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Interval-valued intuitionistic fuzzy multi-criteria model for design concept selection

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ABSTRACT

This paper presents a new approach for design concept selection by using an integrated Fuzzy Analytical Hierarchy Process (FAHP) and an Interval-valued intuitionistic fuzzy modified TOPSIS (IVIF-modified TOPSIS) model. The integrated model which uses the improved score function and a weighted normalized Euclidean distance method for the calculation of the separation measures of alternatives from the positive and negative intuitionistic ideal solutions provides a new approach for the computation of intuitionistic fuzzy ideal solutions. The results of the two approaches are integrated using a reflection defuzzification integration formula. To ensure the feasibility and the rationality of the integrated model, the method is successfully applied for evaluating and selecting some design related problems including a real-life case study for the selection of the best concept design for a new printed-circuit-board (PCB) and for a hypothetical example. The model which provides a novel alternative, has been compared with similar computational methods in the literature.

1. Introduction

As the business world increasingly recognizes the potential of an effective product development for the survival and growth of today's business, the concept, however, can be said to be changing. This is as a result of the increasing competition and the eclectic approach to the development of today's products. For companies to maintain their competitiveness, they sure require strong marketing power, well-integrated organizations as well as effective and efficient capacities in their research and development (R&D) units to develop sustainable and innovative products (Fang & Chyu, 2014). Product development which is recursive and reiterative intellectual process is centered on defining, analyzing, testing, comparing, choosing, specifying, and documenting the developmental strive of new products. It comprises of an information intensive decision-making process that starts with; idea generation and screening, market analysis, product (design) concept, technical implementation, through to commercialization and product pricing (Ulrich, & Eppinger, 2000). Since product design and development decisions are often taken under considerable uncertainty, risk and sometimes under time pressure, it is important to grasp the range of uncertainty and potential risk or consider their impact when making rational and

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product decision (Huanget al. , 2015). This study, however, will focus on the product design concept stage.

The product design concept stage perhaps can be described as one of the most critical stages in product development process. This is the stage where the final decision to select a particular design concept for a given product is made. According to Genget al. (2010), the design decision made in the early phases of new product development is most crucial for determining the success of both the developed product as well as the development process. Design concept evaluation is a complex multi-criteria (group) decision-making (MCDM) process, which involves several imprecise factors/criteria ranging from customer related requirements, product complexity, insufficient information about the design, and the diversity and expertise of the decision makers (DMs). According to Loet al. (2006), DMs preferences often lack the precision and level of confidence required in the concept selection and in most cases contributes to various degrees of uncertainty. Hence, how to cope with these uncertainties becomes critical to the effectiveness of decision-making process.

Several different approaches and methodologies have been proposed in the past to assist design concept evaluation. According to Zhaiet al. (2009), the methods and approaches can be classified into two categories, namely the numerical methods and the non-numerical methods. The non-numerical approach, basically involves the traditional design evaluation approach which includes methods like; concept screening (Ulrich, & Eppinger, 2000) and concept selection and evaluation (Pugh, 1996), while the numerical methods comprises of methods like decision matrixes (King & Sivaloganathan, 1999), quality function deployment (Mariniet al. , 2016), fuzzy set (FS) concepts (Aikhuele & Turan, 2017; Akay et al., 2011; Jenab et al., 2013; Liu 2011), grey relation analysis (Zhai et al., 2009).

Amongst the various numerical methods that have been proposed, the FS application has remained the most widely used approach for design concept evaluation, where this is due to their ability to handle uncertainty, and this has made it a topic of great interest to many researchers. However, in the effort to further address some of those uncertainties that the traditional FS cannot fully handle in the decision–making process. Atanassov (1986) introduced the intuitionistic fuzzy set (IFS) theory. Unlike the traditional FS, the IFS is characterized by a membership and a non-membership function, hence making it more capable of handling vagueness and uncertain information in practice. Extensive literature review has shown, significant increase in the application of the IFS over the past few years mostly for solving multi-criteria decision making (MCDM) problems (Aikhuele & Turan, 2017; Aikhuele & Turan, 2016; Bai, 2013; Chen & Chiou, 2015; Jahromi, 2012; Li, 2005; Linet al. , 2007; Liu & Ren, 2014; Xu, 2014). One of such applications includes the intuitionistic fuzzy technique for order preference by similarity to ideal solution (IF-TOPSIS).

