(\varkappa_a - \varkappa_b),$$

and

$$\frac{v'_2}{m'_2} = -\frac{\dfrac{\partial^2 U}{\partial x\,\partial z}}{\dfrac{\partial^2 U}{\partial y\,\partial z}} + \frac{2K}{\tau}\,\frac{\dfrac{\partial^2 U}{\partial x\,\partial z}}{\dfrac{\partial^2 U}{\partial y\,\partial z}}\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) - \left(1 + \frac{v_2'^2}{m_2'^2}\right)\Delta\alpha_{2I}$$

$$-\frac{v_2'^2}{m_2'^2}\,\frac{m}{4L_2} - \left(\frac{2n_N - n_E - n_W}{4L}\right)_2 + \left(\frac{4LM_a l_a}{m\tau}\right)_2 G\sin\varepsilon(\varkappa_a - \varkappa_b),$$

whence by subtraction, then neglecting quantities under 1/1000 we have

$$\frac{v_1}{m_1} - \frac{v'_2}{m_2'} = \left(1 + \frac{v^2}{m^2}\right)(\Delta\alpha_{2I} - \Delta\alpha_{1I}) + \frac{4LM_a l_a}{m\tau}\,G\sin\varepsilon(\varkappa_a - \varkappa'_a), \tag{21}$$

where for v, m, L, M_a, l_a their mean values are to be taken.

We exchange now the comparative bodies hung on the two half-instruments so that the body with the coefficient $\varkappa'_a$ be hung on beam 1. and that with $\varkappa_a$ on beam 2. We have now for the second series of observations:

$$\frac{v_2}{m_2} - \frac{v'_1}{m'_1} = \left(1 + \frac{v^2}{m^2}\right)(\Delta_{1II} - \Delta\alpha_{2II}) + \frac{4LM_a l_a}{m\tau}\,G\sin\varepsilon(\varkappa_a - \varkappa'_a),$$

and we obtain by addition

$$\left.\begin{aligned}\left(\frac{v_1}{m_1}-\frac{v_2'}{m_2'}\right)+\left(\frac{v_2}{m_2}-\frac{v_1'}{m_1'}\right) &= \frac{8LM_a l_a G\sin\varepsilon}{m\tau}(\varkappa_a-\varkappa_a')\\ &+\left(1+\frac{v^2}{m^2}\right)[(\Delta\alpha_{2I}-\Delta\alpha_{2II})-(\Delta\alpha_{1I}-\Delta\alpha_{1II})].\end{aligned}\right\}\quad(22)$$

and

$$\left.\begin{aligned}\varkappa_a-\varkappa_a' &= \frac{m\tau}{8LM_a l_a G\sin\varepsilon}\left\{\left(\frac{v_1}{m_1}-\frac{v_2'}{m_2'}\right)+\left(\frac{v_2}{m_2}-\frac{v_1'}{m_1'}\right)\right\}\\ &+\frac{m\tau}{8LM_a l_a G\sin\varepsilon}\left(1+\frac{v^2}{m^2}\right)[(\Delta\alpha_{1I}-\Delta\alpha_{1II})-(\Delta\alpha_{2I}-\Delta\alpha_{2II})].\end{aligned}\right|\quad(23)$$

a) *Observations concerning the difference* $\varkappa_{water}+\varkappa_{Cu}$

First series of experiments

On beam 1. of the double gravity variometer was suspended a cylindric brass case filled with water, having 14,14 cm length and 1,16 cm diameter; there was

$$h=21{,}34 \text{ cm},\quad M_a=25{,}447 \text{ g},$$

where the share of water alone was 12,82 g so that $M_{water}=0{,}504\ M_a$,

On beam 2. of the double variometer was suspended a copper cylinder with 6,50 cm length and 0,77 cm diameter.

From 108 observed values were derived the mean values

$$m_1=+6{,}767\ \pm0{,}016,\quad v_1=-2{,}029\ \pm0{,}012,$$

$$m_2'=+6{,}611\ \pm0{,}012,\quad v_2'=-1{,}927\ \pm0{,}005,$$

accordingly

$$\frac{v_1}{m_1}=-0{,}300\pm0{,}012,\qquad \frac{v_2'}{m_2'}=-0{,}291\pm0{,}001.$$

Second series of experiments

On beam 2 of the double gravity variometer was suspended a copper cylinder of 6,40 cm length and 0,77 cm diameter. There was

$$h=21{,}16 \text{ cm},\quad M_a=25{,}441 \text{ g}.$$

On beam 2 was hung a brass case with 14,14 cm and 1,16 cm diameter, filled with water. There was

$$h=21{,}21 \text{ cm},\quad M_a=25{,}809 \text{ g},$$

where the share falling alone on the water was 13,18 g so that

$$M_{water} = 0{,}511 \ M_a.$$

From the observed 92 values were derived the mean values

$$m_1' = +6{,}516 \ \pm 0{,}015, \ v_1' = -1{,}923 \ \pm 0{,}005,$$

$$m_2 = +6{,}786 \ \pm 0{,}010, \ v_2 = -2{,}016 \ \pm 0{,}009,$$

and accordingly

$$\frac{v_1'}{m_1} = -0{,}295 \pm 0{,}001, \quad \frac{v_2}{m_2} = -0{,}297 \pm 0{,}001.$$

Computing $\varkappa_a - \varkappa_a'$ *after formula* (23) we get from the results of the first and second experimental series

$$\left(\frac{v_1}{m_1} - \frac{v_2'}{m_2'}\right) + \left(\frac{v_2}{m_2} - \frac{v_1'}{m_1'}\right) = -0{,}011 \pm 0{,}003,$$

we had further the means

$$n_{1I} = 216{,}4, \ n_{2I} = 594{,}7, \ n_{1II} = 214{,}5, \ n_{2II} = 595{,}3,$$

and accordingly

$$\left(\frac{v^2}{m^2} + 1\right) \{(\Delta\alpha_{1I} - \Delta\alpha_{1II}) - (\Delta\alpha_{2I} - \Delta\alpha_{2II})\} = -0{,}001.$$

We found for the mean value

$$\frac{m\tau}{8LM_a l_a G \sin\varepsilon} = 0{,}3940 \cdot 10^{-6},$$

and so

$$\varkappa - \varkappa' = -0{,}005 \cdot 10^{-6} \ \pm 0{,}001 \cdot 10^{-6}.$$

Supposed that this difference is resulted from the difference $\varkappa_{water} - \varkappa_{Cu}$ alone, so will be, as $M_{water} = 0{,}508 \ M_a$,

$$\varkappa_{water} - \varkappa_{Cu} = -0{,}010 \cdot 10^{-6} \ \pm 0{,}002 \cdot 10^{-6}.$$

Observations concerning the difference $\varkappa_{crystalline} - \varkappa_{cupric\ sulfate}$

First series of experiments

On beam 1 of the double variometer was suspended a cylindric brass case of 12,99 cm length and 1,16 cm diameter filled with crystalline cupric sulfate; here was

$$h = 21{,}22 \text{ cm and } M_a = 25{,}447 \text{ g},$$

from what the share of the cupric sulfate alone was 16,15 g so that $M_{cupric\ sulfate} = 0{,}635 \ M_a$.

On beam 2 was hung a second brass cylinder of 8,01 cm length and 1,16 cm diameter, filled with pieces of electrolytic copper wire; here was

$$h=21{,}23 \text{ cm and } M_a=25{,}810 \text{ g},$$

from what the share falling on the electrolytic copper alone was 18,83 g so that $M_{cu}=0{,}730\ M_a$.

