# Social dilemmas among unequals 

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Direct reciprocity is a powerful mechanism for evolution of cooperation, based on repeated interactions ${ }^{1-4}$. It requires that interacting individuals are sufficiently equal, such that everyone faces similar consequences when they cooperate or defect. Yet inequality is ubiquitous among humans ${ }^{5,6}$ and is generally considered to undermine cooperation and welfare $^{7-10}$. Most previous models of reciprocity neglect inequality ${ }^{11-15}$. They assume that individuals are the same in all relevant aspects. Here we introduce a general framework to study direct reciprocity among unequals. Our model allows for multiple sources of inequality. Subjects can differ in their endowments, their productivities, and in how much they benefit from public goods. We find that extreme inequality prevents cooperation. But if subjects differ in productivity, some endowment inequality can be necessary for cooperation to prevail. Our mathematical predictions are supported by a behavioral experiment where we vary the subjects' endowments and their productivities. We observe that overall welfare is maximized when the two sources of heterogeneity are aligned, such that more productive individuals receive higher endowments. In contrast, when endowments and productivities are misaligned, cooperation quickly breaks down. Our findings have implications for policy-makers concerned with equity, efficiency, and public goods provisioning.

In social dilemmas, overall welfare is maximized if all individuals cooperate, yet each individual prefers to defect ${ }^{16}$. Such dilemmas occur at all levels of human society. They affect families, companies, and nations ${ }^{17,18}$. An extensive body of research has shown that cooperation is more likely when groups are stable and subjects interact repeatedly ${ }^{11-15}$. This mechanism of direct reciprocity, however, presumes that group members have sufficient leverage to influence one another. Subjects need to be able to give appropriate responses. Tit-for-tat can only be effective if it incentivizes others to cooperate. Most previous models of reciprocity assume perfect symmetry between individuals ${ }^{11-15}$. Real groups often exhibit substantial heterogeneity coming from multiple sources ${ }^{5,6}$. Experimental work shows that inequality in players' endowments reduces cooperation ${ }^{7,8}$ and undermines the social structure of a population ${ }^{9}$. Even if subjects start out equally, game dynamics can introduce inequality over time, disfavoring individuals who are more cooperative ${ }^{19}$ (see SI). So far, it has been difficult to predict the effect of heterogeneity on cooperation, especially if subjects vary along multiple dimensions. Here, we propose a general framework to explore how different kinds of heterogeneities interact and affect cooperation.

We consider public good games with $n$ players. In each round, player $i$ receives a fixed endowment $e_{i}$, which can be interpreted as a regular income. After receiving their endowments, players independently decide which fraction $x_{i}$ of their endowment to contribute to the public good. The payoff $u_{i}$ of player $i$ for that round depends on the distribution of endowments, $e_{1}, \ldots, e_{n}$, and on the players' relative contributions, $x_{1}, \ldots, x_{n}$. It is typically assumed that all players have the same endowment and that contributions to the public good are multiplied by a common productivity factor, $r$ (Fig. 1a). Instead, here we consider interactions where players have different endowments, different productivities or where payoffs are nonlinear (Fig. 1b-d).

As a specific example, we consider public good games where each players' contributions are multiplied by an individual factor $r_{i}$ and equally shared among all participants,

$$
\begin{equation*}
u_{i}=\frac{1}{n} \sum_{j=1}^{n} r_{j} e_{j} x_{j}+\left(1-x_{i}\right) e_{i} . \tag{1}
\end{equation*}
$$

The first term represents the payoff derived from the public good, the second term the player's remaining endowment. We interpret the factors $r_{j}$ as the players' productivities and assume $1<r_{j}<n$ for all $j$. Thus, the game is a social dilemma in which individuals have an incentive to free-ride ${ }^{16}$. While we focus on example (1) throughout most of the main text, our findings generalize to arbitrary public good games that satisfy four natural requirements (see Materials and Methods).

If the public good game (1) is played once, defection is the only equilibrium. But for repeated interactions, cooperation can prevail if players adopt conditional strategies such as Tit-for-Tat ${ }^{2}$,

Win-Stay Lose-Shift ${ }^{11}$, or multiplayer-variants thereof ${ }^{20}$. We assume after each round there is another one with probability $\delta$.

To explore the effects of different kinds of heterogeneity, we first characterize when cooperation can be maintained. For a given public good game and a given endowment distribution, we say that full cooperation is feasible if there is a subgame perfect equilibrium in which all players always contribute their entire endowment. In such an equilibrium, players have no incentive to deviate after any history of previous play ${ }^{21}$. In the SI we prove that cooperation is feasible if and only if the strategy Grim is an equilibrium. Grim cooperates unless another player has defected in a previous round ${ }^{3}$. From the equilibrium condition for Grim, it follows that cooperation is feasible in game (1) if and only if for all players $i$ with $e_{i}>0$ we have

$$
\begin{equation*}
\frac{\delta}{n} \sum_{j \neq i} r_{j} e_{j} \geq\left(1-\frac{r_{i}}{n}\right) e_{i} . \tag{2}
\end{equation*}
$$

The expected benefit from the future cooperation of others must exceed the incentive to defect in the present round. For cooperation to be feasible, future losses must outweigh present gains.

Based on this general characterization of when cooperation is feasible, we derive a number of results. First, cooperation is never feasible if there is too much inequality, such that most of the endowment is in the hands of one player (see SI). For linear and symmetric games (Fig. 1), we show that if cooperation is feasible at all, it is feasible for equal endowments (Fig. 2a). However, if the game is asymmetric (Fig. 2b) or non-linear (Fig. 2c), full cooperation may only be feasible when players have unequal endowments. In such a case it can even be optimal to give some players no initial endowment at all.

To gain intuition, consider the case where players differ in productivities, $r_{1}>\ldots>r_{n}$. We find a two-fold advantage of giving higher endowments to more productive players. First, there is a stability advantage: an unequal distribution of endowments makes it easier for full cooperation to be an equilibrium. To see this, assume instead that players receive equal endowments. Then condition (2) implies cooperation is feasible if

$$
\begin{equation*}
\delta \geq \frac{n-r_{n}}{R-r_{n}} \tag{3}
\end{equation*}
$$

Here $R=r_{1}+\ldots+r_{n}$ is the sum of all productivities. For equal endowments, player $n$ with lowest productivity faces the largest temptation to defect, because this player has the highest marginal cost $1-r_{n} / n$ of contributing. This temptation can be counter-balanced by allocating a smaller endowment to player $n$ who then has less gain from withholding, whereas the others have more leverage to retaliate in future rounds. Both effects enhance the stability of cooperation. Second, there is an efficiency advantage of unequal endowments. Because contributions of more
productive players are multiplied by a higher factor, social welfare is maximized when the most productive player obtains the largest share of the initial endowment - subject to the constraint that full cooperation is feasible.

If the game only involves two players, we can compute which endowment distribution is most conducive to cooperation. An endowment distribution is maximally cooperative if it requires the lowest continuation probability $\delta$ for cooperation to be feasible. Using condition (2), we show in the SI that endowments need to be distributed as

$$
\begin{equation*}
\frac{e_{1}}{e_{2}}=\sqrt{\frac{r_{2}\left(2-r_{2}\right)}{r_{1}\left(2-r_{1}\right)}} \tag{4}
\end{equation*}
$$

Equal distribution, $e_{1}=e_{2}$, is only maximally cooperative if players have the same productivities. Otherwise, the more productive player should have a larger share of the endowment.

After exploring under which conditions cooperation is feasible, we study when cooperation can evolve ${ }^{3}$. To make an evolutionary approach computationally tractable, we first consider players who only respond to the outcome of the last round. Moreover, we assume players only choose from a finite set of possible contributions. For example, they may either contribute their full endowment or nothing at all. In that case we refer to the two possible actions as cooperation and defection, respectively. With some small probability $\varepsilon$, players commit errors such that a player who intends to cooperate defects by mistake (and vice versa). Players adopt new strategies over time by comparing their payoff to the payoff they would obtain by using a random alternative strategy. The better the payoff of the alternative strategy, the more likely players switch. We iterate this process for many steps and record the average cooperation rates over time (see Materials and Methods).

Our numerical findings parallel the previous equilibrium results. Cooperation cannot evolve if one of the players receives almost all of the endowment. Moreover, for linear and symmetric games, individuals are most likely to cooperate if everyone receives the same endowment (Fig. 2d). Yet if some players are more productive than others (Fig. 2e), or if the game is non-linear ( $\mathbf{F i g} . \mathbf{2 f}$ ), unequal endowments yield more cooperation and higher payoffs. In all cases, we observe that the strategy Grim is less relevant, because it cannot sustain cooperation in the presence of noise ${ }^{3}$. Instead, cooperation evolves if the strategy Win-Stay Lose-Shift ${ }^{11}$ (WSLS) is an equilibrium (Extended Data Figs. 1-5). WSLS contributes the full endowment in the first round, or if all players made the same relative contribution in the previous round. Otherwise WSLS contributes nothing ${ }^{11,20}$.

In the simulations, a group of defectors is most likely invaded by strategies like Tit-for-Tat.

These conditional cooperators in turn quickly adopt WSLS which is more robust with respect to errors. However, because of stochasticity, any strategy is replaced eventually, even if it is an equilibrium (see SI). Further simulations show that analogous results hold when players choose between more than two discrete contributions each round (Fig. 3), or when strategies are represented by finite-state automata ${ }^{15}$ (Extended Data Fig. 6).

To explore the applicability of these theoretical results, we have designed an online behavioral experiment based on the two player game of Fig. 3. Participants are either equally productive or not. They have the same endowment or not. We obtain five treatments: full equality, endowment inequality, productivity inequality, aligned inequality, and misaligned inequality (Fig. 4a). In the last two treatments, individuals differ in both dimensions. Either the more productive player (aligned) or the less productive player (misaligned) receives the larger endowment. Previous experiments suggests that - in isolation - heterogeneous endowments reduce overall cooperation ${ }^{7,8}$, whereas heterogeneous productivities have a negligible effect ${ }^{22}$. Here we study the interaction of the two heterogeneities in repeated games, for which previous research did not find significant effects ${ }^{23}$.

Based on our evolutionary analysis, we expect aligned inequality to increase welfare and misaligned inequality to reduce it, compared to the case of productivity inequality alone (Fig. 3). The experiment confirms these predictions (Fig. $\mathbf{4 b} \mathbf{- e}$ ). Aligned inequality results in substantially higher contributions than misaligned inequality and generates the highest surplus across all treatments. Under aligned inequality, most high-endowment players match the relative contribution of the low-endowment players. That is, if the low-endowment player gives his full endowment, then so does the high-endowment player, even if her absolute contributions are three times higher. In contrast, contributions under misaligned inequality do not follow a clear norm; often the high-endowment player only matches the co-player's absolute contribution (Extended Data Figs. 7-10 and SI).

We have introduced a general framework to study direct reciprocity among unequals. Our three complementary approaches - equilibrium calculations, evolutionary simulations, and a behavioral experiment - suggest an unexpected benefit of inequality. We show that equal endowments can be detrimental to social welfare if subjects differ along multiple other dimensions, such as productivity or benefit from public goods. In those cases, some inequality can increase both the stability of cooperation and the efficiency of contributions.

Despite these potential benefits, inequality comes with caveats. First, maximizing cooperation requires a delicate balance between different dimensions of heterogeneity. Finding the right
degree of inequality can prove difficult when the players' personal characteristics, such as their productivities, are only known imperfectly. The problem is aggravated by our finding that an excess of inequality is always detrimental. Second, endowment inequality could interfere with institutional solutions to cooperation. For example, when cooperation is maintained through sanctions, heterogeneous groups may disagree on which norm to enforce ${ }^{24}$. Additional problems arise when sanctioning institutions can be corrupted ${ }^{25,26}$, especially when better-endowed individuals can 'play the system'.

Finally, reducing inequality is often considered an important policy objective in itself. Humans dislike inequality ${ }^{27}$ and are sometimes willing to sacrifice their own wealth to guarantee more egalitarian outcomes ${ }^{28,29}$. In addition, inequality often renders successful coordination in social dilemmas difficult, since different actors may disagree on which cooperative equilibrium is fair ${ }^{30}$. However, here we have shown that inequality does not need to render cooperation impossible. When individuals are naturally heterogeneous, moderate inequality can be necessary for cooperation to prevail.

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## Author contributions.

All authors conceived the study, performed the analysis, discussed the results and wrote the manuscript.

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## Main figure legends

Figure 1: Public good games among unequals. We consider social dilemmas in which participants decide how much of their endowment $e_{i}$ to contribute to a public good. Each players' contributions are multiplied by $r_{i}$ and then divided among all players. The players have equal endowments if $e_{1}=e_{2}=e_{3}$. The game is symmetric if players are indistinguishable except for their endowments and contributions. Here, the game is symmetric if $r_{1}=r_{2}=r_{3}$. The game is linear if the payoffs depend linearly on the players' endowments and contributions. Here, the game is linear if the factors $r_{i}$ are constant. a, Most previous work assumes that players have the same endowment, the game is symmetric, and payoffs are linear. b-d, Instead, we allow players to have different endowments, different productivities, and nonlinear payoffs. e, We derive general results for $n$ player games. $\mathbf{f}$, As a special case, we study pairwise interactions.

Figure 2: Feasibility and evolvability of cooperation in public good games among unequals. We consider groups of three players who interact in three different public goods games. In each case, we ask when equal endowments help to maintain cooperation (a-c), or favor its evolution (d-f). The triangles represent the possible ways to distribute the initial endowment among the players. Corners correspond to distributions where one player receives all the endowment. Edges correspond to distributions where one player receives no endowment. The center of the triangle marks equal endowments. 'Group payoff' in the lower panels corresponds to the total payoff across all group members, averaged over $10^{6}$ time steps of an evolutionary simulation. We find that extreme inequality is always detrimental to cooperation. However, when the game is asymmetric or nonlinear, slightly unequal endowments may be necessary for cooperation to be feasible (b, c), and for cooperation to evolve (e, f).

Figure 3: When players differ in their productivities, equal endowments do not maximize contributions. We consider public good games between two players. a, Players either coincide in their productivities, $r_{1}=r_{2}=1.6$, or $\mathbf{b}$, player 1 is more productive, $r_{1}=1.9, r_{2}=1.3$. In each case, we vary player 1's share of the initial endowment. We perform evolutionary simulations for three scenarios, depending on what fraction of the endowment players can contribute: either $\{0,1\}$ or $\{0,1 / 2,1\}$ or $\{0,1 / 3,2 / 3,1\}$. For equal productivities, cooperation is most likely to emerge when both players receive the same endowment. In contrast, when players differ in productivity, the maximum total payoff is achieved when player 1 obtains a larger fraction of the endowment. The position of the maximum is well-approximated by the maximally cooperative endowment distribution (4).

Figure 4: Exploring the effects of multidimensional inequality with a behavioral experiment. a, Based on the two-player game of Fig. 3, we conduct experiments with varying endowments and productivities. There are five conditions: (1) full equality, (2) endowment inequality, (3) productivity inequality, (4) aligned inequality (the more productive player has higher endowment), and (5) misaligned inequality (the more productive player has lower endowment). b-e, For each treatment, we compare the theoretical predictions from evolutionary simulations (grey bars) with the respective average values of the experiment (colored bars). We show the players' relative contributions (upper panels) and the generated surplus (by how much the players' total payoffs exceed their initial endowments, lower panels). Aligned inequality yields high cooperation rates and higher payoffs than other treatments. Colored dots represent individual groups of players; the number of observations for each treatment are 42, 42, 40, 39, 40, respectively. Error bars represent $95 \%$ confidence intervals. Two asterisks indicate experimental differences with $p<0.01$; three asterisks indicate differences with $p<0.001$ (using two-tailed Mann-Whitney tests). See Materials and Methods.


