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Counting states and the Hadron Resonance Gas: Does X(3872) count?

Pablo G. Ortega [a,*], David R. Entem [a], Francisco Fernández [a], Enrique Ruiz Arriola [b]

[a] Grupo de Física Nuclear and Instituto Universitario de Física Fundamental y Matemáticas (IUFFyM), Universidad de Salamanca, E-37008 Salamanca, Spain
[b] Departamento de Física Atómica, Molecular y Nuclear and Instituto Carlos I de Física Teórica y Computacional, Universidad de Granada, E-18071 Granada, Spain

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ABSTRACT

We analyze how the renowned X(3872), a weakly bound state right below the $D\bar{D}^*$ threshold, should effectively be included in a hadronic representation of the QCD partition function. This can be decided by analyzing the $D\bar{D}^*$ scattering phase-shifts in the $J^{PC}=1^{++}$ channel and their contribution to the level density in the continuum from which the abundance in a hot medium can be determined. We show that in a purely molecular picture the bound state contribution cancels the continuum providing a vanishing occupation number density at finite temperature and the $X(3872)$ does not count below the Quark-Gluon Plasma crossover happening at $T\sim 150$ MeV. In contrast, within a coupled-channels approach, for a non vanishing $c\bar{c}$ content the cancellation does not occur due to the onset of the $X(3940)$ which effectively counts as an elementary particle for temperatures above $T\gtrsim 250$ MeV. Thus, a direct inclusion of the $X(3872)$ in the Hadron Resonance Gas is not justified. We also estimate the role of this cancellation in X(3872) production in heavy-ion collision experiments in terms of the corresponding p_T distribution due to a finite energy resolution.

1. Introduction

Counting hadronic states below a certain mass and QCD thermodynamics at finite temperature in a box with a finite volume are intimately related. However, while the counting process requires an individual knowledge of the mass spectrum, thermodynamics generally implies a collective information. Experimentally both pieces of information are obtained by different means; while the single states are determined one by one by spectroscopic measurements and the analysis of hadronic reactions the determination of thermal properties acquires a more macroscopic nature such as in ultra-relativistic heavy ions collisions. Within such context basic objects are occupation numbers and their corresponding transverse momentum and rapidity distributions which are extracted from experiment if assumptions on the fireball freeze-out dynamics are implemented (see e.g. [1] and references therein).

Specifically, the coupling of *any* hadronic state to a heat bath at temperature T is universally given by the Boltzmann factor,

$$Z=\sum_n e^{-M_n/T}=\int dM\rho(M)e^{-M/T}\,. \tag{1}$$

Here M_n mean the QCD (discretized) eigenstates in a finite box which due to confinement are colour neutral and $\rho(M)=N'(M)$ is the density of states where $N(M)$ is the cumulative number of states

$$N(M)=\sum_n \theta(M-M_n)\,. \tag{2}$$

At small temperatures and due to confinement we expect hadronic states to saturate the partition function. Based on the quantum virial expansion in quantum mechanics [2] and quantum field theory [3] a genuine hadronic representation was derived in terms of the S-matrix in the continuum limit, $N(M)=\mathrm{Tr}\log S/2\pi i$ where the cumulative number becomes a real, non-integer, number. In this case, the actual implementation of this approach requires, besides taking the box volume to infinity, consideration of interactions among multiparticle states built from the asymptotic scattering free states. This means that only ground states of the strong interaction (in the confined phase) should be used in constructing the Fock space. At sufficiently low temperatures, lowest masses dominate and one has to successively incorporate π, 2π, 3π, η, K, etc. While two-body states can be described by phase-shifts [2], the three body contribution is a complex problem, making the approach unmanageable without further approximations (see Ref. [4] for a recent and promising attempt to address the $(N>2)$-body problem in a model-independent way). Fortunately, as pointed out

* Corresponding author.
E-mail addresses: pgortega@usal.es (P.G. Ortega), entem@usal.es (D.R. Entem), fdz@usal.es (F. Fernández), earriola@ugr.es (E. Ruiz Arriola).