From 111 observed values were derived the mean values

$$m_1=+6{,}676 \pm 0{,}011,\quad v_1=-1{,}965 \pm 0{,}008,$$

$$m_2'=+6{,}684 \pm 0{,}010,\quad v_2'=-1{,}937 \pm 0{,}006,$$

and accordingly

$$\frac{v_1}{m_1}=-0{,}294\pm 0{,}001,\qquad \frac{v_2'}{m_2'}=-0{,}290\pm 0{,}001.$$

Second experimental series

On beam 1 was suspended a brass cylinder with 8,01 length and 1,16 cm diameter, containing pieces of electrolytic copper wire, and there was

$$h_1=21{,}16 \text{ cm and } M_a=25{,}468 \text{ g},$$

from which the share of the electrolytic copper was 18,49 g so that $M_{Cu}=$ $=0{,}726\ M_a$.

On beam 2 was suspended a 12,99 cm long brass case, with 1,16 cm diameter, filled with crystalline cupric sulfate and there was

$$h=21{,}18 \text{ cm and } M_a=25{,}842 \text{ g},$$

where the share of the cupric sulfate alone was 16,54 g so that $M_{\textit{cryst. cupric sulfate}}=0{,}640\ M_a$.

From the observed 132 values were derived the mean values

$$m_1'=+6{,}635 \pm 0{,}010,\quad v_1'=-1{,}984 \pm 0{,}005,$$

$$m_2=+6{,}613 \pm 0{,}008,\quad v_2=-1{,}923 \pm 0{,}007,$$

and accordingly

$$\frac{v_1'}{m_1'}=0{,}298\pm 0{,}001,\qquad \frac{v_2}{m_2}=-0{,}291\pm 0{,}001.$$

By computing $\varkappa-\varkappa'$ after the formula (23) we obtain

$$\left(\frac{v_1}{m_1}-\frac{v_2'}{m_2'}\right)+\left(\frac{v_2}{m_2}-\frac{v_1'}{m_1'}\right)=+0{,}003\pm 0{,}002,$$

and we had further

$$n_{1I}=216{,}0,\quad n_{2I}=593{,}6,\quad n_{1II}=192{,}8,\quad n_{2II}=592.3,$$

accordingly

$$\left(1+\frac{v^2}{m^2}\right)\{(\Delta\alpha_{1I}-\Delta\alpha_{1II})-(\Delta\alpha_{2I}-\Delta\alpha_{2II})\} = -0{,}010.$$

For the mean value we obtain

$$\frac{m\tau}{8\tau M_a l_a G \sin\varepsilon} = 0{,}3898\cdot 10^{-6},$$

and so

$$\varkappa-\varkappa' = -0{,}003\cdot 10^{-6} \pm 0{,}001\cdot 10^{-6}.$$

Supposed that this difference derives from the difference $\varkappa_{cryst.\ cupric\ sulfate}-$ $-\varkappa_{Cu}$ alone, being $M_{cryst.\ cupric\ sulfate}=0{,}638\ M_a$,
we have

$$\varkappa_{cryst.\ cupric\ sulfate}-\varkappa_{Cu} = -0{,}005\cdot 10^{-6} \pm 0{,}002\cdot 10^{-6}$$

Observations concerning the difference $\varkappa_{solution\ of\ cupric\ sulfate}-\varkappa_{Cu}$

First series of experiments

On the beam 1 of the double gravity variometer was suspended a cylindric brass case of 13,50 cm length and 1,16 cm diameter, the inside of which was plated with silver and filled with solution of cupric sulfate.

The solution contained 20,61 g crystalline cupric sulfate in 49,07 g water, which ratio was according to the solution used by *Heydweiller* with his experiments.[1] We had

$$h=21{,}22\ \text{cm},\quad M_a=25{,}459\ \text{g},$$

from which the share falling to the solution of cupric sulfate alone was 15,38 g so that $M_{sol.\ cupric\ sulfate}=0{,}730\ M_a$.

On beam 2 was hung a second brass cylinder of 8,01 cm length and 1,16 cm diameter, containing pieces of electrolytic copper wire. Here was

$$h=21{,}13\ \text{cm},\quad M_a=25{,}834\ \text{g},$$

from which the share falling on the electrolytic copper alone was 18,85 g, so that $M_{Cu}=0{,}730\ M_a$.

From the 132 observed values we obtained the mean values

$$m_1=+6{,}693\pm 0{,}011,\quad v_1=-2{,}027\pm 0{,}006,$$

$$m\ =+6{,}669\pm 0{,}010,\quad v_2'=-1{,}928\pm 0{,}005,$$

[1] Über Gewichtsänderungen bei chem. und phys. Umsetzungen. Annd. Phys. 5, 1901, p. 394.

and accordingly

$$\frac{v_1}{m_1} = -0{,}003 \pm 0{,}001, \quad \frac{v_2'}{m_2'} = -0{,}289 \pm 0{,}001.$$

Second series of observations

On beam 1 was suspended a brass cylinder of 8,01 cm length and 1,16 cm diameter, containing pieces of electrolytic copper wire; here was

$$h = 21{,}11 \text{ cm}, \ M_a = 25{,}468 \text{ g},$$

where the share of the electrolytic copper alone was 18,49 g so that $M_{Cu} = 0{,}726 \ M_a$.

On beam 2 was suspended a cylindric brass case of 13,50 cm length and 1,16 cm diameter the inside of which was plated with silver and filled with solution of cupric sulfate, here was

$$h = 21{,}22 \text{ cm}, \ M_a = 25{,}833 \text{ g},$$

from which the share of the solution of copper sulfate alone was 15,40 g so that

$$M_{sol.\ cupric\ sulfate} = 0{,}596 \ M_a.$$

From the 132 observed values we obtained the mean values

$$m_1' = +6{,}641 \ \pm 0{,}011, \ v_1' = -1{,}972 \ \pm 0{,}005,$$

$$m_2 = +6{,}766 \ \pm 0{,}010, \ v_2 = -1{,}982 \ \pm 0{,}007,$$

and accordingly

$$\frac{v_1'}{m_1'} = 0{,}297 \pm 0{,}001 \quad \text{and} \quad \frac{v_2}{m_2} = -0{,}293 \pm 0{,}001.$$

Computing $\varkappa - \varkappa'$ *after formula* (23) we obtained

$$\left(\frac{v_1}{m_1} - \frac{v_2'}{m_2'}\right) + \left(\frac{v_2}{m_2} - \frac{v_1'}{m_1'}\right) = -0010 \pm 0{,}002,$$

we had further the means

$$n_{1I} = 192{,}4, \ n_{2I} = 593{,}6, \ n_{1II} = 192{,}8, \ n_{2II} = 593{,}4,$$

accordingly to that the term, which is multiplied by

$$(\Delta\alpha_{1I} - \Delta\alpha_{1II}) - (\Delta\alpha_{2I} - \Delta\alpha_{2II}),$$

is to be disregarded.

We obtained the mean value

$$\frac{m\tau}{8LM_a l_a G \sin\varepsilon} = 0{,}3917 \cdot 10^{-6},$$

and hereby

$$\varkappa - \varkappa' = -0{,}004 \cdot 10^{-6} \pm 0{,}001 \cdot 10^{-6}.$$

By taking on that this difference is arising only from the difference $K_{sol.\ cupr.\ sulfate} - K_{Cu}$ we have, as $M_{sol.\ cupr.\ sulf.} = 0{,}600\ M_a$,

$$\varkappa_{sol.\ cupr.\ sulf.} - \varkappa_{Cu} = -0{,}007 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}.$$

Observations concerning the difference: $\varkappa_{asbestos} - \varkappa_{Cu}$

First experimental series

On beam 1 of the double gravity variometer was suspended a cylindric brass case of 12,99 cm length and 1,16 cm diameter, loaded with asbestos. Here was

$$h = 21{,}31\ \text{cm},\ M_a = 25{,}462\ \text{g},$$

from which the share of asbestos alone was 15,25 so that $M_{asbest} = 0{,}599\ M_a$.