Figure 1: Public good games among unequals.


Figure 2: Feasibility and evolvability of cooperation in public good games among unequals.
Possible
contributions $x \in\{0,1\} \quad x \in\{0,1 / 2,1\} \quad x \in\{0,1 / 3,2 / 3,1\}$


Figure 3: When players differ in their productivities, equal endowments do not maximize contributions.


Figure 4: Exploring the effects of multidimensional inequality with a behavioral experiment.

## Materials and Methods

## General modeling framework.

In the main text we have used the public goods game (1) to illustrate our main findings. However, the framework that we use to study reciprocity in asymmetric social dilemmas is general and can encompass many other examples (as indicated in Fig. 2c,f). Here we briefly introduce our general framework. A full account is provided in the SI.

We consider games with $n$ players. The players' endowments in each round are given by the endowment vector $\mathbf{e}=\left(e_{1}, \ldots, e_{n}\right)$. Endowments are nonnegative, $e_{i} \geq 0$ for all players $i$, and normalized, $e_{1}+\ldots+e_{n}=1$. Given their endowments, players decide which fraction of their endowment they contribute, as summarized by the contribution vector $\mathbf{x}=\left(x_{1}, \ldots, x_{n}\right)$, with $x_{i} \in[0,1]$ for all players $i$. We refer to $x_{i}$ as the player's relative contribution, and to $e_{i} x_{i}$ as the player's absolute contribution. We use the shorthand notation $\mathbf{x}=\mathbf{0}$ if no player contributes to the public good, and $\mathbf{x}=\mathbf{1}$ if all players contribute their full endowment. Given the endowments $\mathbf{e}$ and the relative contributions $\mathbf{x}$ in a given round, player $i$ 's payoff in that round is $u_{i}(\mathbf{e}, \mathbf{x})$. If there are no contributions, players receive their initial endowment $u_{i}(\mathbf{e}, \mathbf{0})=e_{i}$.
We consider public good games that satisfy the following four conditions:
(C) Continuity: The payoff functions $u_{i}(\mathbf{e}, \mathbf{x})$ are continuous in both arguments.
(PE) Positive externalities: As a player with a positive endowment increases her contribution, the payoffs of all other players increase.
(IF) Incentive to free-ride: As a player with positive endowment increases her contribution, her own payoff decreases.
(OC) Optimality of cooperation: As a player with positive endowment increases her contribution, the overall payoff over all members of the group increases.

The first condition of continuity is merely a technical assumption that is useful for some of the analytical results. The other three conditions generalize previous notions of social dilemmas in one-shot games where players can either cooperate or defect ${ }^{16}$.

We note that the above conditions rule out certain threshold public good games, where payoffs increase discontinuously once total contributions exceed a certain threshold ${ }^{31-34}$. In such threshold public good games, cooperation can often emerge even if the game is only played once, because players have an incentive not to fall below the threshold ${ }^{35}$. By considering public good games that satisfy $\mathbf{( C )}-(\mathbf{O C})$, we consider the most stringent case of a social dilemma, where repeated interactions are key to sustain positive contributions. Asymmetric threshold public good games in a one-shot or finite-horizon setting have been studied in Refs. 36-39. Hauser et al ${ }^{40}$ explore the consequences of heterogeneities in the players' background fitness. Finally, Ethan Akin characterizes strategies to maintain cooperation in the asymmetric prisoner's dilemma ${ }^{41}$.

We can classify public good games according to two properties, linearity and symmetry. We say a public good game is linear if payoffs $u_{i}(\mathbf{e}, \mathbf{x})$ are linear in both arguments, $\mathbf{e}$ and $\mathbf{x}$. A public good game is symmetric if players are indistinguishable, except for their endowments and for their contributions. Formally, if $\sigma=\left(\sigma_{1}, \ldots, \sigma_{n}\right)$ is a permutation of the numbers $1, \ldots, n$, and if $\mathbf{e}_{\sigma}$ and $\mathbf{x}_{\sigma}$ are the respectively permuted endowment and contribution vectors, then the public good game is symmetric if $u_{i}(\mathbf{e}, \mathbf{x})=u_{\sigma_{i}}\left(\mathbf{e}_{\sigma}, \mathbf{x}_{\sigma}\right)$ for all permutations $\sigma$, endowments $\mathbf{e}$ and contributions $\mathbf{x}$. That is, if players were to switch roles with respect to their endowments and contributions, their payoffs would change accordingly. In particular, the public good game (1) is symmetric if and only if all players have the same productivity, $r_{1}=\ldots=r_{n}$.

## Equilibrium analysis.

We explore under which conditions full cooperation can be sustained if the public good game is repeated. After each round, there is another round with probability $\delta$. Strategies for the repeated game are rules that tell the player which fraction of the endowment to contribute, depending on the players' endowments and on all previous contributions. If the contribution vector in round $t$ is $\mathbf{x}(t)$, payoffs are given by the weighted average payoff per round, $\pi_{i}=(1-\delta) \sum_{t=0}^{\infty} \delta^{t} u_{i}(\mathbf{e}, \mathbf{x}(t))$.

For a given public good game with payoff function $u=\left(u_{1}, \ldots, u_{n}\right)$, continuation probability $\delta$, and endowment distribution $\mathbf{e}$, we say full cooperation is feasible if there is a subgame perfect equilibrium in which all players contribute their full endowment in every round. In a subgame perfect equilibrium, no player has an incentive to deviate after any given history ${ }^{21}$. It is a refinement of the Nash equilibrium concept: every subgame perfect equilibrium is a Nash equilibrium, but the converse does not need to be true.

We refer to the set of all endowments for which full cooperation is feasible as $E_{u}(\delta)$. In Fig. 2a-c, these sets are illustrated as blue areas within the space of all endowment distributions. In the SI, we characterize these sets for all games that satisfy the conditions (C) - (OC). We show that the following are equivalent (SI, Proposition 1):
(i) Cooperation is feasible for a given endowment distribution $\mathbf{e}$
(ii) The condition $\delta\left(u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{0})\right) \geq u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{1})$ holds for all players $i$ with positive endowment. Here, $\mathbf{1}_{-i}$ is the shorthand notation for a group in which everyone contributes the full endowment, except for player $i$ who contributes nothing.
(iii) Grim is a subgame perfect equilibrium for the endowment distribution $e$.

Based on this general characterization, we prove the following implications.

1. For any given payoff function $u$ and continuation probability $\delta$, there is a threshold $e_{i}^{*}<1$ such that $\mathbf{e} \notin E_{u}(\delta)$ holds for any endowment distribution $\mathbf{e}$ with $e_{i}>e_{i}^{*}$. That is, cooperation is never feasible if one of the players receives an excessive share of the endowment.
2. If the public good game is symmetric and linear, and $E_{u}(\delta) \neq \varnothing$, then $(1 / n, \ldots, 1 / n) \in$ $E_{u}(\delta)$. That is, if full cooperation is feasible in a linear and symmetric public goods game, then it is always feasible when all players receive the same endowment.
3. If the public good game is either asymmetric or non-linear, there are cases for which $E_{u}(\delta) \neq \varnothing$, but $(1 / n, \ldots, 1 / n) \notin E_{u}(\delta)$. That is, in asymmetric or non-linear public good games, full cooperation may only be feasible for unequal endowment distributions.

## Evolutionary analysis.

We have also explored the dynamics that arises if players have not yet settled on a particular equilibrium. Instead, they may begin with randomly initialized strategies, and then learn to use more profitable strategies over time.

To this end, we first consider a simplified strategy space. We assume that the players' contributions in any given round only depend on the outcome of the previous round, as in most previous work on evolution of reciprocity ${ }^{41-63}$. In addition, we assume players can only choose among a fixed finite set $X=\left\{\hat{x}_{1}, \ldots, \hat{x}_{m}\right\}$ of possible contributions. Under these two assumptions, the players' strategies take the form of a vector $\mathbf{p}=\left(p_{0, k}, p_{\mathbf{x}, k}\right)$. The entries $p_{0, k}$ for $1 \leq k \leq m$ give the player's probability to choose contribution level $\hat{x}_{k}$ in the first round, when no previous history is yet available. The other entries $p_{\mathbf{x}, k}$ give the player's probability to choose $\hat{x}_{k}$ in subsequent rounds, conditional on the contribution vector $\mathbf{x} \in X^{n}$ of the previous round. For $\mathbf{p}$ to be a sensible strategy, we require $\sum_{k=1}^{m} p_{0, k}=1$ and $\sum_{k=1}^{m} p_{\mathbf{x}, k}=1$ for all $\mathbf{x}$; that is, the strategy must prescribe an action for any given outcome of the previous round. When all players apply memory- 1 strategies, their payoffs in the repeated game can be computed efficiently by representing the game as a Markov chain. The algorithm is shown in the SI.

In the special case that players can only give their full endowment or nothing at all, we obtain $X=\{0,1\}$. We refer to these two possible actions as 'cooperation' (C) and 'defection' (D). When there are only these two possible contribution levels, we can drop the index $k$ in the definition of a memory-1 strategy and write $\mathbf{p}=\left(p_{0} ; p_{\mathbf{x}}\right)$. Under this notation, $p_{z}$ is now the probability that the player will cooperate in the next round. If the game involves only two players, we obtain the typical format of memory- 1 strategies for the iterated prisoner's dilemma ${ }^{3}, \mathbf{p}=$ $\left(p_{0} ; p_{C C}, p_{C D}, p_{D C}, p_{D D}\right)$. For example, the strategy Grim may be approximated by the memory1 strategy $\mathbf{p}=(1 ; 1,0,0,0)$. In the absence of errors, this memory- 1 strategy cooperates if and only if both players have cooperated in all previous rounds.

Pure memory- 1 strategies have entries that are either zero or one. Given the outcome of the previous round, their action is deterministic. Stochastic memory-1 strategy have entries that can take arbitrary values between zero and one. For given $m$ and $n$, there are finitely many pure
memory-1 strategies, but infinitely many stochastic memory-1 strategies.
For our evolutionary analysis, the players' actions may be subject to implementation errors. That is, if the set of possible contributions is $X=\left\{\hat{x}_{1}, \ldots, \hat{x}_{m}\right\}$ and the player decides to choose the contribution $\hat{x}_{k}$, he will instead make a different contribution $\hat{x}_{l}$ with probability $\varepsilon /(m-1)$. We refer to $\varepsilon$ as the players' error rate. For infinitely repeated games (with $\delta=1$ ), errors have the useful mathematical property that they make the game dynamics ergodic. As a result, the players' payoffs will be independent of the players' contribution in the very first round. In that case, we no longer need to specify a player's initial contribution strategy $p_{0, k}$.

To model how players adapt their memory- 1 strategies over time, we introduce an evolutionary process which we call 'introspection dynamics'. For this process, we again consider $n$ players who interact in an asymmetric public goods game. In each evolutionary time step, one of the players is chosen at random to revise her strategy. To this end, this player $i$ considers a randomly chosen alternative memory- 1 strategy. Suppose the original strategy yields payoff $\pi_{i}$, whereas the alternative strategy yields payoff $\tilde{\pi}_{i}$ (keeping the strategies of the co-players fixed). Then player $i$ switches to the alternative strategy with probability $\rho=\left(1+\exp \left[-s\left(\tilde{\pi}_{i}-\pi_{i}\right)\right]\right)^{-1}$. The parameter $s \geq 0$ represents the 'strength of selection'. In the limiting case $s \rightarrow 0$, the switching probability simplifies to $\rho=1 / 2$, such that players adopt new strategies at random. In the other limiting case $s \rightarrow \infty$, the player only adopts the alternative strategy if it yields at least the payoff of her original strategy. Iterating this updating over many time steps, we obtain an ergodic process on the space of all strategy choices of $n$ players. In particular, we note that this process has no absorbing states. Even if a strategy profile is an equilibrium, there is always a positive chance that one of the players deviates due to chance. For small $n$, the invariant distribution of the evolutionary process can be calculated exactly. For larger $n$, the invariant distribution can be approximated by simulations.

Although memory-1 strategies have been routinely used to explore the evolution of reciprocity ${ }^{41-63}$, it is natural to ask to which extent our results depend on the assumption of oneround memory. To explore this issue, we have repeated all simulations with a more general strategy space. We follow the approach by van Veelen and García ${ }^{15,64,65}$. Players can choose among all strategies that can be represented by finite-state automata over the possible contributions $X$. Finite-state automata contain the previously considered memory-1 strategies as a special case. However, they can also encode strategies with arbitrarily long memory (Extended Data Fig. 6a). For the evolutionary process we assume that when a mutation occurs, four cases can occur. Either (i) the action chosen in a given state changes, (ii) a transition between two states changes, (iii) a new state is added to the finite-state automaton, or (iv) an existing state is removed (see Extended Data Fig. 6b). Simulations show that although absolute cooperation
rates tend to be somewhat lower for this strategy space, all our qualitative predictions remain unchanged (Extended Data Fig. 6c). See SI for details.

## Experimental methods.

For our experiment, we have recruited 436 participants on Amazon Mechanical Turk (AMT) to take part in an interactive game. The experiment was implemented with SoPHIE, an online platform that allows for real-time interaction between AMT participants ${ }^{10,66,67}$.

Participants were matched in pairs, which were randomly assigned to one of the five treatments. For each pair, one participant was randomly determined to adopt role A, whereas the other participant obtained role B. Players received $\$ 1.00$ for participating and could earn a bonus payment depending on their performance in the game. The tokens earned during the game were converted to US dollars at a rate of 800 tokens $=\$ 1.00$. The average bonus participants earned was $\$ 1.70$. After reading the experimental instructions (see SI), all participants had to pass a series of comprehension questions to ensure they understood the consequences of their decisions. All players were anonymous. They were only identified by their player ID ("A" or "B"). Each game consisted of at least 20 rounds. Thereafter, the game was continued with a $50 \%$ probability after each round to avoid end-game effects.

The behavioral experiment is based on the public goods game (1). Prior to the first round, both players were assigned an endowment $e_{i}$ and a productivity value $r_{i}$. The possible values of $e_{i}$ and $r_{i}$ are depicted in Fig. 4a. Once assigned, each participant's $e_{i}$ and $r_{i}$ remained constant throughout the experiment. Both players were informed about their own and the other player's endowment and productivity. Each round, participants decided how much to contribute to the public good. They could contribute any integer between 0 and $e_{i}$. A player's absolute contribution was multiplied by the respective productivity value $r_{i}$. All multiplied contributions were split equally among the players. Participants could not observe the other player's contribution before making their own decision. However, after each round, participants learned each other's contributions as well as the resulting payoffs for each player.

We have analyzed the data using two-tailed non-parametric tests, using pairs of two interacting players as our statistical unit. That is, for each quantity of interest, we calculated the respective average value for each pair of players; then we compared this average value across treatments (Fig. 4) or within each treatment (Extended Data Fig. 7, Extended Data Fig. 10). For comparisons between treatments we use the Mann-Whitney test, whereas for comparisons within a treatment we use the Wilcoxon signed-rank test. In the main text and all figures we report the outcome of each test directly, without correcting for multiple testing. However, all our key findings continue to hold when we apply Bonferroni correction (SI Section 5.3).

The sample size was determined in advance based on similar past research ${ }^{10}$. The number of groups that completed the experiment were $N_{1}=42, N_{2}=42, N_{3}=40, N_{4}=39, N_{5}=40$, for each of the five treatments, respectively. We find no significant differences between groups that completed the experiment and groups in which at least one player dropped out (see SI). For the statistical results presented in the main text, we have only used the first 20 rounds of groups that completed the experiment. However, all our conclusions remain valid if we include dropout groups by using multiple imputation (see Supplementary Tables 1 and 2).

In the SI, we provide a full description of the employed methods, and we report all test statistics and $p$-values. Moreover, we discuss further aspects of our empirical results, such as the game dynamics over time or the distribution of contributions (Extended Data Figs. 7-10).