TOPSIS which is one of the most widely used MCDM methods has found application in several fields with many papers published on its applications. However, due to some of its drawbacks and limitations, many different improvements and modifications have been proposed and implemented, prominently among them is; the Modified-TOPSIS model by Renet al. (2007). The Modified-TOPSIS was presented to improve and solve the ranking reversal issues associated with the traditional TOPSIS methodology. It creates an understanding of the inherent relationship between the Relative closeness (R) value and alternative evaluation. The Modified-TOPSIS model is "described as a process of calculating the distance between the alternatives and the reference points in the D^+ and the D^- plane which results in the construction of the R value to evaluate the quality of the alternative" (Aikhuele & Turan, 2016a; Ren et al., 2007).

To improve the results of the Modified-TOPSIS model and to avoid the bias of using a single distance measure, in this paper, an Interval-Valued Intuitionistic Fuzzy Modified TOPSIS (IVIF-Modified TOPSIS) (Aikhuele & Turan, 2016a) and a Fuzzy Analytical Hierarchy Process (FAHP) model is proposed for design concept evaluation by using partly the algorithm originally presented by Bai (2013), where an improved score function is applied for calculating the separation measure of alternatives. The FAHP model is used to determine the criteria weight. In addressing the bias of using a single separation measure, an additional distance method (i.e. the weighted-normalized-Euclidean-distance method) has been adopted for calculating the separation measure of alternatives where the results from the two approaches are made robust by integrating them using a reflection defuzzification integration formula originally proposed in (Aikhuele & Turan, 2016b). Thereafter the results of the different approaches are compared. The rest of this paper is organized as follows. Section 2 presents briefly the concept of IFS and the Fuzzy Analytical Hierarchy Process. The Interval-Valued Intuitionistic Fuzzy Modified-TOPSIS algorithm is presented in section 3. In Section 4 a numerical case is presented in to explain the proposed methodology. Finally, the conclusions and further works are presented in section 5.

2. Preliminaries

To define the fuzzy nature and complexity of the real world more comprehensively, Atanassov (1986) introduced the IFS, which was extended from the traditional fuzzy set, and further proposed the Interval-Valued Intuitionistic Fuzzy Set (IVIFS) which is the main focus of this study.

Definition 1

Let $\mathrm{D}[0,1]$ be the set of all closed subintervals of the interval [0, 1] and let $\mathrm{X}(\neq \emptyset)$ be a given set. An IVIFS **A** in **X** is expressed as (Bai, 2013);

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle \mid x \in X\}, \tag{1}$$

where $\mu_A: X \to \mathrm{D}[0,1]$, $v_A: X \to \mathrm{D}[0,1]$ with the condition $0 \leq sup\ \mu_A(x) + sup\ v_A(x) \leq 1, \forall x \in X$.

The intervals $\mu_A(x)$ and $v_A(x)$ denote, respectively, the degree of membership and non-membership of the element **x** to the set **A**. Thus, for each $x \in X$ **the** intervals $\mu_A(x)$ and $v_A(x)$ are closed and their lower and upper end points are denoted by $\mu_{AL}(x), \mu_{AU}(x), v_{AL}(x)$ and $v_{Au}(x)$respectively. We can denote the set as;

$$A = \{\langle x, [\mu_{AL}(x), \mu_{AU}(x)],\ [v_{AL}(x), v_{AU}(x)]\rangle \mid x \in X\}, \tag{2}$$

where, $0 \leq \mu_{AU}(x) + v_{AU}(x) \leq 1,\ \mu_{AL}(x) \geq 0,\ v_{AL}(x) \geq 0$

For each element **x**, we can compute the unknown degree (hesitancy degree) of an intuitionistic fuzzy interval of $x \in X$ in **A** which is defined as follows:

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x) = [1 - \mu_{AL}(x) - \mu_{AU}(x), 1 - \mu_{AL}(x) - v_{AL}(x)]. \tag{3}$$

However, if $\mu_A(x) = \mu_{AL}(x) = \mu_{AU}(x)$ and $v_A(x) = v_{AL}(x) = v_{AU}(x)$, then the given IVIFS **A** is reduced to an ordinary IFS. For convenience, the IVIFS can also be expressed as $A = ([a,b],[c,d])$.

