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Passenger service optimization through timetabling with free passenger route choice

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Abstract

Designing a public transport timetable that maximizes passenger service, measured in weighted travel time, is an intricate problem. The weighted travel time depends on the free route choice of passengers. Passenger route choice depends on the timetable. In turn, the timetable that minimizes weighted travel time depends on the route choice of passengers – and therefore requires passenger route choice information. Consequently, a sequential approach where timetables are designed provided pre-fixed passenger assignment to routes, may not find the optimal timetable.

This paper aims to integrate passenger route choice and timetabling. It addresses the problem of designing maximal passenger service public transport timetables in systems with free route choice within a budget for operating costs. Operating costs are defined by the minimal cost vehicle schedule required to operate the timetable.

The proposed methodology integrates a matheuristic for timetabling and vehicle scheduling with a passenger assignment model in an iterative framework, where different forms of integration are evaluated. Focus is on long to medium term timetabling, provided an initial timetable. Results for a realistic case study in the Greater Copenhagen area indicate that our approach consistently leads, at no additional cost, to timetables that represent a reduction in passenger weighted travel time in comparison to both an initial timetable and a non-integrated timetabling method that receives a single passenger assignment as input.

Keywords: Public Transport Bus Timetabling Passenger Route Choice Mixed Integer Linear Programming Matheuristic

1 Introduction

Timetabling consists of assigning specific points in time to a set of events. In bus timetabling, this set of events follows from the service network designing, determining a set of lines, each with a given ordered list of stops and a frequency. The output of the timetabling phase serves as input to the vehicle scheduling problem, that assigns vehicles to specific services in the timetable. Timetabling generally represents a balance between high quality passenger service, and low operating costs. Passenger service depends on the timetable, while the operating costs depend on the vehicle assignment. Passenger service can be expressed in terms of *weighted travel time* (WTT): a weighted sum of the components of *initial waiting time* (IWT), *in-vehicle time* (IVT), and *transfer time* (TrT) Comi et al. (2017). Operating costs are expressed as a function of the number of required vehicles and dead-heading distance.

Integrating passenger route choice and timetabling is an intricate problem, as there is a co-dependency between passenger route choice and timetable design. Indeed, minor changes in the timetable can have large effects on the WTT of routes with a transfer. Especially in dense networks, small changes in the timetable could lead to far larger changes in transfer times, and consequently may cause passengers preferring alternative geographical routes. As a result, providing a fixed passenger assignment as input to the timetabling phase could lead to finding sub-optimal timetables. Furthermore, also the vehicle schedules can be strongly affected by small changes in the timetable. To ensure reasonable operating costs of the resulting timetable, it is vital to consider vehicle scheduling as well.

This paper studies the *Integrated Passenger Assignment, Timetabling and Vehicle Scheduling Problem* (IPAT-VSP) at a tactical level. The objective of the IPAT-VSP is to maximize passenger service in terms of WTT within a budget for operating costs and a set of headway constraints. Input consists of a non-cyclical initial timetable with time-dependent service times, and an *Origin-Destination-Time* (ODT) passenger demand matrix. Timetable decisions consist of shifting departure times of trips, or extending dwell time at transfer stops, with respect to the provided initial timetable. Operating costs are defined by the minimum cost vehicle schedules to operate the timetable.

Key characteristics of passenger route choice as considered in this study are that (i) passengers have free route choice, and (ii) passengers may have different route preferences. Thus passengers with the same origin, destination, and departure time, may still choose different routes. The first is in contrast to the common setting of a central controller assigning passengers to maximize a social optimum. The second is in contrast to assuming all passengers choose the minimum weight path, and reflects that passengers may have different preferences. Specifically, it will ensure that when two almost equivalent paths exist connecting an origin and destination, passengers will be assigned to both. This assumption fits with commonly accepted route choice theory Ben-Akiva et al. (2004).

A matheuristic approach for the IPAT-VSP is proposed that consists of an iterative framework between a passenger route choice model, and an integrated timetabling and vehicle scheduling model. Different forms of integration are evaluated. The implementation of this modular framework combines the integrated timetabling and vehicle scheduling model of Fonseca et al. Fonseca et al. (2018) with the passenger route choice model of Briem et al. Briem et al. (2017), that satisfies the above two key characteristics of free passenger route choice, and different preferences for passengers. The latter also serves as a ground-truth for evaluating the passenger service of any timetable.

A realistic case study representing a large part of the multi-modal public transport network of the Greater Copenhagen Area, Denmark, serves to investigate the value of the IPAT-VSP timetabling approach: (i) in comparison to a the status-quo reflected by an initial timetable; (ii) in case of a change in the line network; and (iii) in case of a change in the OD matrix. Results indicate that including free passenger route choice results in timetables with higher passenger service in all three situations compared to a fixed passenger route choice approach as proposed in Fonseca et al.Fonseca et al. (2018). Moreover, our computational studies, supported by a simple clarifying example, illustrate that in order to find timetables with high passenger service indicating *potentially* interesting transfers is more important than estimating accurate usage of transfers in a current timetable.

To summarize the contributions of this work: (i) we investigate the maximal passenger service timetabling problem in the context of a free passenger route choice; (ii) we propose a modular matheuristic approach for the IPAT-VSP that, in an iterative framework, combines two state of the art models: (1) the IT-VSP matheuristic of Fonseca et al. Fonseca et al. (2018), which maximizes passenger service through minimizing excess transfer time under the assumption of fixed passenger route choice, with (2) the passenger route choice model of Briem et al. Briem et al. (2017), which represents free route choice of passengers; and (iii) we find that the inclusion of free passenger route choice results in timetables with higher passenger service for a realistic case study of the Greater Copenhagen area. Thereby the current study is different from Fonseca et al. (2018) by (a) indicating the value of integrating passenger route choice and timetabling, where Fonseca et al. (2018) assumed the passenger route choice as fixed input; (b) demonstrating the value of this approach for a larger, more complex network case study, and contrasting this against the approach on Fonseca et al. (2018), and (c) the evaluation of the value of the timetabling approach not only in comparison to the status-quo, but also in case of a small network re-design, and a change in passenger demand. The later two would lead to a change in (expected) passenger flows, which this model demonstratively is better capable of handling than the model of Fonseca et al. (2018).

The remainder of this paper is organized as follows: Section 2 reviews previous work on the integration of timetabling and passenger route choice, Section 3 describes the IPAT-VSP and presents a small example, Section 4 describes all components the solution approach for solving the IPAT-VSP, Section 5 describes the case study, Section 6 discusses the results for the case study, and Section 7 shows conclusions and suggestions for future research.

2 Literature Review

Recent years saw an increase in research output that integrates passenger decisions into the optimization models, especially in line planning, timetabling, and delay management models. Schmidt Schmidt (2014) provides an overview on public transport problems integrated with routing decisions.

Schmidt and Schöbel Schmidt and Schöbel (2015) integrate timetabling with passenger route choice minimizing total travel time, and investigate the computational complexity of the problem. Gattermann et al. Gattermann et al. (2016) present a boolean satisfiability problem (SAT) model that integrates periodic timetabling with passenger routing, distributing OD pairs temporally using time slices to make the problem tractable. The model is tested on Germany's long-distance passenger railway network and, for a restricted set of OD pairs, results show better objective values when compared with previous results. Computational time increases considerably when the number of OD pairs increases. Borndörfer et al. Borndörfer et al. (2017) study the integration of passenger routing with periodic timetabling models and propose a variety of models that allow different passenger paths and different objective functions. For a case study in the city of Wuppertal, the authors report a reduction of 1.24% in travel time and 23.57% in transfer waiting time in comparison to a real-world reference solution. Zhu et al. Zhu et al. (2017) present a bi-level model to integrate single line timetabling with passenger routing. The first level determines the headways to minimize total passenger costs (perceived travel time and travel penalties), and the second level determines the passenger arrival times given the headways. The authors use a two stage genetic algorithm to solve hypothetical examples of the problem. Chu Chu (2018) presents a mixed integer program to integrate network design and timetable, while routing passengers through a procedure based on the breadth-first search and path enumeration algorithms. A branch-and-price-and-cut algorithm is presented to solve the problem, and results are presented for example networks. Robenek et al. Robenek et al. (2018) address train timetabling design considering a probabilistic demand forecasting model. The demand elasticities are calibrated using a logit model. The problem is solved using a simulated annealing heuristic and results for a case study of Israeli Railways show revenue increases of 15%. Wu et al. Wu et al. (2019) present a bi-level program to coordinate timetabling and consider passengers' behavior to the timetable modifications by rerouting passengers that in the new timetable have missed transfers. The first level uses a mixed integer non-linear program to design the timetable, minimizing system cost composed by operating and user costs. The second level is a passenger route choice model. The authors use a heuristic algorithm to solve the problem and show results for two small examples: one with 3 lines and 3 transfer stops, and another with 4 lines and 4 transfer stops.

Laporte et al. Laporte et al. (2017) integrate timetabling and vehicle scheduling including special attention to route choice. Their problem designs timetables keeping operating costs (expressed as number of vehicles per line) under a certain budget. The Pareto front of solutions is calculated using an ϵ-constraint solution approach. Also in Liu and Ceder (2017), passenger route choice is integrated with timetabling and vehicle scheduling. The problem is modeled using a bi-objective, bi-level IP formulation, optimizing fleet size and WTT. The authors allow timetable modifications by shifting departure times and initial vehicle schedules are given as input. Deadheading is not allowed, meaning that vehicles are assigned to a single line and can only service trips belonging to that line, which significantly reduces the complexity of the problem. They propose a deficit function based sequential search to solve small examples with up to 4 unidirectional lines, 4 transfer stops, and one hour of operations.

Integration of free passenger route choice with transport optimization problems is also present in other fields. Dumas and Soumis Dumas and Soumis (2008) combine a fleet assignment optimization model with a passenger flow simulation model in an integrated model for the airline booking process. The passenger simulation model in Dumas et al. (2009) describes the passengers' reaction to capacity restrictions and how their itineraries change if their preferred flights are sold out. Cadarso et al. Cadarso et al. (2013) address disruption management in rapid transit networks, integrating an optimization model for timetabling and rolling stock schedules with a model for passengers' behavior. Kroon et al. Kroon et al. (2014) study the integration of free passenger flows in a real-time rolling stock rescheduling model for disruption management. The authors present a heuristic approach that iterates between a simulation model for passenger flows and an optimization model for the rolling stock, updating the objective function of the optimization model at each iteration according to the current passenger flows. Recent research in integration of free passenger route choice in disruption management can be found in Binder et al. (2017), Veelenturf et al. (2017), Wagenaar et al. (2017), Ortega et al. (2018), and Van der

Hurk et al. (2018). Canca et al. Canca et al. (2016) present a mixed integer non linear program to optimize line frequencies (minimizing operating costs and fleet acquisition costs) and simultaneously compute passenger assignments (minimizing average trip time and number of transfers). They use a cutting plane algorithm and present results for a simplification of the Madrid Metropolitan Railway Network.

In comparison to Laporte et al. (2017), we allow a wider set of timetable modifications. We consider free passenger route choice, where passengers may have different preferences, while Fonseca et al. (2018) considered passenger route choice as input. The proposed matheuristic framework is inspired by the framework proposed in Dumas and Soumis (2008) for the airline booking process and Kroon et al. (2014) for disruption management. In comparison to Wu et al. (2019), we consider free route choice for all passengers, while in Wu et al. (2019) only passengers with missed transfers are re-routed. Furthermore, Wu et al. (2019) test the performance of their method in small examples, while we test our method in a much larger real life case study.