On beam 2 was suspended a second brass cylinder of 8,01 cm lenght and 1,16 cm diameter, containing pieces of electrolytic copper wire. Here was

$$h = 21{,}13\ \text{cm},\ M_a = 25{,}834\ \text{g},$$

from what the share of the electrolytic copper alone was 18,85 g so that $M_{Cu} = 0{,}730\ M_a$.

From the 110 observed values were derived the mean values

$$m_1 = +6{,}685 \pm 0{,}012,\ v_1 = -2{,}024 \pm 0{,}006,$$

$$m = +6{,}705 \pm 0{,}009,\ v_2' = -1{,}935 \pm 0{,}004,$$

accordingly

$$\frac{v_1}{m_1} = -0{,}303 \pm 0{,}001 \quad \text{and} \quad \frac{v_2'}{m_2'} = -0{,}289 \pm 0{,}0001.$$

Second series o' experiments

On beam 1 was suspended a brass cylinder of 8,01 cm length and 1,16 cm diameter, containing pieces of electrolytic wire. Here we had

$$h = 21{,}10\ \text{cm},\ M_a = 25{,}469\ \text{g},$$

from which the share falling on the electrolytic copper alone was 18,48 g, accordingly $M_{Cu} = 0{,}726\ M_a$.

On beam 2 was suspended a cylindric brass case of 12,99 cm length and 1,16 diameter, loaded by asbestos. Here was

$$h = 21{,}24\ \text{cm},\quad M_a = 25{,}833\ \text{g},$$

falling on the asbestos alone a weight of 15,25 g so that $M_{asbest} = 0{,}596\ M_a$.

From the observed 106 values were derived the mean values

$$m = +6{,}591 \pm 0{,}012,\ v_1' = -1{,}946 \pm 0{,}005,$$

$$m_2 = +6{,}736 \pm 0{,}013,\ v_2 = -1{,}933 \pm 0{,}008,$$

accordingly

$$\frac{v_1'}{m_1'} = -0{,}295 \pm 0{,}001, \quad \text{and} \quad \frac{v_2}{m_2} = -0{,}287 \pm 0{,}001.$$

Computing $\varkappa - \varkappa'$ after formula (23) we obtain

$$\left(\frac{v_1}{m_1} - \frac{v_2'}{m_2'}\right) + \left(\frac{v_2}{m_2} - \frac{v_1'}{m_1'}\right) = -0{,}006 \pm 0{,}002,$$

and we had the means

$$n_{1I} = 193{,}8, \quad n_{2I} = 592{,}8, \quad n_{1II} = 193{,}8, \quad n_{2II} = 593{,}7,$$

whereby the term multiplied by $(\Delta\alpha_{1I} - \Delta\alpha_{1II}) - (\Delta\alpha_{2I} - \Delta\alpha_{2II})$ is to be disregarded.

We obtain the mean value

$$\frac{m\tau}{8LM_a l_a G \sin\varepsilon} = 0{,}3909 \cdot 10^{-6},$$

and so

$$\varkappa - \varkappa' = -0{,}002 \cdot 10^{-6} \pm 0{,}001 \cdot 10^{-6}.$$

Supposed that this difference derives from the difference $\varkappa_{asbest} - \varkappa_{Cu}$ alone, being $M_{asbest} = 0{,}598\ M_a$, we have

$$\varkappa_{asbest} - \varkappa_{Cu} = -0{,}003 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}.$$

Observations concerning the difference $\varkappa_{tallow} - \varkappa_{Cu}$

First experimental series

On beam 1 of the double gravity variometer was suspended a cylindric brass case of 15,60 cm length and 1,16 cm diameter, filled with pure tallow. The specific density of the tallow used was 0,918 (23,9 C°), and accordingly its mean molecular volume was about 53 times that of the water. It was further

$$h = 21{,}21 \text{ cm}, \quad M_a = 25{,}470 \text{ g},$$

from what the share falling on the tallow alone was 13,78 g, so that $M_{tallow} = 0{,}541\ M_a$.

On beam 2 was suspended a second brass cylinder of 8,01 cm length and 1,16 cm diameter, containing pieces of electrolytic copper wire. Here we had

$$h = 21{,}13 \text{ cm}, \quad M_a = 25{,}834 \text{ g},$$

from what the share of the tallow alone was 18,85 g, so that $M_{cu} = 0{,}730\ M_a$.

From the 118 observed values were derived the mean values

$$m_1 = +6{,}575 \pm 0{,}013, \quad v_1 = -1{,}917 \pm 0{,}012,$$

$$m_2' = +6{,}637 \pm 0{,}013, \quad v_2' = -1{,}877 \pm 0{,}007,$$

and accordingly

$$\frac{v_1}{m_1} = -0{,}292 \pm 0{,}002, \quad \frac{v_2'}{m_2'} = -0{,}283 + 0{,}001.$$

Second experimental series

On beam 1 was suspended a brass cylinder of 8,01 cm length and 1,16 cm diameter, containing pieces of electrolytic copper wire. Here was

$$h = 21{,}10 \text{ cm}, \; M_a = 25{,}469 \text{ g},$$

where the share of the electrolytic copper alone was 18,48 g so that $M_{C_u} = 0{,}726 \; M_a$.

On beam 2 was suspended a cylindric brass case of 15,60 cm length and 1,16 cm diameter, filled with tallow. Here was

$$h = 21{,}16 \text{ cm}, \; M_a = 25{,}847 \text{ g},$$

where the share of the tallow alone was 13,78 g so that $M_{tallow} = 0{,}533 \; M_a$.

We obtained from the 115 observed values the mean values

$$m_1' = +6{,}655 \; \pm 0{,}09, \; v_1' = -1{,}881 \; \pm 0{,}007,$$

$$m_2 = +6{,}831 \; \pm 0{,}005, \; v_2 = -1{,}930 \; \pm 0{,}006,$$

and accordingly

$$\frac{v_1'}{m_1'} = -0{,}283 + 0{,}001 \quad \text{and} \quad \frac{v_2}{m_2} = -0{,}283 \pm 0{,}001.$$

Computing $\varkappa' - \varkappa$ after the formula (23) we obtain

$$\left(\frac{v_1}{m_1} - \frac{v_2'}{m_2'}\right) + \left(\frac{v_2}{m_2} - \frac{v_1'}{m_1'}\right) = -0{,}008 \pm 0{,}003.$$

We had further the means

$$n_{1I} = 195{,}2, \; n_{2I} = 593{,}7. \qquad n_{1II} = 196{,}2, \; n_{2II} = 593{,}9,$$

whereby the term which is multiplied by $(\Delta\alpha_{1I} - \Delta\alpha_{1I}) - (\Delta\alpha_{2I} - \Delta\alpha_{2II})$ is to be neglected.

