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Serial group rings of finite groups. General linear and close groups

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Abstract. For a given p, we determine when the p-modular group ring of a group from $\mathrm{GL}(n,q)$, $\mathrm{SL}(n,q)$ and $\mathrm{PSL}(n,q)$-series is serial.

Introduction

There is a recent progress in classifying finite groups G whose group ring FG over a modular field F is serial. It is shown in [15] that the crucial point in this description is making a list of simple finite groups (and fields of finite characteristics) with this property.

For instance in [14] such a classification is given for symmetric and alternating groups; and [15] provides a list of sporadic simple groups and simple Suzuki groups with this property. Furthermore the first author described in [12] groups in the $\mathrm{PSL}(2,q)$-series whose modular group rings are serial.

In this paper we will continue this line of research by including into considerations all projective special linear groups $\mathrm{PSL}(n,q)$. Despite these groups are the main target of this paper, we have to make a bypass by considering general linear groups $\mathrm{GL}(n,q)$, and also special linear

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groups $\mathrm{SL}(n, q)$. The reason for such a detour is that for general linear groups the structure of Brauer trees of blocks is best known, due to results of Fong and Srinivasan [8, 9]. Namely it is shown there that the Brauer tree of any block of $\mathrm{GL}(n, q)$ is an interval whose exceptional vertex is located at its end.

From general theory it is known (see [1, Sect. 5]) that a block B of a group algebra is serial if and only if its Brauer tree is a star with the exceptional vertex at the center. Thus in the case of the serial p-modular group ring of $\mathrm{GL}(n, q)$ we obtain that all Brauer trees of blocks are intervals with at most two edges and, if a tree has two edges, then the exceptional vertex should have multiplicity one. Furthermore the number of edges in a particular block can be calculated using centralizers and normalizers of defect subgroups. There are rather few cases which are left to analyze, which is achieved in this paper without difficulty.

In most cases descending from $\mathrm{GL}(n, q)$ to $\mathrm{SL}(n, q)$ and then to $\mathrm{PSL}(n, q)$ is a straightforward normal subgroup business, the only difficulty is when p divides $q - 1$. In this case more groups with serial group rings occur, and our analysis is based on [12] or directly by looking at character tables.

There is no doubt that a similar approach applies to all classical groups but, because a myriad of details should be taken into account, we will postpone this to a future paper.

1. Preliminaries

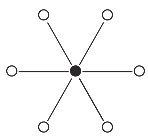
Recall that a module M over a ring R is said to be *uniserial*, if all submodules of M are linearly ordered by inclusion; and M is *serial* if it is a direct sum of uniserial modules. Furthermore R is called a *serial* ring, if R is serial as a right and left module over itself. It is known (see [2, Sect. 32]) that R is serial if and only if there exists a collection $e_1, \ldots, e_n$ of orthogonal idempotents such that each right module $e_i R$ is serial, and the same is true for each left module $R e_j$. For a general theory of serial rings the reader is referred to [19] or recent [4]. Within the class of artinian algebras over a field, the serial rings are also known as *Nakayma algebras* - see [3, Sect. 4.2].

Let G be a finite group and let F be a field of finite characteristic p. If p does not divide the order of G then, by Maschke's theorem, the ring FG is semisimple artinian, hence serial. In this paper we will always assume that *p divides $|G|$*.

Let P denote a p-Sylow subgroup of G. Since (see [2, Theorem 32.3]) artinian serial rings are of finite representation type, it follows from Higman [10] that, if FG is serial, then P is a cyclic group. This gives a necessary condition for seriality, which is not always sufficient: for instance (see [1, p. 123]) the group $\mathrm{SL}(2,5)$ for $p=5$ gives a counterexample.

Furthermore, the seriality of the group ring FG depends on characteristic of F only [6,16]. Thus in this paper (to ease references) we will always assume that F *is algebraically closed*. For instance, it is known (see [18,20] or [13]) that a p-modular group ring of a p-solvable group is serial.