## Parameters used for figures.

Fig. 2a-c shows the region in the endowment space in which full cooperation is feasible. The respective calculations are provided in the $\mathbf{S I}$. The first two columns are based on the linear public goods game (1) with parameters $r_{1}=r_{2}=r_{3}=2, \delta=0.8$ (Fig. 2a) and $r_{1}=2.9, r_{2}=1.5$, $r_{3}=1.1, \delta=0.3$ (Fig. 2b), respectively. The last column considers a non-linear 3-player public goods game with $\delta=0.35$ (Fig. 2c) and payoff function

$$
\begin{equation*}
u_{i}(\mathbf{e}, \mathbf{x})=\frac{1}{2} \cdot \max _{j, k \neq j}\left(e_{j} x_{j}+e_{k} x_{k}\right)+\frac{1}{3} \cdot \sum_{j=1}^{3} e_{j} x_{j}+\left(1-x_{i}\right) e_{i} . \tag{5}
\end{equation*}
$$

This game represents a situation in which the two highest absolute contributions are of particular importance for the public good.

The panels Fig. 2d-f show the outcome of evolutionary simulations. We have systematically varied the players' initial endowments, considering all endowment distributions ( $e_{1}, e_{2}, e_{3}$ ) for which $e_{i} \in\{0.00,0.05, \ldots, 0.95,1.00\}$. We have used the same three payoff functions as in the panels Fig. 2a-c, a continuation probability of $\delta=1$, and strong selection $s=1,000$. Players use stochastic memory-1 strategies without errors, and they either contribute their full endowment or nothing, $X=\{0,1\}$. The evolutionary process was simulated for $10^{6}$ elementary time steps.

Fig. 3 shows simulations as we vary player 1's endowment $e_{1} \in\{0.00,0.05, \ldots, 1.00\}$ and $e_{2}=1-e_{1}$. We use the same productivity values as in the experiment, $r_{1}=r_{2}=1.6$ (left panel) and $r_{1}=1.9, r_{2}=1.3$ (right panel), respectively, and consider the case $\delta=1$ and $s=1,000$. To explore the robustness of our evolutionary findings, we consider three different scenarios, depending on the possible contribution levels in each round, $X_{1}=\{0,1\}, X_{2}=\{0,1 / 2,1\}$, and $X_{3}=\{0,1 / 3,2 / 3,1\}$. Players can choose among all pure memory- 1 strategies, subject to an error rate of $\varepsilon=0.001$. For each value of $e_{1}$, simulations were run for $2 \cdot 10^{6}$ time steps.

Extended Data Figs. 1-4 show further evolutionary results for the pairwise game considered in the behavioral experiment. For these figures, we assume players only choose among pure memory-1 strategies with errors, and they can only contribute their full endowment or nothing in any given round. As a consequence, there are 16 possible strategies $\mathbf{p}=\left(p_{C C}, p_{C D}, p_{D C}, p_{D D}\right)$. For example, the strategy of unconditional defectors is given by $A L L D=(0,0,0,0)$, whereas Win-Stay Lose-Shift takes the form $W S L S=(1,0,0,1)^{3}$. For these 16 strategies, we can compute numerically exact strategy abundances (see SI). Except for the parameters explicitly varied, all payoff parameters are the same as in the five experimental treatments, using a continuation probability $\delta=1$, selection strength $s=1,000$, and error rate $\varepsilon=0.05$.

Extended Data Fig. 5 considers a public goods game where players have the same productivity, but they yield different benefits from the public good. The payoff function is given by

$$
\begin{equation*}
u_{i}(\mathbf{e}, \mathbf{x})=q_{i} \cdot r \sum_{j} x_{j} e_{j}+\left(1-x_{i}\right) e_{i} . \tag{6}
\end{equation*}
$$

Here, $r$ is the players' common productivity and $q_{i} \in[0,1]$ is player $i$ 's share of the public good. The game is a social dilemma only for certain values of $q_{i}$ (see SI for details). We show numerically exact results for pure memory-1 strategies with errors, and possible contribution levels $X=\{0,1\}$. We use the parameters $r=1.6, s=1,000$, and $\varepsilon=0.05$.

For Extended Data Fig. 6, we repeat the simulations shown in Fig. 3 for the case when players can choose their strategies from the set of finite-state automata. We use the same baseline parameters as in Fig. 3. However, simulations are run for longer ( $5 \cdot 10^{6}$ time steps) and the error rate has been set to $\varepsilon=0$ to allow for neutral invasions as in van Veelen and García ${ }^{15,64,65}$.

Fig. 4 and Extended Data Figs. 7 - 10 depict the results of our behavioral experiment. To indicate statistical significance, we either use 1 asterisk (significance at $p=0.05$ ), 2 asterisks ( $p=0.01$ ), or 3 asterisks $(p=0.001)$. As an auxiliary information, we also provide error bars indicating the respective 95\% confidence intervals in Fig. 4 and Extended Data Figs. 7,10. For the theoretical predictions, we have used simulations for stochastic memory-1 strategies and possible contributions $x \in\{0,1\}$. As parameters, we have chosen $s=1,000, \varepsilon=0.001$. As indicated in Fig. 3 and Extended Data Fig. 4, our qualitative predictions are independent of the evolutionary parameters we use, and independent of the possible contribution levels. A detailed description of the methods applied and of the depicted results is provided in the SI.

Code availability. All evolutionary simulations and numerical calculations have been performed with MATLAB R2014A. We provide the respective scripts in the SI. These scripts can be used to compute the players' payoffs, to simulate the introspection dynamics, and to numerically com-
pute the expected dynamics.

Data availability. The experimental data on which Fig. 4 and Extended Data Figs. 7-10 are based on, as well as the STATA file that contains our statistical analysis, is available on
https://osf.io/92jyw/?view_only=35fac40c9ad8406db49684fcb27930bd.

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## Extended data figure legends

Extended Data Figure 1: Realized versus expected trajectories of cooperation. In Fig. 2 and Fig. 3 we have shown how often players cooperate on average. Here we depict evolutionary trajectories over time, for the five treatments considered in our experiment. For this illustration, we assume that players can choose among the 16 pure memory- 1 strategies. The upper panels show five realizations of the introspection dynamics. In the lower panels, we show expected trajectories of the introspection dynamics, which can be derived explicitly (see SI Section 4.3). These expected trajectories represent the cooperation rate over time as we average over many realizations of the process. We observe substantial cooperation in three of the five cases: in the treatments with full equality, productivity equality, and aligned inequality.

Extended Data Figure 2: Under endowment inequality and misaligned inequality, players fail to coordinate on Win-Stay Lose-Shift. Here, we consider the long-run dynamics of the games considered in Extended Data Fig. 1. For each pair ( $\mathbf{p}_{1}, \mathbf{p}_{2}$ ) of pure memory-1 strategies, we can compute how often the respective strategy pair is played according to the invariant distribution of the evolutionary process. $\mathbf{a}, \mathbf{c}, \mathbf{d}$, Under full equality, productivity inequality, or aligned inequality, players typically coordinate on a WSLS equilibrium, as indicated by the colored square in the center of the dotted lines. $\mathbf{b}, \mathbf{e}$, Under endowment inequality or misaligned inequality, players fail to coordinate on a unique equilibrium. Instead, most of the evolving strategies prescribe to defect against the opponent. We note that in those treatments in which players have different endowments, the low-endowment player faces a reduced strength of selection (because this player's endowment is reduced from 0.5 to 0.25 ). As a consequence, his marginal distribution in panels $\mathbf{b}, \mathbf{e}$ is more uniform than the marginal distribution of the high-endowment player.

Extended Data Figure 3: An equilibrium analysis explains why cooperation emerges in only three of the five treatments. Using the same two-player setup as in Extended Data Figs. 1-2, we have explored how much players contribute on average when we simultaneously vary player 1's endowment ( $x$-axis), as well as her productivity $r_{1}$. For each parameter combination, we record the player's total contributions and how often they use WSLS according to the invariant distribution of the evolutionary process (indicated in shades of grey). We compare these evolutionary results with the region where WSLS is an equilibrium (indicated by dashed lines) and with the region where GRIM is an equilibrium (dotted lines), see SI for details. The colored symbols indicate which parameter combinations have been used for the experimental treatments. a-c, For equal productivities, the full equality treatment (1) is in the region where cooperation can evolve, whereas the unequal endowment treatment (2) is not. d-f, For unequal productivities, only the misaligned inequality treatment (5) is outside the region where cooperation can evolve.

Extended Data Figure 4: Robustness of evolutionary results with respect to parameter changes. To explore the robustness of our theoretical predictions, we varied the expected number of rounds played between two players (a), the selection strength (b), and the rate at which players commit an implementation
error (c). While the quantitative results depend on these parameters, the qualitative ordering of the five treatments is the same across all considered scenarios. Except for the parameters explicitly varied on the $x$-axis, all parameters are the same as in Extended Data Figs. 1-2.

## Extended Data Figure 5: Cooperation in an asymmetric game where players derive different payoffs

 from the public good. Instead of considering players who differ in their productivity, here we consider an asymmetric two-player public goods game where players differ in the share of the public good they get (the exact model is specified in the SI). We vary two parameters, player 1's share of the initial endowment, and player 1's share of the public good. For each parameter combination, we record the players' average contributions over the course of the evolutionary process (indicated in grey color). For games in which players get different shares of the public good, we note that the game is only a social dilemma if neither player's share is too large (otherwise that player would always have an incentive to cooperate, no matter what the co-player does). However, if both players get an intermediate share of the public good, full cooperation can again evolve when WSLS is an equilibrium.Extended Data Figure 6: Evolution of cooperation among players using finite-state automata. a, Here we represent finite-state automata for a game between two players in which players can either contribute their full endowment (C) or nothing (D). A finite-state automaton consists of three components: a set of states (represented by big circles), the action played in each state (represented by the color of the circle and the letters ' C ' and ' D '), and a transition rule (represented by arrows; the associated letter shows for which of the co-player's actions the respective arrow is taken). Finite-state automata are able to implement all memory- 1 strategies. In addition, they can encode strategies that depend on arbitrarily long sequences of past actions. b, To model evolution among finite-state automata, we employ the mutation scheme of García and van Veelen ${ }^{15,64}$. When a mutation occurs, either the direction of a random arrow is changed, the action in a randomly chosen state is changed, a random state is removed, or a state is added. c, Using this more general strategy space, we have repeated the simulations in Fig. 3. While overall cooperation rates are slightly lower, all qualitative results remain unchanged.

Extended Data Figure 7: Contributions and payoffs of the two players across treatments. For each of the five experimental treatments, we compare the average contributions and the average payoff of the two players. Grey bars indicate the theoretical prediction based on evolutionary simulations. Colored bars depict the outcome of the experiment. Error bars represent the respective $95 \%$ confidence intervals. Asterisks indicate statistical differences based on two-tailed Wilcoxon signed-rank tests. The number of groups per treatment is $42,42,40,39,40$, respectively. a, $\mathbf{b}$, Under full equality, the two players contribute a similar share of their endowment, and they obtain approximately equal payoffs. Under endowment inequality, both players' cooperation rates are reduced, with the high-endowment player 1's contributions being significantly lower than the contributions of player 2 . c, d, For productivity inequality and aligned inequality, we find no differences in the players' relative contributions. For misaligned inequality, the
relative contributions of the better-endowed but less productive player 2 are drastically reduced. For both aligned and misaligned inequality, the two players earn significantly different payoffs. Nevertheless, the player with the lower payoff in the aligned inequality treatment derives a similar payoff as the two player types under productivity inequality. For details, see SI.

Extended Data Figure 8: Dynamics of cooperation. For each of the five treatments, we show the players' average contributions over the course of the experiment. In all treatments the contributions are relatively stable over time, except for a significant negative trend in the treatment with endowment inequality (see SI for details).

Extended Data Figure 9: Individual cooperation decisions across the five treatments. To analyze the joint contribution decisions of the two players, we plot here how often player 1 has contributed $y_{1}$ tokens while player 2 has contributed $y_{2}$ tokens, for each pair ( $y_{1}, y_{2}$ ). Under full equality, productivity inequality, and aligned inequality, most individual decisions are mutually cooperative. In contrast, under endowment inequality and misaligned inequality, contributions are more scattered. Moreover, in the treatment with misaligned inequality, we observe that a substantial fraction of high endowment players only matches the co-player's absolute contributions. For example, in $12.4 \%$ of the rounds, the low-endowment player contributes all 25 tokens at her disposal, and the high-endowment player contributes the same absolute amount of tokens (corresponding to $1 / 3$ of this player's endowment).

Extended Data Figure 10: Abundance of reciprocal behaviors across the five treatments. To explore whether subjects apply reciprocal strategies, we show the fraction of rounds in which subjects match or exceed their co-player's relative contribution from the previous round. That is, if player 1 has contributed $x \%$ of her endowment in round $t$, we record whether or not player 2 contributes at least $x \%$ of his endowment in round $t+1$. Note that reciprocal strategies do not automatically yield high cooperation rates, because mutually defecting players are also reciprocal. Error bars represent the respective confidence intervals. The asterisks indicate a statistically significant difference, using a two-tailed Wilcoxon signedrank test. Sample sizes are $42,42,40,39,40$, respectively. Generally, we find high levels of reciprocity; only in the treatment with misaligned inequality, the high-endowment low-productivity player 2 exhibits a strongly reduced reciprocity rate. See SI for details.


Extended Data Figure 1: Realized versus expected trajectories of cooperation.


Extended Data Figure 2: When there is endowment inequality or misaligned inequality, players fail to coordinate on Win-Stay Lose-Shift.


Extended Data Figure 3: An equilibrium analysis explains why cooperation emerges in only three of the five treatments.

Full equality


Productivity inequality


Aligned inequality

Extended Data Figure 4: Robustness of our evolutionary results with respect to changes in the model parameters.


Extended Data Figure 5: Cooperation in an asymmetric game where players derive different payoffs from the public good.


Extended Data Figure 6: Evolution of cooperation among players using finite-state automata.


Extended Data Figure 7: Contributions and payoffs of the two players across treatments.


Extended Data Figure 8: Dynamics of cooperation.


Extended Data Figure 9: Individual cooperation decisions across the five treatments.


Extended Data Figure 10: Abundance of reciprocal behaviors across the five treatments.

# Supplementary Information Social dilemmas among unequals 

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## Supplementary Methods and Results

In Section 1, we describe the basic setup of our model. We introduce a general framework to analyze repeated public good games among unequals, and we describe the players' possible strategies and the resulting payoffs. In Section 2, we summarize previous work on asymmetric social dilemmas, and we discuss the novelty of our approach. In Section 3 we analyze under which conditions players can sustain cooperation, depending on the players' endowments and on the shape of the payoff function. We complement this static equilibrium analysis with an evolutionary perspective in Section 4. There, we consider players who adapt their strategies over time. We explore under which conditions players learn to be fully cooperative, assuming that the initial population consists of unconditional defectors. In Section 5 we give a full description of our experimental study, including a more detailed analysis of the subjects' behavior. In Section 6, we critically review some of our model assumptions, and we propose several model extensions for future work. Finally, the appendix contains the proofs of our equilibrium results, the MATLAB code for the simulations, and the game instructions of our behavioral experiment.

## 1 Model description

### 1.1 A general framework for public good games among unequals

In the following, we introduce a general framework for social dilemmas with multiple players who may differ along multiple dimensions. To this end, let us consider a group of $n \geq 2$ players. Initially, each group member $i$ is given an endowment $e_{i} \geq 0$. We refer to the $n$-tuple $\mathbf{e}=\left(e_{1}, \ldots, e_{n}\right)$ as the group's endowment vector. These endowments may be interpreted as the players' regular income, or as the amount of time they have at their free disposal. Without loss of generality, we assume total endowments are normalized such that $e_{1}+\ldots+e_{n}=1$. When players have equal endowments, the endowment vector is $\mathbf{e}=(1 / n, \ldots, 1 / n)$.