We obtain the mean value

$$\frac{m\tau}{8LM_a l_a G \sin \varepsilon} = 0{,}3907 \cdot 10^{-6},$$

and hereby

$$\varkappa - \varkappa' = 0{,}003 \cdot 10^{-6} \pm 0{,}001 \cdot 10^{-6}.$$

Supposed that this difference is resulted from the difference $\varkappa_{tallow} - \varkappa_{Cu}$ alone, we have, as $M_{tallow} = 0{,}537 \; M_a$,

$$\varkappa_{tallow} - \varkappa_{Cu} = -0{,}006 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}.$$

7. Observations made with the purpose to determine the difference $\varkappa - \varkappa'$ concerning the attraction of the Sun

For basis there are serving equation (12) and the considerations preceding it. We shall use the simple gravity variometer, for what we put

$$\Sigma m_a l_a \varkappa_a - \Sigma m_b l_b \varkappa_b = M_a l_a (\varkappa_a - \varkappa_b),$$

and

$$\vartheta = \vartheta_0 - \frac{1}{\tau} \left(f_0 \frac{M}{D^2} \right) M_a l_a (\varkappa_a - \varkappa_b) \sin \zeta \sin A,$$

for the attraction of Sun

$$f_0 \frac{M}{D^2} = 0{,}586,$$

and for the single gravity variometer

$$M_a = 25{,}4 \text{ g}, \quad l_a = 20 \text{ cm}, \quad \tau = 0{,}5035, \quad \vartheta - \vartheta_0 = \frac{n_0 - n}{2464},$$

accordingly

$$n - n_0 = 1\,457\,000 (\varkappa_a - \varkappa_b) \sin \zeta \sin A,$$

and

$$n' - n = 1\,457\,000 (\varkappa_a - \varkappa_b) \, (\sin \zeta' \sin A' - \sin \zeta \sin A),$$

consequently

$$\text{(a)} \qquad \varkappa_a - \varkappa_b = 0{,}6863 \cdot 10^{-6} \frac{n' - n}{\sin \zeta' \sin A' - \sin \zeta \sin A}.$$

Applying this formula it seems to us that we can determine the difference $\varkappa_a - \varkappa_b$ by a sole series of experiments, where the two heterogeneous bodies are separately suspended on the two ends of the torsion balance set into the meridian. But we can expect that the daily oscillation of the balance loaded in this way, deriving eventually from the different gravitation of these different bodies, will be accompanied by such other osciallations with the same period, which are originating in perturbing influences which were not perfectly eliminated.

In order to eliminate, if possible, the influence of these latter ones on the result, we chose the following way for the observations.

In a *first series of observations* there was a platinum cylinder suspended on end *a* of the beam and the scale positions of the beam, as well as the temperatures were read hourly, during two weeks. The values of these readings were then collected and after reduction to the same temperature, the hourly mean values n were computed.

After this, in *a second series of observations* there was suspended on the end *a* a magnalium cylinder and we followed the same way of observation and computation as with the first series of observations. Denoting now the difference $n' - n$ observed during two sections of the day in the first series by

$$(n'-n)_I$$

and by

$$(n'-n)_{II}$$

observed for the same length of two sections of the day in the second series, we compute on the basis of formula (a)

$$\text{(b)} \qquad \varkappa_{\text{magnalium}} - \varkappa_{Pt} = 0{,}6863 \cdot 10^{-6} \frac{(n'-n)_I - (n'-n)_I}{\sin\zeta' \sin A' - \sin\zeta \sin A}.$$

Though $\sin\zeta' \sin A' - \sin\zeta \sin A$ is changing in the time between the first and second experimental series, nevertheless it is certainly satisfactory to introduce here in the calculation the mean value of this quantity.

First series of observations

On the end a of the single gravity variometer was suspended a platinum cylinder of 6,01 cm length and 0,5 cm diameter and the beam was set in the meridian, with its end a pointing to the North. Here was

$$h = 21{,}24 \text{ cm}, \quad M_a = 25{,}421 \text{ g}.$$

The observations took place between June 18 and July 2, 1908, and they were taken hourly, from which the hourly mean values, as well as their departure from the total mean were formed.

Second series of observations

On the end a of the single gravity variometer a magnalium cylinder of 11,91 cm length and 1,01 diameter was suspended and the beam was set in the meridian with its end a pointing to the North. Here was

$$h = 21{,}24 \text{ cm and } M_a = 25{,}362 \text{ g}.$$

The observations were taken between July 21 and August 4, 1908, and arranged so as in the first experimental series.

For the computation for $\varkappa_{magnalium} - \varkappa_{Pt}$ those n and n' values were used, for which $\zeta = 90°$, i. e., the reading was taken at sunrise and sunset.

Approximately we take for the value n at sunrise the mean of two hourly values taken at sunrise at $4^h\,0^m$ and $5^h\,0^m$ a. m. and for n' the mean of two values read at $7^h\,0^m$ and $8^h\,0^m$ p. m., at sunset, i. e., when $A = -120°$ and $A' = +120°$.

We had from the readings in the first series

$$n' - n = -0{,}062,$$

and in the second series

$$n' - n = -0{,}046.$$

Would we base our calculations on a sole series of observations, that is, to apply formula (a), we had

$$\varkappa_{magnalium} - \varkappa_{Pt} = -0{,}018 \cdot 10^{-6}.$$

With exclusion of perturbing influences causing fluctuations at daily periods i. e. using the results of both series of observations and formula (b) we obtain more correctly

$$\varkappa_{magnalium} - \varkappa_{Pt} = +0{,}006 \cdot 10^{-6}.$$

In a more complete evaluation of the observed material, and in a desirable extension of it, unfortunately, we were hindered by shortness of time.

8. On observations to decide whether an absorption of the gravitation by an intermediate body is taking place.

With our former considerations is very closely connected the question whether the attraction exerted by a body A to an other body B be depending on a third body C lying between them, more particularly, whether for the attraction an absorbing capacity be attributable to bodies. For, if this would be the case, so had bodies of different shape and size to be differently attracted by an other one. Surely, this attraction should depend on the orientations of the single parts of the attracted body with respect to the attracting one, the front parts of the attracted body would then modify the attraction of the parts lying in the background. In this comprehension the above described observations concerning the attraction of different bodies can be considered as serving for settling the question, though the possibility of a direct experiment is not precluded.

I don't think, however, on experiments, like those of *Messrs. L. V. Austin and C. B. Thwing*[1], who endeavoured by interposing some cm thick layers of water, lead, and mercury to measure the effect exerted on the position of a torsion balance, what was deflected by the attraction of masses weighing several kilograms.

Experiments of this sort, carried out with utmost care, hardly can lead to more accurate results than those presented by the aforeasaid Gentlemen in their paper of 1897, proving that the influence of the attraction by those interposed layers is less than 1/500 of it. A result like this is much easier obtainable by considering that a balance, though subject to the much greater attraction of the Earth, undergoes no perceptible change in its equilibrium position, when layers of the above mentioned sort would be put beneath one of its scale-pans, with due protection, the accuracy attainable by this latter instrument could be increased even to the millionth of the weight. And still much more accurate results can we attain with the torsion balance.

We have carried out experiments of this sort, namely with the gravitational compensator already in 1902[2].

[Those experiments have of course, the character of trials, and if we still make known them here, we should like to consider them as preliminary tests. The time required to perform them satisfactorily, expecially to build more complete instruments, was not at our disposal.

The instrument was very like that described by Eötvös, therefore we need not discuss here its particularities.]

[1] An experimental research on gravitational permeability, Phys. Rev., 5, 1897, p. 294.
[2] S. treatise IV (58). [*The title is given in the foot-note on p. 126.]

The brass spheres, 50 g each, fixed to both ends of a 50 cm long torsion balance beam enclosed besides a metal tube for protection also by arrangements for compensation (S. Fig. 7. and 8.)