3 The IPAT-VSP

The objective of the IPAT-VSP is to find the maximum passenger service timetable T^* of all feasible timetables $T \in \mathcal{T}$, which we define as the timetable with minimum total weighted travel time. Let $l \in L$ be the set of directed lines, where each line is defined by a sequence of stops $s \in S$, with S the set of all stops. Let a trip i represent a vehicle servicing all stops of a line l once. Each line l is associated with a frequency, leading to a set T_l of trips in the timetable for this line. The timetable is defined by the set of all trips $T = \bigcup_{l \in L} T_l$ and $T_{l'} \cap T_{l''} = \emptyset$ for all $l', l'' \in L, l' \neq l''$. The timetabling problem is to for all trips $i \in T$ assign arrival and departure times to all stops of the trip $s \in S_i$.

Passenger service, measured in weighted travel time, is defined by the route choice of passengers, and depends on the timetable. A (passenger) route choice model calculates a *passenger assignment* (PA) provided a timetable T and an origin-destination-departure-time matrix ODt. A passenger assignment consists of a set of paths P, where each path $p \in P$ is associated with a number of passengers that select this path. A path $p \in P$ represents an ordered list of trips $i \in T$ that connect the origin to the destination of the passenger in time and space, where the departure time of the first trip, from the origin stop of the passenger, is not before the departure time of the passenger. The path specifies per trip the boarding and departure stop of the passenger, where when disembarking a trip i passengers have either arrived at their destination, or will transfer to a trip of another line $l \in L$.

The the total weighted travel time, that follows from the passenger assignment, is defined as:

$$\mathrm{WTT} = \sum_{p \in P} w_p \cdot (\alpha \cdot \mathrm{IWT}_p + \mathrm{IVT}_p + \beta \cdot \mathrm{TrT}_p)$$

where WTT is the total weighted travel time of the passenger assignment, w_p the number of passengers that will select path p, IWT_p the initial waiting time of path p, IVT_p the in-vehicle time of path p, and TrT_p the transfer penalty of path p. We assume $\alpha, \beta \geq 1$.

The IPAT-VSP is a bi-level optimization problem: (i) the operator aims to find the maximum passenger service timetable within an operating budget, and (ii) the passengers aim to find their individual best paths in this timetable. Here we assume that (ii) represents free route choice of passengers. In other words, the social optimum for all passengers may not be equal to the individual optimum of each passenger. In addition, we assume that passengers may have different preferences. The later will ensure that when two paths exist of (almost) equal WTT, some passengers will prefer the one path, others the other path. As a consequence of these properties, it is not straight forward to translate timetabling with passenger free route choice to the context of a mathematical optimization model, where e.g. minimizing total WTT would lead to a social optimum, rather than an individual optimum. Furthermore, timetabling itself is already a computationally hard problem, thus the integration of the two is expected to be intractable as well.

Many models, like Fonseca et al. (2018), assume a PA as input. Specifically, Fonseca et al. (2018) assumed as input the number of passengers per *transfer location*, where a transfer location is defined by a specific (feeder) trip $i \in T$, at a stop $s \in S$, to another (receiver) line $l \in L$, so $i \not\in T_l$. The problem is that the number of passengers that will use such a transfer location, depends on the quality of the transfer (that is, excess waiting time above of the minimum transfer time), as well as the availability of alternative paths. Both transfer times and quality of alternative paths depend on the timetable. As the timetable is unknown, the demand per transfer location is unknown, and the input of a non-optimal PA may lead to the selection of non-optimal timetables. Moreover, while the objective to minimize excess transfer time (transfer time longer than the minimum required time to transfer), as in Fonseca et al. (2018), will lead to minimizing total WTT in case of *fixed* route choice; when passengers have free route choice and may change their routes, *higher* total weighted transfer times may actually represent solutions with *lower* total WTT. The example in Section 3.1 will illustrate this.

The example of Section 3.1 shows that even perfect information on the PA of an initial timetable cannot guarantee the finding of the optimal passenger service timetable. Specifically, long transfer times in an initial timetable can cause these transfer locations to be overlooked when they are not used in the initial timetable. We hypothesize that identifying these *potential beneficial transfer locations*, rather than including the set of *current* transfer locations, can lead to finding timetables with higher passenger service. We demonstrate by a set of case studies that indeed our matheuristic approach is able to find timetables with higher passenger service than (1) models assuming a static passenger assignment as input (specifically, than Fonseca et al. (2018)), and (2) integration where input consists only of the passenger assignment for an initial, or current, timetable. In section 4 we will present our modular math-heuristic approach to the IPAT-VSP.

3.1 Example

Consider the example in Figure 1, with three bus lines (1, 2, 3) and four stops (A, B, C, D). Consider that all bus lines have a headway of 20 minutes. Information on travel times is indicated along the edges, and a minimum transfer time of 4 minutes is required to guarantee a successful transfer, as indicated by the curved arrows. Three *transfer opportunities* exist in this network: at B where lines 1 and 3 meet, at C where lines 1 and 2 meet, and at D where lines 2 and 3 meet.

This example is used to (1) illustrate that when the timetabling model receives a PA input that is different from the PA of the optimal timetable, the timetabling model may not find the optimal timetable; and (2) that although given a fixed PA, minimizing the weighted sum of (excess) transfer time, as in Fonseca et al. (2018), will lead to minimizing total WTT, this is not true when the final PA is different from the input PA. Even more, timetables with a *higher* weighted sum of (excess) transfer time could be associated to timetables with a *lower* total WTT, and thus higher passenger service.

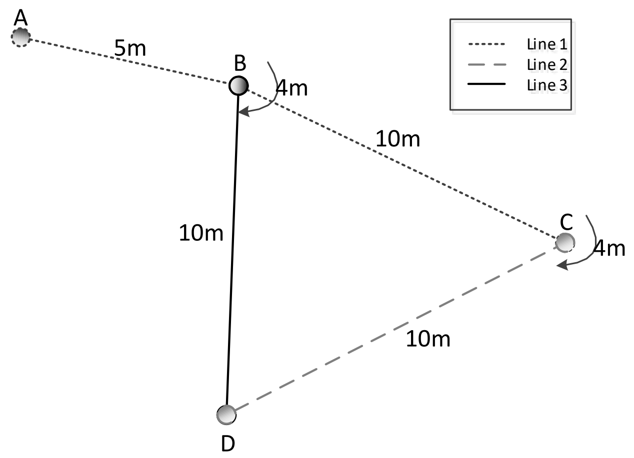

Figure 1: Example of a public transport network with three lines and four stops

Passengers traveling from A to D have two routing options: (i) traveling with line 1 from A to C, then transfer to line 2 to travel towards D; and (ii) traveling with line 1 from A to B, then transferring to line 3 to travel towards D. The passenger service is reflected in the weighted travel time, which is calculated as WTT $= \alpha \cdot \mathrm{IWT} + \mathrm{IVT} + \beta \cdot \mathrm{TrT}$, where $\alpha, \beta \geq 1$. We may focus on IVT and TrT alone, as for both (i) and (ii) the IWT (dependent on the frequency of the first line) is equal. The IVT of (i) is 5+10+10=25, which is longer than the IVT of (ii) 5+10=15. Passengers will however prefer the longer IVT of route (i) when $25+\beta \cdot \mathrm{TrT}_{(i)} < 15 + \beta \cdot \mathrm{TrT}_{(ii)}$.

Consider a timetable where the transfer from line 1 to 2 is perfectly synchronized at C, with no excess transfer time. Moreover, line 3 departs from B one minute before the arrival of line 1 at B, resulting in 19 minutes of transfer time at this location. For a value of $\beta = 3$ even under free route choice all passengers will prefer route (i) with a WTT=25+3·4=37 over route (ii) with a WTT=15+3·19=72. However, passenger service could be increased in this example if timetabling decisions would reduce the transfer time from line 1 to line 3 at B in route (ii) such that $15 + \beta \cdot \mathrm{TrT}_{(ii)} < 25 + \beta \cdot 4$. This alternative has much lower IVT in exchange for a transfer penalty and some additional waiting time, as required for the transfer.

(1): Indeed, in our example reducing the transfer time at stop B could improve passenger service by attracting passengers to route (ii). However, as in the current PA no passengers are using this transfer location, the timetabling model has no incentive to improve the synchronization of the two lines at this location. This shows that a PA different from the optimal PA could prevent finding the optimal timetable.

(2): The reduction in transfer time at B could improve total WTT already at a *positive excess transfer time* (when $15 + \beta \cdot \mathrm{TrT}_{(ii)} < 25 + \beta \cdot 4$, with $\beta \geq 1$). However, this would lead to an increase in total weighted excess transfer time of the model in comparison to passengers only using the perfect synchronized transfer at C. In fact, passengers only transferring at the perfect synchronized transfer in C leads to a minimal objective of 0 minutes excess transfer time, which could suggest the optimal timetable is found. This is true if route choice was fixed. However, as it is not fixed, the total WTT would be lower when there is a low transfer time at B, even if the transfer is not perfectly synchronized.

4 Solution method

This paper proposes a modular matheuristic, the `MHeuPA`, to solve the IPAT-VSP. The objective is to maximize passenger service in terms of minimizing WTT by modifying an initial timetable under the assumption of free passenger route choice and respecting a budget on operating costs. It does so by integrating an integrated timetabling and vehicle scheduling model, the `MHeu` (Section 4.1), with a passenger route choice model, the `PTTA` model (Section 4.2). As the approach is modular, alternative models for timetabling and passenger route choice could be used as well.

The two main points that need to be taken into account for solving the IPAT-VSP, that result from the assumption of free route choice, are:

1. passenger volumes per transfer location in the current timetable may not represent the passenger volumes per transfer location in the maximal passenger service timetable
2. higher objective values of the model of Fonseca et al. Fonseca et al. (2018) could actually be associated with lower overall WTT, and thus higher passenger service.

These two points were illustrated by the example in Section 3.1. To address these two points, (1) different passenger routings, by changing the waiting cost parameter in the `PTTA` model of Briem et al. Briem et al. (2017), are tested in the integration, and (2) timetable quality in terms of WTT is *always* calculated by the passenger route choice model.

Figure 2 outlines the IPAT-VSP matheuristic, the `MHeuPA`, that links the IT-VSP matheuristic (`MHeu`) of Fonseca et al. Fonseca et al. (2018) with the passenger route choice model of Briem et al. Briem et al. (2017). Section 4.3 will discuss this algorithm in detail.

Input consists of an initial timetable $\mathcal{T}_0$ defining departure and arrival times for the set of all timetabled trips T, an ODt matrix ODt representing passenger demand over time, a stop criterion $stopCriterion$, and a waiting cost function `CostF` for the `PTTA` (Section 4.3.1). These cost functions are the key to how the route choice model and timetabling model are working together to solve the IPAT-VSP. We compare different cost functions. Initialization consists of the `PTTA` calculating passenger assignment for an initial timetable according to the realistic waiting cost value (ϕ^R), and determining the operating cost budget by calculating the minimum cost vehicle schedule by running a *Multi Depot Vehicle Scheduling Problem* (MDVSP) for the initial timetable.

