Title: AJAE Appendix for A General Equilibrium Theory of Contracts in Community Supported Agriculture

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Appendix

Proposition 1.

Given complete markets with no arbitrage, the price relationship between yield and weight contracts is given by:

$$q_m^w c_m^w - q_{jm}^y c_{jm}^y = \Delta c_{jm} \cdot E_Q\left[p_m^*\right] - Cov_Q\left[y_{jm}, p_m^*\right].$$

Proof of Proposition 1.

The proposition follows from adding and subtracting identical terms and simplifying, by factoring out the yield-price covariance, and by substituting the forward price of the crop for its risk-adjusted expectation.

$$\text{(A1)}\qquad \begin{aligned} q_{jm}^y c_{jm}^y &= E_Q\left[y_{jm} p_m^*\right] \\ &= E_Q\left[y_{jm} p_m^*\right] + E_Q\left[y_{jm}\right] E_Q\left[p_m^*\right] - E_Q\left[y_{jm}\right] E_Q\left[p_m^*\right] + q_m^w c_{jm}^w - q_m^w c_{jm}^w \\ &= Cov_Q\left[y_{jm}, p_m^*\right] - \left(c_{jm}^w - E_Q\left[y_{jm}\right]\right) q_m^w + q_m^w c_{jm}^w \\ &= Cov_Q\left[y_{jm}, p_m^*\right] - q_m^w \Delta c_{jm} + q_m^w c_{jm}^w \qquad \square \end{aligned}$$

Proposition 2.

Assuming risk-averse farmers and system parameters such that contract sales are non-negative $\left(b^{w*} \geq 0, b^{y*} \geq 0\right)$, then:

Claim 2.1. Positive price-revenue covariance implies farmers may specialize or diversify contract offerings. Formally, $\text{Cov}\left[y_m p_m^*, p_m^*\right] > 0 \Rightarrow b^{y*} \in [0,1), b^{w*} \geq 0$.

Claim 2.2. Negative price-revenue covariance and positive sales of weight contracts can only co-exist if farmers simultaneously oversell yield contracts (above 100%). Formally, $b^{w*} > 0 \cap \text{Cov}\left[y_m p_m^*, p_m^*\right] < 0 \Rightarrow b^{y*} > 1$.

Claim 2.3. Except as in Claim 2.2, non-positive price-revenue covariance implies zero sales of weight contracts and partial or zero sales of yield contracts. Formally, $\text{Cov}\left[y_m p_m^*, p_m^*\right] \leq 0 \cap b^{y*} < 1 \Rightarrow b^{w*} = 0$.

Claim 2.4. Positive price-revenue covariance implies optimal choices for b^{y*} and b^{w*} will diverge in response to changes in the price-revenue covariance. Formally, $\partial b^{w*} / \partial \text{Cov}\left[y_m p_m^*, p_m^*\right] > 0$ and $\partial b^{y*} / \partial \text{Cov}\left[y_m p_m^*, p_m^*\right] \leq 0$. Marginal effects of price changes are ambiguous without further assumptions.

Proof of Proposition 2.

Proof of Claims 2.1, 2.2 and 2.3 follows directly from examination of the truth table in table A1, which shows a "Y" if such a combination of $\left(b^{y*}, b^{w*}\right)$ *may* satisfy the first order conditions (Equations 8-12), or an "N" otherwise. For example, we know that $b^{y*} \neq 1$ by contradiction. $b^{y*} = 1$ implies Equation 12 is non-negative, which implies $b^{w*} = 0$. Then $b^{w*} = 0$ and $b^{y*} = 1$ together imply Equation 11 is non-negative, which in turn implies that $b^{y*} = 0$, a contradiction.

Claim 2.2 is an extreme case, essentially involving contract speculation, and the level of negative covariance making it possible may not exist given other individual- and crop-specific parameters of the problem such as preferences and price-yield dependence. Claim 2.4 follows from evaluation of comparative statics with respect to the price-revenue covariance, treated as a parameter. $\text{Cov}\left[y_m p_m^*, p_m^*\right] > 0$ leads to negative off-diagonal terms in the Hessian matrix arising from the farmer's expected utility problem. Solving for comparative statics via the Implicit Function Theorem yields the result. □

Proposition 3.

Assuming diversification means division of the farm into IID crops as described above, we compare the choices of diversified farmers against those of mono-crop farmers, all else held equal:

Claim 3.1. Diversification will cause some farmers to move from an interior solution (positive sales) in both contracts to a corner solution (zero sales) in at least one.