Each of these two arrangements applied on both ends consists of a cylindric metal case of 5 cm diameter encircling the protecting tube. The metal cases bear two oppositely lying cylinder quadrants (compensating masses), made of cast-lead, which are resting on horizontal shafts, by the aid of which the angle of inclination φ of the middle line K K of the quadrants can be altered against the horizon. The dimensions of the quadrants are: inner-radius 2,5 cm, outer radius 12 cm, thickness, i. e. the distance between the two boundary planes

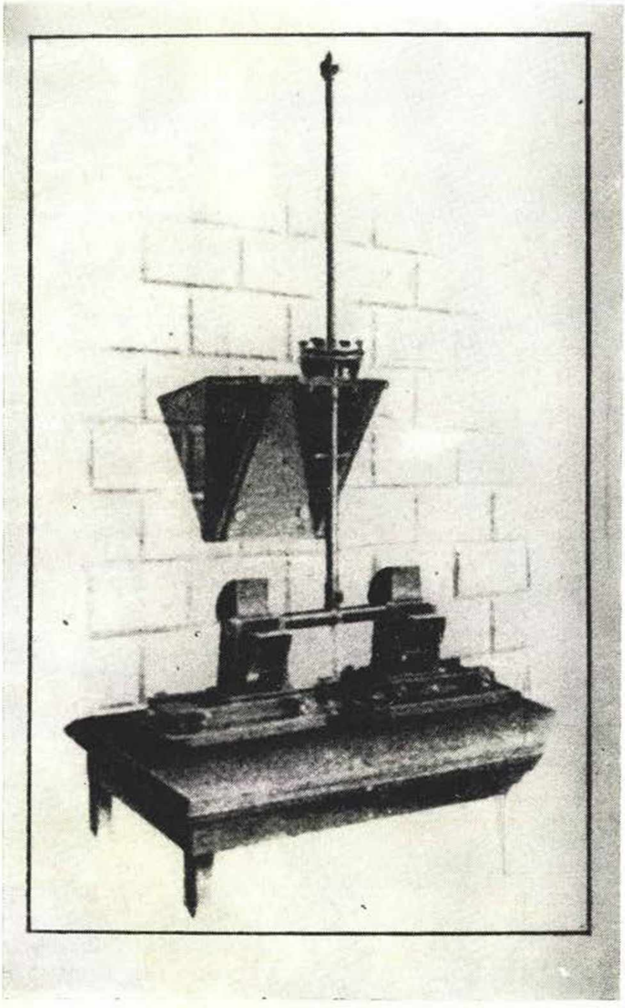

Fig. 7.

9,5 cm. The ends of the balances, more precisely the brass spheres fixed to them are oscillating in the centre of each compensating quadrant pair.

The centre P of the spheres on the balance ends should be in the axis of rotation C of the compensators, when the adjustment of the instrument is perfect. But, as perfection cannot be achieved, we have shown in the figur 8. the points P and C as detached from each other, and we shall accordingly denote the coordinates of P by ξ, ζ, relating to the coordinate-system X, Z passing through C.

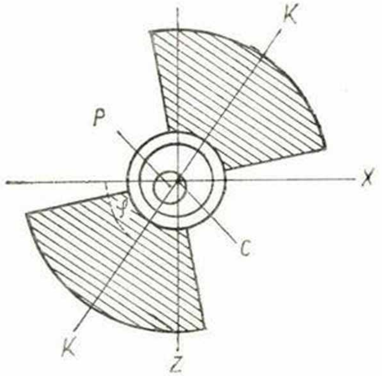

Fig. 8.

In the present case the instrument was steadily used so that both compensators had the same position against the beam ends enclosed by them. The figure shows the cross-section of the compensator and the spheres oscillating in it so as they appear to an observer who standing at one end looks toward the axis of rotation, and the figure shows just so for the observer who standing at the other end is looking also toward the axis of rotation.

In this case we can express the torque exerted on the beam by the attraction of the compensators in the following form:

$$F = A\,\xi + B\cos\varphi + \xi\, C\cos 2\varphi + \xi D\sin 2\varphi .$$

In the course of the investigations in question the compensators were set nto four different positions, distant from each other by right angles, namely

position I	position II
$\varphi = 45°$	$\varphi_2 = 135°$
$F_1 = A\xi_1 + B\cos\frac{\pi}{4} + \zeta D$	$F_2 = A\xi_2 - B\cos\frac{\pi}{4} - \zeta D$
position III	position IV
$\varphi_3 = 225°$	$\varphi_4 = 315°$
$F_3 = A\xi_2 - B\cos\frac{\pi}{4} + \zeta D$	$F_4 = A\xi_4 + B\cos\frac{\pi}{4} - \zeta D.$

If we suppose that the attraction of the Earth's masses acting upon the masses of the torsion balance is affected by the masses of the compensators like absorbing bodies, to the above torque F is adding an other torque Φ, wich is oriented forwards or backwards, according to the position of the compensators.

Thinking namely the Earth divided into two halves by a vertical plane, which passes through the balance beam, so will the action of one half of the Earth pass through the compensator, while that of the other half will not (s.

fig. 9.). Each of these halves produces a horizontal component of the attraction, the value of which as referred to the unit mass is G/π, disregarding a possible absorption, and it is directed to the side where the attracting half-Earth is lying. For the case of absorption the attraction of the half-Earth affected by it is to take for

$$\frac{G}{\pi}(1-\mu)$$

where μ is depending on the absorbing capacity of the intermediate body, further on its shape, size and position.

In this manner, the action of both half-Earth, results in an horizontal component directed to that side where the absorption is smaller. Denoting by m the mass of one sphere on the balance-beam, by l its radius of rotation, we have the torques in the four positions of the balance resulting from the one-side absorption, as follows:

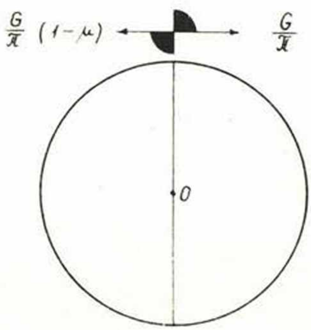

Fig. 9.

position I

$$\Phi_1 = -2ml\frac{G}{\pi}\mu$$

position II

$$\Phi_2 = +2ml\frac{G}{\pi}\mu$$

position III

$$\Phi_3 = -2ml\frac{G}{\gamma}\mu$$

position IV

$$\Phi_4 = +2ml\frac{G}{\pi}\mu.$$

We admit the position of equilibrium as being accomplished partly by the sum of the torques F and Φ, partly by the torques acting against the torsion. We express this latter in the form $\tau\vartheta_0+\tau\vartheta$ where ϑ_0 means the position of the beam, when $\xi=0$ and $\vartheta_0+\vartheta$ represents the total angle of torsion. Writing further

$$\xi = l\vartheta$$

we obtain for the conditions of equilibrium in the four positions I–IV:

$$\tau\vartheta_0+\tau\vartheta_1 = Al\vartheta_1 + B\cos\frac{\pi}{4} + \zeta D - 2ml\frac{G}{\pi}\mu,$$

$$\tau\vartheta_0+\tau\vartheta_2 = Al\vartheta_2 - B\cos\frac{\pi}{4} - \zeta D + 2ml\frac{G}{\pi}\mu,$$

$$\tau\vartheta_0+\tau\vartheta_3 = Al\vartheta_3 - B\cos\frac{\pi}{4} + \zeta D - 2ml\frac{G}{\pi}\mu,$$

$$\tau\vartheta_0+\tau\vartheta_4 = Al\vartheta_4 + B\cos\frac{\pi}{4} - \zeta D + 2ml\frac{G}{\pi}\mu.$$

Subtracting the sum of the second and fourth equations from the sum of the first and third one, we obtain

$$(\tau - Al)(\vartheta_1 + \vartheta_3 - \vartheta_2 - \vartheta_4) = 4\zeta D - 8ml\frac{G}{\pi}\mu.$$

Observing by the aid of mirror and scale, denoting the scale reading by n and the distance between scale and mirror by L, expressed in scale divisions we have

$$n_1 + n_3 - n_2 - n_4 = \frac{8LD\zeta}{\tau - Al} - \frac{16Lml}{\tau - Al}\frac{G}{\pi}\mu.$$