The core of the method consists of three blocks executed iteratively. First a `PTTA` is run, where the waiting cost function `CostF` may have waiting cost values different from ϕ^R. This is done to generate different passenger assignments than the ones that would be generated with ϕ^R, in an attempt to obtain timetables with higher

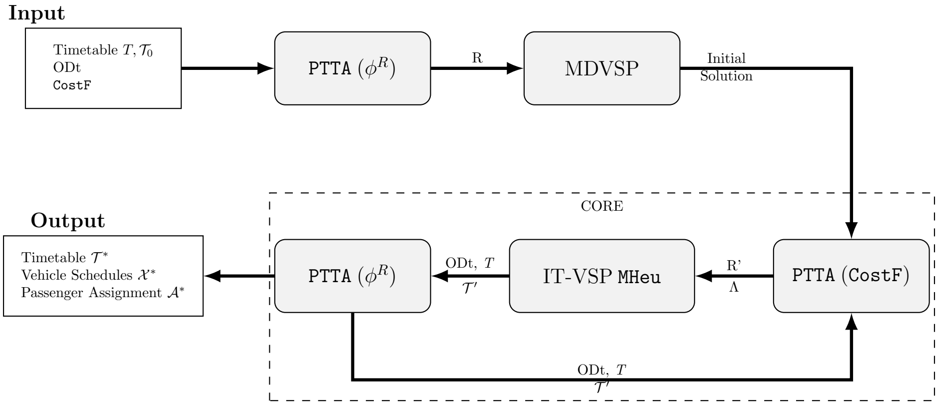

Figure 2: Flow diagram of the IPAT-VSP matheuristic approach

passenger service. The PTTA provides the IT-VSP with a set of passenger transfer demands R' and a vector Λ of passengers on board at each stop. Next, the IT-VSP MHeu is solved maximizing passenger service in terms of minimizing excess transfer time and additional in-vehicle time resulting from a possible extension of vehicle dwell time. The timetable is rescheduled within a budget for operating costs.

The resulting new timetable $\mathcal{T}'$ is evaluated in terms of passenger service according to the PTTA using ϕ^R. The iterative procedure stops after the maximum running time is reached, and outputs the computed timetable with highest passenger service $\mathcal{T}^*$ (the one with lowest WTT), the vehicle schedules $\mathcal{X}^*$ that cover that timetable and respect the budget constraint, and the passenger assignment $\mathcal{A}^*$ associated with timetable $\mathcal{T}^*$ computed by PTTA. Note that the framework is modular in that any passenger route choice model could be used instead of PTTA, and the timetabling module IT-VSP could be replaced by a different model.

4.1 Integrated timetabling and vehicle scheduling – the IT-VSP matheuristic approach

Input to the IT-VSP consists of an initial timetable for the set of all trips $i \in T$, passenger route choice information, a budget for the operating costs, and costs and parameters related with the case study, such as allowed headways, maximum dwell times, or turnaround times. The passenger route choice information consists of a set of transfer opportunities R, where each $r \in R$ defines a transfer stop, a transfer-from trip $i \in T$, a desired to transfer to line l, and a number of passengers that are expected to make this transfer. Furthermore, P contains the expected on-board passengers per trip $i \in T$ at stop $s \in S_i$, Λ_{is}. The passenger route choice information is computed in the *Public Transport Traffic Assignment* (PTTA) model (Section 4.2), which requires a timetable as input.

The objective of the IT-VSP is to minimize a weighted sum of passenger costs incurred by extending dwell times at stops for passengers on-board, and passenger costs incurred when transferring. Passengers incur transfer time costs when transfers are above the minimum transfer time. Transfers below the minimum transfer time are infeasible.

Decision variables consists of: a) binary assignment variables $x_{ijk} \in \{0,1\}$ storing which vehicles are assigned to which trips, b) departure and arrival time variables τ^d_{is} and $\tau^a_{is} \in \mathbb{Z}^+_0$ for each trip $i \in T$ and each stop $s \in S_i$, c) excess transfer time variables $\gamma_r \in \mathbb{R}^+_0$, which store the amount of excess transfer time for passengers using each transfer location $r \in R$, d) binary transfer variables $\alpha_{ijs} \in \{0,1\}$, which indicate which trip $j \in T$ passengers of transfer location $r = (i, l_j, s) \in R$ embark, and e) dwell time variables $\delta_{is} \in \mathbb{Z}^+_0$, which store the number of minutes of dwell time added to trip $i \in T$ at stop $s \in J_i$.

Constraints ensure the assignment of all expected transferring passengers to specific transfers, ensure headway constraints to be within a specific range, and limit the amount of added dwell time per stop and per trip, in addition to feasibility constraints for both the timetable and the vehicle schedule. Trips can only have additional dwell time and not less dwell time than in the initial timetable, since the dwell time in the initial timetable is considered to be the minimum dwell time. In fact, for the case study addressed in this paper, most trips have a dwell time of zero at all visited stops. Output consist of a timetable and a vehicle schedule.

The full mathematical model is presented in Appendix 1. This is an extended version of the model in Fonseca et al. Fonseca et al. (2018). This extended version allows to explicitly include the effect of extended dwell time for on-board passengers.

The IT-VSP is solved by a matheuristic approach based on the MILP formulation described above, which we denote by `MHeu`. A heuristic is necessary since solving real-life instances of the IT-VSP directly with a general solver is not feasible, due to the size of the instances making the problem intractable. The `MHeu` solves the IT-VSP by iteratively solving a sub-problem IT-VSP(T') that only allows timetable modifications for a subset of trips $T' \subseteq T$, but still solves the full vehicle scheduling problem. The heuristic stores the solution with lowest weighted travel time encountered so far.

4.2 Passenger route choice model – the PTTA model

We measure passenger service using the `PTTA` to evaluate the WTT of a timetable. Furthermore, the `PTTA` provides input to the `MHeu` in terms of the set of transfer opportunities R and the number of passengers on board Λ_{is}.

Input to the passenger route choice model `PTTA` is a timetable T, a set of possible transfer locations, the minimum required transfer time for a transfer to be feasible, and an ODt matrix. Output of the `PTTA` are the set of transfer opportunities R, the number of passengers on board for each trip $i \in T$ and each stop $s \in S_i$, Λ_{is}, and the WTT of the resulting passenger assignment.

To evaluate the quality of a timetable, cost parameters are set to use *perceived arrival times* (PAT) where the actual arrival time is weighted with a factor of 1, waiting time is weighted with a factor of 2, and each transfer receives an additional penalty of 5 minutes on top of the waiting time. When the `PTTA` model serves as input to the timetabling heuristic IT-VSP, different waits for the waiting time component are evaluated. In the route choice set generation, maximum difference in PAT is set to $\Delta_{\max} = 15$ minutes. As random utility model we use the linear decision model that was used in Briem et al. (2017).

The `PTTA` model is used to estimate which routes are likely to be chosen by passengers in a system with free route choice, as well as the expected number of passengers per route. The underlying model and algorithm to compute these routes were first presented in Briem et al. (2017). Conceptually, the `PTTA` model is a sequential route choice model. This means, that decisions are not made based on complete routes, but one journey leg at a time. Given a passenger, a current location, and a destination, the model specifies which step is probably taken next by the passenger in order to reach the destination. The probability for every possible next step is determined using a random utility model, with the utility being influenced by several cost functions, such as travel time, waiting time, and number of transfers. The `PTTA` model iteratively repeats this process until every passenger reached its destination, thereby compiling the complete routes used by the passengers.

For a given destination and journey leg, the random utility model in the `PTTA` characterizes the likelihood of the journey leg being used as next leg of a route leading to the destination. Thus, the first step of computing the overall passenger assignment in the `PTTA` model consists of computing the utilities for all pairs of possible journey legs and destinations. The PAT at the destination when using the specific leg in turn determines the utility of a leg (for a given destination). The PAT is a linear combination, which besides the actual arrival time, factors in all criteria that effect the route choice. For this work, the PAT is the weighted sum of the actual arrival time, the number of transfers, and the time spent waiting for the next trip. An important aspect of the `PTTA` model is the algorithm that allows for an efficient computation of PATs for all pairs of journey legs and destinations. To this end, PAT values are computed for one destination at a time. For a given destination, the PATs of all possible journey legs are computed iteratively, sorted by time in decreasing order. A detailed description of the process can be found in Briem et al. (2017). The benefit of processing the journey legs in decreasing order of time is that for a given leg the PATs of all possible journey continuations are already known. Thus, the PAT of every leg can be computed quite efficiently. If the leg itself ends at the destination then the PAT of this leg is given by its actual arrival time (since a single leg does not comprise transfers by definition). If the leg does not end at the destination, then the route has to be continued with another leg. In this case the PAT is given as the PAT of the following leg (which is already known), plus the additional cost (transfer time, waiting time) to connect the legs.

After all PATs have been computed, the actual passenger route choice is determined using a simulation approach. For every passenger, a sequence of decisions is made based on a random utility model. Each of these decisions determines the next journey leg the passenger takes towards the destination. The choice set for this decision is determined on basis of the PAT values. It consists of the leg with the lowest PAT as well as all other legs, such that the difference of PATs in the choice set does not surpass a certain, user-defined limit ($\Delta_{\max}$). The utility of each leg ℓ in the choice set is then defined as $\max(0, \min_{\ell' \neq \ell}(\mathrm{PAT}_{\ell'}) - \mathrm{PAT}_{\ell} + \Delta_{\max})$. Finally, the probability of each leg in the choice set can be obtained using a random utility model. Proportional to these probabilities one leg is chosen, e.g. the passenger is assigned to this leg as part of his route. This process is repeated until all passengers have been assigned to full routes, reaching their destinations. In order to obtain a distribution of several routes that could be used by a passenger (alongside with their respective probabilities), several virtual passenger can be simulated for every actual passenger.

4.3 IPAT-VSP matheuristic – the `MHeuPA`

Algorithm 1 presents the pseudo code for the `MHeuPA` approach.

Algorithm 1 : `MHeuPA`

Input: T, $\mathcal{T}_0$, ODt, $stopCriterion$, ϕ^R, `CostF`

Initialization procedure:

1: $(\mathcal{A}_0, R_0, \Lambda_0) \leftarrow \texttt{PTTA}(\mathcal{T}_0, ODt, \phi^R)$
2: $(\mathrm{X}_0,\ \mathcal{T}_0) \leftarrow$ solve IT-VSP`(1)-(21)` using R_0 and Λ_0 and with departure and arrival times (τ^d_{is}, τ^a_{is}) fixed to $\mathcal{T}_0$ for all $i \in T$
3: $\mathcal{S}_0 = (\mathrm{X}_0,\ \mathcal{T}_0, \mathcal{A}_0)$
4: $\mathcal{S}^* = \mathcal{S}_0$
5: $\eta = 0$

Iterative procedure:

6: **while** $stopCriterion$ not reached **do**
7:     $\eta = \eta + 1$
8:     $T' \leftarrow \texttt{selectTrips}(\mathcal{S}_{\eta-1})$
9:     $(\mathcal{A}', R', \Lambda') \leftarrow \texttt{PTTA}(\mathcal{T}_{\eta-1}, ODt, \texttt{CostF})$
10:     $(\mathrm{X}_\eta,\ \mathcal{T}_\eta) \leftarrow$ solve IT-VSP`(1)-(21)` using R' and Λ' and with departure and arrival times τ^d_{is}, τ^a_{is} fixed to $\mathcal{T}_{\eta-1}$ for all $i \in T \setminus T'$
11:     $(\mathcal{A}_\eta, R_\eta, \Lambda_\eta) \leftarrow \texttt{PTTA}(\mathcal{T}_\eta, ODt, \phi^R)$
12:     $\mathcal{S}_\eta = (\mathrm{X}_\eta,\ \mathcal{T}_\eta, \mathcal{A}_\eta)$
13:     **if** $\texttt{WTT}(\mathcal{S}_\eta) < \texttt{WTT}(\mathcal{S}^*)$ **then**
14:         $\mathcal{S}^* \leftarrow \mathcal{S}_\eta$
15:     **end if**
16: **end while**
17: **return** $\mathcal{S}^*$

Steps 1-5 are the initialization procedure. The algorithm starts by calculating an assignment for the initial timetable $\mathcal{T}_0$ in step 1, using the `PTTA` with ϕ^R. The set of transfer opportunities R_0 and the vehicle occupancy Λ_0 computed by the `PTTA` are used as input to solve the MDVSP in step 2 without allowing timetable modifications, thus the departure and arrival times of all trips $i \in T$ from/to stop $s \in S_i$, τ^d_{is}, τ^a_{is}, are fixed to $\mathcal{T}_0$ (meaning that these trips will have arrival and departure times at all stops visited equal to the ones in the initial timetable $\mathcal{T}_0$). An initial solution $\mathcal{S}_0$ is defined in step 3, composed by vehicle schedules X_0, the initial timetable $\mathcal{T}_0$, and the initial assignment $\mathcal{A}_0$, and since this is the only solution so far, in step 4 it is also saved as the current best solution in terms of WTT. The iterative procedure is described in Lines 6 - 16, which runs until the stop criterion $stopCriterion$ is met.