Each player can choose which fraction $x_{i}$ of her endowment she wishes to contribute to the public good, with $0 \leq x_{i} \leq 1$. We call the $n$-tuple $\mathbf{x}=\left(x_{1}, \ldots, x_{n}\right)$ the group's contribution vector. Given an endowment vector $\mathbf{e}$ and a contribution vector $\mathbf{x}$, player $i$ 's absolute contribution to the public good is $e_{i} \cdot x_{i}$. We use the shortcut notation $\mathbf{x}=\mathbf{1}$ for the contribution vector where each player contributes her full endowment. We write $\mathbf{x}=\mathbf{0}$ when each player contributes nothing at all. For two contribution vectors $\mathbf{x}=\left(x_{1}, \ldots, x_{n}\right)$ and $\mathbf{x}^{\prime}=\left(x_{1}^{\prime}, \ldots, x_{n}^{\prime}\right)$, we write $\mathbf{x} \geq \mathbf{x}^{\prime}$ if $x_{i} \geq x_{i}^{\prime}$ for all players $i$.

The payoffs of the public good game may depend on the endowments, the contributions, and on personal characteristics of the players (such as a player's productivity, or the degree to which a player benefits from publicly provided goods). Specifically, given the distribution of endowments e, and the relative contributions $\mathbf{x}$, players yield the payoff $u(\mathbf{e}, \mathbf{x}) \in \mathbb{R}^{n}$, where $u$ denotes the public good's payoff function. The $i$-th component $u_{i}(\mathbf{e}, \mathbf{x})$ corresponds to the payoff of player $i$. We assume that when no player contributes, players end up with their initial endowments, $u_{i}(\mathbf{e}, \mathbf{0})=e_{i}$ for all players $i$. We refer to the sum over all individual payoffs, $U(\mathbf{e}, \mathbf{x}):=\sum_{i=1}^{n} u_{i}(\mathbf{e}, \mathbf{x})$, as the group payoff.

In the following, we will mostly be interested in situations in which there is a tension between individual and collective incentives to contribute. To formalize this tension, we consider public good games with the following four properties.
(C) Continuity: Payoffs change continuously in the players' endowments and contributions.

The payoff function $u(\mathbf{e}, \mathbf{x})$ is continuous in both its arguments, $\mathbf{e}$ and $\mathbf{x}$.
(PE) Positive externalities: Players prefer their co-players to make higher contributions.
If $\mathbf{x}$ and $\mathbf{x}^{\prime}$ are two contribution vectors such that $x_{i}=x_{i}^{\prime}$ and $x_{j} \geq x_{j}^{\prime}$ for all $j \neq i$ then $u_{i}(\mathbf{e}, \mathbf{x}) \geq$ $u_{i}\left(\mathbf{e}, \mathbf{x}^{\prime}\right)$ for all endowment distributions $\mathbf{e}$. In this case, the strict inequality $u_{i}(\mathbf{e}, \mathbf{x})>u_{i}\left(\mathbf{e}, \mathbf{x}^{\prime}\right)$ holds if and only if there is at least one co-player $j$ with $e_{j}>0$ and $x_{j}>x_{j}^{\prime}$.
(IF) Incentives to free-ride: Players have an incentive to lower their own contributions.
If $\mathbf{x}$ and $\mathbf{x}^{\prime}$ are two contribution vectors such that $x_{i}<x_{i}^{\prime}$ and $x_{j}=x_{j}^{\prime}$ for all $j \neq i$ then $u_{i}(\mathbf{e}, \mathbf{x}) \geq$ $u_{i}\left(\mathbf{e}, \mathbf{x}^{\prime}\right)$ for all endowment distributions $\mathbf{e}$. In this case, the strict inequality $u_{i}(\mathbf{e}, \mathbf{x})>u_{i}\left(\mathbf{e}, \mathbf{x}^{\prime}\right)$ holds if and only if player $i$ 's endowment is positive, $e_{i}>0$.
(OC) Optimality of cooperation: From a collective perspective, higher contributions are preferred. If $\mathbf{x}$ and $\mathbf{x}^{\prime}$ are two contribution vectors such that $\mathbf{x} \geq \mathbf{x}^{\prime}$ then $U(\mathbf{e}, \mathbf{x}) \geq U\left(\mathbf{e}, \mathbf{x}^{\prime}\right)$. In this case, the strict inequality $U(\mathbf{e}, \mathbf{x})>U\left(\mathbf{e}, \mathbf{x}^{\prime}\right)$ holds if and only if there is a player $i$ with $e_{i}>0$ and $x_{i}>x_{i}^{\prime}$.

The first condition of continuity is mainly required for technical reasons; it will be useful to show that full cooperation cannot be sustained when initial endowments are too unequal. The other three properties extend previous definitions of cooperation in symmetric social dilemmas ${ }^{1,2}$ to the case of asymmetric public good games. In any game that satisfies (IF), players have an incentive to contribute as little as possible. Without repeated interactions, such games have a unique Nash equilibrium according to which no player contributes, $\mathbf{x}=\mathbf{0}$.

We note that the four properties introduced above rule out certain threshold public good games, where payoffs suddenly increase when payoffs exceed a predefined threshold ${ }^{3-8}$. In such games, cooperation can already be sustained in one-shot interactions, because people have an incentive not to fall below the threshold. In contrast, here we are interested in games where the social dilemma is as stringent as possible, and where repeated interactions are key to maintain mutual cooperation.

In Fig. 1 of the main text we have classified different public good games according to two dimensions, linearity and symmetry. A public good game is linear if a player's payoff is a linear function of her own endowment and of the absolute contributions of all other group members,

$$
\begin{equation*}
u_{i}(\mathbf{e}, \mathbf{x})=\sum_{j}^{n} c_{i j} x_{j} e_{j}+e_{i} \tag{1}
\end{equation*}
$$

The constants $c_{i j}$ determine to which extent player $j$ 's absolute contribution to the public good affects player $i$ 's payoff. We note that property ( $\mathbf{( P E )}$ requires $c_{i j}>0$ for all co-players $j \neq i$, whereas prop-
erty (IF) requires $c_{i i}<0$ for all players $i$. The property (OC) requires $c_{1 j}+\ldots+c_{n j}>0$ for all players $j$. Property (C) is satisfied since linear functions are continuous.

A public good game is symmetric if players do not differ in their personal characteristics, such that any payoff differences between players can entirely be explained by differences in their endowments and contributions. To formalize this notion of symmetry, let $\sigma$ be a permutation of the numbers $(1, \ldots, n)$. For a given endowment vector $\mathbf{e}$ and contribution vector $\mathbf{x}$, let $\mathbf{e}_{\sigma}:=\left(e_{\sigma(1)}, \ldots, e_{\sigma(n)}\right)$ and $\mathbf{x}_{\sigma}:=\left(x_{\sigma(1)}, \ldots, x_{\sigma(n)}\right)$ be the permuted endowment and contribution vectors. We call the payoff function $u$ symmetric if

$$
\begin{equation*}
u_{i}(\mathbf{e}, \mathbf{x})=u_{\sigma(i)}\left(\mathbf{e}_{\sigma}, \mathbf{x}_{\sigma}\right) \text { for all } \mathbf{e}, \mathbf{x}, \sigma . \tag{2}
\end{equation*}
$$

That is, if players were to switch roles with respect to their endowments and their contributions, then their payoffs are switched accordingly. In particular, for symmetric payoff functions the group payoff is independent of the players' identities, $U(\mathbf{e}, \mathbf{x})=U\left(\mathbf{e}_{\sigma}, \mathbf{x}_{\sigma}\right)$.

### 1.2 Repeated games

Herein, we consider individuals that do not interact once with each other, but repeatedly over several rounds. Specifically, we assume that after each round of the public goods game, there is another round with probability $\delta$, with $0<\delta<1$. It follows that the expected number of rounds follows a geometric distribution with mean $1 /(1-\delta)$. Player $i$ receives the same endowment $e_{i}$ in every round. However, the player's contribution $x_{i}$ may change from one round to the next, and the player's decision how much to contribute in a given round may depend on previous outcomes of the game. Players make these decisions according to their strategies. A strategy for player $i$ is a rule that tells the player how to choose her contribution $x_{i}(t)$ in round $t$, depending on the players' endowments $\mathbf{e}$ and depending on all players' previous contribution decisions.

For example, the strategy ALLD sets $x_{\text {ALLD }}(t)=0$ in every round, independent of the players' endowment and independent of the outcome of previous rounds. Another example is the strategy Grim. A Grim player contributes her full endowment $x_{\text {Grim }}(0)=1$ in the initial round. In all subsequent rounds, she continues to contribute her full endowment $x_{\text {Grim }}(t)=1$, provided that all players have done so in all previous rounds. Otherwise, if at least one player has contributed less than his full endowment in some previous round, a Grim player contributes nothing, $x_{\text {Grim }}(t)=0$.

In this way, the players' strategies determine which fraction of the endowment the players contribute in each round. If the contributions at time $t$ are given by the vector $\mathbf{x}(t)=\left(x_{1}(t), \ldots, x_{n}(t)\right)$, the players' payoffs for the repeated game are defined as the weighted average

$$
\begin{equation*}
\pi:=(1-\delta) \sum_{t=0}^{\infty} \delta^{t} u(\mathbf{e}, \mathbf{x}(t)) \tag{3}
\end{equation*}
$$

We will sometimes consider the limiting case that there is always another round, and $\delta \rightarrow 1$. In that case,
the players' payoffs according to Eq. (3) approach the average payoff per round,

$$
\begin{equation*}
\pi=\lim _{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} u(\mathbf{e}, \mathbf{x}(t)), \tag{4}
\end{equation*}
$$

provided this limit exists. Herein, we will often consider players with bounded memory such that the existence of the limit in Eq. (4) is guaranteed ${ }^{9}$.

## 2 Related literature

### 2.1 Related work in evolutionary game theory

Traditionally, most models on the evolution of cooperation study the behavior of individuals who have the same personal attributes, the same incentives to cooperate, and the same strategy set to choose from ${ }^{10}$. Only a few models have considered the effect of different sources of asymmetry. Here we provide a review of this previous work.

Bergstrom et al ${ }^{11}$ consider a one-shot model of public good provision in which players decide how much of their wealth they want to spend either on a private or a public good. In their model, individual preferences are such that total contributions to the public good in equilibrium are always positive; in particular, mutual defection is not an equilibrium. Provided all players contribute, a sufficiently weak redistribution of wealth has no effect at all because players adjust their public good contributions accordingly. However, equalizing the initial wealth of players may decrease total contributions if it involves transfers from contributors to non-contributors.

The effect of asymmetric cooperation costs has also been studied in the context of the volunteer's dilemma ${ }^{12,13}$. In the volunteer's dilemma, it takes one group member to pay a cost for everyone in the group to derive some benefit. Asymmetric costs can facilitate coordination when players can make their choice in continuous time ${ }^{13}$ : in equilibrium, the player with the lowest cost volunteers without delay.

Closer to our work, inequality in the players' endowments and their wealth have been considered in models of the collective risk-dilemma ${ }^{8,14,15}$. In the collective-risk dilemma, a group of players needs to collect a fixed target sum in order to avoid dangerous climate change ${ }^{3}$. If they fail to meet the required threshold, they lose their remaining endowment with some exogenous loss probability. Vasconcelos et al ${ }^{14}$ consider a model in which players are either 'rich' or 'poor', and in which players can decide whether or not they wish to contribute a fixed fraction of their endowment to the public pool. In this model, heterogeneity can promote cooperation, but only in cases when poor players imitate the strategies of the rich (and vice versa). In Abou Chakra et al ${ }^{8}$, players differ along two dimensions, their wealth and their loss probability. Players who have more to lose - either because they are wealthier or because they have a higher loss probability - tend to make higher contributions.

Finally, Akin ${ }^{16}$ considers memory- 1 strategies in the asymmetric repeated prisoner's dilemma without discounting. Building on the recently developed theory of zero-determinant strategies ${ }^{17-22}$, he characterizes all strategies that can be used to sustain mutual cooperation.

We add to this evolutionary game theory literature in the following way:
(i) We provide a general framework that is applicable to social dilemmas with an arbitrary number of players and continuous degrees of cooperation.
(ii) We allow for various different dimensions of heterogeneity. For example, players can simultaneously differ in their endowments, productivities, or their benefit from the public good. We predict how these different forms of inequality interact.
(iii) We explore direct reciprocity in strict social dilemmas, while most of the previous work had its focus on inequality in one-shot games, or in games with a finite horizon.
(iv) In our applications, the threat of future retaliation is key to sustain cooperation. The logic of reciprocity seems to suggest that cooperation is most likely if all players have the same endowment, yet we show that this does not need to be the case.
(v) We employ three different, but complementary approaches: equilibrium calculations, individualbased simulations, and a behavioral experiment. Each approach is based on slightly different assumptions, yet they all yield the same qualitative results.

### 2.2 Related experimental work

The public goods game is one of the classical paradigms to study the dynamics of cooperation in behavioral experiments. The vast literature on this topic suggests that individual contributions depend on many different aspects, including the net cost of contributions, communication opportunities, and thresholds, see the reviews of Ledyard ${ }^{23}$ and Zelmer ${ }^{24}$. In the following, we summarize previous experimental work in which participants either experienced only one kind of inequality, or in which different kinds of inequality interacted.

Effect of one-dimensional inequality. Several studies have explored the origins and consequences of endowment inequality in public good games. For example, Buckley and Croson ${ }^{25}$ study a linear public goods game in groups in which half of the players have 25 tokens per round, whereas the other half has 50 tokens per round. They find that both kinds of players contribute roughly the same absolute amount (that is, players with a higher endowment make a lower relative contribution). In a similar experiment, Hargreaves Heap et al ${ }^{26}$ compare the contributions of homogeneous and heterogeneous groups. Their participants either have 20,50 , or 80 tokens, and they interact in groups of size 3. Players who are either given 20 or 50 tokens play consistently, independent of whether they are in a homogeneous or in a heterogeneous group. However, 80 -token players contribute significantly less in mixed groups, compared to homogeneous groups in which everyone receives 80 tokens. In the experiment of Cherry et al ${ }^{27}$, players either earn their initial endowments in a quiz, or the endowments are randomly assigned by the experimenter. This study finds that the origin of the endowment has no effect, whereas heterogeneity decreases the players' contributions. Similarly, in the experiment of Kesternich et al ${ }^{28}$, subjects are either assigned random endowments, or they earn their endowments by outperforming their peers in a 'slider
task' that requires effort. Again, when contributions are voluntary, the origin of endowment seems to have a minor effect, although highly endowed subjects tend to contribute less if they have earned their endowment in the effort task.

Gächter et $\mathrm{al}^{29}$ explore the consequences of endogenous inequality. In their linear public goods game, all players are equipped with the same starting capital. However, a player's accumulated wealth in round $t$ is his endowment for round $t+1$. The study finds that inequality (as measured by the Gini coefficient) is increasing over time. Moreover, there is substantial variation in inequality between groups. In a similar experiment, Cardigan et $\mathrm{al}^{30}$ study contributions in public good games when a player's payoff in odd rounds serves as the player's endowment in the subsequent even rounds. They find that under certain conditions, this "carryover" can increase contributions in odd rounds. Nishi et al ${ }^{31}$ consider pairwise prisoner's dilemma games among players on a dynamical network. Their treatments differ in the degree of inequality, and in whether players are informed about their co-player's accumulated wealth. As their main result they find that heterogeneity has a negligible impact on cooperation per se, but visible wealth inequality has a negative impact ${ }^{32}$.