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soon after [3] the role of narrow resonances [5] and effective elementarity [6] was shown to reduce the thermodynamics of QCD in the confined phase to a Hadron Resonance Gas (HRG), where the hadronic states are identified and counted *one by one* effectively entering the partition function as single particle states.[1] In the mid 60's Hagedorn analyzed the mass-level density $\rho(M) = N'(M)$ and, conjecturing the validity of the HRG, *predicted* the bulk of states at higher masses, which later on were experimentally confirmed [7]. The more recent updates in [8,9] proposed to use directly $N(M)$ as the relevant quantity, which features explicitly the notion of counting as shown in Eq. (2). Overall, resonance widths (in the Breit–Wigner approximation) have the effect of reshuffling the mass distribution around the resonance mass value and hence increasing, regularizing, i.e. making it smooth, and "de-quantizing" this quantity [10,11].

The commonly accepted reference for hadronic states is the Particle Data Group (PDG) table [12], a compilation reflecting a consensus in the particle physics community whose cumulative number $N_{\rm PDG}(M)$ has most spectacularly been checked by the computation of the trace anomaly, $\epsilon - 3P = T^5 \partial_T (\log Z/T^3)/V$, on the lattice [13–15] at temperatures $T \lesssim 200$ MeV below the crossover to the Quark-Gluon Plasma (QGP) phase. It is worth noting that this agreement between the WB [13,15] and the HotQCD [14] lattice collaborations and with the HRG has come after many years of frustration and controversy. Width effects reflect the mass reshuffling by increasing the trace anomaly and agree still within the lattice uncertainties [10,11] (see e.g. Ref. [16] for a pedagogical exposition and overview).

These results suggest that *all* states listed by the PDG should also be counted in the cumulative number as genuine contributions to the QCD partition function and hence directly included in the HRG. However, in a remarkable and forgotten paper Dashen and Kane pointed out the possibility that not all hadron states should be counted on a hadronic scale [17] as they become fluctuations in a mass-spectrum coarse grained sense. The deuteron, a $J^{PC} = 1^{++}$ np composite, was prompted as a non-controversial example where the weak binding effect is compensated by the nearby np continuum yielding an overall vanishing contribution. The basic idea was that certain interactions do not generate new states but simply reorder the already existing ones (see [18] for an explicit figure of the cumulative number in the deuteron channel).

The possibility of having loosely bound states near the charm threshold, i.e. Charm Molecules, was envisaged long ago [19]. Actually, the discovery of the state $X(3872)$ in 2003 by the Belle Collaboration in the exclusive $B^\pm \to K^\pm \pi^+\pi^- J/\psi$ decay [20] has initiated a new era in hadronic spectroscopy. This state decays through the $J/\psi\rho$ and $J/\psi\omega$ channels which are forbidden for a $c\bar{c}$ configuration and has $J^{PC} = 1^{++}$ as concluded by the LHCb Experiment by means of the five-dimensional angular analysis of the process $B^+ \to K^+ X(3872)$ with $X(3872) \to J/\psi \rho^0 \to J/\psi \pi^+\pi^-$ [21]. As a natural consequence this state has entered the PDG with a current binding energy of $B_X \equiv M_X - M_{D^0} - M_{\bar{D}^{0*}} = 0.01(18)$ MeV [12].

The proliferation of new X,Y,Z states (see [22] for a recent review) and their inclusion in the PDG poses the natural question whether or not these states have some degree of redundancy in order to build the hadron spectrum. The possibility that this might happen for some weakly bound X,Y,Z states has been suggested recently [16,18]. In the present paper we analyze this issue for the renowned $X(3872)$ case by analyzing for the first time $D\bar{D}^*$ scattering and show that the answer to this question depends on the particular dynamics of the system. This is particularly relevant as recently the p_T distribution of the $X(3872)$ in pp collisions have been determined both theoretically [23,24] and experimentally by CMS [25] and ATLAS [26] and the possible implications on the molecular content have been examined [27]. Our results apply specifically to $X(3872)$ production in heavy-ion collisions, for which no experiments exist yet.