We say that the Brauer tree of a block is a *star* if it has no path of length more than 2. Here is a typical shape of a star with the exceptional vertex in the center:



A useful criterion for checking seriality is given by the following.

Fact 1 (see [1, Sect. 5] or [7, Corollary VII.2.22])**.** A modular group ring $R=FG$ is serial if and only if for each block B of R its Brauer tree is a star whose exceptional vertex (if any) is located in the center.

Thus a satisfactory description of groups with serial group rings depends on the supply of information on Brauer trees of blocks, which is not always readily available.

In some cases the seriality can be lifted from normal subgroups. Suppose that B is a block of the group algebra FG; H is a normal subgroup of G and b is a block of FH. A definition of the notion that B *covers* b can be found in [1, Sect. 14]. For instance if H contains a p-Sylow subgroup of B, then the principal block B_0 of G covers the principal block b_0 of H.

Fact 2 (see [7, Theorem 6.2.7])**.** 1) Suppose that a block B of G covers a block b of H where H contains a defect group of B. Then B is serial if and only if b is serial.

2) Suppose that F is a field of characteristic p and let H be a normal subgroup of G whose index $|G/H|$ is coprime to p. Then the ring FG is serial if and only if FH is serial.

Suppose that B is a block of a modular group ring FG with a cyclic defect group D and let e denote the number of edges in the Brauer tree of B. For instance the defect group of the principal block B_0 equals P. By $C_G(D)$ we denote the centralizer of D in G; and $N_G(D)$ is the normalizer of D.

Fact 3 (see [1, Sect. 5, Theorem 1])**.** The number of edges e in the Brauer tree of a block B divides the order of the factor group $N_G(D)/C_G(D)$, hence divides $p-1$. Furthermore the multiplicity of the exceptional vertex equals $(|D|-1)/e$.

For the principal block B_0 the number of edges e equals to the order $|N_G(P)/C_G(P)|$.

We will need one more technical result. Recall that $O_{p'}$ denotes the largest normal subgroup of G consisting of elements whose order is coprime to p. We say that an element $g \in G$ is in the *kernel* of a block B if g acts trivially on every indecomposable projective module in B.

Fact 4 (see [7, Lemma IV.4.12])**.** The kernel of the principal block of G equals $O_{p'}$.

2. General linear group

In this section we will describe serial rings of general linear groups $\mathrm{GL}(n,q)$ over finite fields with q elements.

Theorem 1. *Let $G = \mathrm{GL}(n,q)$, $n \geqslant 2$ and let F be a field of characteristic p dividing the order of G. Then the group ring FG is serial if and only if one of the following holds.*

1) $n=2$ and $p=q$ equal 2 or 3.

2) $n=2,3$, $p=3$ and $q \equiv 2,5 \pmod 9$.

For instance $\mathrm{GL}(3,2) \cong \mathrm{PSL}(2,7)$ and, for any field of characteristic 3, the group ring of this group is serial.

Recall that the order of $\mathrm{GL}(n,q)$ equals $q^{n(n-1)/2} \cdot (q-1) \cdot \ldots \cdot (q^n-1)$. Thus if p divides the order of G, then either $p \mid q$ or p divides q^k-1 for some $k=1,\ldots,n$.

We will divide the proof of Theorem 1 in two parts. The case of the defining characteristic $p \mid q$ is easy.

Lemma 1. *Let $q=p^r$, $G=\mathrm{GL}(n,q)$ and F is a field of characteristic p. The group ring FG is serial if and only if $n=2$, $r=1$ and p equals 2 or 3.*

Proof. If $n = 3$ then the matrices $\left(\begin{smallmatrix} 1&1&0\\ 0&1&0\\ 0&0&1 \end{smallmatrix}\right)$ and $\left(\begin{smallmatrix} 1&0&0\\ 0&1&0\\ 0&1&1 \end{smallmatrix}\right)$ generate a subgroup $C_p \times C_p$, hence P is not cyclic; and we argue similarly for $n \geqslant 4$.