Each iteration η starts by selecting in step 8 the subset of trips $T' \subset T$ to modify, being κ the number of trips selected. Trips in T' are allowed modifications in arrival and departure times (shifts and stretches), while all other trips $i \in T \setminus T'$ remain fixed to the timetable in solution $\mathcal{S}_{\eta-1}$. In step 9, an assignment of passengers is calculated for the current timetable $\mathcal{T}_{\eta-1}$ according to `CostF`, generating transfer opportunities R' and vehicle occupancy Λ'. A new timetable $\mathcal{T}_\eta$ and vehicles schedules X_η are calculated in step 10, solving the restricted IT-VSP(T'), with τ^d_{is}, τ^a_{is} fixed to $\mathcal{T}_{\eta-1}$ for all $i \in T \setminus T'$, optimizing the transfer opportunities R' considering the vector Λ' of passengers on board. The realistic assignment (obtained using ϕ^R) of passengers $\mathcal{A}_\eta$ to the new timetable is calculated in step 11 by running the realistic `PTTA`. The iteration solution $\mathcal{S}_\eta$ is set in step 12. Steps 13-15 save $\mathcal{S}_\eta$ as the best solution $\mathcal{S}^*$ if the weighted travel time associated with it is lower than the weighted travel time associated with the current best solution. The IPAT-VSP concludes in step 17 by returning the best solution $\mathcal{S}^*$ found once the stop criterion *stopCriterion* is met.

4.3.1 Waiting cost functions

As demonstrated in the example in Section 3.1, the objective is to find all *potentially beneficial transfer locations*, that is, transfer locations that, with a good synchronization of the transfer, could be used by passengers. Whether the transfer location will be used by passengers, depends on the quality of the set of alternative paths available, and therefore cannot be determined per transfer location independently. Different waiting cost functions are used in a desire to find all potentially beneficial transfer locations.

Each run of the `MHeuPA` uses exactly one of five different waiting cost functions `CostF`. These functions change the weight attributed to the waiting costs when computing a new passenger assignment to serve as input to the IT-VSP `MHeu`.

- Realistic (`Realistic`): this waiting cost function runs the `PTTA` model with the realistic value ϕ^R for the waiting costs at every iteration. It reflects a base-case where the PA of a current timetable is provided as input to the timetabling module.
- No waiting costs (`NoCosts`): this waiting cost function runs the `PTTA` model without waiting costs at every iteration. This will lead to passengers selecting the path with the minimal IVT.
- Linear ascending costs (`LinAsc`): this waiting cost function increases the waiting costs in the `PTTA` model per iteration of the `MHeuPA`. Assuming a total running time of $maxT$ seconds, the `PTTA` iterations in the first $maxT/10$ seconds use a waiting costs parameter of 0. In the remainder of the iterations, the waiting costs increase linearly until the realistic value is achieved by the end of the experiment. The waiting costs at each iteration can be calculated using

$$WC = \frac{\phi t}{maxT}$$

  where t is the current total running time.
- Random waiting costs (`Random`): this waiting cost function runs the `PTTA` model with random waiting costs at every iteration of the `MHeuPA`, with a value between 0 and the realistic waiting costs $\phi = 2$, with a uniform distribution.
- Random and linear ascending costs (`RandLinAsc`): this waiting cost function combines the `Random` and `LinAsc` waiting cost functions. In the initial two thirds of the computational time, the `PTTA` model iterations use a random value between 0 and the realistic value for the waiting costs. In the last third, it uses linear ascending costs, calculated using

$$WC = \frac{\phi(t - 2maxT/3)}{maxT - 2maxT/3}$$

  where t is the current total running time.

The advantage of a lower waiting time costs is that transfer locations that currently have high waiting time, but would provide low in-vehicle time paths, will attract passengers – and thus provide an incentive to the timetabling model to improve the synchronization of trips at these transfer locations. The downside is that in dense networks it is unlikely that *all* transfer locations will be able to provide perfect transfers. Thus, the real passenger assignment is likely to be different from the zero or small waiting cost assignment; and therefore the trade-offs made in the timetable may be non-optimal due to erroneous numbers of expected passengers per transfer location, and expected passengers on-board.

Additionally we define `Route_Fixed` as running the `MHeuPA` with a fixed routing of passengers, using then the `PTTA` to evaluate the solutions in terms of WTT only. The results using `Route_Fixed` correspond to the IT-VSP `MHeu` results evaluated in WTT, and are included in this paper to enable a comparison with Fonseca et al. Fonseca et al. (2018).

Finally, any timetable is evaluated by running an independent passenger assignment of `PTTA` with a ground-truth costs for waiting time. In our experiments, we will use the value of 2, which was selected in Briem et al. (2017).

5 Case study

For the experimental section, we focus on a subset of the public transport network in the Greater Copenhagen area. We consider 8 bi-directional express-bus lines, referred to as S-Bus lines. In comparison to regular bus lines, these are faster and with fewer stops, acting mainly as a complement to the local urban trains (S-Train) across and radially. Figure 3 depicts a geographical representation of the network, which includes not only the S-Bus and S-Train lines but also two bi-directional Metro lines, one bi-directional train line, and one high-frequency bus line that connects the city center to the airport. The public transport agency Movia, which is responsible for the planning of buses in the region of Zealand, provided data for the case study. We allow timetable modifications by shifts and stretches to the trips in the S-Bus lines, while all other lines in the case study operate according to a fixed timetable. The vehicle scheduling component of the IPAT-VSP is solved solely for the S-Bus trips.

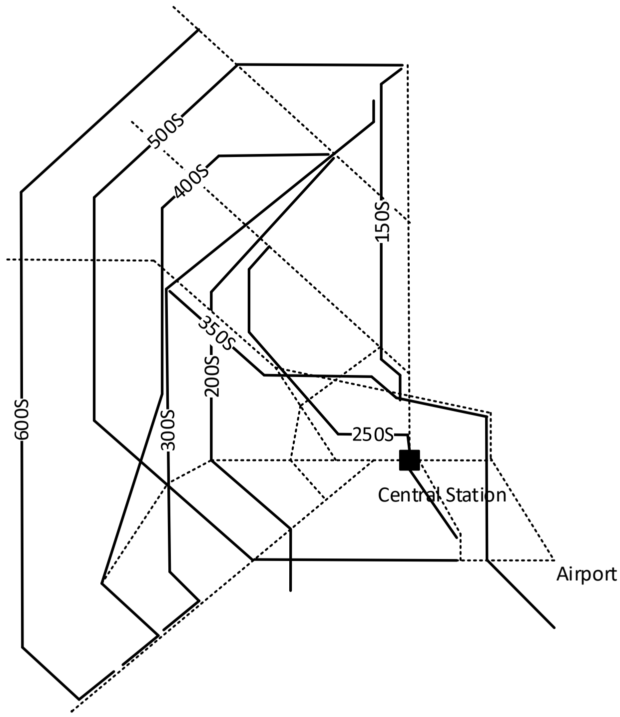

Figure 3: Geographic representation of the case study network. The thick lines represent the S-Bus network, while the dashed lines represent the S-Train, train, Metro and fixed bus lines

The data input for the IPAT-VSP is composed by: (i) a *initial timetable* for all S-Bus lines; (ii) a *fixed timetable* for all other lines in the network; (iii) a *distance matrix* with all distances between stops and depots; (iv) an *ODt matrix* describing passenger demand for the full network; (v) *costs and parameters* specific to the case study: minimum and maximum turnaround times, minimum transfer times at stops, vehicle operating costs, fixed costs per vehicle schedule created, travel time and waiting time costs for passengers, driving speed for vehicles while deadheading, maximum deadhead distance, maximum added dwell time per trip and per stop, and depot capacities.

The timetables used in input components (i) and (ii) are publicly available and the distance matrix (iii) was obtained using geographical data. The ODt matrix (iv) was provided by Rapidis[1]. It describes minute-by-minute passenger demand between the stations in our network. The ODt contains 164,333 entries representing 170,117 passengers. As for costs and parameters (v), these were estimated together with Movia. We use estimates of operating waiting time, distance, and schedule costs expressed in Danish kroner (DKK, 1 euro is equivalent to approximately 7.5 DKK), which together define the operating costs of a solution. Travel times, initial waiting times, and transfer times are weighted by value of time (VOT) factors of respectively 100, 300, and 300 DKK. We used value of time studies developed at the Center for Transport Analytics at the Technical University of Denmark as inspiration for these values[2].

[1]Rapidis is a Danish company that develops tools for planning in Transportation and Logistics. website: `http://www.rapidis.com/`

[2]Center for Transport Analytics website: `http://www.cta.man.dtu.dk/`

Table 1 shows information about all lines included in the case study network. The first column indicates the name of the line, followed by the mode of transport, the number of stops with transfer opportunities, the number of trips, an indication of whether the trips in the line are part of the timetabling design or not, the minimum headway in the initial timetable, and the maximum headway in the initial timetable.

Table 1: Lines in the case study network

Line name	Mode	Stops	Trips	Timetabling	Min headway	Max headway
150S	S-Bus	5	256	yes	4	21
200S	S-Bus	7	186	yes	6	23
250S	S-Bus	7	161	yes	8	24
300S	S-Bus	8	230	yes	5	23
350S	S-Bus	12	304	yes	4	22
400S	S-Bus	7	140	yes	7	21
500S	S-Bus	8	167	yes	7	31
600S	S-Bus	7	141	yes	4	34
A	S-Train	10	202	no	10	20
B	S-Train	8	202	no	10	20
Bx	S-Train	4	8	no	20	20
C	S-Train	8	205	no	10	20
E	S-Train	7	200	no	9	21
F	S-Train	3	374	no	5	10
H	S-Train	12	115	no	20	20
KB	Train	6	246	no	3	32
M1	Metro	7	487	no	2	12
M2	Metro	6	450	no	2	12
5C	Bus	8	542	no	4	4

For each trip $i \in T_l$ at each stop $s \in J_i \cup st_i$, minimum and maximum headways, h^-_{is} and h^+_{is}, are calculated based on the scheduled headways between trip i and its immediate precedent trip in the line, trip $i-1$, as indicated in Table 2. Notice that in our case study the minimum scheduled headway is 4 minutes. In this case, the minimum and maximum headways allowed will be 3 and 5 minutes respectively.