2. Counting states and their abundance

For an elementary and free state with g-degrees of freedom and mass m in a medium with temperature T the average density of particles is given by

$$\bar{n} = \frac{\langle N \rangle_T}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{g}{e^{\sqrt{k^2+m^2}/T} + \eta} = \frac{T^3}{2\pi^2} \sum_{n=1}^{\infty} g \frac{(-\eta)^{n+1}}{n} \left(\frac{m}{T}\right)^2 K_2(nm/T)\,, \tag{3}$$

where $K_2(x)$ is the modified Bessel function and $\eta = \mp 1$ for bosons/fermions respectively.[2] In the case of composite particles or two-body interacting particles, according to the quantum virial expansion [2,3] the effects of interactions can be expressed in terms of scattering phase shifts

$$n(T) = \int \frac{d^3p}{(2\pi)^3} dm \frac{g}{e^{\sqrt{p^2+m^2}/T} + \eta} \rho(m)\,, \tag{4}$$

where

$$\rho(m) = \frac{1}{\pi} \frac{d\delta}{dm}\,. \tag{5}$$

For a narrow resonance with mass m_R and width $\Gamma_R \to 0$ the phase-shift can be described by a Breit–Wigner shape $\delta(m) = \tan^{-1}[(m - m_R)/\Gamma_R]$ so that $\delta'(m) \to \pi\delta(m - m_R)$, and their contribution becomes that of an elementary particle with mass m_R [5]. For instance, in the case of $\pi\pi$ scattering in the isovector channel the contribution is given by the corresponding ρ resonance. Interestingly, cancellations among different $\pi\pi$ and πK channels have been reported [28–31] implying, for instance, that the lowest 0^{++} isoscalar state, quoted as the $f_0(500)$ in the PDG and also known as the σ meson cancels the isotensor contribution, i.e. $\delta_{00} + 5\delta_{0,2} = 0$ within uncertainties for $\sqrt{s} \le 900$ MeV [32]. This is essentially a cancellation between the attraction in the $I = 0$ channel generating the resonance and a repulsive in the $I = 2$ channel possibly triggered by the finite pion-size generating a hard core.

Here, we address a different type of cancellation unveiled by Dashen and Kane [17], namely the fact that for a certain type of loosely bound state, the contribution may effectively vanish. For completeness, let us review briefly their argument. The cumulative number in a given channel in the continuum with threshold $M_{\rm th}$ is

$$N(M) = \sum_n \theta(M - M_n^B) + \frac{1}{\pi} \sum_{\alpha=1}^{K} [\delta_\alpha(M) - \delta_\alpha(M_{\rm th})]\,. \tag{6}$$

Here the bound states masses M_n^B have been explicitly separated from scattering states written in terms of the eigenvalues of the S-matrix, i.e. $S = U {\rm Diag}(\delta_1, \dots, \delta_K) U^\dagger$ with U a unitary transformation for K-coupled channels. With this definition we have $N(0) = 0$,

[1] This way one handles, e.g., three body interactions as two-step processes mediated by resonant scattering; if $2\pi \to \rho$, then $3\pi \to \pi\rho \to \omega, A_1$ and so on.

[2] In practice the Boltzmann approximation (i.e., just keeping $n = 1$) is sufficient for low temperatures.

and in the single channel case, in the limit of high masses $M \to \infty$ becomes

$$N(\infty) = n_B + \frac{1}{\pi}[\delta(\infty) - \delta(M_{\rm th})] = 0 \quad (7)$$

due to Levinson's theorem which is the statement that the total number of states does not depend on the interaction. In the NN channel where $M_{\rm th} = 2M_N$ the appearance of the deuteron changes rapidly at $M = 2M_N - B_d$ by one unit so that $N(2M_N - B_d + 0^+) - N(2M_N - B_d - 0^+) = 1$, but when we increase the energy this number decreases slowly to zero at about pion production threshold $N(2M_N + m_\pi) - N(2M_N - B_d - 0^+) \sim 0$. This features are depicted in Ref. [18] for $\sqrt{s}$ up to 3.5 GeV. A direct consequence of this is that the deuteron abundance at hadronic temperatures will be almost zero! This effect is explicitly seen in the np virial coefficient at rather low temperatures [33].

3. The X(3872) and $D\bar{D}^*$ Scattering in the molecular picture

While $X(3872)$ is most naturally defined as a pole of the $D\bar{D}^*$ scattering amplitude, to our knowledge the physically meaningful phase-shifts have never been explicitly analyzed. Actually, the QCD evidence for $X(3872)$ on the lattice has been pointed out [34] by analyzing the energy shifts on a finite volume by means of the Lüscher's formula where the connection to $D\bar{D}^*$ scattering is established.