In ranking the Interval-Valued Intuitionistic Fuzzy Set (IVIFS), Bai, (2013), introduced the improved score function, which is based on the unknown degree for calculating the separation measure of alternatives and for converting the Interval-Valued Intuitionistic Fuzzy numbers (IVIFNs) to representative crisp value, the formula of the improved score function is given below;

$$I(A) = \frac{a + a(1-a-c) + b + b(1-b-d)}{2}, \tag{4}$$

2.1. Fuzzy-Analytical Hierarchy Process (FAHP)

The fuzzy AHP which was proposed by Van Laarhoven and Pedrycz in 1983 can be described as an extension of the Analytic Hierarchy Process (AHP), which stands as an excellent multi-criteria decision-making tool for solving both quantitative and qualitative problems. The fuzzy AHP is unique for its ability to deal with fuzziness and vagueness of linguistic judgments by establishing an effective prioritization. The fuzzy AHP method was borne out of the inability of the AHP to deal with imprecision and subjectiveness in the pair-wise comparison process (Aikhuele & Turan, 2017a; Badizadeh & Khanmohammadi, 2011). The application of fuzzy AHP, allows complex multi-criteria decisions problems to be structured into hierarchical descending order from an overall objective to various criteria, sub-criteria and so on until the lowest level, where the decision alternatives or selection choices are laid down at the last level of the hierarchy. There are different approaches for solving fuzzy AHP-based model; however, this study will be concerned only with Chang's extent analysis approach. The main philosophy behind this theory and approach have been expressed and supported with real case applications in (Chang, 1996). This method uses linguistic variables to express the comparative judgments given by decision makers. The approach can be represented using the following notations (Kumar & Singh, 2012);

Let $X = \{x_1, x_2, \dots, x_n\}$ represent an object set and $G = \{g_1, g_2, \dots, g_m\}$ a goal set. In the method proposed by Chang (Chang, 1996), each object is taken and extent analysis is performed for each goal respectively. Thus, **m** extent analysis values for each object can be obtained with the following:

$$M_{gi}^1, M_{gi}^2, \cdots, M_{gi}^m, \quad i = 1, 2, \cdots, n \tag{5}$$

where all the M_{gi}^j (**j** = **1**, 2… **m**) are triangular fuzzy numbers (TFNs) and are denoted as (*l,m,u*) for conveniences (see Table 1).

The computational steps for Chang's extent analysis approach are described below;

(i) Compute the value of the fuzzy synthetic extent with respect to the **i**th object according to

$$S_i = \sum_{j=1}^{m} M^i{}_{gi} \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M^i{}_{gi}\right]^{-1} \tag{6}$$

where $\sum_{j=1}^{m} M^i{}_{gi}$ is obtained by performing the fuzzy addition operation of **m** extent analysis values for a particular matrix such that

$$\sum_{j=1}^{m} M_{gi}^i = \left(\sum_{j=1}^{m} l_j \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right) \tag{7}$$

and $\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M^i{}_{gi}\right]^{-1}$ is given by

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gj}^i\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right) \tag{8}$$

(ii) Compute the degree of possibility of $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$, where the degree of possibility between the two fuzzy synthetic extents is given in Eq. (7), and can be equivalently expressed as Eq. (9);

$$\begin{aligned} V(M_2 \geq M_1) &= \sup[\min\{\mu_{M_1}(x), \mu_{M_2}(y)\}] \\ V(M_2 \geq M_1) &= hgt(M_1 \cap M_2) = \mu_{M_2}(d) \end{aligned} \tag{9}$$

where d is the ordinate of the highest intersection point D between μ_{M_2} and μ_{M_2}.

$$\mu_{M_2}(d) = \begin{cases} 1; & if\ M_2 \geq M_1 \\ 0; & if\ \ l1 \geq U2 \\ \dfrac{l_1 - u_2}{(M_2 - u_2) - (M_1 - l_1)}; & otherwise \end{cases} \tag{10}$$

In comparing **M** 1 and **M** 2, the values of $V(M_2 \geq M_1)$ and $V(M_1 \geq M_2)$ are required.

(iii) Compute the degree of possibility for a convex fuzzy number M to be greater than convex fuzzy numbers **M** $_i$ (**i** = 1, 2,…, **k**) and is defined as;

$$V(M \geq M_1, M_2, \cdots, M_k) = V[(M \geq M_1)\text{and}(M \geq M_2)\text{ and} \cdots \text{and } (M \geq M_k)] = \min V(M \geq M_i), \quad i = 1,2,\cdots,k. \tag{11}$$

Assume that $d'(C_i) = \min V(S_i \geq S_K)\ \ k = 1,2, \dots, n; k \neq i.$ then the weight vector is given by;

$$W' = \{\, d'(C_1),\ d'(C_2), \dots,\ d'(C_n)\}^T \tag{12}$$

where $C_1, C_2, \dots, C_n$ is n criteria.