Table 2: Allowed headway variations based on scheduled headways

Scheduled headway (m)	Minimum and maximum headway variation (m)
$= 4$	+/- 1
≤ 12	+/- 2
≤ 20	+/- 3
≥ 21	+/- 4

The maximum dwell time added at each stop is 3 minutes (i.e., $w^+_{is} = 3, i \in T, s \in J_i$), meaning that a maximum of 10 minutes of dwell time can be added in total to a trip (i.e., $w = 10$). The added dwell time is deducted from the buffer in the turnaround time at the end of the trip. The shifts allowed in each trip departure time were created based on the initial timetable for each bus line. Considering consecutively timetabled trips $(i-1), i, (i+1) \in T_l$ and with departure time from the first stop $d_{i-1,st_i}, d_{i,st_i}, d_{i+1,st_i}$ respectively, the lower and upper shift limits for trip i are calculated with the expressions

$$d^-_{i,st_i} = d_{i,st_i} - \left\lfloor \frac{d_{i,st_i} - d_{i-1,st_i} - 1}{2} \right\rfloor \quad , \quad d^+_{i,st_i} = d_{i,st_i} + \left\lfloor \frac{d_{i+1,st_i} - d_{i,st_i}}{2} \right\rfloor$$

ensuring that trips can never overtake each other in the timetable.

As they are not part of the input, vehicle schedules that cover the initial timetable for the S-Bus trips are calculated using an MDVSP model. The solution consists of 205 vehicle schedules that cover the 1585 S-Bus trips. It uses constraints (3)-(5) of the mathematical model in Section 4.1. The initial timetable considers time dependent service times, but the mathematical model uses constant deadhead speeds along the day. Trips from different lines can be included in the same vehicle schedule, which is known as *interlining*, thus allowing deadheading between consecutive trips in a schedule. The maximum deadhead distance is 15 kilometers (i.e., $u = 15$), the minimum turnaround time is 12 minutes (i.e., $q^- = 12$), and the maximum turnaround time is 30 minutes (i.e., $q^+ = 30$).

6 Computational experiments

This section evaluates the performance of the MHeuPA through a set of computational experiments for the case study of the Greater Copenhagen Area described in Section 5. Results of the MHeuPA for different waiting cost functions (Section 4.3.1) are compared to the fixed passenger route choice model of Fonseca et al. Fonseca et al. (2018), the IT-VSP MHeu. Note that results between this paper and Fonseca et al. (2018) cannot be directly compared as the current case study represents a larger network, with new detailed passenger demand information that was not available yet during the Fonseca et al. Fonseca et al. (2018) study; and secondly due to a different measure of passenger service, which in this paper is represented as WTT calculated by the passenger route choice model of Briem et al. Briem et al. (2017). Route_Fixed is an exact representation of the model of Fonseca et al. Fonseca et al. (2018) in this new setting. Thus, the comparison between the MHeuPA and the Route_Fixed demonstrates the value of including free passenger route choice.

We evaluate our approach in the following three situations:

- *In comparison to an initial timetable representing the current timetable for our case study area* (Section 6.1). This case study is similar to the setting of Fonseca et al. (2018), and therefore allows the most direct comparison between fixed and free passenger route choice. This section presents a detailed analysis of the results for the different components of weighted travel time, benefits specifically for transferring passengers, and the resulting vehicle schedules.
- *In case of a change in the public transport network* (Section 6.2). A change in the public transport network results in a timetabling situation where one would expect a change in passenger route choice. This situation is simulated by offsetting the timetables of one, or a set, of public transport lines in the network, such that headway constraints and time-dependent vehicle travel times are still respected, but transfers are likely offset.
- *In case of a change in passenger demand* (Section 6.3). A change in the passenger demand matrix could lead to a change the relevance of transfer opportunities: making some transfer opportunities more important than others; for instance in case of a special event. This could also make it important to consider free passenger route choice. We assume that, whatever the change in demand, sufficient capacity is available, as measures to increase capacity on routes (e.g. longer vehicles, or higher frequencies) are not part of the timetabling decisions considered in this paper.

The algorithm is implemented in C++ and uses CPLEX version 12.6 to solve the mathematical program at each iteration. All experiments were conducted on HPC servers, using Intel Xeon E5-2660 v3 2.60GHz processors, and 8 computation cores. Each iteration uses CPLEX warm-start to start from the previous solution. Presented are average results over five runs with each setting, with a 3 hour computation time limit. Fixed parameters are the number of trips selected per iteration $\kappa = 350$, the maximum running time per iteration $\psi = 30$, and the realistic value for waiting costs $\phi = 2$. The values for the parameters κ and ψ are based on the computational results of Fonseca et al. (2018) and taking into account that the current case study is larger both in terms of network and OD matrix, while the value for ϕ is based on the findings of Briem et al. (2017).

The solution quality is expressed in terms of weighted travel time (WTT) and its components: in-vehicle time (IVT), initial waiting time (IWT), and transfer time (TrT). We also compare the operating costs (OpC) across experiments. To compare the solutions obtained with the MHeuPA with the initial timetable and with the solutions obtained with the Route_Fixed, we use percentage improvements to initial and percentage improvements to Route_Fixed. For $x = \{\texttt{WTT}, \texttt{IVT}, \texttt{IWT}, \texttt{TrT}, \texttt{OpC}\}$ and $\overline{f}_x(S)$ being the x-type average cost of a solution S over n runs, we calculate the percentage improvement to the initial timetable as

$$\frac{\overline{f}_x(\mathcal{S}_{\texttt{MHeuPA}}) - f_x(\mathcal{S}_{\texttt{Initial}})}{f_x(\mathcal{S}_{\texttt{Initial}})} \times 100\%$$

and the percentage improvement to the solutions obtained with the Route_Fixed as

$$\frac{\overline{f}_x(\mathcal{S}_{\texttt{MHeuPA}}) - \overline{f}_x(\mathcal{S}_{\texttt{Route\_Fixed}})}{\overline{f}_x(\mathcal{S}_{\texttt{Route\_Fixed}})} \times 100\%$$

since there is only one solution for the initial timetable but n solutions for the Route_Fixed (one for each run). A negative percentage for x corresponds to a reduction of costs in the MHeuPA solution in comparison to the initial timetable or to the Route_Fixed solutions. Since we use the budget version of the IT-VSP MHeu in all experiments, we keep the operating costs under a budget. We consider as budget the operating costs obtained by solving the MDVSP for the initial timetable.

6.1 Results for the base scenario

In this section, we present the results for the base scenario: initial timetable and base ODt matrix. We present results in terms of WTT and different WTT components, operating costs, timetable modifications and characteristics of vehicle schedules. Additionally we show a convergence analysis for the different waiting cost functions considered and histograms of WTT variations.

Table 3 shows the WTT results for the different waiting cost functions in absolute WTT values, percentage improvement to initial timetable, and percentage improvement to Route_Fixed solution. We present results both for the full ODt matrix and for a version of the ODt matrix that considers only passengers that, given the public transport network lines, will have to transfer at least one time.

Table 3: WTT results for the base scenario for 1) the full ODt matrix and 2) for the transferring passengers only

	Full ODt Matrix			Zoom on transferring passengers		
Solution	WTT (DKK)	Improv. to Base (%)	Improv. to Route_Fixed (%)	WTT (DKK)	Improv. to Base (%)	Improv. to Route_Fixed (%)
initial timetable	14,952,021	-	-	11,281,416	-	-
Route_Fixed	14,844,516	-0.72	-	11,153,787	-1.13	-
Realistic	14,832,916	-0.80	-0.08	11,138,733	-1.26	-0.13
NoCosts	**14,805,563**	**-0.98**	**-0.26**	**11,110,820**	**-1.51**	**-0.39**
LinAsc	14,810,788	-0.94	-0.23	11,114,746	-1.48	-0.35
Random	14,811,079	-0.94	-0.23	11,113,400	-1.49	-0.36
RandomLinAsc	14,811,457	-0.94	-0.22	11,112,630	-1.50	-0.37

The results in Table 3 support the hypothesis that inclusion of free route choice leads to timetables with higher passenger service, and that using alternative cost models, to find potentially beneficial transfer locations, also enable finding timetables with higher passenger service. Indeed, all MHeuPA solutions improve passenger service in comparison to Route_Fixed solutions by 0.08% to 0.26% in the full ODt matrix and by 0.13% to 0.39% in the zoom on transferring passengers. The NoCosts is the best performing waiting cost function, both for the full ODt matrix and for transferring passengers only, while all alternative cost functions improve on input from the Realistic assignment model. All approaches improve passenger service in terms of WTT in relation to the initial timetable. The improvement in passenger service stems mainly from improved WTT for passengers with a transfer in their path, which is observed when comparing the results for the full ODt matrix with the results for transferring passengers only.

Figure 4 shows the convergence of WTT for the three best CostF functions LinAsc, NoCosts, and RandomLinAsc. The horizontal axis shows the algorithm total computational time in minutes and the vertical axis shows the percentage reduction in WTT in comparison to the initial timetable. The figure shows that the LinAsc waiting cost function has a steeper initial decline in WTT, while NoCosts finds the overall minimum WTT from around 120 minutes of computational time. This indicates that the LinAsc waiting cost function allows to find good timetables fast, but in the long run it is better to use the NoCosts waiting cost function. For all waiting cost functions, the improvements in WTT decline after the first 1.5 hours of computational time.

Table 4 shows for all waiting cost functions the average improvement in WTT expressed in DKK, the percentages of passengers better and worse off, the percentage of passengers better off by more than 10 DKK of WTT, and the percentage of passengers worse off by more than 10 DKK of WTT. The value of 10 DKK of WTT is equivalent to 6 minutes of in-vehicle time or 2 minutes of excess transfer time or initial waiting time.

Table 4 shows that the NoCosts waiting cost function achieves the best average improvement in WTT per passenger, with a value of -1.45 DKK, which follows from the results in Table 3. The percentage of passengers better and worse off is similar across waiting cost functions, with more passengers being better off than worse off when compared with the initial timetable. Specifically for differences in WTT larger than ±10%, the percentages of passengers better off are higher than the percentages of passengers worse off, as evidenced by the last two

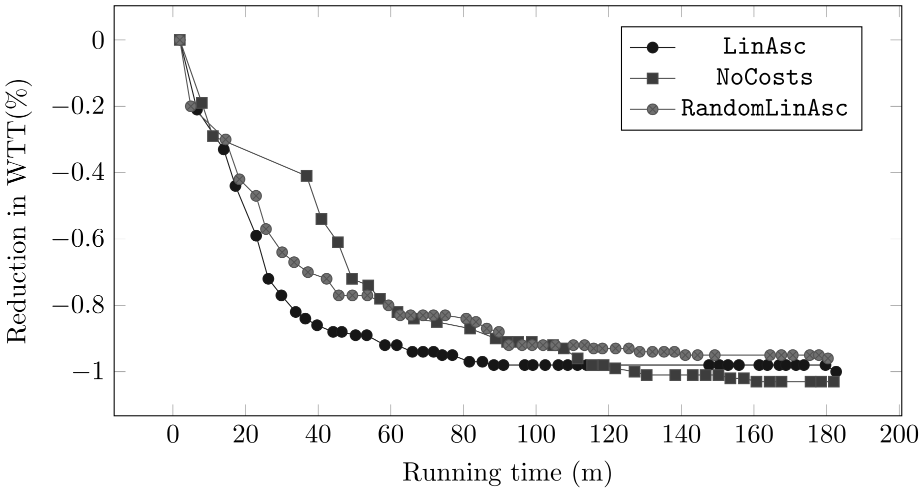

Figure 4: WTT convergence of the `LinAsc`, `NoCosts`, and `RandomLinAsc` solutions over time

Table 4: Passenger WTT improvements for the base scenario

Solution	Avg. improvement (DKK)	Pax better off (%)	Pax worse off (%)	Perc of pax with WTT reduction >10 DKK (%)	Perc of pax with WTT increase >10 DKK (%)
`Route_Fixed`	-1.12	34.37	28.52	10.23	6.84
`Realistic`	-1.21	35.41	29.64	12.43	8.17
`NoCosts`	-1.45	35.20	31.19	13.22	9.64
`LinAsc`	-1.40	35.72	30.79	13.40	9.28
`Random`	-1.39	35.86	30.94	13.54	9.40
`RandomLinAsc`	-1.39	36.03	30.94	13.74	9.59

columns in Table 4. Although the `NoCosts` waiting cost function achieves the best average improvement, it is not the one that achieves the highest percentage of passengers better off, with the `RandomLinAsc` surpassing the 36% mark. If the objective is to maximize the percentage of passengers better off by a certain threshold or to minimize the percentage of passengers worse off it might be preferable to use other waiting cost functions than the `NoCosts` waiting cost function for this instance.