Thus it remains to consider the case $n = 2$.

If $r \geqslant 2$, it is easily checked that P is not cyclic, hence we may assume that $p = q$. Because $p - 1$, the index of $\mathrm{SL}(2, p)$ in $\mathrm{GL}(2, p)$, is coprime to p, it follows from Fact 2 that the seriality of group rings of $\mathrm{GL}(2, p)$ and $\mathrm{SL}(2, p)$ is equivalent.

If $p \geqslant 5$ we conclude from [1, p. 124] that the Brauer tree of the principal block B_0 of the group $H = \mathrm{SL}(2, p)$ is an interval with at least 3 edges, hence the ring FH (and then FG) is not serial. It remains to consider the case $p = 2, 3$.

If $p = 2$, then $G = \mathrm{GL}(2, 2) \cong S_3$ is 2-nilpotent, hence the ring FG is serial.

Similarly for $p = 3$ the group $\mathrm{GL}(2, 3)$ has order 48 and is 3-solvable, hence FG is serial. □

Thus we may assume that p does not divide q. Let d be the order of q modulo p, i.e. the least d such that $p \mid q^d - 1$. By the assumption we have $1 \leqslant d \leqslant n$, and clearly $d \mid p - 1$.

We will show that d cannot be very small (otherwise the p-Sylow subgroup P of G is not cyclic) and cannot be very large (otherwise the Brauer tree of the principal block has too many edges).

The description of normalizers and centralizers of p-Sylow subgroups of $\mathrm{GL}(n, q)$ is well known (see [21, 22]). We will add some explanations to ease reader's task.

Lemma 2. *1) P is cyclic if and only if $n < 2d$.*

2) If $n < 2d$ then the factor group $N_G(P)/C_G(P)$ has order d.

Proof. Consider the Galois field $\mathbb{F}_{q^d}$ as a vector space (of dimension d) over $\mathbb{F}_q$ with a basis $v_1, \dots, v_d$. Let z be nonzero element of $\mathbb{F}_{q^d}$. Then $zv_i = \sum_j z_{ij} v_j$ for some $z_{ij} \in \mathbb{F}_q$. The mapping $z \mapsto (z_{ij})$ defines an embedding of the multiplicative group of $\mathbb{F}_{q^d}$ into $\mathrm{GL}(d, q)$. The image of a generator of $\mathbb{F}^*_{q^d}$ gives us a matrix $x \in \mathrm{GL}(d, q)$ of order $q^d - 1$.

Write $q^d - 1 = p^a \cdot s$ such that p and s are coprime, hence $y = x^s$ generates the p-Sylow subgroup P of order p^a.

1) If $n \geqslant 2d$, then one could insert in $\mathrm{GL}(n, q)$ two copies of $\mathrm{GL}(d, q)$ as 1 through d, and $d + 1$ through $2d$ diagonal blocks. It follows easily that P is not cyclic.

On the other hand, if $n < 2d$ then, comparing the sizes, we see that P can be chosen inside $\mathrm{GL}(d, q)$ embedded in the upper left 1 through d corner of $\mathrm{GL}(n, q)$, and therefore is generated by y.

2) It is known (see [22]) that the centralizer of P is generated by x, hence has order $q^d - 1$. Furthermore (see [21, Lemma 4.6]) the normalizer of P is generated over $C_G(P)$ by an element of order d.

For our purposes it suffices to find an element which normalizes P and has order d modulo the centralizer. This can be achieved as follows.

Suppose that the action of x on the basis is given by a matrix $A = (a_{ij})$, $a_{ij} \in \mathbb{F}_q$: $xv_i = \sum_j a_{ij} v_j$. Applying the Frobenius morphism $x \mapsto x^q$ on $\mathbb{F}_{q^d}$ we obtain $x^q v_i^q = \sum_j a_{ij} v_j^q$. It follows that the action of x^q in the basis v_i^q is given by the same matrix A.