Moreover, several studies have explored how heterogeneity interacts with different mechanisms for the maintenance of cooperation. These mechanisms include communication ${ }^{33,34}$, voting ${ }^{35,36}$, institutional sharing rules ${ }^{28,37}$, mandatory minimum contributions ${ }^{38}$, or sanctions ${ }^{29,36,39}$. For example, the experiment by Reuben and Riedl ${ }^{39}$ explores the interaction of endowment inequality and costly punishment. There are $2 \times 4=8$ treatments. The treatments differ in whether or not subjects are able to punish each other after each round, and in the type of endowment or benefit inequality considered. This study finds that without punishment, contributions steadily decline across all groups, such that any difference between groups is blurred. In contrast, when punishment is possible, contributions are high in all group types. Moreover, depending on the type of inequality, different contribution norms are enforced.

Without any additional mechanisms, the existing data suggests that unequal endowments tend to decrease contributions, whereas the effect of unequal productivities is ambiguous. Our own experiment agrees with this view: when the two players are equally productive, unequal endowments diminish overall cooperation. Conversely, when players have equal endowments, differences in the players' productivity have a negligible effect on contributions. Interestingly, in most of the aforementioned experiments on asymmetric public good games, the number of rounds is fixed and known to the participants. For such repeated social dilemmas with a predefined horizon, backward induction suggests that participants should not cooperate at all. In contrast to this prediction, a complete unravelling of contributions is rarely observed in repeated social dilemma experiments ${ }^{40}$. In such cases, our theoretical framework for indefinitely repeated games may provide a more accurate prediction of the qualitative behavior in asymmetric public good games, even if subjects are informed about the exact length of the game ex ante.

Effect of interacting inequalities. The interaction of different forms of inequality has received comparably little attention, with two exceptions. First, in one treatment of Chan et al ${ }^{33}$, the authors allow for two sources of heterogeneity. In that treatment, players could differ in their endowments and in the
benefit they derived from the public good (the treatment only allows for what we would call aligned inequality). The authors use a non-linear payoff function, and parameters have been chosen such that mutual defection cannot arise as an equilibrium. As a consequence, their experiment does not meet the conditions of a strict social dilemma, as discussed in Section 1. Nevertheless, their results are in line with our prediction: they find that in the absence of communication, aligned inequality can help cooperation.

Second, van Gerwen et al ${ }^{41}$ explore cooperative behavior in a contextualized public goods game between three subjects. Subjects may differ in their 'skills' (akin to our endowment), and in whether or not they have received 'training' (akin to our differences in productivities). In addition, groups may differ in how many rounds subjects interact with each other. Groups either remain stable for six consecutive rounds ('stable teams'), or they are formed anew after each round ('altering teams'). For comparison with our study, which is based on direct reciprocity, the stable team treatments of van Gerwen et al are most relevant. For those treatments, the authors find that more skilled subjects (i.e., players with higher endowment) tend to make higher absolute contributions. Interestingly, however, a player's contribution is neither affected by his training, nor is there a significant interaction between training and skills. Moreover, on the group level van Gerwen et al find no difference in contributions based on whether training is provided to high-skill or low-skill subjects (see the 'High' and 'Low' condition in their Figure 2). That is, they find no difference between aligned and misaligned inequality as we do. There are two reasons that might explain why their conclusions differ:

1. The comparably short duration of their experiment: Their stable teams only interact for six rounds. Moreover, the total number of rounds is commonly known, leading to a significant end-game effect (as confirmed by their Table 4). The horizon of the stable team experiments may thus be too short for reciprocity to unfold.
2. The framing of their experiment: To increase external validity, subjects in van Gerwen et al are provided with strongly contextualized instructions. For example, subjects learn that their training is a gift by the employer. The authors themselves expect that these contextualized instructions increase the contributions of those individuals who receive training. Such framing effects might explain why groups with aligned inequality did not outperform groups with misaligned inequality. The framing might also explain why some statistical relationships became significant when altering teams were considered instead of stable teams.

In sum, we add to the previous experimental literature on public good games in the following way.
(i) We provide a general theoretical framework to explain reciprocal behavior in repeated social dilemmas. Our framework applies to arbitrary public good games. Players may be symmetric or asymmetric, and their payoffs may be linear or nonlinear.
(ii) We experimentally confirm previous results on the isolated effects of endowment and productivity inequality.
(iii) We are first to show that in repeated games, endowment and productivity inequality interact nontrivially. If the two inequalities are aligned, overall welfare is maximized. If they are misaligned, cooperation is reduced substantially.

## 3 Equilibrium analysis

Whereas one-shot public good games only allow for equilibria with zero contributions, repeated games can give rise to alternative equilibria ${ }^{9,42}$. In the following, we are particularly interested in the subgame perfect equilibria in which players contribute their full endowment in every round. A strategy profile is called a subgame perfect equilibrium if, after any previous history of play, no player has an incentive to deviate from her strategy ${ }^{43}$. The subgame perfect equilibrium concept represents an equilibrium refinement of the Nash equilibrium: every subgame perfect equilibrium is also a Nash equilibrium, while the converse does not need to hold ${ }^{44}$.

### 3.1 Feasibility of cooperation

We say that full cooperation is feasible in a given public good game if there is a subgame perfect equilibrium in which all players contribute their full endowment in every round. For a given payoff function $u$ and continuation probability $\delta$, let $E_{u}(\delta)$ denote the set of all endowment distributions that allow for full cooperation,

$$
\begin{equation*}
E_{u}(\delta)=\left\{\mathbf{e}=\left(e_{1}, \ldots, e_{n}\right) \mid \text { Full cooperation is feasible for the given endowment distribution } \mathbf{e}\right\} . \tag{5}
\end{equation*}
$$

In the upper triangles in Fig. 2, the respective sets $E_{u}(\delta)$ are represented by blue-shaded areas. In the following Proposition, we give a simple characterization of the set $E_{u}(\delta)$ for arbitrary public good games. We use the notation $\mathbf{x}=\mathbf{1}_{-i}$ to refer to the contribution vector where all players but player $i$ give their full endowment, whereas player $i$ contributes nothing.

Proposition 1. Consider a public good game with payoff function $u$ and continuation probability $\delta<1$, and let $\mathbf{e}$ denote some endowment distribution. The following are equivalent:

1. $\mathbf{e} \in E_{u}(\delta)$
2. The following condition holds for all players $i$ with $e_{i}>0$,

$$
\begin{equation*}
\delta\left(u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{0})\right) \geq u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{1}) \tag{6}
\end{equation*}
$$

3. The strategy profile where all players apply the strategy Grim is a subgame perfect equilibrium for the given endowment distribution $\mathbf{e}$.

All proofs for the results in this section are in the Appendix. The above Proposition says that if full cooperation can be sustained at all for a given endowment distribution $\mathbf{e}$, then it can be sustained with the strategy Grim. In most applications, the stability of Grim can be checked easily. Using Proposition 1, we can show that $E_{u}(\delta)$ exhibits the usual monotonic dependence on the continuation probability $\delta$.

Proposition 2. Consider a public good game with payoff function $u$.

1. Suppose $\delta$ and $\delta^{\prime}$ are two continuation probabilities with $\delta<\delta^{\prime}$. Then $E_{u}(\delta) \subset E_{u}\left(\delta^{\prime}\right)$.
2. There is a $\delta^{\prime}<1$ such that $E_{u}(\delta) \neq \emptyset$ for all $\delta \geq \delta^{\prime}$.

That is, the set of all endowment distributions for which cooperation is feasible increases with the continuation probability. If a given endowment distribution allows for full cooperation for a given continuation probability, then it also allows for full cooperation for all larger continuation probabilities. Moreover, cooperation is always feasible if only the continuation probability $\delta$ is sufficiently large.

Using Proposition 1 we can also show that cooperation is never feasible if initial endowments are too strongly biased in favor of one player.

Proposition 3. Consider an arbitrary public good game with continuation probability $\delta<1$ and endowment distribution $\mathbf{e}$. For any player $i$, there is an $\varepsilon_{i}>0$ such that if player $i$ 's endowment exceeds $e_{i}=1-\varepsilon_{i}$ then full cooperation is infeasible.

The result of Proposition 3 also becomes apparent in Fig. 2 of the main text. In all triangles depicted, there is a neighborhood around the corners of the triangle where cooperation is not feasible. Intuitively, full cooperation in repeated games can only be sustained if any deviation in the present round can be punished by withholding contributions in future rounds. However, if the initial endowments are too concentrated such that one player is given an excessive share, her co-players cease to have enough leverage to prevent her from free riding.

It may appear that a similar argument could also show that an equal endowment distribution is generally most conducive to cooperation. In the following sections we show that this conjecture is true when the public good game is linear and symmetric, whereas it is false if payoffs are nonlinear or asymmetric.

### 3.2 Linear and symmetric public good games

A linear public goods game (1) satisfies the symmetry condition (2) if and only if the payoff function has the form

$$
\begin{equation*}
u_{i}(\mathbf{e}, \mathbf{x})=c_{1} \sum_{j \neq i} x_{j} e_{j}-c_{2} x_{i} e_{i}+e_{i} . \tag{7}
\end{equation*}
$$

Here, the constant $c_{1}$ measures by how much player $i$ 's payoff is affected by co-player $j$ 's contribution (due to symmetry, this constant is independent of both $i$ and $j$ ). Similarly, $c_{2}$ measures to which extent player $i$ 's payoff depends on her own contribution; again, due to symmetry $c_{2}$ is independent of $i$. The three properties (PE), (IF), and (OC) then imply $c_{1}>0, c_{2}>0$ and $(n-1) c_{1}-c_{2}>0$, respectively. In the main text we have considered scenarios where total contributions of all players are multiplied by a joint productivity factor $r$, with $1<r<n$, and then equally shared among all group members. This standard formulation of public goods games can be recovered as a special case of game (7) when we set $c_{1}=r / n$ and $c_{2}=1-r / n$.

By applying Proposition 1 to the linear and symmetric public goods game (7), we find that cooperation is feasible for a given endowment distribution $\mathbf{e}$ if and only if

$$
\begin{equation*}
\delta c_{1}\left(1-e_{i}\right) \geq c_{2} e_{i} \text { for all players } i \text { with } e_{i}>0 \tag{8}
\end{equation*}
$$

Equivalently, cooperation is feasible for the endowment distribution $\mathbf{e}$ if and only if

$$
\begin{equation*}
\delta \frac{c_{1}}{c_{2}} \geq \frac{\max \left\{e_{i}\right\}}{1-\max \left\{e_{i}\right\}} . \tag{9}
\end{equation*}
$$

Condition (9) indicates that it is always the player with the highest initial endowment who has the largest temptation to deviate from full cooperation.

Now, suppose that cooperation is feasible for some endowment distribution, such that $E_{u}(\delta) \neq \emptyset$. Thus, there needs to be an endowment distribution $\mathbf{e}=\left(e_{1}, \ldots, e_{n}\right)$ that satisfies inequality (9). But since $\max \left\{e_{i}\right\} \geq 1 / n$ and since the map $z \mapsto z /(1-z)$ is monotonically increasing for $0<z<1$, it then follows that also the equal endowment distribution $(1 / n, \ldots, 1 / n)$ satisfies inequality (9). That is, if full cooperation is feasible at all in some given linear and symmetric public good game, then it can always be achieved by assigning equal endowments to all players.

By plugging the equal endowment distribution $\mathbf{e}=(1 / n, \ldots, 1 / n)$ into condition (9), we can conclude that cooperation is feasible in a linear and symmetric public goods game if and only if $\delta \geq \delta^{*}$ with $\delta^{*}:=c_{2} /\left((n-1) c_{1}\right)$. In particular, since $(n-1) c_{1}-c_{2}>0$ it follows that $\delta^{*}<1$, and thus cooperation is always feasible if the public goods game is repeated sufficiently often. We can summarize these observations as follows.

Proposition 4. Consider a linear and symmetric public goods game with payoff function (7).

1. If full cooperation is feasible for some arbitrary endowment distribution, it is also feasible when players have equal endowments. That is, if $E_{u}(\delta) \neq \emptyset$ for some $\delta$, then $(1 / n, \ldots, 1 / n) \in E_{u}(\delta)$.
2. Full cooperation is feasible, that is $E_{u}(\delta) \neq \emptyset$, if and only if $\delta \geq \frac{c_{2}}{c_{1}} \frac{1}{n-1}$.

This first result in Proposition 4 is also apparent in Fig. 2a: the center of the triangle which represents the point of equal endowments is always an element of the blue area where full cooperation is feasible, provided $\delta$ is sufficiently large for this blue area to exist.

### 3.3 Asymmetric linear public good games

As an example of a public goods game that is linear but asymmetric, we have considered the following example in the main text,

$$
\begin{equation*}
u_{i}(\mathbf{e}, \mathbf{x})=\frac{1}{n} \sum_{j} r_{j} x_{j} e_{j}+\left(1-x_{i}\right) e_{i} \tag{10}
\end{equation*}
$$

We have interpreted $r_{j}$ as the productivity of player $j$ 's contributions to the public good. The payoff function (10) satisfies the two properties (IF) and (OC) if and only if the individual productivities satisfy $1<r_{j}<n$. If that is the case, property ( $\mathbf{P E}$ ) holds automatically, whereas property $(\mathbf{C})$ is trivially satisfied for all public good games of the form (10).

When we apply Proposition 1 to the asymmetric public goods game (10), we yield the following
necessary and sufficient condition for the feasibility of cooperation:

$$
\begin{equation*}
\frac{\delta}{n} \sum_{j \neq i} r_{j} e_{j} \geq\left(1-\frac{r_{i}}{n}\right) e_{i} \text { for all players } i \text { with } e_{i}>0 \tag{11}
\end{equation*}
$$

In particular, for equal endowments, $\mathbf{e}=(1 / n, \ldots, 1 / n)$, full cooperation is feasible if and only if

$$
\begin{equation*}
\delta \geq \frac{n-\min \left(r_{j}\right)}{\sum_{j=1}^{n} r_{j}-\min \left(r_{j}\right)} . \tag{12}
\end{equation*}
$$

Condition (12) indicates that when players are given equal endowments, the player with the the lowest productivity will face the largest temptation to deviate (because this player faces the highest marginal cost $1-r_{i} / n$ of contributing). This observation suggests that when players differ in their productivities, cooperation may be easier to sustain when players also differ in their endowments.

In Fig. 2b of the main text, we have provided a simple example. There, we have considered a 3player public goods game with $r_{1}=2.9, r_{2}=1.5, r_{3}=1.1$, and continuation probability $\delta=0.3$. A straightforward computation of the conditions (11) shows that when players have equal endowment, full cooperation is infeasible, because both players 2 and 3 (the players with comparably low productivity) have an incentive to deviate. In contrast, if we redistribute endowments such that $\mathbf{e}=(4 / 5,1 / 5,0)$, the feasibility condition (11) is satisfied for all players. Interestingly, according to this distribution most of the initial endowment goes to the most productive player 1 , and player 3 does not receive any endowment at all. However, player 2's share of the endowment, $e_{2}=1 / 5$, is sufficient to still incentivize player 1 to cooperate.

Moreover, we note that even if full cooperation were feasible with equal endowments, players may still fare better when a higher share of the endowment is given to the more productive player. To illustrate this point, consider the above public good game with $r_{1}=2.9, r_{2}=1.5, r_{3}=1.1$, but with the continuation probability increased to $\delta=0.5$. In that case, cooperation is also feasible for an equal endowment distribution. However, in that equilibrium, each player's payoff is $\left(r_{1}+r_{2}+r_{3}\right) / 9 \approx 0.61$, which is smaller than the mutual cooperation payoff for $\mathbf{e}=(4 / 5,1 / 5,0)$, yielding a payoff of $\left(4 r_{1}+r_{2}\right) / 15 \approx 0.87$. We can summarize these observations as follows.