For better understanding how the `MHeuPA` solutions improve the WTT for passengers, Figures 5 and 6 show histograms of changes in WTT in the best solutions obtained by the `Route_Fixed` and by the `MHeuPA`. The horizontal axis shows the change in WTT experienced by passengers and the vertical axis shows the absolute number of passengers that experience changes in each interval. The histogram is divided in two figures due to the difference in magnitude of the number of passengers.

Figures 5 and 6 show that in the `MHeuPA` solution more passengers experience high decreases in WTT, especially in the interval $[-70, -20[$. From Figure 6, it is clearly observed that the amount of passengers on the left hand side of the histogram (passengers better off) is larger than the one on the right hand side (passengers worse off), which is linked to the results in Table 4. However, Figure 5 shows that in the `Route_Fixed` solution more passengers experience smaller changes in WTT than in the `MHeuPA` solution, between -10 DKK and 10 DKK. Additionally, Figures 5 and 6 show that the `Route_Fixed` solution has less passengers worse off than the `MHeuPA` solution, which is explained by existing in general less changes in the timetable. The timetables of the `MHeuPA` represent a different trade-off between passenger groups, and although not a strict improvement for all passengers; the disbenefits for some passengers are offset by the benefits for a larger group of other passengers.

Table 5 shows the results in terms of each of the components of WTT, for the same set of experiments as in Tables 3 and 4. The table contains the absolute values in DKK of in-vehicle time (IVT), initial waiting time (IWT), and transfer time (TrT), along with their percentage improvements in relation to the initial timetable.

The results in Table 5 show that all components of WTT are improved in relation to the initial timetable, as evidenced by the negative percentages. Most of the improvement in WTT comes from the improvement in transfer times, with decreases ranging from -2.59% to -3.17% compared to decreases of -0.13% to -0.34% in in-vehicle time and of -0.04% to -0.40% in initial waiting time. This is expected, since transfer time is the WTT

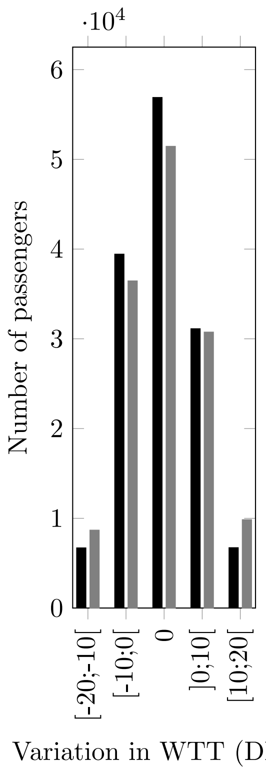

Figure 5: Histogram of variation in WTT for the `Route_Fixed` solution and for the best solution obtained with our algorithm

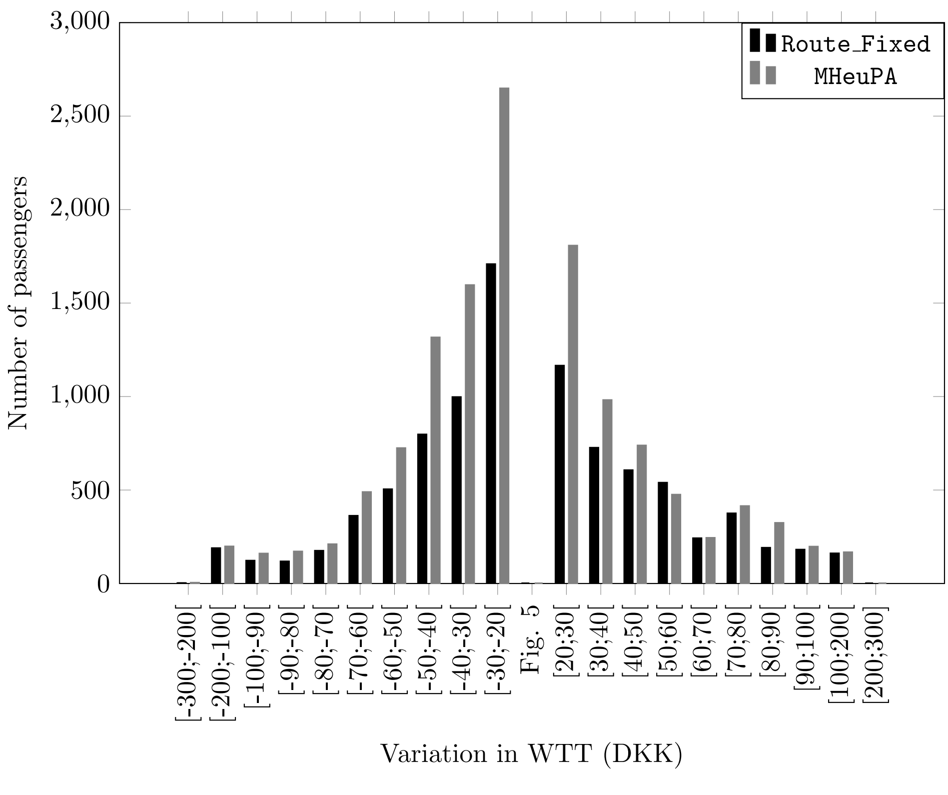

Figure 6: Histogram of variation in WTT for the `Route_Fixed` solution and for the best solution obtained with our algorithm

Table 5: WTT details for the base scenario

Solution	IVT (DKK)	Improv to Base (%)	IWT (DKK)	Improv to Base (%)	TrT (DKK)	Improv to Base (%)
initial timetable	7,399,710	-	4,043,178	-	3,509,133	-
`Route_Fixed`	7,390,300	-0.13	4,036,102	-0.18	3,418,115	-2.59
`Realistic`	7,389,179	-0.14	4,041,434	-0.04	3,402,302	-3.04
`NoCosts`	7,374,506	-0.34	4,026,984	-0.40	3,404,074	-2.99
`LinAsc`	7,378,596	-0.29	4,034,440	-0.22	3,397,752	-3.17
`Random`	7,378,806	-0.28	4,033,539	-0.24	3,398,735	-3.15
`RandomLinAsc`	7,379,542	-0.27	4,033,954	-0.23	3,397,962	-3.17

component that is specifically considered in the IT-VSP objective function. Among the `MHeuPA` results, the `NoCosts` waiting cost function obtains the smaller reduction in TrT, with a value of -3.00%, but obtains the higher reductions in IVT and IWT, respectively -0.34% and -0.40%. Improvements in IVT show that passenger routes actually change in comparison to the initial timetable, because trips can only have additional dwell time and not less dwell time than in the initial timetable. This indicates that passengers are able to find better routes to travel from origin to destination, spending less time in-vehicle. Improvements in IWT are incidental since the IT-VSP objective does not include the effect of timetable modifications on IWT. Trips are shifted to cater for transfer synchronization, but in the process also IWT could be reduced.

Figure 7 visualizes the excess transfer time (transfer time minus the minimum required transfer time for a feasible transfer) experienced by passengers in the initial timetable and in the best `MHeuPA` solution. The horizontal axis shows the excess transfer time in minutes and the vertical axis shows the percentage of transferring passengers that experiences that value of excess transfer time.

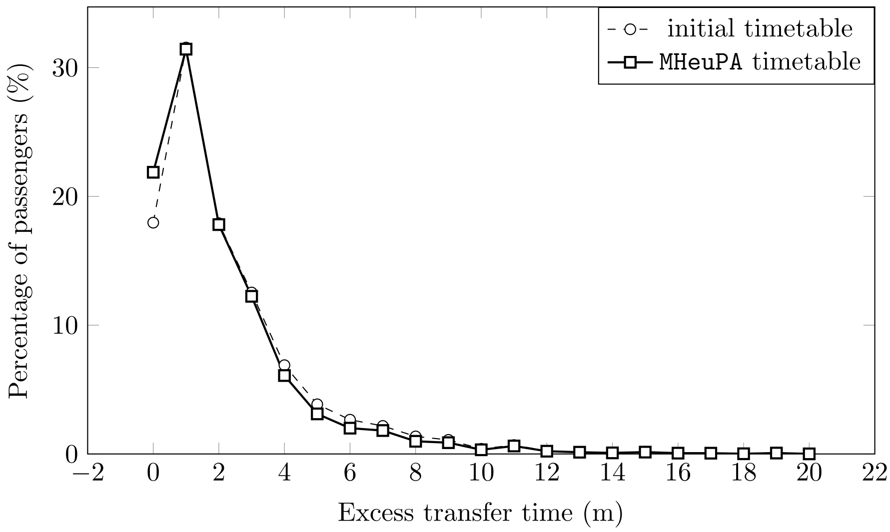

Figure 7: Comparison of excess transfer time in the initial timetable and in the best `MHeuPA` solution timetable

Figure 7 shows that in the `MHeuPA` timetable there are significantly more passengers experiencing perfectly synchronized transfers, with 0 minutes of excess transfer time. A total of 22% of transferring passengers experience perfectly synchronized transfers in the `MHeuPA` solution, while in the initial timetable this value is 18%. For all other excess transfer time values, the initial timetable has more passengers experiencing each value. Furthermore, the average excess transfer time decrease from 2.29 m to 2.06 m in the `MHeuPA` solution, for more than 100,000 transferring passengers.

Table 6 shows results for the same set of experiments from an operating perspective. The table contains information on absolute value of operating costs in DKK, percentage improvement in relation to the initial timetable, percentage of trips modified by shifts only, stretches only, and both, average added stretches, average added shifts, number of vehicle schedules, and average schedule duration in minutes.

The results in Table 6 show that all waiting cost functions use less operating costs than the budget of the initial timetable, but also considerably decrease them, with percentages between 1.64% and 2.19%. The solutions obtained with the `MHeuPA` shift 52.8% to 56.6% of the trips, add stretches to 2.6% to 4.0% of the trips,

Table 6: **Operating costs, timetable and vehicle schedules for the 8 line case and for the base OD matrix**

Solution	OpC (DKK)	OpC improv to Base (%)	Trips with shifts only (%)	Trips with stretches only (%)	Trips with shifts and stretches (%)	Avg added dwell (m)	Avg added shift (m)	Number Schedules	Avg schedule duration (m)
initial timetable	323,744	-	-	-	-	-	-	205	784
`Route_Fixed`	317,417	-1.95	48.3	3.8	15.4	2.3	1.7	200	802
`Realistic`	317,608	-1.90	52.8	3.3	14.2	2.4	2.0	201	800
`NoCosts`	318,124	-1.74	55.1	3.8	14.0	2.2	1.9	201	798
`LinAsc`	318,437	-1.64	56.6	2.6	13.6	2.3	2.0	201	798
`Random`	316,754	-2.16	54.0	4.0	17.1	2.3	2.0	200	802
`RandomLinAsc`	316,649	-2.19	54.4	3.8	16.6	2.4	2.1	200	803

and add both shifts and stretches to 13.6% to 17.1% of the trips. The values of added dwell time and added shifts are averages over the total number of trips with added stretches and added shifts respectively. On average, just 2 minutes of dwell time are added to modified trips. Regarding the vehicle schedules, the `Route_Fixed` and `MHeuPA` solutions use 200 to 201 schedules to cover all trips, while the base solution uses 205. Furthermore, schedules are on average longer in the `Route_Fixed` and `MHeuPA` solutions with values ranging between 798 and 803 minutes, compared to the 784 minutes in the initial timetable. This means that the `Route_Fixed` and `MHeuPA` solutions use resources more efficiently, with vehicles covering on average more trips.