When the weight vector is normalized we have;

$$W = \{d(C_1), d(C_2), \dots, d(C_n)\} \tag{13}$$

where W is not a fuzzy number.

3. The IVIF-Modified TOPSIS and FAHP Algorithm

In this section, the algorithm for the proposed integrated model is expressed using a stepwise procedure. The implementation steps which is partly from Bai, (Bai, 2013) algorithm have been modified to suit the present study.

Step 1: Set up a group of Decision Makers (DMs). With their opinion construct the interval-valued intuitionistic fuzzy decision matrix $(\widetilde{\text{D}})$ of the alternatives (A_i) with respect to the criteria (C_i), using linguistic variables and the interval-valued intuitionistic fuzzy number (IVIFN) (see Table 1) $\boldsymbol{x_{ij}} = ([\boldsymbol{a_{ij}}, \boldsymbol{b_{ij}}], [\boldsymbol{c_{ij}}, \boldsymbol{d_{ij}}]),\ i = 1,2,\dots,m;\ \boldsymbol{j = 1,\dots,n}$

Step 2: Convert the interval-valued fuzzy decision matrix $\boldsymbol{D_{mxn}(x_{ij})}$to the improved score matrix $\boldsymbol{R_{mxn}(I_{ij}(a_{ij}))}$(see Eqs. (14-15)

$$\boldsymbol{R_{mxn}(I_{ij}(a_{ij}))} = \begin{bmatrix} \boldsymbol{I_{11}(x_{11})} & \boldsymbol{I_{12}(x_{12})} & \dots & \boldsymbol{I_{1n}(x_{1n})} \\ \boldsymbol{I_{22}(x_{22})} & \boldsymbol{I_{22}(x_{22})} & \cdots & \boldsymbol{I_{2n}(x_{2n})} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{I_{m1}(x_{m1})} & \boldsymbol{I_{m2}(x_{m2})} & \cdots & \boldsymbol{I_{mn}(x_{mn})} \end{bmatrix} \tag{14}$$

$$\boldsymbol{D_{mxn}(x_{ij})} = \begin{bmatrix} ([\boldsymbol{a_{11}}, \boldsymbol{b_{11}}], [\boldsymbol{c_{11}}, \boldsymbol{d_{11}}]) & ([\boldsymbol{a_{12}}, \boldsymbol{b_{12}}], [\boldsymbol{c_{12}}, \boldsymbol{d_{12}}]) & \dots & ([\boldsymbol{a_{1n}}, \boldsymbol{b_{1n}}], [\boldsymbol{c_{1n}}, \boldsymbol{d_{1n}}]) \\ ([\boldsymbol{a_{21}}, \boldsymbol{b_{21}}], [\boldsymbol{c_{21}}, \boldsymbol{d_{21}}]) & ([\boldsymbol{a_{22}}, \boldsymbol{b_{22}}], [\boldsymbol{c_{22}}, \boldsymbol{d_{22}}]) & \cdots & ([\boldsymbol{a_{2n}}, \boldsymbol{b_{2n}}], [\boldsymbol{c_{2n}}, \boldsymbol{d_{2n}}]) \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ ([\boldsymbol{a_{m1}}, \boldsymbol{b_{m1}}], [\boldsymbol{c_{m1}}, \boldsymbol{d_{m1}}]) & ([\boldsymbol{a_{m2}}, \boldsymbol{b_{m2}}], [\boldsymbol{c_{m2}}, \boldsymbol{d_{m2}}]) & \cdots & ([\boldsymbol{a_{mn}}, \boldsymbol{b_{mn}}], [\boldsymbol{c_{mn}}, \boldsymbol{d_{mn}}]) \end{bmatrix} \tag{15}$$