Because in the original basis this action is given by A^q, we conclude that $UAU^{-1} = A^q$, where U is the transition (from v_i^q to v_i) matrix. Then the conjugation by U defines an automorphism of order d on the subgroup generated by x. It follows that this action induces on P an automorphism ψ of the same order.

Namely, let $\psi(y) = y^q$ and suppose that $y^{q^k} = y$ for some k. Plugging $y = x^s$ we obtain $x^{(q^k-1)s} = 1$, therefore $q^d - 1 = p^a \cdot s$ divides $(q^k - 1)s$. It follows that p^a divides $q^k - 1$, and hence d divides k, by the choice of d. $\square$

Now we complete the proof of Theorem 1 by showing the following.

Proposition 1. *Let $G = \mathrm{GL}(n, q)$ and F is a field of characteristic p dividing the order of G but not dividing q. Then the group ring FG is serial if and only if $n = 2, 3$, $p = 3$ and $q \equiv 2, 5 \pmod 9$.*

Proof. We may assume that the p-Sylow subgroup P of G is cyclic. By the item 1) of Lemma 2 it follows that $n/2 < d \leqslant n$, where d is the order of q modulo p.

Suppose first that $d > 2$. If $p = 2$ it follows (since p does not divide q) that q is odd, therefore p divides $q - 1$ and $d = 1$, a contradiction. Thus we may assume that $p > 2$.

By Fact 3 and the item 2) of Lemma 2 the Brauer tree of the principal block B_0 of G has $e = d$ edges. Furthermore [8, Prop. 4] implies that this tree is an interval. Since $d > 2$, this block is not serial.

Thus we are left with the case $d = 2$, in particular p divides $q^2 - 1$. The definition of d yields that p does not divide $q - 1$ and hence divides $q + 1$. Again the Brauer tree of the principal block of G is an interval with $e = 2$ edges, whose exceptional vertex is located at its end. By Fact 3 the multiplicity of this vertex is $(|P| - 1)/2$. If $|P| > 3$ this bock is not serial.

Thus we may assume that $|P| = 3$, which clearly yields $p = 3$ and $q \equiv 2, 5 \pmod 9$ (otherwise 9 divides the order of P). It follows that the principal block is serial.

We prove that, in this case, any non-principal block B of G is also serial. Namely, by Fact 3 the number of edges, e, of this block divides $p - 1 = 2$. If $e = 1$ then this block contains only one Brauer character, hence serial. If $e = 2$ then the multiplicity of the exceptional vertex equals $(3 - 1)/2 = 1$, hence this block is also serial. □

Note that in the proof of the implication $\Rightarrow$ in Theorem 1 we used only that the principal block B_0 of $\mathrm{GL}(n, q)$ is serial.

3. Special linear and projective special linear groups

In this section we will consider the seriality of group rings of special linear groups $\mathrm{SL}(n, q)$ and projective special linear groups $\mathrm{PSL}(n, q)$. The answer turns out to be the same for both series; and the proofs go in parallel.

Recall that $\mathrm{SL}(n, q)$ is a normal subgroup of $\mathrm{GL}(n, q)$ of index $q - 1$. Furthermore $\mathrm{PSL}(n, q)$ is obtained from $\mathrm{SL}(n, q)$ by factoring out the center Z whose order equals $(n, q - 1)$. Note also that, except of $\mathrm{PSL}(2, 2)$ and $\mathrm{PSL}(2, 3)$, $\mathrm{PSL}(n, q)$ is a simple group.

To avoid long sentences we will divide the classification theorem in two cases: when p divides $q - 1$ and when it is not. In the former case the answer is the same as in Theorem 1.

