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REFLECTION AND TRANSMISSION IN THE IONOSPHERE CONSIDERING COLLISIONS IN A FIRST APPROXIMATION

A. Yesil and M. Aydoğdu

Department of Physics
Faculty of Arts and Sciences
Firat University
Elazığ 23119, Turkey

A. G. Elias †

CONICET
Consejo Nacional de Investigaciones Cientificas y Tecnicas
Argentina

Abstract—Reflection and transmission coefficients (R and T) of high frequency waves propagating in the ionosphere are studied taking collisions into account. This was done approximating the expression $(1 + Z^2)^{-1}$ in the refractive index using binomial expansion and neglecting terms of order higher than Z^2, where Z is the ratio between the electron collision frequency and the wave frequency. R and T height profiles were assessed using the International Reference Ionosphere, IRI, to estimate the ionosphere plasma parameters. Although no significant differences are found between the estimation with and without collisions, the method employed to include collisions may be useful for other purposes where collisions should be taken into account.

1. INTRODUCTION

Reflection and transmission are important phenomena in the study of radio wave propagation in the ionosphere [1, 2], especially in the case of vertical sounding. The reflection and transmission coefficients, R and T respectively, are assessed from the refractive index, which is obtained from Maxwell's equation. Assuming a plane wave solution,

† Also with Dpto. de Fisica, Facultad de Ciencias Exactas y Tecnologia, Universidad Nacional de Tucuman, Argentina

where the velocity and the fields vary as $\exp[i(\mathbf{k}.\mathbf{r} - \omega t)]$, a general wave equation for electromagnetic waves can be written as:

$$n^2\boldsymbol{E} - \boldsymbol{n}(\boldsymbol{n}.\boldsymbol{E}) - \left[I + \frac{i}{\varepsilon_0 \omega}\sigma\right].\boldsymbol{E} = 0 \tag{1}$$

where $\mathbf{n}(= \mathbf{k}c/\omega)$ is the refractive index, $\mathbf{k}$ the wave vector, $\mathbf{E}$ the electric field, $\mathbf{I}$ the unit matrix, σ the conductivity tensor and ε_0 the free space electric permittivity coefficient.

Equation (1) is the basic dispersion relation from where $\mathbf{n}$ can be obtained in terms of plasma parameters.

At high frequencies, the ratio between the electron collision frequency (ν) and the wave frequency (ω), that is $Z(= \nu/\omega)$, is small and can often be neglected. However, small differences become relevant when higher accuracy is needed. In the present work, a first approximation is made to include collisions in the calculation of n. Then, the height profile of the reflection and transmission coefficients for a high frequency vertical traveling wave is assessed and then compared to the same profile but without considering collisions.

2. REFRACTIVE INDEX CONSIDERING COLLISIONS IN A FIRST ORDER APPROXIMATION

A vertical electromagnetic wave is considered, which travels in the z direction in the ionosphere. Figure 1 shows the assumed geographic coordinates and geomagnetic field. The z-axis is vertical with its origin located on the ground. The x and y-axis are geographic eastward and northward respectively, in the northern hemisphere. Being I and d the magnetic dip and declination angles respectively, the geomagnetic field in terms of its components is $\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$, where $B_x = B\cos(I)\sin(d)$, $B_y = B\cos(I)\cos(d)$, $B_z = -B\sin(I)$.

The travelling electromagnetic wave presents a component propagating in a direction perpendicular to the magnetic field and other along the magnetic field. In the first case we have the ordinary (O) and extraordinary (X) waves with a refractive index (n_O and n_X) given by Equations (2) and (3) respectively [3, 4]:

$$n_O^2 = 1 - \frac{X}{1+Z^2} + iZ\frac{X}{1+Z^2} \tag{2}$$

$$\begin{aligned} n_X^2 = {} & 1 - \frac{X\left[(1-X)\left(1-X-Y^2\cos^2 I\cos^2 d\right)+Z^2\right]}{\left[1-X-Z^2-Y^2\cos^2 I\cos^2 d\right]^2 + Z^2\left[2-X\right]^2} \\ & + iZ\frac{X\left[(1-X)^2+Z^2+Y^2\cos^2 I\cos^2 d\right]}{\left[1-X-Z^2-Y^2\cos^2 I\cos^2 d\right]^2 + Z^2\left[2-X\right]^2} \end{aligned} \tag{3}$$

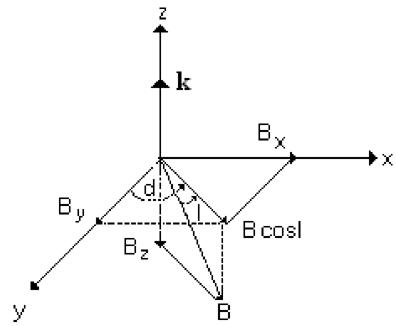

Figure 1. Geometry of Earth's magnetic field, $\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$, and the wave vector, $\mathbf{k}$, of the traveling electromagentic wave. I and d are the magnetic dip and declination angles respectively.