## Proposition 5.

1. There are asymmetric linear public good games such that $E_{u}(\delta) \neq \emptyset$, but $(1 / n, \ldots, 1 / n) \notin E_{u}(\delta)$.
2. There are asymmetric linear public good games such that even if full cooperation is feasible for equal endowments, unequal endowments allow for higher equilibrium payoffs for every player.

While the above Proposition shows that some inequality can improve the stability of cooperation, it does not clarify which kind of inequality is most conducive to cooperation. The following result shows that the conditions for the feasibility of cooperation are most easily satisfied if the players' endowments match their productivities. The proof is given in the Appendix.

Proposition 6. Consider the public goods game with payoff function (10), and assume without loss of generality that players are ordered according to their productivities, $r_{1} \leq r_{2} \leq \ldots \leq r_{n}$. Let $\mathbf{e}^{\prime}=$ $\left(e_{1}^{\prime}, \ldots, e_{n}^{\prime}\right)$ be some endowment vector, and let $\mathbf{e}^{\prime \prime}=\left(e_{1}^{\prime \prime}, \ldots, e_{n}^{\prime \prime}\right)$ be the permuted endowment vector such that the ordering of the entries in $\mathbf{e}^{\prime \prime}$ is aligned with the players' productivities; that is, $\mathbf{e}^{\prime \prime}=\mathbf{e}_{\sigma}^{\prime}$ for some permutation $\sigma$ and $e_{1}^{\prime \prime} \leq e_{2}^{\prime \prime} \leq \ldots \leq e_{n}^{\prime \prime}$. If $\mathbf{e}^{\prime} \in E_{u}(\delta)$, then $\mathbf{e}^{\prime \prime} \in E_{u}(\delta)$.

Proposition 6 states that if full cooperation can be sustained for some endowment distribution, then it can always be sustained by redistributing the players' endowments such that more productive players have higher endowments.

So far, we have been concerned with asymmetric games in which players have differed in their productivity. Alternatively, we may also consider games in which players are equally productive, but they differ in the extent to which they benefit from publicly provided goods. Such games may be represented by the payoff function

$$
\begin{equation*}
u_{i}(\mathbf{e}, \mathbf{x})=q_{i} \cdot r \sum_{j} x_{j} e_{j}+\left(1-x_{i}\right) e_{i} . \tag{13}
\end{equation*}
$$

The coefficient $r$ with $1<r<n$ represents the players' common productivity. The coefficient $q_{i}$ determines the relative value of the public good to player $i$. Without loss of generality, we may assume that these coefficients are normalized such that $q_{1}+\ldots+q_{n}=1$. In order for the game to qualify as a social dilemma, (PE) requires that $q_{i}>0$ for all players $i$, whereas (IF) requires $q_{i}<1 / r$.

Again, it is straightforward to construct examples of a public good game with payoff function (13) such that full cooperation can only be sustained for unequal endowment distributions. According to Proposition 1, cooperation is feasible for a given endowment distribution $\mathbf{e}$ if and only if

$$
\begin{equation*}
\delta q_{i} r\left(1-e_{i}\right) \geq\left(1-q_{i} r\right) e_{i} \text { for all players } i \text { with } e_{i}>0 . \tag{14}
\end{equation*}
$$

For equal endowments, $\mathbf{e}=(1 / n, \ldots, 1 / n)$, it follows that full cooperation is feasible if and only if

$$
\begin{equation*}
\delta \geq \frac{1-r \cdot \min \left(q_{i}\right)}{(n-1) r \cdot \min \left(q_{i}\right)} \tag{15}
\end{equation*}
$$

Similar to before, under equal endowments it is the player with the highest marginal cost to contribute who is most likely to deviate (here, the player with the smallest benefit from the public good). To prevent this player from deviating, it may again be effective to redistribute initial endowments, such that players with a stronger incentive to contribute get a larger share of the endowment.

As a numerical example, let us consider a 3-player public goods game with $q_{1}=0.1, q_{2}=0.3$, $q_{3}=0.6, r=1.5$ and $\delta=0.5$. A straightforward computation shows that when players have equal endowments, condition (14) is violated for players 1 and 2 . Instead, the inequalities in (14) hold for all players if endowments are redistributed such that $\mathbf{e}=(0,1 / 5,4 / 5)$.

We note, however, that games in which players derive unequal benefits from the public good differ in
two important aspects from games where players have unequal productivities. First, when players derive unequal benefits, player $i$ 's payoff in the fully cooperative equilibrium is $\pi_{i}=q_{i} r$, and thus independent of the initial endowments. That is, provided players still achieve full cooperation, a redistribution of endowments has no payoff effects. Second, in the 3-player game with different productivities, everyone preferred the mutual cooperation payoff for the endowment distribution $(0,1 / 5,4 / 5)$ to the mutual defection payoff when players have equal endowments. In the 3-player game where players derive different benefits from the public good, this does not need to be the case anymore. In the example discussed in the previous paragraph, player 1 yields a payoff of $1 / 3$ if all players are given an equal endowment and everyone defects. In contrast, player 1 yields the lower payoff of 0.15 if the initial endowment distribution is $\mathbf{e}=(0,1 / 5,4 / 5)$ and everyone cooperates. That is, even if a redistribution of endowments from $(1 / 3,1 / 3,1 / 3)$ to $(0,1 / 5,4 / 5)$ favors mutual cooperation, players may no longer unanimously agree to such a redistribution.

### 3.4 An example of a nonlinear symmetric public goods game

As a simple example of a nonlinear but symmetric public goods game, Fig. 2c,f of the main text illustrates the case of a game with payoff function

$$
\begin{equation*}
u_{i}(\mathbf{e}, \mathbf{x})=c_{1} \cdot \max _{j, k \neq j}\left(e_{j} x_{j}+e_{k} x_{k}\right)+c_{2} \cdot \sum_{j=1}^{n} e_{j} x_{j}+\left(1-x_{i}\right) e_{i}, \tag{16}
\end{equation*}
$$

for constants $c_{1}, c_{2}>0$. Due to the first term on the right hand's side, the payoff function $u$ is piecewise linear but not linear for $n \geq 3$. This game reflects a situation in which the two highest absolute contributions are particularly important for the success of the public good. Payoff function (16) satisfies


Given that the payoff function (16) provides an extra benefit to the two highest absolute contributions, an equal distribution of endowments may no longer be optimal for groups of size $n \geq 3$. According to Proposition 1, full cooperation can be sustained for equal endowments if and only if

$$
\begin{equation*}
\delta \geq \frac{1-c_{2}}{2 c_{1}+(n-1) c_{2}} . \tag{17}
\end{equation*}
$$

When players have unequal endowments, and without loss of generality $e_{1} \geq e_{2} \geq \ldots \geq e_{n}$, Proposition 1 yields the following condition for full cooperation,

$$
\begin{equation*}
\delta \geq \max \left\{\frac{\left(1-c_{1}-c_{2}\right) e_{1}+c_{1} e_{3}}{c_{1}\left(e_{2}+e_{3}\right)+c_{2}\left(1-e_{1}\right)}, \frac{\left(1-c_{2}\right) e_{3}}{c_{1}\left(e_{1}+e_{2}\right)+c_{2}\left(1-e_{3}\right)}\right\} . \tag{18}
\end{equation*}
$$

Condition (18) indicates that if any player has an incentive to deviate from full cooperation, it is either the player with the highest endowment, or the player with the third-highest endowment (i.e., the first player whose contribution is not multiplied by the factor $c_{1}$ ). By comparing the conditions (17) and (18), we obtain the following result.

Proposition 7. Consider a nonlinear public good game between $n \geq 3$ players with payoff function (16). Let $\mathbf{e}^{\prime}$ be the equal endowment distribution. Let $\mathbf{e}^{\prime \prime}$ be a distribution where the full endowment is equally split among two players only; that is, there are players $i, j$ such that $e_{i}^{\prime \prime}=e_{j}^{\prime \prime}=1 / 2$ and $e_{k}^{\prime \prime}=0$ for all other $k$. Then for all continuation probabilities $\frac{1-c_{1}-c_{2}}{c_{1}+c_{2}} \leq \delta<\frac{1-c_{2}}{2 c_{1}+(n-1) c_{2}}$ we have $\mathbf{e}^{\prime} \notin E_{u}(\delta), \mathbf{e}^{\prime \prime} \in E_{u}(\delta)$.

The content of Proposition 7 is illustrated in Fig. 2c, where we have considered the case $n=3$ with $c_{1}=1 / 2, c_{2}=1 / 3$. For these parameter values, equal endowments can maintain cooperation for $\delta \geq 0.4$, whereas splitting the endowment between two players can maintain cooperation for $\delta \geq 0.2$. For the figure, we have used the value $\delta=0.35$; in that case, cooperation is feasible in three separate regions, centered around the three points where two players equally split the full endowment.

### 3.5 Maximally cooperative endowment distributions

In the remainder of this section, we wish to introduce a notion for endowment distributions that are most conducive to cooperation. To this end, let us first formally define the minimal continuation probability compatible with full cooperation. For a given public good game with payoff function $u$, we define this minimal continuation probability $\delta_{u}^{*}$ as

$$
\begin{equation*}
\delta_{u}^{*}:=\inf \left\{\delta \in[0,1] \mid E_{u}(\delta) \neq \emptyset\right\} . \tag{19}
\end{equation*}
$$

Because of Proposition 2, this minimal continuation probability is well-defined and satisfies $0<\delta_{u}^{*}<1$. Using this minimal continuation probability we introduce the following notion of maximally cooperative endowment distributions.

Definition. For a given public good game $u$ we say that an endowment distribution $\mathbf{e}^{*}=\left(e_{1}^{*}, \ldots, e_{n}^{*}\right)$ is maximally cooperative if $\mathrm{e}^{*} \in E_{u}\left(\delta_{u}^{*}\right)$.

That is, an endowment distribution $\mathbf{e}^{*}$ is maximally cooperative if it requires the lowest continuation probability for cooperation to be feasible. Due to Proposition 2, it follows that such a distribution is most conducive to cooperation in the following sense: If cooperation is feasible for any endowment distribution for a given $\delta$, then it is guaranteed to be feasible when players receive the maximally cooperative endowment distribution $\mathbf{e}^{*}$.

To find the maximally cooperative endowment distribution, one needs to identify a pair ( $\mathbf{e}^{*}, \delta^{*}$ ) such that all inequalities (6) are satisfied; moreover, for any other such pair (e, $\delta$ ) it needs to be true that $\delta \geq \delta^{*}$. Because the left hand's side of the inequalities (6) may involve products of $e_{i}$ and $\delta$, this generally yields a nonlinear optimization problem. In the following we solve this optimization problem for the special case of a pairwise game with a linear payoff function.

Proposition 8. Consider a public good game between two players with linear payoff function u given
by Eq. (1), such that

$$
\begin{align*}
& u_{1}(\mathbf{e}, \mathbf{x})=c_{11} x_{1} e_{1}+c_{12} x_{2} e_{2}+e_{1}  \tag{20}\\
& u_{2}(\mathbf{e}, \mathbf{x})=c_{21} x_{1} e_{1}+c_{22} x_{2} e_{2}+e_{2},
\end{align*}
$$

for constants $c_{11}, c_{22}<0$ and $c_{12}, c_{21}>0$. Then the minimal continuation probability $\delta_{u}^{*}$ and the maximally cooperative endowment distribution $\mathbf{e}^{*}=\left(e_{1}^{*}, e_{2}^{*}\right)$ are given by

$$
\begin{equation*}
\delta_{u}^{*}=\sqrt{\frac{c_{11} c_{22}}{c_{12} c_{21}}} \quad \text { and } \quad \frac{e_{1}^{*}}{e_{2}^{*}}=\sqrt{\frac{c_{12} c_{22}}{c_{11} c_{21}}} \tag{21}
\end{equation*}
$$

In the main text, we have considered the example where players have different productivities, such that $c_{i j}=r_{j} / n$ for $j \neq i$ and $c_{i i}=r_{i} / n-1$. In that special case, the maximally cooperative endowment distribution according to Eq. (21) becomes

$$
\begin{equation*}
\frac{e_{1}}{e_{2}}=\sqrt{\frac{r_{2}\left(2-r_{2}\right)}{r_{1}\left(2-r_{1}\right)}} . \tag{22}
\end{equation*}
$$

In particular, it follows that an equal endowment distribution $e_{1}=e_{2}$ is maximally cooperative if and only if the two players coincide in their productivities. If players differ in their productivities, the more productive player is supposed to yield a higher endowment according to the maximally cooperative endowment distribution.

Similarly, for the game with asymmetric benefits considered in Section 3.3, we can set $c_{i j}=q_{i} r / n$ and $c_{i i}=q_{i} r / n-1$. In this case, the maximally cooperative endowment distribution becomes

$$
\begin{equation*}
\frac{e_{1}}{e_{2}}=\sqrt{\frac{q_{2}\left(2-r q_{2}\right)}{q_{1}\left(2-r q_{1}\right)}} \tag{23}
\end{equation*}
$$

Again, we conclude that equal endowments are only maximally cooperative if $q_{1}=q_{2}$, such that both players derive the same benefit from the public good. Otherwise, if there is a player who derives a larger benefit, this player is supposed to receive the larger endowment.

## 4 Evolutionary analysis

Whereas the previous results have clarified when full cooperation is feasible, based on static equilibrium considerations, here we explore whether cooperation can actually evolve if players have not yet settled on a particular equilibrium. To make such an evolutionary approach computationally tractable, we first focus on a a reduced strategy space.

### 4.1 Memory-one strategies

Compared to the previous equilibrium analysis, the strategy space considered in the following is based on two simplifying assumptions. First, we assume that in each round, players only choose among a finite
set $X$ of contribution levels, $X=\left\{\hat{x}_{1}, \ldots, \hat{x}_{m}\right\}$. In the most simple case that $X=\{0,1\}$, we refer to the two possible choices as 'defection' (D) and 'cooperation' (C), respectively. Second, we assume that players use memory-one strategies: a player's decision to cooperate in the next round may depend on the outcome of the previous round, but it is independent of all earlier rounds. Memory-one strategies have the major advantage that the payoffs according to Eqs. (3) and (4) can be computed efficiently, as we describe further below.

Memory-one strategies can be represented by a vector $\mathbf{p}=\left(p_{0, k}, p_{\mathbf{x}, k}\right)$, with entries in the unit interval $[0,1]$. The entries $p_{0, k}$ for $1 \leq k \leq m$ denote the probability that in the first round, the player contributes a fraction $\hat{x}_{k}$ of the endowment . The remaining entries $p_{\mathbf{x}, k}$ encode the conditional probability that the player contributes $\hat{x}_{k}$ in any subsequent round, given that the contribution vector of the previous round was $\mathbf{x} \in X^{n}$. For the vector $\mathbf{p}$ to be a sensible strategy, we require that in each round, the respective probabilities sum up to one, $\sum_{k} p_{0, k}=1$ and $\sum_{k} p_{\mathbf{x}, k}=1$ for all contribution vectors $\mathbf{x} \in X^{n}$. If all entries of $\mathbf{p}$ are either equal to 0 or equal to 1 , we say the memory-one strategy is deterministic or pure. Otherwise the strategy is called stochastic.