The weak binding of the $X(3872)$ has suggested in the early studies a purely molecular nature. It is instructive to analyze scattering within a purely hadronic picture of contact interaction [35], with the hope that short distance details can be safely ignored.[3] If we take an interaction of the form $V_0(k',k) = C_0 g(k') g(k)$, the phase shift is given by (see e.g. Ref. [36]),

$$\begin{aligned} p\cot\delta_0(p) &= -\frac{1}{V_0(p,p)}\left[1 - \frac{2}{\pi}\int_0^\infty dq \frac{q^2}{p^2 - q^2} V_0(q,q)\right] \\ &= -\frac{1}{\alpha_0} + \frac{1}{2} r_0 p^2 + \ldots \end{aligned} \quad (8)$$

where in the last line a low momentum Effective Range Expansion (ERE) has been carried out, identifying α_0 with the scattering length and r_0 with the effective range. Fixing $\alpha_0 = 3.14$ fm and $r_0 = 1.25$ fm (see next Section) we get the phase shift and using Eq. (6) we get the cumulative number including the continuum states depicted in Fig. 1 compared with the case where *only* the X(3872) is considered.[4] This illustrates the point made by Dashen and Kane [17] in the case of the $X(3872)$, showing that in the molecular picture the state *does not* count in the $D\bar{D}^*$ continuum on coarse mass scales of about $\Delta M_{D\bar{D}^*} \sim 200$ MeV.[5]

4. The X(3872) and $D\bar{D}^*$ Scattering in the cluster quark model picture

The multichannel scattering problem with confined intermediate states was initiated after the first charmonium evidences

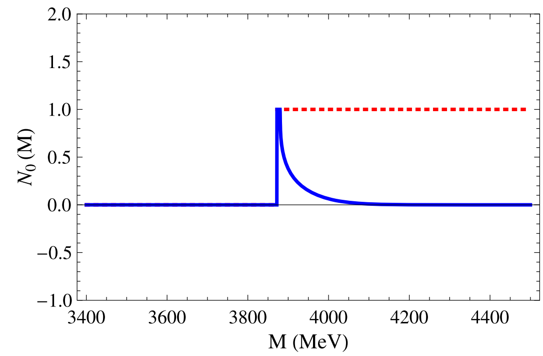

Fig. 1. (Color online.) Cumulative number in the 1^{++} channel as a function of the $D\bar{D}^*$ mass (in MeV) for the $X(3872)$ *only* (dotted, red) and the full contribution including the continuum (full, blue).

based on the decomposition of the Hilbert space as $\mathscr{H} = \mathscr{H}_{c\bar{c}} \oplus \mathscr{H}_{D\bar{D}}$ [37,38]. In the multichannel case with permanently confined channels, Levinson's theorem is modified [37] by subtracting the number of bound states of the purely confining potential, n_c, so that $N(\infty) = n_c$ in Eq. (6).

A coupled-channels calculation which included such decomposition was addressed in Ref. [39], performed in the framework of the constituent quark model (CQM) proposed in Ref. [40]. This CQM has been extensively used to describe the hadron phenomenology both in the light [41] and the heavy quark sectors [42,43]. In Ref. [39], the $X(3872)$ resonance together with the $X(3940)$ have been explained as two $J^{PC} = 1^{++}$ states, being the $X(3872)$ basically a $D\bar{D}^*$ + h.c. molecule with a small amount of 2^3P_1 $c\bar{c}$ state while the $X(3940)$ is a mixture with more than 60% of $c\bar{c}$ structure. Actually, in the absence of mixing, $X(3940)$ becomes a pure $c\bar{c}$ state, and the only confined state in the $J^{PC} = 1^{++}$ channel. The aim of Ref. [39] (extended in Ref. [44]) was to study the $J^{PC} = 1^{++}$ sector including the effect of the closest $c\bar{c}$ states in the dynamics of the $D\bar{D}^*$ channel. For simplicity, we will consider the $D^{(*)}$ mesons as effectively stable, due to their narrow width, and we will only consider the isospin-zero $D\bar{D}^*$ channel, as the $c\bar{c} - DD^*$ coupling mechanism occurs solely in $I = 0$. The isospin breaking coming from the $D^{(*)\pm} - D^{(*)0}$ mass differences does introduce a sizable $I = 1$ component in the wave function of the $X(3872)$ [44], but we have checked that it does not alter the conclusions reached in this work.