Table 1
Intuitionistic Fuzzy Numbers for approximating the linguistic variable

Linguistic terms	Interval-valued intuitionistic fuzzy Number	Triangular Fuzzy Numbers (TFN)
Very low (VL)	([0.1, 0.3], [0.25, 0.4])	(0.1, 0.25, 0.3)
Low (L)	([0.2, 0.55], [0.3, 0.55])	(0.2, 0.3, 0.55)
Good (G)	([0.3, 0.6], [0.45, 0.65])	(0.3, 0.45, 0.6)
High (H)	([0.5, 0.7], [0.6, 0.7])	(0.5, 0.6, 0.7)
Excellent (EX)	([0.6, 0.9], [0.75, 1.0])	(0.6, 0.75, 0.9)

$$d^{+}{}_{i}(A^{+},\boldsymbol{A_i}) = \sqrt{\sum_{i=1}^{n}\left[w_j\,(1-\left(\boldsymbol{I_{ij}}\left(\boldsymbol{x_{ij}}\right)\right)\right]^2}$$
$$d^{-}{}_{i}(A^{-},\boldsymbol{A_i}) = \sqrt{\sum_{i=1}^{n}\left[w_j\,\left(\boldsymbol{I_{ij}}\left(\boldsymbol{x_{ij}}\right)\right)\right]^2} \tag{17}$$

$$d^{+}{}_{i}(A^{+},\boldsymbol{A_i}) = (\frac{1}{4}\sum_{j=0}^{n} w_j\left(\left(a_{ij}-a_j\right)^2+\left(b_{ij}-b_j\right)^2+\left(c_{ij}-c_j\right)^2+\left(d_{ij}-d_j\right)^2\right.$$
$$\left.+\left(\pi^{l}{}_{ij}-\pi^{l}{}_{j}\right)^2+\left(\pi^{u}{}_{ij}-\pi^{u}{}_{j}\right)^2\right)^{1/2}$$
$$d^{-}{}_{i}(A^{-},\boldsymbol{A_i}) = (\frac{1}{4}\sum_{j=0}^{n} w_j\left(\left(a_{ij}-a_j\right)^2+\left(b_{ij}-b_j\right)^2+\left(c_{ij}-c_j\right)^2+\left(d_{ij}-d_j\right)^2\right.$$
$$\left.+\left(\pi^{l}{}_{ij}-\pi^{l}{}_{j}\right)^2+\left(\pi^{u}{}_{ij}-\pi^{u}{}_{j}\right)^2\right)^{1/2} \tag{18}$$

where $\pi^{l}{}_{ij} = 1-b_{ij}-d_{ij},\ \pi^{u}{}_{ij} = 1-a_{ij}-c_{ij}, \pi^{l}{}_{j} = 1-b_j-d_j$ and $\pi^{u}{}_{j} = 1-a_j-c_j$

Step 6. To combine the distance separation measure proposed in this study, the new reflection defuzzification integration formula is applied as shown in equation (19) for both the positive and negative distance points respectively.

$$D^{+}{}_{i}(A^{+},\ \boldsymbol{A_i})_{\text{total}} = \alpha_1 d^{+}{}_{i}(A^{+},\ \boldsymbol{A_i}) + \alpha_2 d^{+}{}_{i}(A^{+},\ \boldsymbol{A_i})$$
$$D^{-}{}_{i}(A^{-},\boldsymbol{A_i})_{\text{total}} = \alpha_1 d^{-}{}_{i}(A^{-},\boldsymbol{A_i}) + \alpha_2 d^{-}{}_{i}(A^{-},\boldsymbol{A_i}) \tag{19}$$

where $\alpha_1+\alpha_2 = 1$

Step 7. Set a point, say B as the optimized ideal references point $(d_i(A,\ \boldsymbol{A_i})$, for the alternatives that is; B (*min* $d(A^{+},\ \boldsymbol{A_i})$**,** max$d(A^{-},\ \boldsymbol{A_i})$ **).** Then calculate the distances from each alternative. The relative closeness R_i to the ideal solution is calculated using the equation,

$$R_i = \sqrt{[(d(A^{+},\quad \boldsymbol{A_i}), -min\, d(A^{+},\quad \boldsymbol{A_i}))^2 + (d(A^{-},\quad \boldsymbol{A_i}), -max\, d(A^{-},\quad \boldsymbol{A_i})\ ^2]} \tag{20}$$

Step 8. Rank the alternatives in increasing order. However, if there are two alternatives say A_1 and A_2, with $R_1 = R_2$ where$1 \neq 2$, then R_i is calculated using equation (21) to choose the better one with the smaller R_i value for model.

$$R_i = (d(A^{+},\quad \boldsymbol{A_i}), -min\, d(A^{+},\quad \boldsymbol{A_i})) \tag{21}$$

4. Numerical Example

In this section, we demonstrate the computational process of the proposed IVIF-Modified TOPSIS and FAHP algorithm using a practical problem in literature and a real-life product design assessment problem.