When all $n$ players use memory-one strategies $\mathbf{p}^{1}, \ldots, \mathbf{p}^{n}$, the resulting payoffs can be calculated using a Markov chain approach. The states of this Markov chain are the $m^{n}$ possible contribution vectors $\mathrm{x} \in X^{n}$ that determine the outcome of a given round. If $\mathbf{x}$ is the contribution vector of the previous round, the probability that the next round's contribution vector is $\mathbf{x}^{\prime}=\left\{\hat{x}_{k_{1}}, \ldots, \hat{x}_{k_{n}}\right\}$ can be calculated as $m_{\mathbf{x}, \mathbf{x}^{\prime}}=p_{\mathbf{x}, k_{1}}^{1} \cdot \ldots \cdot p_{\mathbf{x}, k_{n}}^{n}$. The corresponding transition probabilities can be assembled in a matrix $M=\left(m_{\mathbf{x}, \mathbf{x}^{\prime}}\right)$. In addition, we need to specify the probability to observe each contribution vector $\mathbf{x}^{\prime}=$ $\left\{\hat{x}_{k_{1}}, \ldots, \hat{x}_{k_{n}}\right\}$ in the very first round. This probability is given by $v_{\mathbf{x}^{\prime}}^{0}=p_{0, k_{1}}^{1} \cdot \ldots \cdot p_{0, k_{n}}^{n}$. Given the initial distribution $\mathbf{v}^{0}=\left(v_{\mathbf{x}}^{0}\right)$ and the transition matrix $M=\left(m_{\mathbf{x}, \mathbf{x}^{\prime}}\right)$, we calculate

$$
\begin{equation*}
\mathbf{v}=(1-\delta) \mathbf{v}^{0} \cdot(I-\delta M)^{-1} \tag{24}
\end{equation*}
$$

where $I$ is the $m^{n} \times m^{n}$ identity matrix. The entries of the vector $\mathbf{v}=\left(v_{\mathbf{x}}\right)$ give the weighted probability to observe each possible outcome $\mathbf{x} \in X^{n}$ over the course of the repeated game. For $\delta \rightarrow 1$, the vector $\mathbf{v}$ approaches a left eigenvector of $M$ with respect to the eigenvalue 1 . Given the vector $\mathbf{v}$, we can calculate the players' payoffs according to Eqs. (3) and (4) as

$$
\begin{equation*}
\pi=\sum_{\mathbf{x} \in X^{n}} v_{\mathbf{x}} \cdot u(\mathbf{e}, \mathbf{x}) . \tag{25}
\end{equation*}
$$

For our evolutionary analysis, we suppose that the players' decisions can be subject to noise. That is, in any given round there is some small probability $\varepsilon>0$ that a player implements a wrong move. For example, a player might contribute less than he actually wanted, or he may cooperate although he actually wanted to defect. As a consequence, if the player's actual memory-1 strategy is $\mathbf{p}$, his effective strategy becomes $(1-\varepsilon) \cdot \mathbf{p}+\varepsilon /(m-1) \cdot(\mathbf{1}-\mathbf{p})$. This assumption ensures that the transition matrix $M$ is positive (that is, each entry of $M$ is positive). In the limit $\delta \rightarrow 1$, the presence of noise thus implies that the invariant distribution $\mathbf{v}$ is independent of the outcome of the initial round. For the simulations in
the main text, we have used $\varepsilon=0.001$ throughout. Other error rates give similar results, provided that errors are sufficiently rare (Extended Data Fig. 4).

### 4.2 A model of introspection dynamics

In the following we introduce a simple framework to model how players adopt new memory-one strategies over time. When games are symmetric, it is common to consider a process where players tend to imitate strategies of more successful players ${ }^{45}$, or where successful players produce more offspring ${ }^{46}$. For games among unequals, however, such processes may yield counterintuitive results, because a strategy that is successful for one player may be disadvantageous for another. Alternatively, to study the dynamics in asymmetric matrix games, past work has considered players who are drawn from different populations, and who only adopt strategies that are successful within their own population ${ }^{47,48}$. However, for games in large groups such a multi-population approach seems unrealistic. In many of our applications, different players may interact with each other although they do not stem from different well-defined populations. To model the strategy dynamics among unequal players, we thus introduce a novel evolutionary process. Our process is based on the idea that occasionally, players may come up with new strategies, and that they compare their payoff so far with the payoff they could have got if they had used the new strategy all along. We call this process introspection dynamics.

Specifically, in a group of $n$ players, we assume that in each evolutionary time step one of the players is chosen at random. This player $i$ then picks a random new strategy of her available strategy set (while the co-players' strategies remain fixed). If the player's previous strategy yielded a payoff $\pi_{i}$, whereas the new strategy would have yielded the payoff $\tilde{\pi}_{i}$, we assume that the player switches to the new strategy with probability

$$
\begin{equation*}
\rho_{s}\left(\pi_{i}, \tilde{\pi}_{i}\right)=\frac{1}{1+\exp \left[-s\left(\tilde{\pi}_{i}-\pi_{i}\right)\right]} \tag{26}
\end{equation*}
$$

The parameter $s \geq 0$ reflects the strength of selection. In the boundary case $s=0$, the switching probability simplifies to $\rho=1 / 2$, such that strategy changes occur at random. In the limit of strong selection, $s \rightarrow \infty$, the player only adopts the new strategy if it yields at least the payoff of the old strategy.

In Figs. 2-4 of the main text, we have shown simulation results for this introspection dynamics. There, we have assumed that initially all players use ALLD. Then we have iterated the above described elementary updating process, assuming that subjects can choose among all stochastic memory-1 strategies and that payoffs are computed by Eq. (25). By iterating this process over many time steps $t$, we yield a sequence of strategies $\left(\mathbf{p}^{1}(t), \ldots, \mathbf{p}^{n}(t)\right)$, where $\mathbf{p}^{i}(t)$ denotes the memory-1 strategy employed by player $i$ after $t$ time steps. Based on this sequence, we can calculate the average cooperation rates and payoffs of all players over the entire simulation. The respective MATLAB implementation is provided in the Appendix.

### 4.3 Introspection dynamics for players with finitely many strategies

If players employ pure memory-one strategies, such that the total number of strategies $k$ is finite, we can use an exact method to compute expected trajectories of the introspection dynamics. The introspection dynamics takes the form of a discrete Markov chain with $k^{n}$ possible states $P=\left(\mathbf{p}^{1}, \ldots, \mathbf{p}^{n}\right)$. The transition probability from state $P$ to state $\tilde{P}$ is given by

$$
w_{P, \tilde{P}}=\left\{\begin{array}{cl}
\frac{1}{n(k-1)} \cdot \rho_{s}\left(\pi_{i}, \tilde{\pi}_{i}\right) & \text { if } \mathbf{p}^{i} \neq \tilde{\mathbf{p}}^{i}, \text { and } \mathbf{p}^{j}=\tilde{\mathbf{p}}^{j} \text { for all } j \neq i .  \tag{27}\\
1-\sum_{\tilde{\mathbf{p}}^{i} \neq \mathbf{p}^{i}} \frac{1}{n(k-1)} \cdot \rho_{s}\left(\pi_{i}, \tilde{\pi}_{i}\right) & \text { if } P=\tilde{P} \\
0 & \text { otherwise. }
\end{array}\right.
$$

The transition probabilities according to Eq. (27) indicate that within each time step, at most one player can change her strategy. This player is chosen randomly (a particular player $i$ is chosen with probability $1 / n$ ), and then given the chance to pick a new strategy (each remaining strategy has the same probability $1 /(k-1)$ ). This strategy is then adopted with probability $\rho_{s}\left(\pi_{i}, \tilde{\pi}_{i}\right)$. Again, we can collect all transition probabilities in a $k^{n} \times k^{n}$ matrix $W=\left(w_{P, \tilde{P}}\right)$. If the initial state of the population is given by the vector $\mathbf{y}(0)=\left(y_{P}(0)\right)$, then the expected strategy distribution at time $t$ is $\mathbf{y}(t)=\mathbf{y}(0) \cdot W^{t}$. Because the evolutionary process is ergodic for all finite selection strengths $s$, the distribution $\mathbf{y}(t)$ converges to a unique invariant distribution $\mathbf{y}=\left(y_{P}\right)$ that solves $\mathbf{y}=\mathbf{y} \cdot W$. Its entries $y_{P}$ reflect how often we are to observe the population in state $P=\left(\mathbf{p}^{1}, \ldots, \mathbf{p}^{n}\right)$ over an evolutionary timescale.

For the evolutionary results depicted in our Extended Data Figures, we have employed this Markov chain approach for groups of size $n=2$. In that case, each player can choose among $k=2^{5}=32$ pure memory-1 strategies (when $\delta<1$ ), or among $k=2^{4}=16$ strategies (when $\delta=1$, such that the cooperation probability in the first round can be neglected). Extended Data Fig. 1 is based on the five scenarios considered in the behavioral experiment. It compares realized cooperation trajectories of introspection dynamics as defined in Section 4.2 to the expected trajectories defined in this section. Extended Data Fig. 2 depicts the invariant distribution y for the same five scenarios. In three cases, we observe that players predominantly settle at a state where both players employ Win-Stay Lose-Shift (WSLS, see next subsection). To calculate the invariant distribution $\mathbf{y}$, we have used MATLAB to solve the homogeneous linear system $\left(I_{256}-W^{\prime}\right) \mathbf{y}=0$. Here, $I_{256}$ is the $2^{4} \cdot 2^{4}=256$-dimensional identity matrix, and $W^{\prime}$ refers to $W$ transposed. Extended Data Fig. 3 shows the players' average contributions and how often they employ WSLS, again computed from the invariant distribution y of the process. Extended Data Fig. 4 illustrates how the players' average payoffs according to the invariant distribution $\mathbf{y}$ change as we vary the expected number of rounds $1 /(1-\delta)$, the selection strength $s$, or the error rate $\varepsilon$. Finally, in Extended Data Fig. 5 we present evolutionary results when players are engaged in a public goods game with asymmetric benefits, with payoffs given by Eq. (13). The MATLAB implementation to generate the respective data is provided in the Appendix.

### 4.4 Analysis of Win-Stay Lose-Shift

Equilibrium conditions for Win-Stay Lose-Shift. For our evolutionary results, we have assumed that the players' decisions are subject to noise. Under such conditions, a homogeneous group of Grim players may no longer be able to maintain full cooperation: if one of the Grim players fails to cooperate due to an error, the whole group switches to defection for the remainder of the game. Instead, the evolutionary results depicted in Extended Data Fig. 1 - Extended Data Fig. 5 suggest that for high continuation probabilities, full cooperation only emerges as the strategy Win-Stay Lose-Shift (WSLS) becomes predominant.

While previous studies have explored this strategy for symmetric social dilemmas with discrete actions ${ }^{49-52}$, we can extend the definition of WSLS for general $n$-player public goods games with continuous contribution levels as follows. A WSLS player contributes her full endowment in the first round, or if all players with a positive endowment have contributed the same share of their endowment in the previous round. Otherwise, if people have differed in their relative contributions in the last round, a WSLS player contributes nothing. For a two-player game with two discrete contribution levels, this definition reduces to the usual definition of WSLS, $\mathbf{p}=\left(p_{0} ; p_{C C}, p_{C D}, p_{D C}, p_{D D}\right)=(1 ; 1,0,0,1)$.

In contrast to Grim, a population of WSLS players is robust with respect to noise. If one of the players deviates from full cooperation due to an error, all players collectively defect the next round, and then revert to mutual cooperation in all subsequent rounds.

The following Proposition characterizes when WSLS is an equilibrium. The proofs of the results in this section are again given in the Appendix.

Proposition 9. Consider a public good game with payoff function $u$, continuation probability $\delta<1$, and endowment distribution $\mathbf{e}$. Then the strategy profile where all players apply WSLS is a subgame perfect equilibrium if and only if

$$
\begin{equation*}
\delta\left(u_{i}(\mathbf{e}, \mathbf{1})-u_{i}(\mathbf{e}, \mathbf{0})\right) \geq\left(u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{1})\right) \text { for all players } i \text { with } e_{i}>0 . \tag{28}
\end{equation*}
$$

We note that Proposition 9 does not make any restrictions on the possible deviation strategies. When condition (28) holds, there is no incentive to deviate from WSLS even if players have access to highermemory strategies, or when they choose among continuous contribution levels.

When we compare the two Propositions 1 and 9 , we note that the critical threshold $\delta$ for WSLS to be an equilibrium is strictly larger than the respective threshold for Grim. As a consequence, the set of endowment distributions for which WSLS is an equilibrium, referred to as $E_{u}^{W}(\delta)$, is a strict subset of the set $E_{u}(\delta)$ of endowments where Grim is an equilibrium. This relationship is also illustrated in Extended Data Fig. 3, where the region $E_{u}^{W}(\delta)$ (indicated by dashed lines) is strictly contained in the region $E_{u}(\delta)$ (indicated by dotted lines). As shown in that figure, high cooperation rates only evolve in the parameter region where WSLS is a subgame perfect equilibrium. The following Proposition shows that the sets $E_{u}^{W}(\delta)$ and $E_{u}(\delta)$ have similar qualitative properties.

## Proposition 10.

1. Consider a public goods game with payoff function $u$, and let $\delta, \delta^{\prime}$ be two continuation probabilities with $\delta<\delta^{\prime}$. Then $E_{u}^{W}(\delta) \subset E_{u}^{W}\left(\delta^{\prime}\right)$.
2. If $E_{u}^{W}(\delta) \neq \emptyset$ for some public goods game with linear and symmetric payoff function $u$ and for some continuation probability $\delta$, then $(1 / n, \ldots, 1 / n) \in E_{u}^{W}(\delta)$.
3. There are public good games with an asymmetric payoff function $u$ such that $E_{u}^{W}(\delta) \neq \emptyset$ for some continuation probability $\delta$, but $(1 / n, \ldots, 1 / n) \notin E_{u}^{W}(\delta)$.
4. Consider a pairwise game with linear payoff function (1). If $E_{u}^{W}(\delta) \neq \emptyset$ for some given continuation probability $\delta$, then $\mathbf{e}^{*} \in E_{u}^{W}(\delta)$, where $\mathbf{e}^{*}$ is the maximally cooperative endowment distribution as characterized by Eq. (21).

To the extent that the evolution of cooperation is linked to the performance of WSLS, the above results complement the static results from the previous section. Cooperation should more readily evolve as the continuation probability $\delta$ increases; in symmetric and linear games, cooperation is most likely to evolve under an equal endowment distribution; and in asymmetric games, an uneven distribution of endowments is more favorable to the evolution of cooperation. These predictions based on the stability of WSLS are further supported by our numerical results in Fig. 2, Fig. 3, and Extended Data Fig. 1 Extended Data Fig. 5.

Evolutionary stability of Win-Stay Lose-Shift. The conditions for subgame perfection in Eq. (28) guarantee that when all players adopt WSLS, no player can gain a higher payoff by deviating. However, they do not imply that WSLS is evolutionarily stable in the sense of Maynard Smith ${ }^{53}$. After all, neutral deviations that leave the deviating player with exactly the same payoff may still be possible.

There is by now a rather extensive literature on whether evolutionarily stable strategies for repeated games exist at all ${ }^{54-63}$. For the repeated prisoner's dilemma without errors, Selten and Hammerstein have noted that Tit-for-Tat is not evolutionarily stable because it can be neutrally invaded by ALLC ${ }^{54}$. Boyd and Lorberbaum have generalized this result to show that in the absence of errors, no pure strategy is evolutionarily stable ${ }^{55}$. In particular, they show that once a first neutral mutant gains a sizable proportion of the population, a second mutant strategy can be selectively favored to invade. Similar results apply to mixtures of pure strategies ${ }^{56}$ and to stochastic strategies ${ }^{57}$. Taking a dynamic perspective, van Veelen and Garcia have shown in a series of papers that cooperation rates in evolving populations tend to oscillate ${ }^{59-61}$. For any resident population, there always exist neutral 'stepping stone' mutations out of equilibrium. Over time, cooperation comes and goes.

However, these results do not mean that evolutionary stability in repeated games is generally impossible. Under the realistic assumption that players sometimes commit errors, Boyd has shown that a strategy is evolutionarily stable if it is a 'strong perfect equilibrium' ${ }^{62}$. That is, in a game with discounted payoffs, a strategy is evolutionarily stable if it is a strict best response to itself after any possible history. Intuitively, errors guarantee that any possible history is reached with positive probability. This general reachability in turn prohibits neutral mutants to exist that only differ from the resident off the
equilibrium path. Based on this insight, we can reformulate Proposition 9 to gain the following result on the evolutionary stability of WSLS.