We adopt the coupled-channels formalism described already in Ref. [44] and decompose the hadronic state as

$$|\Psi\rangle = \sum_\alpha c_\alpha |\psi_\alpha\rangle + \sum_\beta \chi_\beta(P) |\phi_A \phi_B \beta\rangle, \quad (9)$$

where $|\psi_\alpha\rangle$ are $c\bar{c}$ eigenstates of the two body Hamiltonian, ϕ_M are $q\bar{q}$ eigenstates describing the A and B mesons, $|\phi_A \phi_B \beta\rangle$ is the two meson state with β quantum numbers coupled to total J^{PC} quantum numbers and $\chi_\beta(P)$ is the relative wave function between the two mesons in the molecule.

In this formalism, in addition to the direct meson-meson interaction due to the exchange of pseudo-Goldstone bosons at $q\bar{q}$ level described by the aforementioned CQM [40], with parameters updated at Ref. [42] for the heavy quark sectors, two- and four-quark configurations are coupled using the $^3P_0$ model [45,46], the same transition mechanism that, within our approach, allows us to compute open-flavor meson strong decays. This model assumes that the transition operator is

$$T = -3\sqrt{2}\gamma' \sum_\mu \int d^3p\, d^3p'\, \delta^{(3)}(p + p') \times$$

[3] Isospin effects have been considered in [35] where the coupling of the X to the neutral and charged components is very similar. Here we will ignore the effect and take an average value for the binding.

[4] We use the Gaussian regulator $g(k) = e^{-k^2/\Lambda^2}$ and obtain $C_0 = -1.99$ fm and $\Lambda = 2.05$ fm^{-1}. The pole in the scattering amplitude is at $k_X = i0.43$ fm^{-1} corresponding to $M_X = 3868$ MeV. Note that we disregard isospin effects, see Ref. [35] otherwise. Other smooth regulators give similar results.

[5] The resemblance with the deuteron case is striking, see Ref. [18] for $\sqrt{s}$ up to 3.5 GeV, where mass scales are about a half, $M_N \sim M_D/2$ and $M_d \sim M_X/2$ as in the $X(3872)$. So, the coarse mass scale here is $\Delta M_{NN} \sim \Delta M_{D\bar{D}^*}/2 \sim 100$ MeV.

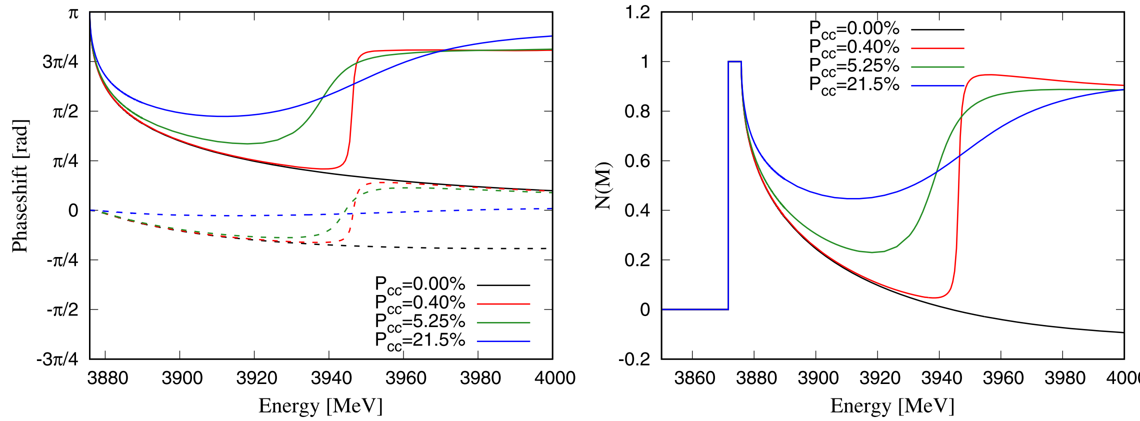

Fig. 2. (Color online.) Left panel: S- (solid) and D-wave (dashed) phase-shifts in radians as a function of the $D\bar{D}^*$ invariant mass. Right panel: Cumulative number in the $X(3872)$ channel as a function of the $D\bar{D}^*$ mass.

$$\times \left[\mathscr{Y}_1\left(\frac{p-p'}{2}\right) b^\dagger_\mu(p) d^\dagger_\nu(p')\right]^{C=1,I=0,S=1,J=0}, \tag{10}$$

where μ ($\nu = \bar{\mu}$) are the quark (antiquark) quantum numbers and $\gamma' = 2^{5/2}\pi^{1/2}\gamma$ with $\gamma = \frac{g}{2m}$ is a dimensionless constant that gives the strength of the $q\bar{q}$ pair creation from the vacuum. From this operator we define the transition potential $h_{\beta\alpha}(P)$ within the $^3P_0$ model as [47]

$$\langle \phi_A \phi_B \beta | T | \psi_\alpha \rangle = P\, h_{\beta\alpha}(P)\, \delta^{(3)}(\vec{P}_{\text{cm}}). \tag{11}$$