Proposition 2. *Let G is one of the groups $\mathrm{SL}(n, q)$ or $\mathrm{PSL}(n, q)$, $n \geqslant 2$. Let F be a field of characteristic p such that p does not divide $q - 1$. Then the ring FG is serial if and only if one of the following holds.*

1) $n = 2$ and $p = q$ equal 2 or 3.

2) $n = 2, 3$, $p = 3$ and $q \equiv 2, 5 \pmod 9$.

Proof. Since p does not divide $q - 1$, by Fact 2, we conclude that the seriality of group rings of $\mathrm{SL}(n, q)$ and $\mathrm{GL}(n, q)$ is equivalent. Applying Theorem 1 we obtain the desired conclusion for $\mathrm{SL}(n, q)$.

Thus we may assume that $G = \mathrm{PSL}(n, q)$. If 1) or 2) holds true then the group ring R of $\mathrm{SL}(n, q)$ is serial. Since G is a factor group of this group, it follows that the group ring of G is a factor ring of R, therefore is also serial.

Thus we may assume that the group ring of $\mathrm{PSL}(n, q)$ is serial and we need to show that either 1) or 2) holds true.

By Fact 4 the principal block b_0 of $\mathrm{SL}(n,q)$ has Z in its kernel, and therefore coincides with the principal block of $\mathrm{PSL}(n,q)$. Furthermore, because $\mathrm{SL}(n,q)$ contains the p-Sylow subgroup of $\mathrm{GL}(n,q)$ it follows by Fact 2 that the principal block B_0 of $\mathrm{GL}(n,q)$ is serial.

Now the result follows from the proof of Theorem 1 (see a remark at the end of Section 2). □

Now we consider the remaining case $p \mid q-1$. In this case serial rings occur more often than in the GL-case (cp. Theorem 1).

Proposition 3. *Let G be one of the group $\mathrm{SL}(n,q)$ or $\mathrm{PSL}(n,q)$, $n \geqslant 2$ and let F be a field of characteristic p dividing $q-1$. The group ring FG is serial if and only if $n=2$ and $p \neq 2$.*

Proof. If $n \geqslant 3$ then it is easily seen that p-Sylow subgroups of G are not cyclic. Thus we may assume that $n=2$.

If $G = \mathrm{PSL}(2,q)$ then FG is serial if and only if $p \neq 2$ [12].

Thus we may assume that $G = \mathrm{SL}(2,q)$. If $p=2$ then the group ring FG is not serial. Indeed, otherwise, being a factor ring of FG, the group ring of $\mathrm{PSL}(2,q)$ would be serial, a contradiction.

It remains to consider the case $p>2$ and we have to prove that the group ring of FG is serial. Observe that, if q is even, then $\mathrm{SL}(2,q) \cong \mathrm{PSL}(2,q)$, hence the ring is serial. Thus we assume that q is odd. In this case the center Z of $\mathrm{SL}(2,q)$ consists of matrices $\pm I$, where $I = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$.

For the remaining part of the proof we need the character table of $G = \mathrm{SL}(2,q)$ — see Table 1.

In the table, $1 \leqslant l \leqslant (q-3)/2$, $1 \leqslant m \leqslant (q-1)/2$, $\varepsilon = (-1)^{(q-1)/2}$, ρ is a primitive $(q-1)$-th root of 1, and σ is a primitive $(q+1)$-th root of 1.

Let ν be a generator of the group $\mathbb{F}_q^*$. Denote $\gamma = \left(\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}\right)$, $\delta = \left(\begin{smallmatrix} 1 & 0 \\ \nu & 1 \end{smallmatrix}\right)$, $\alpha = \begin{pmatrix} \nu & 0 \\ 0 & \nu^{-1} \end{pmatrix}$. So, the order of α is $q-1$. The group G contains also an element β of order $q+1$. Moreover, two columns for the classes of $\gamma' = -I \cdot \gamma$ and $\delta' = -I \cdot \delta$ are omitted (to save space in the table). The values of any irreducible character χ of G on these classes are obtained by the formulas $\chi(\gamma') = \chi(\gamma)\chi(-I)/\chi(I)$ and $\chi(\delta') = \chi(\delta)\chi(-I)/\chi(I)$. Since $p \mid q-1$, only the sixth column of the table contain p-singular elements.