In the case along the magnetic field, we have the circularly polarized waves with a refractive index (n_p) given by Equation (4) [3, 4]:

$$n_p^2 = 1 - \frac{X\left[1 \mp Y \sin I\right]}{\left[1 \mp Y \sin I\right]^2 + Z^2} + iZ\frac{X}{\left[1 \mp Y \sin I\right]^2 + Z^2} \tag{4}$$

where the signs $(-)$ and $(+)$ correspond to right and left hand polarization respectively.

Equations (2), (3) and (4) are written in terms of the magnetoionic parameters $X(= \omega_P^2/\omega^2)$, $Y(= \omega_C/\omega)$ and $Z(= \nu/\omega)$, where ω_P and ω_C are the plasma frequency and the electron gyrofrequency respectively. From these equations' from, it is possible to write the refractive index as $n^2 = (\mu + i\chi)^2 = M + iN$. The real part of n (μ) becomes then:

$$\mu^2 = \frac{1}{2}\left[\left(M^2 + N^2\right)^{1/2} + M\right] \tag{5}$$

For HF waves $Z^2 \ll 1$, so the expression $(1 + Z^2)^{-1}$ in the refractive indices can be approximated by $(1 - Z^2)$, using binomial expansion and neglecting terms of order higher than Z^2. By using these approximations, μ results for the ordinary, extraordinary and circularly polarized waves, as follows [4]:

$$\mu_o^2 \approx (1 - X) + Z^2\frac{X\,(4 - 3X)}{4\,(1 - X)} \tag{6}$$

$$\mu_x^2 \approx \frac{(1 - X)^2 - Y^2 \cos^2 I \cos^2 d}{1 - X - Y^2 \cos^2 I \cos^2 d}$$

$$+Z^2 \frac{X^2\left[(1-X)^2+Y^2\cos^2 I\cos^2 d\right]^2}{4\left[1-X-Y^2\cos^2 I\cos^2 d\right]^3\left[(1-X)^2-Y^2\cos^2 I\cos^2 d\right]} \quad (7)$$

$$\mu_p^2 \approx (1-X') + Z'^2\frac{X'(4-3X')}{4(1-X')} \quad (8)$$

with $X' = X/[1-Y\sin(I)]$ and $Z' = Z/(1-Y\sin I)$. These equations hold for $X < 1$ and $X' < 1$.

The collision frequency ν is the sum $\nu_{ei} + \nu_{en}$, where ν_{ei} and ν_{en} are the electron-ion and the electron-neutral collision frequencies respectively. According to Rishbeth and Garriott [5] these frequencies are given by:

$$\nu_{ei} = N\left[59 + 4.18\log\left(\frac{T_e^3}{N}\right)\right] \times 10^{-6}T_e^{-3/2} \text{ [m.k.s.]} \quad (9)$$

$$\nu_{en} = 5.4 \times 10^{-16}N_nT_e^{1/2} \text{ [m.k.s.]} \quad (10)$$

N is the electron density, N_n the neutral particle density, and T_e the electron temperature.

3. ASSESSMENT OF REFLECTION AND TRANSMISSION COEFFICIENTS

If **k** is perpendicular to the incident plane, the well known expressions for the reflections (R) and transmissions (T) coefficients are

$$R = \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2} \quad (11)$$

$$T = \frac{4n_1n_2}{(n_1 + n_2)^2} \quad (12)$$

where $R + T = 1$. The wave propagation depends on the real part of the refractive index [6], and Equations (11) and (12) become

$$R = \frac{(\mu_1 - \mu_2)^2}{(\mu_1 + \mu_2)^2} \quad (13)$$

$$T = \frac{4\mu_1\mu_2}{(\mu_1 + \mu_2)^2} \quad (14)$$

The calculation of R for a vertical HF wave propagating in the ionosphere was done in the present work for the geographic coordinates