Using the latter coupling mechanism, the coupled-channels system can be expressed as a Schrödinger-type equation,

$$\sum_\beta \int \left(H_{\beta'\beta}(P',P) + V^{\text{eff}}_{\beta'\beta}(P',P)\right)\times \\ \times \chi_\beta(P) P^2 dP = E\chi_{\beta'}(P'), \tag{12}$$

where $H_{\beta'\beta}$ is the Resonating Group Method (RGM) Hamiltonian for the two-meson states obtained from the $q\bar{q}$ interaction. The effective potential $V^{\text{eff}}_{\beta'\beta}$ encodes the coupling with the $c\bar{c}$ bare spectrum, and can be written as

$$V^{\text{eff}}_{\beta'\beta}(P',P;E) = \sum_\alpha \frac{h_{\beta'\alpha}(P') h_{\alpha\beta}(P)}{E - M_\alpha}, \tag{13}$$

where M_α are the masses of the bare $c\bar{c}$ mesons.

In the cluster quark model picture the interaction between quarks contains a tensor force due to pion exchange. Besides, the effective potential $V^{\text{eff}}_{\beta'\beta}$ mixes different partial waves. Therefore, the S-matrix couples S and D waves,

$$S^{J1} = \begin{pmatrix} \cos\epsilon_j & -\sin\epsilon_j \\ \sin\epsilon_j & \cos\epsilon_j \end{pmatrix} \begin{pmatrix} e^{2i\delta^{1j}_{j-1}} & 0 \\ 0 & e^{2i\delta^{1j}_{j+1}} \end{pmatrix} \\ \times \begin{pmatrix} \cos\epsilon_j & -\sin\epsilon_j \\ \sin\epsilon_j & \cos\epsilon_j \end{pmatrix}. \tag{14}$$

From here we define the T-matrix

$$S^{JS} = 1 - 2ikT^{JS}. \tag{15}$$

The S and D eigen phase-shifts are shown in Fig. 2 together with the result for the cumulative number. The outstanding feature is the turnover of the function as soon as a slightly non-vanishing $c\bar{c}$ content in the $X(3872)$ is included, unlike the purely molecular picture. The steep rise in the phase shift corresponds to a resonant state located at a mass $M \sim 3945$ MeV and may be identified with the $X(3940)$ which in the purely molecular picture would disappear as the $c\bar{c}$ spectrum would decouple from the $D\bar{D}^*$ scattering. Thus, the raise in the 1^{++} channel is not due to the $X(3872)$ but to the onset of the $X(3940)$ resonance. The PDG values for X(3940) $M = 3942(9)$ MeV and $\Gamma = 37^{+27}_{-17}$ MeV [12] suggests indeed a non-vanishing mixing and $P_{\bar{c}c} = 5 - 25\%$ for the X(3872). Moreover, we have checked that the S-wave phase-shift asymptotically approaches π (due to the bound X(3940)-state of the purely confined channel) and hence $N(\infty) = \pi$ in agreement with the modified Levinson's theorem [37].

Table 1
$X(3872)$ $c\bar{c}$ probability, scattering length and effective range for the S-wave as a function of the dimensionless constant γ of the $^3P_0$ transition operator. The mass of the $D\bar{D}^*$ bound state $X(3872)$ is fixed at 3871.7 MeV. The mass and width of the $X(3940)$ resonance is also shown (PDG values are [12] $M = 3942(9)$ MeV and $\Gamma = 37^{+27}_{-17}$ MeV).