In particular, the cyclic group $\langle\alpha\rangle$ contains a generator y of a p-Sylow subgroup P of G.

It is easy to show (see [5, p. 230]) that $C_G(y) = \langle\alpha\rangle$ and $N_G(y) = \langle\alpha, \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)\rangle$. Hence $|N_G(P)/C_G(P)| = 2$. In particular, the number of

Classes	I	$-I$	γ	δ	α^l	β^m
Number of classes	1	1	1	1	$\frac{q-3}{2}$	$\frac{q-1}{2}$
Size of classes	1	1	$\frac{q^2-1}{2}$	$\frac{q^2-1}{2}$	$q(q+1)$	$q(q-1)$
1_G	1	1	1	1	1	1
ψ	q	q	0	0	1	-1
χ_i $(i = 1, \ldots, \frac{q-3}{2})$	$q+1$	$(-1)^i \times$ $\times(q+1)$	1	1	$\rho^{il} + \rho^{-il}$	0
$\theta_j (j = 1, \ldots, \frac{q-1}{2})$	$q-1$	$(-1)^j \times$ $\times(q-1)$	-1	-1	0	$-(\sigma^{jm} + \sigma^{jm})$
$\xi_1,\ \xi_2$	$\frac{q+1}{2}$	$\frac{\varepsilon(q+1)}{2}$	$\frac{1\pm\sqrt{\varepsilon q}}{2}$	$\frac{1\mp\sqrt{\varepsilon q}}{2}$	$(-1)^l$	0
$\eta_1,\ \eta_2$	$\frac{q-1}{2}$	$-\frac{\varepsilon(q-1)}{2}$	$\frac{-1\pm\sqrt{\varepsilon q}}{2}$	$\frac{-1\mp\sqrt{\varepsilon q}}{2}$	0	$(-1)^{m+1}$

Table 1. The character table of $\mathrm{SL}(2,q)$, q is odd [5, p. 228]

edges in the principal block B_0 of G equals 2, furthermore the number of edges in any block of G divides 2.

Observe that θ_j, η_1 and η_2 have value 0 on the class of α. By [17, Theorem 4.4.14], these characters belong to blocks of defect zero. It follows that these blocks contain only one irreducible ordinary character, hence is serial.

Furthermore it is easily checked (using [11, Theorem 2.1.8]) that the Steinberg character ψ belongs to the principal block B_0. Looking at the values on p-singular elements (and using cross-naught business — see [11, Chap. 2]) we see that the Brauer tree of B_0 is an interval with 2 edges having 1_G and ψ at its ends. Thus if there is an exceptional vertex it should be located at the center of this interval (in fact certain characters χ_i will occupy the center making an exceptional vertex there).

Because each character χ_i has the largest possible degree, it follows from [11, Lemma 2.1.22] that such a character cannot occur at the end of an interval of length 2. Thus the only possibility for such an interval is to have ξ_1 at one end, ξ_2 at another end, and some characters χ_i in between. But this block is clearly serial.

In fact such a block exists if $q \equiv 1 \pmod 4$; otherwise each non-principal block contains at most one modular character (i.e. its Brauer tree has at most one edge).

By this we have established that the group ring of $\mathrm{SL}(2,q)$ is serial if $2 \neq p \mid q-1$, hence finished the proof of the proposition. $\square$

Prepositions 2 and 3 completely describe groups of $\mathrm{SL}(n,q)$ and $\mathrm{PSL}(n,q)$-series whose p-modular group rings are serial.

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