For the evaluation of the observations, according to the apparatus used, we took the sufficiently approximating values:
$L = 1315$ scale division, $m = 30$ g, $1 = 25$ cm, $G = 982$ cgs, $\tau - Al = 0{,}103$ cgs.
The last quantity was determined by deflection experiments of the compensator-beam. Using those values we have

$$n_1 + n_3 - n_2 - n_4 = \frac{8LD}{\tau - Al}\zeta - 47890 \cdot 10^6 \mu$$

Although we could the factor of ζ quite easily determine from the dimensions of the apparatus, we determined it rather from observations so that we observed two values of the quantity $n_1 + n_3 - n_2 - n_4$ due to different values of ζ. We have so:

$$(n_1' + n_3' - n_2' - n_4') - (n_1 + n_3 - n_2 - n_4) = \frac{8LD}{\tau - Al}(\zeta' - \zeta).$$

Such change in the value of ζ can be easily operated and measured by sinking or lifting the compensator, what is resting on plate screws. From these kinds of experiments we obtained, if ζ is expressed in cm,

$$\frac{8LD}{\tau - Al} = 608,$$

and so

$$n_1 + n_3 - n_2 - n_4 = 608\zeta - 47\,890 \cdot 10^6 \mu,$$

from which

$$\mu = \frac{n_2 + n_4 - n_1 - n_3}{47890 \cdot 10^6} + \frac{608\zeta}{47890 \cdot 10^6}.$$

The numerical values of this formula are proving the high accuracy obtainable with the determination of μ, but on the other hand they point to difficulties to be surmounted. They are consisting not only in the protection from perturbing effects, which are doubly effective in the case of a high accuracy like this, but expecially also in that the influence of the term multiplied by ζ should be possibly avoided or confidently determined.

With our experiments the mounting of the apparatus in a cellar of uniform temperature afforded the necessary protection and by the aid of cathetometers we succeeded in adjusting the compensators so that ζ differed from zero by less then 1/500 cm. We carried out three series of observations under these circumstances, already some years ago; the readings are collected in the following short table.

Readings

Position		17 April	20 April	23 April
I.	n_1	246,2	264,0	266,2
II.	n_2	247,4	264,6	268,0
III.	n_3	246,3	263,8	267,1
IV.	n_4	246,0	262,5	266,6
I.	n_1	246,0	263,9	265,9

Omitting the term proportional to ζ we computed μ from the observations

17th April $$\mu = -\frac{1}{47890 \cdot 10^6} \cdot 1{,}0,$$

20th ,, $$\mu = -\frac{1}{47890 \cdot 10^6} \cdot 0{,}6,$$

23th ,, $$\mu = +\frac{1}{47890 \cdot 10^6} \cdot 1{,}4.$$

Considering that an inaccuracy of 1/50 mm = 1/500 in the adjustment of ζ should cause an error slightly over one unit, values of μ differing from zero by about one unit can be ascribed to this imperfection. As far as it is permissible, on the basis of a few experiments we may assert, that μ, *i. e., the diminution of the Eart's attraction effected by the interposed compensating quadrants was less, than a fifty thousand millionth part of it.*

Experiments, like these, should be many times repeated to become conclusive, moreover their accuracy should be increased, what is attainable by according dimensions of the apparatus in order to get rid of the influence of the factor of ζ. [The compensating masses should be placed at a greater distance from the beam. Unfortunately we found no time for that as yet.]

Examining the meaning of the results concerning μ, we think ourselves absolved from the trouble of a more accurate calculation of this quantity having the supposition that the absorption is proportional to the radiated section, and so the question is to fix a limiting value. But we have the right to state that those sections of the straight lines, which coming from points of the half-Earth pass through the compensator mass before reaching the attracted sphere, have an average length over 5 cm. We may say, therefore, that the attraction of the Earth passing through a 5 cm thick layer of lead is not affected by an absorption which is over the fifty thousand millionth of its value.

For a 1 m thick layer of lead their limit would be 1/2500 million and for the absorption through the whole length of the Earth's diameter about 1/400. But supposing that the absorption be proportional to the mass passed through, according to our experiments *the absorption of the whole Earth along its diameter should be less than about 1/800.*

Observations on ebb and flow and its generating forces show however, that this limit for an eventual absorption of the gravitation caused by the whole Earth along one of its diameters is even smaller.

We can convince ourselves of this in the simplest way by considering the vertical forces due to the attraction of Sun and Moon at two points of the Earth, where $\zeta=0$ and $\zeta=\pi$.

In place of the value

$$-Z = 2f\frac{M}{D^2}\frac{a}{D}$$

of the force without absorption, we have now

$$-Z = 2f\frac{M}{D^2}\frac{a}{D}+\mu f\frac{M}{D^2},$$

or

$$-Z = 2f\frac{M}{D^2}\frac{a}{D}\left(1+\mu\frac{D}{2a}\right),$$

where μ means the part of the attraction absorbed by the Earth's mass along the length of its diameter.

In this way we have to write for the tide caused by the sun:

$$-Z = 2f\frac{M}{D^2}\frac{a}{D}(1+11800\,\mu)$$

and for the lunar tide:

$$-Z = 2f\frac{M'}{D'^2}\frac{a}{D'}(1+30{,}14\mu).$$

Would μ touch the limit 1/1600 found by our torsion balance observations, we had for the Sun

$$-Z = 2f\frac{M}{D^2}\frac{a}{D}(1+7{,}4),$$

and for the Moon

$$-Z = 2f\frac{M'}{D'^2}\frac{a}{D}(1+0{,}002).$$

In this case the solar tide should be magnified about eight-times, while the lunar tide should be hardly noticeable.

Even the roughest observations of the tidal phenomena contradict a supposition like that. But one might think on the observation of the tidal forces in order to determine the value μ or at least to fix more exactly the limit, which this value cannot surpass.

Namely the proportion of the tidal force of the Sun to that of the Moon according to our previous considerations is

$$\frac{Z}{Z'} = \frac{Z_0}{Z'_0}(1+11770\mu)$$

where Z_0/Z'_0 represent the theoretical value of this proportion when $\mu=0$.

Many years of observations of tidal phenomena do us entitle to say that at least the magnitude of the solar tides does not surpass that of the lunar tides, by that is given us the prove that the attraction of sun along an Earth's diameter has no more loss than a tenthousandth part of it. We obtain this result by the aid of our last formula if we put there

$$\frac{Z}{Z'} = 1 \quad \text{and} \quad \frac{Z_0}{Z'_0} = \frac{1}{2{,}2}.$$

More accurate results of this sort are well to be expected from observations of the tidal force.

[We have already such observations at our disposal. We have in our hands the fair work of *O. Hecker* ,,Beobachtungen an Horizontal-Pendeln usw." (Veröffentlichung des k. preuss. Geodetischen Institutes. Neue Folge No. 32, 1907), which is reach in observations and in very interesting conclusions derived from those.

The observations were taken on two horizontal-pendulums, and the last results are collected on pp. 31 and 32 of the treatise in the following formulas

Pendulum I	computed attraction of the Moon	$0''00922 \cos(2t-305°{,}5)$
	observed lunar wave	$0''00622 \cos(2t-285°{,}4)$
Pendulum II	computed attraction of the Moon	$0''00900 \cos(2t-48°{,}7)$
	observed lunar wave	$0''00543 \cos(2t-63°{,}2)$
Pendulum I	computed attraction of the Sun	$0''00399 \cos(2t-305°{,}5)$
	observed solar wave	$0''00244 \cos(2t-273°{,}6)$
Pendulum II	computed attraction of the Sun	$0''00389 \cos(2t-48°{,}7)$
	observed solar wave	$0''00585 \cos(2t-48°{,}3)$

We don't want to deal here more closely with the satisfactory agreement of the computed and observed phases, we are rather interested in the ratio of the amplitudes for Sun and Moon. Denoting the amplitudes by A_s and A_m we obtain

$$\text{Pendulum I} \begin{cases} \dfrac{A_s}{A_m} \text{ computed} = 0{,}432 \\[2ex] \dfrac{A_s}{A_m} \text{ observed} = 0{,}392 \end{cases}$$

$$\text{Pendulum II} \begin{cases} \dfrac{A_s}{A_m} \text{ computed} = 0{,}432 \\[2ex] \dfrac{A_s}{A_m} \text{ observed} = 1{,}077 \end{cases}$$

The good accord between the computed and observed values for pendulum I makes us to take the limit of μ lower than before. The pity is that pendulum II hinders us to do so with full conscience, though Prof. *Hecker* emphasizes several times in his paper that pendulum II suffered by many disturbances and it was even less accurate in its dates.