Claim 3.2. Effects of diversification on farmers who maintain an interior solution cannot be signed without further assumptions; this includes the extreme case of speculation under large negative covariance.

Claim 3.3. Diversification will cause farmers with $b^{w*} = 0$ to double the amount held back from yield contracts, $\left(1 - b^{y*}\right)$, or else move from $b^{y*} \leq 0.5$ to $b^{y*} = 0$. That is, these farmers will have b^{y*} unambiguously decrease.

Claim 3.4. Diversification will cause farmers with $b^{y*} = 0$ to decrease the amount of weight contracts, b^{w*}, to a new interior solution, or else move to $b^{w*} = 0$ from $b^{w*} \leq 0.25 \cdot \text{Cov}\left[yp^*\right] / \text{Var}\left[p^*\right]$.

Proof of Proposition 3.

Claims 3.1, 3.3 and 3.4 follow directly from examination of the first order conditions resulting from combining Equations 8-10 and 14-15. The resulting truth table of possible parameter combinations satisfying the first order conditions is identical to that in the proof of Proposition 2. The defining difference is the scaling of marginal effects on variance by a factor of 0.5. Since marginal effects on the mean are unchanged in our example, the effect of scaling marginal effects on variance is to require them to double (via changing choices, b^{y*} and b^{w*}) in order to satisfy the necessary first order conditions. If we consider the space of all parameters where mono-crop farmers satisfy the necessary conditions for an interior choice, then the modified first-order conditions are satisfied by strictly fewer of the farmers if they are diversified. Claim 3.2 follows from examining the comparative statics effects as in the proof of Proposition 2. Given our assumptions, the discrete effect of moving from mono-crop to diversified can be decomposed as the integral of marginal effects over a scale factor moving from one to 0.5, where the scale factor applies to all variances and covariances in farmer's problem. Namely, letting γ be the scale factor, Equations 11 and 12 become:

(A2) $$\frac{\partial\sigma^2}{\partial b^y} = (1-\gamma)\left(-2\left(1-b^y\right)\mathrm{Var}\left[yp^*\right] + b^w\mathrm{Cov}\left[yp^*, p^*\right]\right), \text{ and}$$

(A3) $$\frac{\partial\sigma^2}{\partial b^w} = (1-\gamma)\left(2b^w\mathrm{Var}\left[p^*\right] - \left(1-b^y\right)\mathrm{Cov}\left[yp^*, p^*\right]\right),$$

where $\gamma = 0$ indicates mono-crop production and $\gamma = 0.5$ equates to our diversification example. With this setup, the discrete changes in a farmer's optimal choices from one interior solution to another can be expressed as the integral over marginal changes with respect to the scale parameter, γ :

(A4) $$\Delta b^{y*} = \int_0^{0.5} \frac{\partial b^{y*}}{\partial \gamma} \mathrm{d}\gamma \text{ , and } \Delta b^{w*} = \int_0^{0.5} \frac{\partial b^{w*}}{\partial \gamma} \mathrm{d}\gamma \text{ .}$$

Here, we recognize that for interior solutions, $\partial^2 EU / \partial b^y \partial \gamma < 0$ and $\partial^2 EU / \partial b^w \partial \gamma < 0$. If the price-revenue covariance is positive then the inverse Hessian matrix has alternating signs, so applying the Implicit Function Theorem leads to indeterminate signs of the comparative statics results without further assumptions. The comparative statics also have indeterminate signs in the case of interior solutions with extreme negative covariance, because the sign of the off-diagonal term in the inverse Hessian matrix is not known ex ante. □

Table A1. Possible Contracting Choices as a Function of Covariance.

	$\text{Cov}\left[y_m p_m^*, p_m^*\right] > 0$		$\text{Cov}\left[y_m p_m^*, p_m^*\right] = 0$		$\text{Cov}\left[y_m p_m^*, p_m^*\right] < 0$	
	$b^{w*} = 0$	$b^{w*} > 0$	$b^{w*} = 0$	$b^{w*} > 0$	$b^{w*} = 0$	$b^{w*} > 0$
$b^{y*} = 0$	Y	Y	Y	N	Y	N
$b^{y*} \in (0,1)$	Y	Y	Y	N	Y	N
$b^{y*} = 1$	N	N	N	N	N	N
$b^{y*} \geq 1$	N	N	N	N	N	Y