$\gamma(^3P_0)$	$\mathscr{P}_{c\bar{c}}$ [%]	α_0 [fm]	r_0 [fm]	M [MeV]	Γ [MeV]
0.00	0.00	3.14	1.21	3947.43	0.00
0.05	0.40	3.14	1.20	3946.29	1.38
0.10	1.82	3.11	1.17	3943.06	5.88
0.16	5.25	3.05	1.10	3938.56	15.18
0.20	14.25	2.88	0.85	3937.09	37.93
0.23	21.50	2.73	0.63	3947.05	56.03

In Ref. [39] the $^3P_0$-model γ parameter of Eq. (10) was constrained via strong decays in the charmonium spectrum. However, in the present study we analyze the effect of adiabatically connecting the $c\bar{c}$ spectrum and the $D\bar{D}^*$, so we will vary γ from zero to the value used in Ref. [39], maintaining the mass of the bound state fixed at the experimental 3871.7 MeV by consequently adapting the strength of the direct meson-meson interaction. Besides this re-scaling we take exactly the parameters of Ref. [39]. The $X(3940)$ and the S-wave effective range expansion parameters, are given in Table 1 for different γ values, where for the coupled-channels version of Eq. (8) we follow Ref. [48] adapted to the present situation. These values should be compared with the lattice results [34] for $m_\pi = 266$ MeV of $\alpha_0 = 1.7(4)$fm and $r_0 = 0.5(1)$fm extracted from finite volume calculations, bearing in mind that they found a binding energy of -11 ± 7 MeV below the $D^0\bar{D}^{0*}$ threshold.

5. Finite temperature and X(3872) production

Finally, we turn now to the consequences for finite temperature calculations. The level density and the corresponding occupation number (relative to the elementary one) are shown in Fig. 3 as

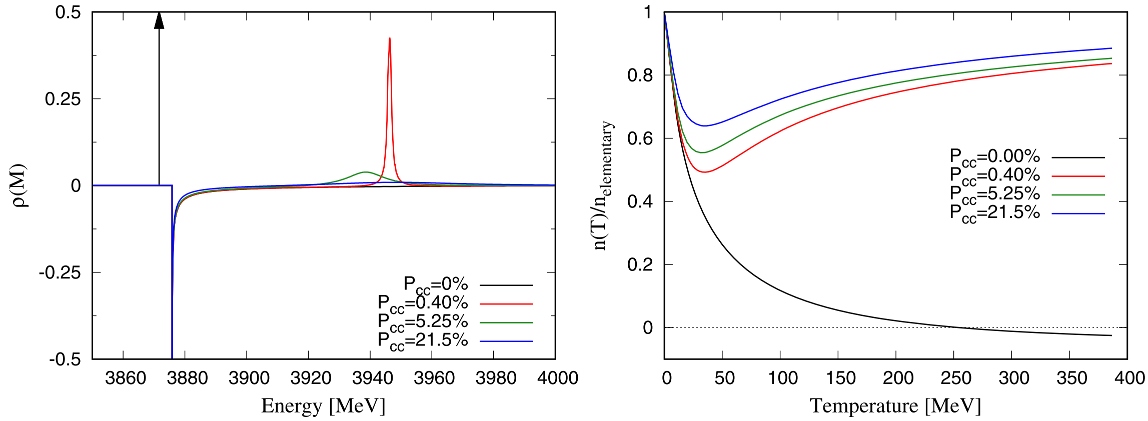

Fig. 3. (Color online.) Left panel: Total Level density $\rho(M)$ (Eq. (5)) of the $D\bar{D}^*$ in the $J^{PC}=1^{++}$ channel as a function of the mass. The arrow indicates the contribution of the $X(3872)$ bound state, which is a Dirac delta $\delta(m-m_X)$. Right panel: Occupation number $n(T)$ of the $D\bar{D}^*$ in the $J^{PC}=1^{++}$ channel, as a function of the temperature T (in MeV), with respect to the contribution of the $X(3872)$ assuming it is an elementary particle and no continuum contribution (Eq. (3)).

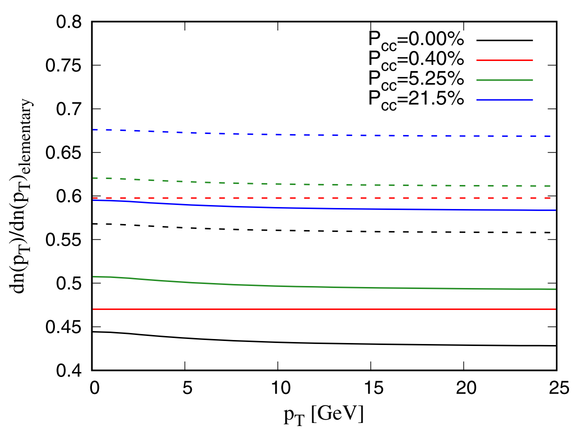

Fig. 4. (Color online.) Relative p_T distribution (see Eq. (16)) of the $J^{PC}=1^{++}$-channel with a binning of $\Delta m = 2B_X$ MeV (dashed) $\Delta m = 5B_X$ MeV (solid) for different $P_{\bar{c}c}$ content.