Case 1: A hypothetical example originally presented by Ye, (2009) is modified to demonstrate the computational process of the IVIF-Modified-TOPSIS algorithm.

Let us consider a decision-making problem for the selection of a preferred Naval vessel from a group of candidates; S_1, S_2, S_3 and S_4 as a reference for a new design. The expert has to make a decision according to the following, Performance (C_1), Economy (C_2) and Appearance (C_3). If the weight of the criteria is calculated using FAHP model $W = \{0.35, 0.25, 0.40\}$. Then the Vessel is evaluated using the proposed algorithm with respect to the criteria. Following the implementation step of the algorithm the decision matrix $\boldsymbol{D_{mxn}(x_{ij})}$ is determined as shown in the matrix below.

$$\boldsymbol{D_{4x3}(x_{ij})} = \begin{bmatrix} ([0.4,0.5],[0.3,0.4]) & ([0.4,0.6],[0.2,0.4]) & ([0.1,0.3],[0.5,0.6]) \\ ([0.6,0.7],[0.2,0.3]) & ([0.6,0.7],[0.2,0.3]) & ([0.4,0.7],[0.1,0.2]) \\ ([0.3,0.6],[0.3,0.4]) & ([0.5,0.6],[0.3,0.4]) & ([0.5,0.6],[0.1,0.3]) \\ ([0.7,0.8],[0.1,0.2]) & ([0.6,0.7],[0.1,0.3]) & ([0.3,0.4],[0.1,0.2]) \end{bmatrix}$$

Using the improved score function (equation (4)) the interval-valued intuitionistic fuzzy decision matrix $D_{mxn}(x_{ij})$ is converted to the improved score matrix $R_{mxn}\left(I_{ij}\left(a_{ij}\right)\right)$ as show in the matrix.

$$\boldsymbol{R_{mxn}\left(I_{ij}\left(a_{ij}\right)\right)} = \begin{bmatrix} 0.5350 & 0.5800 & 0.2350 \\ 0.7100 & 0.7100 & 0.6850 \\ 0.5100 & 0.6000 & 0.6800 \\ 0.8200 & 0.7400 & 0.5200 \end{bmatrix}$$

By using equation (17) we compute the improved score function-based separation measures $\boldsymbol{(d^+_i(A^+, A_i)_{isf}}$ and $\boldsymbol{(d^-_i(A^-, A_i)_{isf}}$ $\boldsymbol{(i = 1,2,3,4)}$, the weighted normalized Euclidean distance method for the separation measures $\boldsymbol{(d^+_i(A^+, A_i)_{wn}}$ and $\boldsymbol{(d^-_i(A^-, A_i)_{wn}}$ $\boldsymbol{(i = 1,2,3,4)}$, is calculated using equation (18) and finally the results are integrated using the reflection defuzzification integration formula $\boldsymbol{(d^+_i(A^+, A_i)_t}$ and $\boldsymbol{(d^-_i(A^-, A_i)_t}$ given in equation (19). The results of the computations are shown in Table 5.

Table 5
Results of the separation measures

	$(d^+_i(A^+,A_i)_{is}$	$(d^-_i(A^-,A_i)_{is}$	$(d^+_i(A^+,A_i)_{wn}$	$(d^-_i(A^-,A_i)_{wn}$	$(d^-_i(A^-,A_i)_t$	$(d^-_i(A^-,A_i)_t$
A_1	0.362	0.255	0.5748	0.2448	0.937	0.500
A_2	0.177	0.410	0.5545	0.2845	0.732	0.695
A_3	0.236	0.358	0.5548	0.2623	0.791	0.621
A_4	0.212	0.400	0.6494	0.2406	0.862	0.640

Finally, the results for the relative closeness of the alternatives to the ideal solution are calculated using Eq. (20). The individual approaches are calculated and finally, they are compared with that of the traditional TOPSIS model as shown in Table 6.