But using the dates of this pendulum II alone, we arrive to the result stated before that the loss of the Sun's attraction suffered along an Earth's diameter is less than one tenthousandth part. This result is ten times more accurate than that received by the gravitational compensator, but we should point again to the fact, that the observations made by that instrument were only of preliminary character and they promise us a much higher degree of accuracy, if carried out more carefully.]

9. Experiments with radioactive substances

Investigations on radioactive substances were carried out in two directions, at first, concerning the proportion of their masses to the attraction of Earth exerted on them, secondly about the question if these substances do exert an absorption on the attraction or even a specific attraction or repulsion.

a) *Observations concerning the proportion of mass and attraction*

We carried out experiments with a preparative of radium which derived from the *Curie* Laboratory and was put at our disposal. The total weight of this specimen closed in a small tube was 0,200 g, containing 0,100 g pure $RaBr_2$, with 1 500 000 – fold activity of that of metallic uranium. This specimen was available to us only for a short while at the beginning of this work, that is why we could perform our observations but by the first procedure.

The small tube containing the radium was carefully fixed in the midle of a closed brass tube, and suspended on the end of beam, the observations were then carried out in the same manner as with the magnalium and wood.

We had to consider at this time, that the suspended mass M_a was not homogeneous, as only a 1/250 part of it was consisting of $RaBr_2$.

The direct determination concerned an average coefficient $\varkappa_a$ of the attraction of the total mass M_a, where the coefficient $\varkappa_{Ra}$ of the attraction of the radium bromide added only a contribution with 1/250 of M_a. In this way, if we attribute the value $\varkappa_a - \varkappa_{Pt}$ to the effect of the radium specimen alone, we have to put

$$\varkappa_{Ra} - \varkappa_{Pt} = 250(\varkappa_a - \varkappa_{Pt}).$$

First series of experiments

On end a of beam 1 of the double gravity variometer was suspended the closed brass tube of 9,62 cm length and 0,90 cm diameter, containing the radium specimen. There was

$$M_a = 25,396 \text{ g}, \quad h = 21,55 \text{ cm}.$$

From 15 observed values we obtained the mean value

$$m = +6,566 \pm 0,028,$$

and from 43 values

$$v = -1,736 \pm 0,008.$$

Second series of experiments

For this purpose are serving the experiments made with the same apparatus for the determination of $\varkappa_{wood} - \varkappa_{Pt}$ (s. p. 136.), where the end a was loaded with a platinum cylinder.

The results of that series of experiments were

$$m' = +6,595 \pm 0,016, \qquad v' = -1,754 + 0,011.$$

For the computation of $\varkappa_a - \varkappa_{Pt}$ *after formula (17)* we take the mean values

$$M_a = 25,396\ g \quad \text{and} \quad \frac{\tau}{4LM_a l_a G \sin \varepsilon} = 0,1178 \cdot 10^{-6}.$$

In the N-position we had

$$n = 191,5 \text{ and } n' = 191,3,$$

whence $n - n' = +0,2$, consequently $m(\Delta\alpha - \Delta\alpha')$ is to be neglected. Here was

$$\frac{h - h'}{h} = +0,017 \quad \text{and} \quad v\frac{h - h'}{h} = -0,029$$

so that we obtain

$$\varkappa_a - \varkappa_{Pt} = 0,005 \cdot 10^{-6} \pm 0,002 \cdot 10^{-6}.$$

But computing after formula (20) so as in the case of magnalium and wood, we obtain

$$\varkappa_a - \varkappa_{Pt} = -0,001 \cdot 10^{-6} \pm 0,002 \cdot 10^{-6}$$

and at last after our preceding determination:

$$\varkappa_{Ra} - \varkappa_{Pt} = -0,25 \cdot 10^{-6} \pm 0,50 \cdot 10^{-6}.$$

b) *Observations concerning a specific mechanical effect of specimens of radium*

In addition we want to report here on experiments carried out by us years ago (1904) with the purpose to discover possible mechanical effects of radium specimens on the torsion balance beam.

The researches have lead us in the domain travelled by the treatise of *Mr. Robert Geigel:* „On absorption of gravitational energy by radioactive substances[1]. After the remarks made shortly by *Mr. W. Kaufmann* on the work of *Mr. R. Geigel*[2], the publication of any further remarks seemed us superfluous at that time; we think, however, that our experiments are worth mentioning in the frame of the present treatise.

[1] R. Geigel, Ann. d. Phys. 10, 1903, p. 429.
[2] R. Geigel, Ann. d. Phys. 10, 1903, p. 894.

With the experiments we used 10 mg of a radium specimen; its activity was about million – fold that of uranium metal.

The radium specimen was closed in a small tube, of 4,5 cm length, 0,5 cm outer diameter, and 0,66 g weight.

Experiment no. 1

After reading the equilibrium position of the gravity variometer, the small tube containing the radium specimen was placed into the housing of the torsion balance and set there up on a light frame of wire so that the tube was lying at the end *b* of the beam, parallel to the platinum cylinder inserted to the beam, and in the same height.

The radium tube was put once to one side, then to the other side of the oscillating platinum cylinder, and the position of equilibrium was read in each case. The distance *H* between platinum cylinder and tube could be figured out from the scale reading where the oscillating beam was repulsed by the tube.

At a distance of $H = 50$ sc. division $= 4{,}05$ mm the tube pushed away the platinum cylinder by 1,8 scale divisions, according to a force *P*, the value of which is easily calculated from

$$\frac{lP}{\tau} = \frac{n' - n}{2L},$$

i. e.,

$$\frac{20P}{0{,}5} = \frac{1{,}8}{2464}.$$

Repulsing force $P = 0{,}000\,018_2$.

With repetition of the experiment we obtained:

repulsing force $P = 0{,}000\,018_8$.

The pulverized radium specimen was lying during the experiments dispersed on the base of the tube, about 2 mm deeper than the axis of the platinum cylinder.

Experiment no. 2

All was arranged so as with experiment no. 1, with the difference, that the radium tube was lifted above the platinum cylinder by about 3 mm. At $H = 41$ sc. div. $= 3{,}2$ mm the tube attracts the platinum cylinder by 2,5 scale divisions, with the

force of attraction $P = 0{,}000\,025_3$.

Experiment no. 3

Instead of the tube containing the radium specimen we placed into the housing of the instrument an empty glass tube having the same form and size as that one with experiment no. 1. Traces of a repulsion were showing themselves not exceeding the value $P = 0{,}000\,001$, and it seemed to be decreasing in time.

We repeated these experiments several times and proved by them the result first found. The simplest, but rather light-minded interpretation would be the supposition of a specific attractive force of the radioactive substance, according to the attraction found by experiment no. 2, moreover an absorption of the Earth's attraction by the substance, what could cause the repulsion found by experiment no. 1. Experiment no. 3., where such substance was not present, looks rather to corroborate interpretations like this, but we have to bear in mind that the glass tube here used replaced the radium tube with respect only to its mass-attraction, and not to other effects, especially its heat effect.