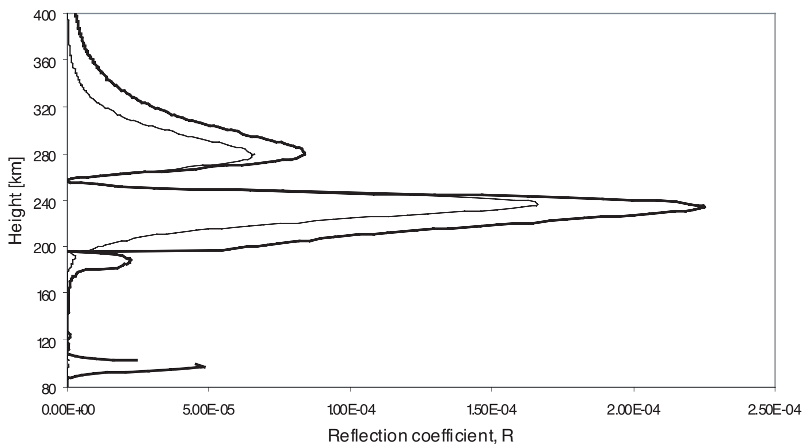

Figure 2. Reflection coefficients profile for the ordinary wave without collisions (thin line) and with collisions (enhanced line).

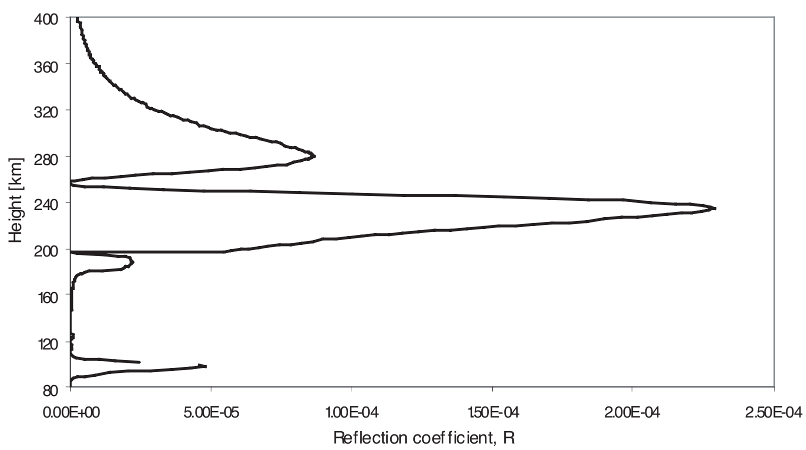

Figure 3. Reflection transmission coefficients profile for the extraordinary wave with and without collision (the curves coincide).

of Elazig ($40°N$, $39°E$), $I = 55°$, and $d = 3°E$. The plasma parameters were obtained from the International Reference Ionosphere, IRI (available at http://modelweb.gsfc.nasa.gov/ionos/iri.html) for December at 12:00 LT. T is obtained as 1-R, so it will not be shown in the figures.

The calculations were done for the 80 km–380 km height range, iterating with a 1 km step.

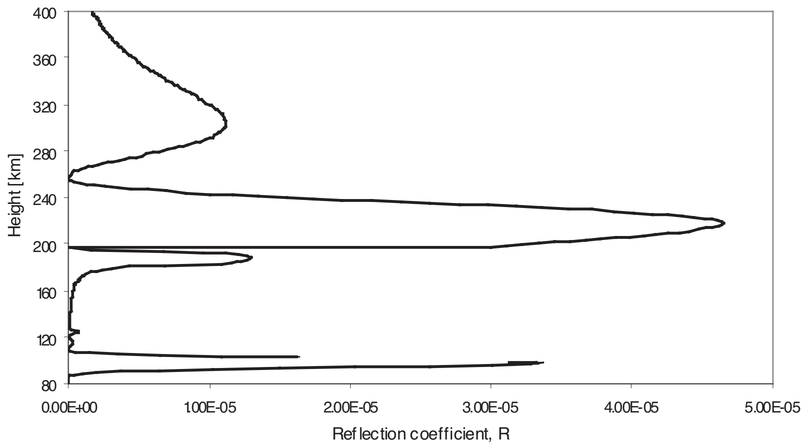

Figure 4. Reflection transmission coefficients profile for the circularly polarized wave with and without collision (the curves coincide).

Figures 2, 3 and 4 show the R profile for the ordinary, extraordinary and circularly polarized waves, when collisions in the ionosphere are considered and when they are neglected. Differences in R values can be noticed only for the ordinary wave. In the ordinary and extraordinary case, maximum R values occur at $\sim 235\,\mathrm{km}$ that is the ionosphere peak height (hmF2). For the polarized wave, R peaks at $\sim 220\,\mathrm{km}$.

4. CONCLUSIONS

From our results, in the case of high frequency waves travelling vertically in the ionosphere, as already known, there is no need in considering collision for R and T calculations, at least to a degree of 10^4. However, the procedure may be useful in the assessments of other ionosphere variables such as the refractive index itself, where collisions are usually neglected [7–9].

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