Table 6
Comparison of Results of the different measures

	Proposed model $(\boldsymbol{R_i})$	Rank	improved score $(\boldsymbol{R_i})$	Rank	weighted normalized Euclidean $(\boldsymbol{R_i})$	Rank	TOPSIS model	Rank	Ye, (2009)	Rank
A_1	0.277	4	0.242	4	0.045	3	0.413	4	0.155	4
A_2	0.013	1	0.000	1	0.000	1	0.698	1	0.433	1
A_3	0.091	2	0.079	3	0.022	2	0.653	2	0.312	3
A_4	0.131	3	0.037	2	0.105	4	0. .603	3	0.365	2

Case 2

In the case study, we consider a real life design concepts problem for the selection of a printed circuit board (PCB) concept for a proposed new car. The preferred PCB design concept is selected from a group of candidate (A_1, A_2, A_3, and A_4) with respect to the criteria; Mass and size (C_1), Ergonomics (C_2), Simple assembly (C_3), Easy handling (C_4), Easy maintenance (C_5), Few production errors (C_6), Cost (C_7), Fewer spec controls (C_8), Safety Standard (C_9), Fulfills environmental standard (C_{10}), Attractive design (C_{11}), and Modifiable (C_{12}) which have a weight values $\omega = \{0.086, 0.086, 0.084, 0.083,\ 0.086, 0.079, 0.081, 0.084, 0.079, 0.081, 0.084, 0.088\}$, respectively, which have been calculated using the FAHP, then we select the preferred PCB design concept from the group of candidates using the proposed model. The overall experts aggregated final preference judgment is given in Table 7.

Table 7

Decision matrix for the proposed fuzzy model

	A_1	A_2	A_3	A_4
C_1	([0.20, 0.48], [0.33, 0.53])	([0.40, 0.65], [0.50, 0.65])	([0.30, 0.53], [0.43, 0.58])	([0.20, 0.48], [0.33, 0.53])
C_2	([0.37, 0.57], [0.48, 0.6])	([0.47, 0.80], [0.65, 0.88])	([0.43, 0.72], [0.55, 0.70])	([0.20, 0.48], [0.33, 0.53])
C_3	([0.43, 0.67], [0.55, 0.85])	([0.27, 0.58], [0.4, 0.60])	([0.17, 0.47]. [0.28, 0.75])	([0.20, 0.48], [0.33, 0.63])
C_4	([0.33, 0.62], [0.45, 0.68])	([0.37, 0.63], [0.50, 0.62])	([0.33, 0.62], [0.45, 0.78])	([0.23, 0.57], [0.35, 0.53])
C_5	([0.30, 0.53], [0.43, 0.63])	([0.53, 0.77], [0.65, 0.68])	([0.30, 0.58], [0.43, 0.50])	([0.37, 0.63], [0.50, 0.58])
C_6	([0.27, 0.52], [0.38, 0.58])	([0.53, 0.77], [0.65, 0.90])	([0.23, 0.57], [0.63, 0.35])	([0.43, 0.67], [0.55, 0.67])
C_7	([0.43, 0.72], [0.55, 0.55])	([0.43, 0.70], [0.58, 0.65])	([0.37, 0.63], [0.58, 0.50])	([0.33, 0.62], [0.45, 0.67])
C_8	([0.37, 0.57], [0.48, 0.78)	([0.37, 0.57], [0.48, 0.88])	([0.17, 0.40], [0.32, 0.67])	([0.23, 0.50], [0.38, 0.63])
C_9	([0.30, 0.58], [0.43, 0.6])	([0.47, 0.78], [0.60, 0.60])	([0.20, 0.55], [0.30, 0.48])	([0.40, 0.65], [0.50, 0.57])
C_{10}	([0.33, 0.62], [0.45, 0.65])	([0.37, 0.57], [0.48, 0.85])	([0.30, 0.60], [0.45, 0.55])	([0.47, 0.73], [0.60, 0.65])
C_{11}	(0.30, 0.58], [0.43, 0.63])	([0.10, 0.30], [0.25, 0.60])	([0.23, 0.43], [0.37, 0.65])	([0.43, 0.67], [0.55, 0.78])
C_{12}	(0.40, 0.65], [0.50, 0.58])	([0.27, 0.58], [0.40, 0.68])	([0.33, 0.62], [0.45, 0.60])	([0.37, 0.68], [0.50, 0.73])

Using the improved score function (equation (4)) the interval-valued intuitionistic fuzzy decision matrix $D_{mxn}(x_{ij})$ is converted to the improved score matrix $R_{mxn}\left(I_{ij}\left(a_{ij}\right)\right)$ as show in the Table 8.