The overall improvement in WTT in comparison to the initial timetable is approximately 1%, of which 0.25% is due to the inclusion of free passenger route choice. The 0.25% is equivalent to a daily reduction of approximately 40,000 DKK when expressed as value of time. Due to the budget constraints, these savings come at no additional operating costs, and in fact allow a reduction of operational costs, as demonstrated in Table 6.

6.2 Designing timetables - changes in the public transport network

In this section, we analyze solutions obtained when we start from a randomized timetable. Note that given a line plan and frequency, we can always create an initial timetable. It may however be of bad quality. This section demonstrates that also provided such a "bad" starting point, the `MHeuPA` can be used for constructing a timetable. Thus in principle it is suitable for designing new timetables. Results for the best performing waiting cost functions from the previous section (`NoCosts`, `LinAsc`, and `RandomLinAsc`) are compared to solutions obtained with the `Route_Fixed` (fixed passenger route choice). We test four different scenarios:

1. offsetting the timetable for line 350S only, which is the line that transports the largest volume of passengers in the network;
2. offsetting the timetables for lines 250S, 300S, and 400S, which are the lines with largest volumes of passenger transfers;
3. offsetting the timetables for lines 350S, 500S, and 600S, which are the lines with largest volumes of passenger transfers involving a bus trip;
4. offsetting the full S-Bus network timetable.

When designing the timetable for one line, we start from its trips in the initial timetable. We then shift all trips of that line by a random amount of time between 0 and double the minimum periodicity of that line. The minimum periodicity of a line in the initial timetable is between 13% and 41% of its maximum periodicity, so in most cases the timetable modifications will allow attaining a timetable close to the initial timetable. Each direction in the offset line is shifted by a different random amount of time. Timetable modifications by shifts and stretches are allowed for all S-Bus lines, in all scenarios. Offsetting the timetable may also change the vehicle schedules obtained when solving the MDVSP. In order to enable a fair comparison between results in this section and in the previous section, we use as budget for the operating costs the same budget used before, i.e. the operating costs obtained by solving the MDVSP for the initial timetable. Table 7 shows results for all four scenarios in terms of absolute WTT and DKK savings in comparison to the `Route_Fixed` solutions. Each scenario is associated with an α value, which is the percentage increase in WTT of the offset initial timetable in comparison to the initial timetable.

The results in Table 7 show that all `MHeuPA` solutions have a lower WTT than the solutions obtained with the `Route_Fixed`, similarly to what is observed in Section 6.1. Again, the `NoCosts` waiting cost function is

Table 7: **WTT results for the four scenarios of designing timetables**

Offset lines (α)	350S (0.07)		250S, 300S, 400S (0.16)		350S, 500S, 600S (0.13)		All S-Bus network (0.39)	
Solution	WTT (DKK)	Savings ict Route_Fixed (DKK)	WTT (DKK)	Savings ict Route_Fixed (DKK)	WTT (DKK)	Savings ict Route_Fixed (DKK)	WTT (DKK)	Savings ict Route_Fixed (DKK)
Route_Fixed	14,858,155	-	14,862,350	-	14,886,838	-	14,928,154	-
NoCosts	14,813,879	-44,276	14,817,685	-44,665	14,840,451	-46,388	14,879,730	-48,424
LinAsc	14,818,344	-39,811	14,832,089	-30,261	14,843,607	-43,231	14,882,466	-45,689
RandomLinAsc	14,822,558	-35,596	14,832,355	-29,995	14,840,764	-46,074	14,894,508	-33,646

the one that shows the largest decreases in WTT. By comparing Tables 3 and 7, we see that the solutions in Table 7 also have a lower WTT than the initial timetable of Section 6.1, despite starting from a worse timetable (evidenced by all α values being positive). Solutions obtained in the previous sections with the same waiting cost functions are better in terms of WTT by 0.05% to 0.56% than solutions obtained in this section, since they start from a timetable with more transfer synchronization and therefore lower WTT.

6.3 Changes in passenger demand

In this section, we test the MHeuPA for different ODt matrices than the base matrix. We test two different scenarios:

1. a random variation in the base ODt matrix (Random ±10%);
2. an event simulation ODt matrix (Event Simulation).

The random variation scenario was generated by varying OD hourly demand in the base ODt matrix randomly by a value between -10% and 10%. As for the Special Event Simulation scenario, we selected three stations in the city center and simulated a two hour event happening between 6p.m. and 8p.m. Consequently, we increase by 50% all OD pairs in the base ODt towards these three stations with departure time during the two hours prior to the event. We also increase by 50% all OD pairs in the base ODt originating from these three stations in the two hours after the event.

Table 8 shows the results for the two scenarios, in absolute WTT and percentage improvement in relation to the Route_Fixed solutions. Similarly to Section 6.2, each scenario is associated with an α value, indicating the percentage increase of the initial solution in comparison to the initial timetable for the base ODt matrix.

Table 8: **WTT results for the two scenarios of changing the OD matrices**

OD Variation (α)	Random ± 10% (0.03)		Special Event Simulation (0.35)		initial timetable (Table 3) (14,952,021 DKK)	
Solution	WTT (DKK)	Improv. to Route_Fixed (%)	WTT (DKK)	Improv. to Route_Fixed (%)	Improv. in Random (%)	Improv. in Event (%)
Route_Fixed	14,857,357	-	14,906,759	-	-0.63	-0.30
NoCosts	14,814,032	-0.29	14,861,172	-0.31	-0.92	-0.61
LinAsc	14,825,444	-0.21	14,867,111	-0.27	-0.85	-0.57
RandomLinAsc	14,822,207	-0.24	14,865,578	-0.28	-0.87	-0.58

From Table 3, the initial timetable has a WTT of 14,952,021 DKK. We observe that, despite the changes in passenger demand, the MHeuPA is able to obtain solutions that have lower WTT than the initial timetable. Furthermore, the integration with a PTTA model proved to be beneficial, evidenced by the solutions with lower WTT obtained with the MHeuPA solutions in comparison to the Route_Fixed solutions. The NoCosts waiting cost function once again outperforms the other cost functions, with a reduction in WTT of -0.29% in the random ODt scenario and of -0.31% in the event simulation scenario, in comparison to the Route_Fixed solutions. Furthermore, the NoCosts is also the best performing waiting cost function when comparing with the initial timetable, with WTT reductions of -0.92% in the random ODt scenario and of -0.61% in the special event simulation scenario.

We acknowledge that, in general, in case of a large event it is important to evaluate if there is capacity to transport the higher volumes of passengers. Since our approach does not take into account capacity constraints, it is out of the scope of the current work to consider the analysis of capacity restrictions. The purpose of the above described cases is to demonstrate new timetables can *also* be found in case of a change in demand scenarios. Thus, the `MHeuPA` is suitable to use in a wide range of situations: improving on the current timetable, dealing with a change in the network, dealing with a change in passenger demand.

7 Conclusions and future research

This paper addresses the problem of maximizing passenger service through timetabling under the assumptions of free passenger route choice within a fixed budget for operating costs at a tactical level. Free route choice implies that passengers follow their individually preferred path, rather than one that optimizes a social optimum, and that passengers with the same origin, destination, and departure time may have different preferences. The latter ensures that in case two equivalent routes exist, passengers are assumed to use both.

The proposed matheuristic for the IPAT-VSP combines two state-of-the-art models: the integrated timetabling and vehicle scheduling model of Fonseca et al. (2018) with the passenger route choice model of Briem et al. (2017). Provided an initial timetable and an ODt matrix describing passenger demand over time, the objective of the `MHeuPA` is to maximize passenger service, expressed as weighted travel time, through modifications of the timetable. These modifications consist of changes in the starting time of trips (shifts), and addition of dwell time (stretches) at transfer stops, in comparison to the initial timetable within a set of headway constraints and a budget on operating costs. Operating costs are defined by the minimum cost vehicle schedules for a timetable, which problem is simultaneously solved during the timetabling procedure.

A realistic case study focused on timetabling bus lines in the context of the multi-modal network of the Greater Copenhagen area illustrates that (i) including free passenger route choice leads to timetables with higher passenger service than assuming fixed passenger route choice such as in Fonseca et al. (2018), (ii) that the indication of *potentially* interesting transfers for passengers results in timetables with a higher passenger service than providing the timetabling model information on the precise passenger route choice on the current timetable, and (iii) that benefits of including free passenger route choice can be found in comparison to the current timetable of our case study area, in case of a change in the network, and in case of a change in passenger demand. The latter also suggests that the proposed `MHeuPA` approach could be used to design new timetables in case of changes in the network, e.g. due to planned maintenance, or in case of an expected change in the demand matrix, e.g. due to special events.

Although the higher passenger service in our case study results from a trade-off between passenger groups, the increase in service results foremost from a sizable decrease in WTT for a large group of passengers that offsets the increase in WTT for others. Overall improvement in WTT in comparison to the initial timetable is approximately 1%, of which 0.25% is due to the inclusion of free passenger route choice. The 0.25% is equivalent to a daily reduction of approximately 40,000 DKK when expressed as value of time. Due to the budget constraints, these savings come at no additional operating costs.

In summary, this paper contributes to the field of timetabling and public transport planning by studying integrated maximal passenger service timetabling and vehicle scheduling in the context of a realistic free passenger route choice model representing free route choice of passengers; demonstrating that the inclusion of free passenger route choice leads to timetables with higher passenger service and that the indication of potential important transfers for passengers is more important than providing a timetabling model with accurate information on the passenger route choice in a current, initial timetable.

Future research may focus on a further integration of passenger route choice decisions into the timetabling and vehicle scheduling model; or on extending the timetabling procedure to include decisions on stops per line and frequency, which have a major influence on passenger service but are currently generally fixed in the previous planning stage of line planning and network design. Moreover, future research could focus on finding exact lower bounds for the maximal passenger service timetabling problem.