In order to suppress every doubt about this very important question we examined the effect of a glass tube, which was like the radium tube not only with respect to form and mass, but also to the heat it was steadily radiating.

Into the glass tube was sealed a short piece of platinum wire, of 0,04 mm diameter and 1,41 ohm electrical resistance, and it was heated by a current of according intensity.

At first, careful comparison was made of the quantities of heat radiated in the same time by the tube heated by current and the radium tube. A comparison like this performed by thermoelectric method rendered us the result, that the radium specimen radiated 0,169 g calorie in an hour according to 0,0118 ampere intensity. The tube with the platinum wire was then placed into the inside of the housing, while the current was conducted to it through carefully packed holes.

Experiment no. 4

At $H=32$ sc. division = 2,4 mm appeared a repulsion of 1,8 sc. div. so that there was a

repulsing force $P=0{,}000\,018$,

equal to the repulsing force found by experiment no. 1.

Experiment no. 5

The torsion balance beam was lowered by about 3 mm. Here appeared an attraction, as with experiment no. 2,

attracting force $P=0{,}000\,024_8$.

With the repetition of experiments no. 4. and no. 5. applying a higher intensity of current of $i=0{,}0250$ amp, when platinum cylinder and heating wire were in the same height (experiment no. 4.), we found a repulsion by 9 sc. division, and an attraction up to touch when the wire was in a higher position. (experiment no. 5.)

We believe to have satisfactorily proved, that attraction and repulsion appearing in experiments no. 1. and no. 2. were not effected by a specific mechanical action of the radium specimen, not even by absorption of the Earth's attraction, but they were solely caused by thermic effects, exerting mainly a mechanical action as a result of warming up the air.

The radium specimen of about 10 mg weight exerted, thus, on the torsion balance beam at a distance of about 4 mm certainly no specific attractive or repulsive force, the value of which could touch one unit of 10^{-6} order. There was also no recognizable trace of an absorbing effect on the Earth's attraction

10. *Grouping of the results*

1. Observations after Eötvös' method

Considering in *Newton's* formula

$$P = f\frac{mm'}{r^2}$$

the quantity f as depending on the nature of the attracted body and putting

$$f = f_0(1+\varkappa),$$

the results of our observations can be represented by values of the quantity $\varkappa$ computed from those. We show there the values $\varkappa - \varkappa_{Pt}$ found, together with the mean errors of the determinations, where

$$\varkappa_{Pt} = 0, \text{ if } f_{Pt} = f_0.$$

Magnalium	$+0{,}004 \cdot 10^{-6} \pm 0{,}001 \cdot 10^{-6}$
Snake-wood	$-0{,}001 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}$
Copper	$+0{,}004 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}$
Water	$-0{,}006 \cdot 10^{-6} \pm 0{,}003 \cdot 10^{-6}$
Crystalline cupric sulfate	$-0{,}001 \cdot 10^{-6} \pm 0{,}003 \cdot 10^{-6}$
Solution of cupric sulfate	$-0{,}003 \cdot 10^{-6} \pm 0{,}003 \cdot 10^{-6}$
Asbestos	$+0{,}001 \cdot 10^{-6} \pm 0{,}003 \cdot 10^{-6}$
Tallow	$-0{,}002 \cdot 10^{-6} \pm 0{,}003 \cdot 10^{-6}$

The mean values found for $\varkappa - \varkappa_{Pt}$ are smaller in four cases, slightly greater in three cases than their average errors, and equal in one case.

The probability of a value different from zero for the quantity $\varkappa$ even in these cases is vanishingly little, as a review of the according observational data shows quite long sequences with uniform departure from the average, the influence of which on the average could only be annulled by much longer series of observations.

Among the bodies subjected to the observations are to be found those with very different specific gravities, molecular gravities, molecular volumes, and also with different states and structures.

We believe to have the right to state that $\varkappa$ relating to the Earth's attraction does not reach the value of $0{,}005 \cdot 10^{-6}$ for any of those bodies.

In connection with the question if the attraction would change following a chemical reaction or dissolution taken place in the attracted bodies, we obtained even smaller limits.

For *Landolt's* silver sulfate – ferrous sulfate reaction we found namely

$$\varkappa_{before} - \varkappa_{after} = 0{,}000 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6},$$

and for the solution of cupric sulfate in water after the proportion given by *Heydweiller*

$$\varkappa_{before} - \varkappa_{after} = 0{,}002 \cdot 10^{-6} \pm 0{,}002 \cdot 10^{-6}$$

2. Observations in the meridian

For the Sun's attraction we obtained

$$\varkappa_{magnalium} - \varkappa_{platinum} = +0{,}006 \cdot 10^{-6}$$

3. *Observations concerning an absorption-like influencing or attraction through intermediate bodies.*

By experiments with the gravitational compensator it was shown, that a 5 cm thick lead layer causes no absorption touching a value of $0{,}00002 \cdot 10^{-6}$. Accordingly:

the absorption through a 1 m thick lead plate is less than $0{,}0004 \cdot 10^{-6}$ times the Earth's attraction, absorption through the Earth along one of its diameters is $< 1/800$ part of the Earth's attraction.

4. *Observations on radioactive substances*

From experiments with a radium specimen of 0,20 g weight we obtained

$$\varkappa_{RaBr_2} - \varkappa_{Pt} = 0{,}25 \cdot 10^{-6} \pm 0{,}50 \cdot 10^{-6}.$$

From experiments with other specimens we noticed,

a) that the specimen exerts on a platinum cylinder of 30 g weight laying at 4 mm distance from it no specific attraction or repulsion what would reach one unit of the order 10^{-6};

b) that this specimen causes no noticeable absorption of the Earth's attraction.

We can express in a few words the final results of our work.

We have carried out long series of observations, the accuracy of which is surpassing all previous ones, but we were not able in any case to discover noticeable departures from the law of the proportionality of inertia and gravity.[1]

(Manuscript received on February 27, 1922)

[1] The measurements carried out by Dr. J. Renner (Math. u. Naturwiss. Anz. d. Ung. Akademie d. Wiss, vol. LIII, 1935, pp. 542 – 570, in Hungarian, with German résumé) in 1935, in the Eötvös-Institute after the third procedure fully affirmed the conviction of the authors expressed in the introduction of this treatise, that the accuracy of measurements of this sort can be still considerably increased. He succeeded in that by careful choice of the torsion balance and excellent torsion wires, as well as by perfect elimination of the disturbing effects of variations in temperature. He was able to show that in the case of platinum, brass, glass drop, smashed glass drops, paraffine, ammonium fluoride, manganese copper alloy, and bismuth the difference of the gravitational constants does not surpass in any case the average error $0{,}52 \cdot 10^{-9}$ of the measurements, i. e., the value 1 : 2 000 000 000, in one case (brass – bismuth) it remained even under 1 : 5 000 000 000.

As we can hardly expect any further increase of the measuring accuracy and the discovery of a specific gravitation, this fundamental problem of physics appears to be definitely solved. – And still remains here the question to be answered: how do things stand with live matter? Newton had already investigated corn and wood; tallow as an organic stuff was drawn here in the measurements, but we are thinking on real living organisms, actively developing by cell-division, which were never chosen for objects of similar investigations. The difficulties would somewhat increase, probably the attainable accuracy somewhat decrease, and a negative result may be foreseen. Anyhow, inertia and gravitation are both inseparable universal properties of matter; besides, they let themselves compare with $10^{-8} - 10^{-9}$ accuracy. Therefore it would be doubtless of certain natural philosophical significance to make sure, that even in this regard there is no difference between living and lifeless matter.