Table 8

Improved score matrix

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
A_1	0.385	0.449	0.380	0.418	0.413	0.416	0.482	0.398	0.428	0.428	0.420	0.470
A_2	0.448	0.335	0.417	0.445	0.429	0.344	0.440	0.370	0.460	0.378	0.248	0.394
A_3	0.426	0.428	0.315	0.387	0.457	0.439	0.468	0.314	0.417	0.443	0.359	0.443
A_4	0.385	0.385	0.361	0.420	0.455	0.440	0.421	0.377	0.474	0.445	0.404	0.410

By using Eq. (17) we compute the improved score function-based separation measures $(\boldsymbol{d^+}_{\boldsymbol{i}}(\boldsymbol{A^+},\ \boldsymbol{A_i})_{\boldsymbol{isf}}$ and $(\boldsymbol{d^-}_{\boldsymbol{i}}(\boldsymbol{A^-},\ \boldsymbol{A_i})_{\boldsymbol{isf}}\ (\boldsymbol{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4})$, the weighted normalized Euclidean distance method for the separation measures $(\boldsymbol{d^+}_{\boldsymbol{i}}(\boldsymbol{A^+},\ \boldsymbol{A_i})_{\boldsymbol{wn}}$ and $(\boldsymbol{d^-}_{\boldsymbol{i}}(\boldsymbol{A^-},\ \boldsymbol{A_i})_{\boldsymbol{wn}}\ (\boldsymbol{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4})$, is calculated using equation (18) and finally the results are integrated using the reflection defuzzification integration formula $(\boldsymbol{d^+}_{\boldsymbol{i}}(\boldsymbol{A^+},\ \boldsymbol{A_i})_{\boldsymbol{t}}$ and $(\boldsymbol{d^-}_{\boldsymbol{i}}(\boldsymbol{A^-},\ \boldsymbol{A_i})_{\boldsymbol{t}}$ given in Eq. (19). The results of the computations are shown in Table 9.

Table 9

Results of the separation measures

	$(\boldsymbol{d^+}_{\boldsymbol{i}}(\boldsymbol{A^+}, \boldsymbol{A_i})_{\boldsymbol{is}}$	$(\boldsymbol{d^-}_{\boldsymbol{i}}(\boldsymbol{A^-}, \boldsymbol{A_i})_{\boldsymbol{is}}$	$(\boldsymbol{d^+}_{\boldsymbol{i}}(\boldsymbol{A^+}, \boldsymbol{A_i})_{wn}$	$(\boldsymbol{d^-}_{\boldsymbol{i}}(\boldsymbol{A^-}, \boldsymbol{A_i})_{wn}$	$(\boldsymbol{d^-}_{\boldsymbol{i}}(\boldsymbol{A^-}, \boldsymbol{A_i})_{t}$	$(\boldsymbol{d^-}_{\boldsymbol{i}}(\boldsymbol{A^-}, \boldsymbol{A_i})_{t}$
A_1	0.167	0.123	0.586	0.510	0.753	0.633
A_2	0.177	0.115	0.629	0.541	0.806	0.656
A_3	0.172	0.119	0.601	0.516	0.773	0.635
A_4	0.170	0.120	0.594	0.523	0.764	0.643

Finally, the results for the relative closeness R_i , $(i = 1,2,3,4)$ to the ideal solution is calculated using Eq. (20), the results is given as;

$R_1 = 0.023,\ R_2 = 0.053,\ R_3 = 0.029,$ and $R_4 = 0.017$, therefore the ranking orders for the four candidates are in the form (increasing order) $A_4 < A_1 < A_3 < A_2$), obviously, A_4 is the best candidate according to the model.

5. Conclusion

This paper has presented a new approach for design concept selection using an integrated FAHP and the IVIF-modified TOPSIS model. The integrated model which has used the improved score function originally proposed by Bai (2013) and the weighted normalized Euclidean distance method for the calculation of the separation measures of alternatives from the positive and negative intuitionistic ideal solutions provides a whole new approach for the computation of intuitionistic fuzzy ideal solutions. The application of the weighted normalized Euclidean distance method along with the improved score function is mainly to avoid the bias of using a single separation distance measure or the confusion in determining the specific separation distance measure that is fittest. The results of the two approaches have been integrated using reflection defuzzification integration formula. To ensure the feasibility and rationality of the proposed integrated model, it was applied for evaluating and selection of some design related problems including the real-life case study for the selection of the best concept design for a new printed-circuit-board (PCB). Finally, it was also applied for a modified hypothetical example which was compared with similar some computational methods in the literature. In the future, we hope to apply the proposed model to MCDM problems in other domains.

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