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Appendix 1 - IT-VSP mathematical model

Sets	
S	Set of all stops
L	Set of all directed lines
$T = \{1, \dots, n\}$	Set of all timetabled trips
$T_l \subseteq T$	Subset of all trips in the directed line $l \in L$
$T^1 \subseteq T$	Set of all trips which are the first in their directed line
$S_i \subseteq S$	Set of all stops visited by trip $i \in T$
$J_i \subseteq S_i$	Set of all intermediate stops visited by trip $i \in T$, i.e., $J_i = S_i \setminus \{st_i, et_i\}$
R	Set of all transfer opportunities, each defined by a triplet (i, l, s): passengers disembarking trip $i \in T$ at stop $s \in J_i \cup \{et_i\}$ with the intent of embarking a trip $j \in T_l$ of line $l \in L$ such that $l \neq l_i$ and $s \in J_j \cup \{st_j\}$
K	Set of all depots
I	Set of all compatible trips, $I = \{(i, j) \vert i, j \in T : i \neq j, Dist(et_i, st_j) \leq u, a^-_{i,et_i} + q^- + b_{ij} \leq d^+_{j,st_j}, a^+_{i,et_i} + q^+ + b_{ij} \geq d^-_{j,st_j}\}$
V_k	Set of nodes, which contains a node for each trip $i \in T$, as well as for depot $k \in K$ which is denoted $n + k$, thus $V_k = T \cup \{n + k\}$
A_k	Set of arcs, including deadhead trips, pull-out trips, and pull-in trips, thus $A_k = I \cup (\{n + k\} \times T) \cup (T \times \{n + k\})$
$G_k = (V_k, A_k)$	Graph associated with depot $k \in K$
Q^D	Set of all deadhead triplets $Q^D = \{(i, j, k) : k \in K, (i, j) \in I\}$
Q^O	Set of all pull-out triplets $Q^O = \{(n + k, j, k) : k \in K, j \in T\}$
Q^H	Set of all pull-in triplets $Q^H = \{(i, n + k, k) : i \in T, k \in K\}$
Q	Set of all compatible triplets (i, j, k), representing a vehicle from depot $k \in K$ covering the pair of trips $(i, j) \in A_k$. $Q = Q^D \cup Q^O \cup Q^H$
$T(Q)$	Set of all pairs of trips $i, j \in T$ for which a triplet involving i and j exists, $T(Q) = \{(i, j) \vert i, j \in T : \exists (i, j, k) \in Q\}$.
Parameters	
l_i	Directed line of trip $i \in T$
t_i	Total travel time of trip $i \in T$ in the initial timetable
$st_i \in S_i$	Start terminal of trip $i \in T$
$et_i \in S_i$	End terminal of trip $i \in T$
h^-_{is}, h^+_{is}	Minimum and maximum headways, respectively, in relation to the timetabled headways, for each trip $i \in T$ at each stop $s \in J_i \cup \{st_i\}$
$d^-_{i,st_i}, d^+_{i,st_i}$	Minimum and maximum departure shift from the first station for trip $i \in T$, defined in relation to its departure time in the initial timetable
w^-_{is}	Dwell time in the initial timetable of a trip $i \in T$ at stop $s \in J_i$
w^+_{is}	Maximum allowed dwell time of a trip $i \in T$ at stop $s \in J_i$
w	Upper bound on the total added dwell time to all stops of any trip
Λ_{is}	Number of passengers that are on board (and will continue on board) when trip $i \in T$ arrives at stop $s \in J_i$
a^-_{is}, a^+_{is}	Earliest and latest arrival times of trip $i \in T$ at stop $s \in J_i \cup \{et_i\}$, determined by the possible timetable modifications
d^-_{is}, d^+_{is}	Earliest and latest departure times of trip $i \in T$ from stop $s \in J_i \cup \{st_i\}$, determined by the possible timetable modifications
f_r	Number of passengers requesting transfer $r \in R$
e_r	Minimum transfer time for transfer $r \in R$
q^-, q^+	Minimum and maximum turnaround times
b_{ij}	Driving time between et_i and st_j
v_k	Number of vehicles available at depot $k \in K$
$Dist(i, j)$	Distance between the end terminal of trip $i \in T$, et_i, and the start terminal of trip $j \in T$, st_j
u	Maximum deadhead distance
c_{ijk}	Operating cost associated with servicing triplet $(i, j, k) \in Q$. The cost c_{ijk} of triplet $(i, j, k) \in Q$ is equal to the deadhead time b_{ij} multiplied by a driving cost per time unit; if $(i, j, k) \in Q^O$, c_{ijk} also includes a fixed cost for creating a new schedule, corresponding to the fixed cost for using a vehicle.
c^{DW}	Operating cost per minute of extra dwell time

c^{OB}	Cost per minute of extra dwell time per on board passenger
c^{TR}	Cost per minute of excess transfer time at transfers per passenger
Decision variables	
$x_{ijk} \in \{0,1\}$	1 if and only if a vehicle from depot k travels from node i directly to node j, 0 otherwise
$\tau^d_{is} \in \mathbb{Z}^+_0$	Departure time of trip $i \in T$ from stop $s \in J_i \cup \{st_i\}$
$\tau^a_{is} \in \mathbb{Z}^+_0$	Arrival time of trip $i \in T$ at stop $s \in J_i \cup \{et_i\}$
$\gamma_r \in \mathbb{R}^+_0$	Excess transfer time for passengers using transfer location $r \in R$
$\alpha_{ijs} \in \{0,1\}$	1 if and only if passengers of transfer location $r = (i, l_j, s) \in R$ embark trip $j \in T$, 0 otherwise
$\delta_{is} \in \mathbb{Z}^+_0$	Minutes of dwell time added to trip $i \in T$ at stop $s \in J_i$

We define a directed line $l \in L$ as a sequence of stops $s \in S$ visited by a vehicle, the set of all trips $T = \bigcup_{l\in L} T_l$ and $T_{l'} \cap T_{l''} = \emptyset$ for all $l', l'' \in L, l' \neq l''$. Passengers are assumed to transfer to the earliest feasible trip $j \in T_l$. A transfer r from trip i to line l at stop s, $r = (i, l, s) \in R$, is feasible when the minimum transfer time for transfer $r \in R$, e_r, is not greater than the difference between the departure time of trip $j \in T_l$ from stop s and the arrival time of trip i at stop s. The turnaround time should generally be in the interval $[q^-, q^+]$. Buffer time added to the trip in the form of dwell time is subtracted from the minimum turnaround time q^-. Each vehicle used in a feasible solution covers a sequence of compatible trips and must return to the depot from which it departed. Two trips $i, j \in T$ are compatible if the following three conditions hold: (a) $Dist(et_i, st_j)$ is smaller than u; (b) The sum of a^-_{i,et_i}, q^-, and b_{ij} is smaller or equal to d^+_{j,st_j}; and (c) the sum of a^+_{i,et_i}, q^+, and b_{ij} is greater or equal to d^-_{j,st_j}. The α_{ijs} variables indicating transfer opportunities are defined only for a set $W = \{(i,j,s) | i,j \in T, s \in S : r = (i,l,s) \in R, j \in T_l, i \neq j, a^-_{is} + e_r \leq d^+_{js}, a^+_{is} + e_r + 1.5h_l \geq d^-_{js}\}$, where h_l is the largest frequency observed for line $l \in L$ throughout the day. This improves the tractability of the model by reducing the number of α_{ijs} variables created, without imposing any practical constraints, since at least one transfer to a trip in $l \in L$ will be available given the timetable modifications.

The MILP formulation for the IT-VSP is:

$$\min \sum_{i\in T}\sum_{s\in J_i} c^{OB}\Lambda_{is}\delta_{is} + \sum_{r\in R} c^{TR} f_r \gamma_r \tag{1}$$

$$\text{s.t.} \sum_{(i,j,k)\in Q} c_{ijk}x_{ijk} + \sum_{i\in T}\sum_{s\in J_i} c^{DW}\delta_{is} \leq B \tag{2}$$

$$\sum_{(i,j,k)\in Q} x_{ijk} = 1 \qquad i \in T \tag{3}$$

$$\sum_{(i,j,k)\in Q} x_{ijk} - \sum_{(j,i,k)\in Q} x_{jik} = 0 \qquad k \in K \quad j \in V_k \tag{4}$$

$$\sum_{(i,j,k)\in Q^O} x_{ijk} \leq v_k \qquad k \in K \tag{5}$$

$$d^-_{i,st_i} \leq \tau^d_{i,st_i} \leq d^+_{i,st_i} \qquad i \in T \tag{6}$$

$$0 \leq \tau^d_{is} - \tau^a_{is} - w^-_{is} \leq w^+_{is} \qquad i \in T \quad s \in J_i \tag{7}$$

$$\sum_{s\in J_i} \delta_{is} \leq w \qquad i \in T \tag{8}$$

$$\delta_{is} = \tau^d_{i,s} - \tau^a_{i,s} - w^-_{is} \qquad i \in T \quad s \in J_i \tag{9}$$

$$h^-_{is} \leq \tau^d_{is} - \tau^d_{i-1,s} \leq h^+_{is} \qquad l \in L \quad i \in T_l : i \notin T^1 \quad s \in J_i \cup \{st_i\} \tag{10}$$

$$\tau^a_{i,et_i} + b_{ij} + q^- - \sum_{s\in J_i}\delta_{is} - M\Big(1 - \sum_{(i,j,k)\in Q} x_{ijk}\Big) \leq \tau^d_{j,st_j} \qquad (i,j) \in T(Q) \tag{11}$$

$$M \sum_{\substack{k\in T_l:(i,k,s)\in W,\\ k\leq j}} \alpha_{iks} \geq \tau^d_{js} - \tau^a_{is} - e_r \qquad r \in R \quad (i,j,s) \in W \tag{12}$$

$$\tau^d_{js} - \tau^a_{is} - e_r \geq M(\alpha_{ijs} - 1) \qquad r \in R \quad (i,j,s) \in W \tag{13}$$

$$\sum_{j\in T_l:(i,j,s)\in W} \alpha_{ijs} = 1 \qquad r = (i,l,s) \in R \tag{14}$$

$$\tau^d_{js} - \tau^a_{is} - e_r - M(1 - \alpha_{ijs}) \leq \gamma_r \qquad r \in R \quad (i,j,s) \in W \tag{15}$$

$$x_{ijk} \in \{0,1\} \qquad (i,j,k) \in Q \tag{16}$$

$$\tau^d_{is} \in \mathbb{Z}_+ \qquad i \in T \quad s \in J_i \cup \{st_i\} \tag{17}$$

$$\tau^a_{is} \in \mathbb{Z}_+ \qquad i \in T \quad s \in J_i \cup \{et_i\} \tag{18}$$
$$\delta_{is} \in \mathbb{Z}_+ \qquad i \in T \quad s \in J_i \tag{19}$$
$$\gamma_r \in \mathbb{R}_+ \qquad r \in R \tag{20}$$
$$\alpha_{ijs} \in \{0,1\} \qquad (i,j,s) \in W \tag{21}$$

The objective function (1) minimizes a weighted sum of passengers' costs. The first term refers to on board passenger costs incurred when adding dwell time to trips. The second term refers to costs associated with excess transfer times.

Constraints (2) impose an upper bound (budget) on the operating costs. The first term considers driving operating costs for deadhead, pull out, and pull in trips, and the second term considers operating costs associated with additional dwell times. Constraints (3) - (5) model classical MDVSP constraints: assignment constraints (3) guarantee coverage of each trip $i \in T$ by including it in exactly one vehicle schedule; flow conservation constraints (4) on trip and depot nodes guarantee the continuity of the vehicle schedules created; and capacity constraints (5) limit the number of pull-out trips to the maximum number of vehicles available at each depot $k \in K$.

Constraints (6) - (10) model timetable modifications: constraints (6) force lower and upper shift bounds on the departure time from the first stop of each trip; constraints (7) impose a maximum added dwell time at each stop of a trip; constraints (8) bound the total added dwell time to all intermediate stops of a trip; constraints (9) define the δ_{is} variables to the added dwell time in the corresponding intermediate stop $s \in J_i$ of trip $i \in T$; and constraints (10) model the minimum and maximum headways between each trip $i \in T$ and its precedent trip in the same directed line at each stop $s \in J_i \cup \{st_i\}$. Linking constraints (11) relate the vehicle scheduling and the timetable modification parts of the problem. These guarantee that if trips i and j are serviced consecutively by the same vehicle, then the vehicle has time to deadhead from et_i to st_j without violating the minimum turnaround time q^-.

Linking constraints (12) and (13) relate the transfer variables α_{ijs} and the departure and arrival times of trips: constraints (12) ensure that passengers arriving from trip i at stop s transfer to one of the trips j, such that $(i, l(j), s) \in R$, if the arrival and departure times allow the transfer to take place; constraints (13) prevent variable α_{ijs} from taking value 1 whenever passengers do not have enough time to transfer from trip i to trip j at stop s, where $(i, l(j), s) \in R$. Constraints (14) impose that each transfer location is performed by transferring to exactly one trip $j \in T_l$. Constraints (15) define the values of γ_r variables to the excess transfer times, determining this value for each transfer location based on the selected transfers. Constraints (16)-(21) define the range of all sets of decision variables.