functions of the invariant mass (left) and the temperature (right). As we see that the cancellation between the bound state and the continuum only happens for zero $c\bar{c}$ probability content, when the $c\bar{c}$ spectrum is decoupled from the $D\bar{D}^*$ scattering. However, note that the non-vanishing occupation number is merely due to the resonant reaction $D\bar{D}^* \to X(3940) \to D\bar{D}^*$. This is exactly the same feature observed in $\pi\pi$ scattering in the 1^{--} channel to the $\pi\pi \to \rho \to \pi\pi$ resonant reaction [28–30].

Of course, one may wonder what is the range of applicability of the present calculation, particularly as a function of the temperature. At higher temperatures effects of hadron dissociation sets in, accompanied by the explicit emergence of the quarks and gluons degrees of freedom. The hadronic state representation would then, presumably, break down. This is supported by recent lattice calculations, when combinations of higher order fluctuations are computed [49] and found to vanish for hadrons (in the Boltzmann approximation) but not for quarks, and is found to be non-vanishing for $T > 154$ MeV. Our Fig. 3 vividly shows that the effect is quite visible before hadron dissociation, and should thus be relevant in the study of production and absorption of $X(3872)$ in a hot medium such as the one generated in heavy ion collisions [1].

The real experiments in pp-collisions uses a finite binning step $\Delta m = 3$ MeV [25] and $\Delta m = 1.5$ MeV [26]. We note that this is 10–15 times *much larger* than the binding energy of $B_X = 0.01(18)$ MeV quoted by the PDG [12]. Therefore, any signal contains a contamination of continuum and bound states in the 1^{++} channel and it is foreseeable that future experiments in heavy ion collisions will implement a similar Δm.

Actually, the p_T distribution at mid-rapidity of a fireball at rest [50] stemming from an invariant mass distribution $\rho(m)$ binned with step Δm in the notation of Eq. (3) is given by

$$\frac{d\bar{n}(p_T)}{dm_T^2} = \int_{\Delta m} \rho(m)dm \sum_{n=1}^{\infty} \frac{gm_T}{(2\pi)^2}\frac{(-\eta)^{n+1}}{n} K_1\left(\frac{nm_T}{T}\right) \tag{16}$$

where $m_T^2 = p_T^2 + m^2$ and the integral extends over $M_X \pm \Delta m/2$. The result of the ratio of the finite-Δm binned to the elementary p_T distribution is shown in Fig. 4 for $T = 200$ MeV. Neglecting isospin effects we have in the model $B_X = 4$ MeV, so that we take $\Delta m = 2B_X$ and $\Delta m = 5B_X$ to illustrate the situation. As we see, the effect is dramatic in the strength which is reduced by almost 50% and is saturated when the binning is larger than $\Delta m = 5B_X$. We also see that the p_T dependence is not affected much in a wide range. In a future publication we will provide a more comprehensive analysis including current freeze-out models, such as blast-wave or Hubble-like expansion patterns which might realistically be tested with future heavy ion X(3872) production experiments. This would require, in particular, a fine tuning of parameters of Ref. [39] to account for the most recent PDG figures [12].

6. Conclusions

The production and absorption of $X(3872)$ in high energy heavy ion collisions [51] or the time evolution of the $X(3872)$ abundance in a hot hadron gas [52] has been investigated recently in an attempt to pin down its structure from its behavior in the Quark-Gluon Plasma (QGP). Abundances depend on the nature of the state. These studies echo an opposite strategy with similar studies of J/Ψ where the melting of this very well known state is used to diagnose the QGP. Our calculation shows that a possible signal for $X(3872)$ abundance might in fact be erroneously confused with the $X(3940)$ as a non-vanishing occupation number of the $D\bar{D}^*$ spectrum in the 1^{++} channel at temperatures above the crossover to the QGP phase. Below this temperature, the $X(3872)$ does not count and should not be included in the Hadron Resonance Gas. The Dashen–Kane effect extends also to $X(3872)$ production and detection in heavy ions collisions and more generally in any production process where the experimental resolution exceeds the binding energy.

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