Proposition 11. Consider a public good game with payoff function $u$, continuation probability $\delta<1$, and endowment distribution $\mathbf{e}$ with $e_{i}>0$ for all players $i$. Moreover, suppose the inequality (28) is strict,

$$
\begin{equation*}
\delta\left(u_{i}(\mathbf{e}, \mathbf{1})-u_{i}(\mathbf{e}, \mathbf{0})\right)>\left(u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{1})\right) \text { for all players } i . \tag{29}
\end{equation*}
$$

Then there exists a threshold $\varepsilon_{0}>0$ such that WSLS is evolutionarily stable for all error rates $0<\varepsilon<\varepsilon_{0}$.
When the conditions of Proposition 11 are met, any deviation in a group of WSLS players leads to a strictly lower expected payoff for the deviating player. As a consequence, any such deviation is disfavored by the evolutionary process.

Two remarks are in order. First, we stress that Proposition 11 again does not restrict the possible mutant strategies. Even if players are permitted to react to arbitrarily long histories of past play, any deviation from WSLS leaves them with a strictly lower payoff. Second, we note that for any finite selection strength, the evolutionary process we have considered is ergodic. That is, no matter which strategy profile the group currently adopts, there is always a chance that one of the players decides to switch his strategy, even if the new strategy yields a strictly lower payoff. The process is guaranteed to leave any strategy profile eventually. Realized cooperation trajectories will thus exhibit oscillations, even if WSLS is evolutionarily stable (see Extended Data Fig. 1 for some typical examples of such trajectories).

Nevertheless, the conditions in Proposition 11 are important, as they affect how often players cooperate on average over an evolutionary timescale. Once WSLS is evolutionarily stable, a group of WSLS players can only be left due to a rare chance event. Depending on the strength of selection, a considerable number of elementary introspection events may be necessary until one player deviates.

### 4.5 Evolution among players with finite state automata

So far we have explored the evolution of cooperation when players can choose among all memory-1 strategies. Memory-1 strategies have been routinely used in previous work as they are convenient to analyze ${ }^{17-22}$. However, the emergence of reciprocity in evolutionary simulations can be sensitive to the considered strategy space ${ }^{50,64,65}$ and to the applied mutation scheme ${ }^{66,67}$. It is thus natural to ask to which extent our evolutionary results depend on the assumption of memory-1 players. To explore this issue in more detail, we consider the evolutionary dynamics when players have access to a vastly larger strategy set.

Strategies represented by finite state automata. In the following, we adapt the framework of van Veelen and Garcia ${ }^{59-61}$ to the asymmetric games studied herein. For simplicity, we describe the framework for pairwise games only, and we assume players use pure strategies without errors.

As before, players can choose among a finite set of contribution levels $X=\left\{\hat{x}_{1}, \ldots, \hat{x}_{m}\right\}$. To make their decisions, players use finite state automata $\mathcal{A}=(\Omega, \lambda, \mu)$. The first component $\Omega=\left\{\omega_{1}, \ldots, \omega_{k}\right\}$ is the set of possible states of the automaton. The second component $\lambda: \Omega \rightarrow X$ determines how much the player contributes in each state. Finally, the third component $\mu: \Omega \times X \rightarrow \Omega$ is the transition function. Depending on the automaton's present state and on the co-player's contribution, it determines the player's state in the next round. Without loss of generality, we assume that the finite state automaton is initially in the first state $\omega_{1}$.

The memory- 1 strategies considered in the previous sections can all be represented as finite state automata. As an example, let us assume that players can either contribute their full endowment or nothing, $X=\{0,1\}$. Again, we identify these two actions with cooperation (C) and defection (D), respectively. Suppose a player uses the memory-1 strategy $\mathbf{p}=\left(p_{0} ; p_{C C}, p_{C D}, p_{D C}, p_{D D}\right)$, and for simplicity assume that initially the player cooperates, $p_{0}=1$. Then $\mathbf{p}$ is equivalent to a finite state automaton $\mathcal{A}_{\mathbf{p}}=\left(\Omega_{\mathbf{p}}, \lambda_{\mathbf{p}}, \mu_{\mathbf{p}}\right)$ with elements defined by

$$
\Omega_{\mathbf{p}}=\left(\omega_{1}, \omega_{2}\right), \quad \lambda_{\mathbf{p}}\left(\omega_{1}\right)=C \text { and } \lambda_{\mathbf{p}}\left(\omega_{2}\right)=D, \quad \mu_{\mathbf{p}}(\omega, x)= \begin{cases}\omega_{1} & \text { if } p_{\omega, x}=1  \tag{30}\\ \omega_{2} & \text { if } p_{\omega, x}=0\end{cases}
$$

We illustrate this representation of memory-1 strategies as finite state automata in Extended Data Fig. 6a. There, we depict three particular examples, AllD, Tit-for-Tat, and Win-Stay Lose-Shift.

While every memory-1 strategy can be represented as a finite state automaton, the set of strategies representable by finite state automata is considerably richer. As an example of a strategy that is not memory-1, Extended Data Fig. 6a provides a finite-state representation of Tit for two Tats ${ }^{68}$. More generally, it can be shown that in fact all possible pure strategies for the repeated prisoner's dilemma can be approximated arbitrarily closely by finite state automata ${ }^{60}$.

To calculate the players' payoffs, suppose player 1 uses an automaton with $k_{1}$ states and player 2 uses an automaton with $k_{2}$ states. Because strategies are pure and players do not commit errors, the game dynamics is deterministic. In particular, it takes at most $k_{1} \cdot k_{2}$ rounds until the two players enter an infinite loop. From that moment on, their actions repeat themselves indefinitely. For the simulations shown in Extended Data Fig. 6c, we assume no discounting, and the players' payoffs are thus simply defined as their average payoff per round over all rounds of the loop. Similar results can be obtained in discounted games. In the Appendix, we provide the MATLAB implementation that we have used.

Introspection dynamics for players with finite state automata. To model the evolutionary dynamics, we assume that initially both players use the AllD automaton depicted in Extended Data Fig. 6a. In each time step, one of the players is then chosen at random to experiment with a mutation $\tilde{\mathcal{A}}$ of his present automaton $\mathcal{A}$. Following van Veelen and Garcia ${ }^{59,60}$, we allow for four different types of mutations (see also Extended Data Fig. 6b),
(i) Mutation in the state transitions ('Change an arrow'). When this mutation occurs, one input ( $\omega, x$ ) of the transition function is randomly chosen. Then its output $\mu(\omega, x)$ is changed randomly, such
that all alternative states in $\Omega$ have the same probability $1 /(k-1)$ to become the new output.
(ii) Mutation in the chosen contribution level ('Change an action'). When this mutation occurs, one state $\omega \in \Omega$ is randomly chosen. Then the chosen contribution level $\lambda(\omega)$ in that state is changed randomly, such that all alternative contribution levels have the same probability $1 /(m-1)$ to be chosen.
(iii) Mutation in the number of states ('Remove a state'). When this mutation occurs, one state $\omega \in \Omega$ is randomly chosen. This state is then removed from $\Omega$. All transitions that lead into this state are randomly and independently redirected. All remaining states have the same probability to be the new target of these redirected transitions.
(iv) Mutation in the number of states ('Add a state'). When this mutation occurs, a new state $\omega^{\prime}$ is added to the set $\Omega$. For each pair $\left(\omega^{\prime}, x\right)$, the outgoing transitions $\mu\left(\omega^{\prime}, x\right)$ are determined randomly, such that all states in $\Omega$ have the same probability to be the target (including the new state). In addition, one of the existing transitions among the previous states is randomly chosen, and randomly redirected towards the new state. That is, a pair $(\omega, x)$ with $\omega \neq \omega^{\prime}$ is randomly chosen, and the value of $\mu(\omega, x)$ is changed to $\omega^{\prime}$.

As in the case of memory- 1 strategies, we assume that the player then compares his previous payoff $\pi_{i}$ with the payoff $\tilde{\pi}_{i}$ he could have got when using the mutated automaton $\tilde{\mathcal{A}}$. The player switches to the mutated automaton with probability $\rho_{s}\left(\pi_{i}, \tilde{\pi}_{i}\right)$, as defined in Eq. (26). This elementary updating process is then again iterated for $T$ time steps. This yields two sequences $\left(\mathcal{A}_{1}(0), \ldots, \mathcal{A}_{1}(T)\right)$ and $\left(\mathcal{A}_{2}(0), \ldots, \mathcal{A}_{2}(T)\right)$, where $\mathcal{A}_{i}(t)$ is the automaton used by player $i$ after $t$ time steps. Based on this sequence, we calculate the average cooperation rate over the entire simulation.

We note that the above mutation scheme is capable of producing every possible finite state automaton over the set of possible contributions $X$. Like van Veelen and Garcia ${ }^{59,60}$, we assume for the simulations that the four types of mutations mentioned occur with probabilities $17.5 \%, 17.5 \%, 35 \%$, and $30 \%$, respectively. We obtain similar results when different probabilities are used (however, for computational reasons, it is useful to assume that the removal of a state is more likely than adding a new state).

Simulation results. To compare the evolutionary dynamics among players with finite state automata with our previous results on memory-1 strategies, we have re-run the simulations of Fig. $\mathbf{3}$ in the main text. That is, we consider two scenarios, depending on whether the players' productivities are equal or unequal ( $r_{1}=r_{2}=1.6$ compared to $r_{1}=1.9>1.3=r_{2}$ ). In each scenario, we run separate simulations for different endowment distributions, $e_{1} \in\{0,0.1, \ldots, 0.9,1\}$. The results are shown in Extended Data Fig. 6c.

Compared to Fig. 3, we first observe that the simulations based on finite state automata yield somewhat lower cooperation rates overall. This reduced level of cooperation seems to be partly driven by the differences in the mutation process. In the memory-1 space, we have assumed that every possible
strategy is equally likely to be the next mutant. In particular, an AllD player may switch to Tit for Tat within one time step. In contrast, when players use finite state automata, an AllD player can only arrive at Tit-for-Tat through a sequence of consecutive elementary mutations. This sequence is easily interrupted once a mutation leads to the removal of a state. As a consequence, we observe that players with finite state automata tend to defect unconditionally more often than memory-1 players.

However, all qualitative results reported before remain unchanged. When players are equally productive, we observe that cooperation is most abundant when players obtain equal endowments. Conversely, when players have different productivities, cooperation rates reach a maximum when the more productive player obtains more of the endowment. Again, the position of this maximum is reasonably approximated by the maximally cooperative endowment distribution as computed in Eq. (22).

Overall, the results in Fig. 3 and Extended Data Fig. 6 suggest that our evolutionary results on the effect of endowment inequality are not sensitive to the considered strategy space. Provided that the strategy space is kept constant, equal endowments promote cooperation when players are equally productive. Unequal endowments prove beneficial when players differ in their productivity.

## 5 Experimental analysis

### 5.1 Methods and procedures

Recruitment and sample size. To explore the validity of our theoretical results, we have recruited $N=436$ participants on Amazon Mechanical Turk (AMT). AMT is an online labor market which is routinely used for psychological and economic studies ${ }^{69-71}$. Participants on AMT receive a base payment for participation: the base pay was $\$ 1.00$ in our experiment. In addition, participants may receive further payments, known as a "bonus payment", based on their performance in the game. To earn a bonus, participants could earn tokens during the game which were converted to US dollars at the end of the game at a rate of 800 tokens $=\$ 1.00$; the average bonus participants earned was $\$ 1.70$.

For our experimental study, participants were recruited to interact with each other in pairs. We implemented the online experiment with SoPHIE, an online platform that allows for real-time interaction between AMT participants ${ }^{72-74}$. The sample size was determined in advance based on similar past research ${ }^{74}$. We recruited at least 80 participants per condition because we expected that the effect size for our key predictions would be moderate and that the dropout rate would be about $30 \%$, as previously observed in similar interactive online experiments ${ }^{73,74}$. In the end, dropouts were lower than expected (about 7\%; for a detailed dropout analysis, see Section 5.3).

The experiment was approved by the Harvard University Institutional Review Board (Protocol Number 14306).

Instructions and comprehension. Participants entered the study, read the instructions and had to pass several comprehension questions before they could continue (the instructions as well as the questions are provided in the Appendix). If they failed one of the comprehension questions, the relevant instructions
were shown again so that participants could try again. Only when all questions have been answered successfully were participants eligible to continue.

After reading the instructions and passing the comprehension questions, participants entered a waiting room. Once at least two participants had arrived, they were paired for the duration of the experiment. If no other participant arrived within 10 minutes, unpaired participants were redirected to an exit screen, and thanked for their attention and patience. They received the respective $\$ 1.00$ base payment for participating in the study but no further bonus payment.

Parameters and decisions each round. We implemented five different treatments based on the linear public good games (7) and (16). The treatments differed in how the initial endowment was distributed among the players, and in how productive the players' contributions were (i.e., by which factor a player's absolute contributions were multiplied). Specifically, we have used the following parameters (see also Fig. 4a):

| Treatment | Endowment $e_{i}$ |  | Productivity $r_{i}$ |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Player 1 | Player 2 | Player 1 | Player 2 |
| (1) Full equality | 50 | 50 | 1.6 | 1.6 |
| (2) Endowment inequality | 75 | 25 | 1.6 | 1.6 |
| (3) Productivity inequality | 50 | 50 | 1.9 | 1.3 |
| (4) Aligned inequality | 75 | 25 | 1.9 | 1.3 |
| (5) Misaligned inequality | 25 | 75 | 1.9 | 1.3 |

The parameter values for the treatments (1) - (3) were chosen such that $(i)$ the total size of the endowment is constant $\left(e_{1}+e_{2}=100\right)$, and (ii) the maximum achievable group payoff is the same across conditions. The parameter values for the treatments (4) and (5) then follow from symmetry considerations. We note that all parameter values are in line with the four properties $(\mathbf{C})-(\mathbf{O C})$, as introduced in Section 1.

Every pair of participants interacted in at least 20 rounds of the public goods game. To avoid endgame effects, players knew that in each round after the 20th round, there was a $50 \%$ probability that there would be another round. From this distribution, the average number of rounds was 21.0 with a range between 20 and 29. Taking into account that due to dropouts some groups did not complete the game, the average number of rounds actually played was 20.3 with a range between 1 and 29 .

In each round, both participants had to decide simultaneously how much to contribute to the public good. Participants could contribute any integer value between 0 and $e_{i}$, the participant's respective endowment. After both participants made their decisions, they received all relevant information on the outcome of this round: each participants' absolute contributions, their productivity factor, the total public good provided by each participant, and the returns from the public good per participant. Participants then continued to the next round of the game until the end of the game. When the game had ended, participants were thanked for their participation, and they received their payment.

Statistical methods. For all our main experimental results, depicted in Fig. 4 and Extended Data Figs. 7-10, we used non-parametric tests (Mann-Whitney test for between-treatment comparisons, and Wilcoxon signed-rank test for within-treatment comparisons). For these tests, we considered groups of two players as our statistical unit. We used two-tailed tests throughout. For between-treatment comparisons, we restrict ourselves to comparisons where the players' productivity is kept fixed but their endowments may vary. That is, we compare the two treatments (1) and (2), and we separately compare the treatments (3), (4), and (5). In this way our results indicate, for given productivities of the players, how a redistribution of endowments would affect the players' contributions to the public good.

When we report results on the players' contributions, we consider their relative contributions (i.e., which fraction of their endowment the players contribute, corresponding to the variable $x$ in our theoretical model). As a consequence, we note that even when we find that a high-endowment player gives relatively less than a low-endowment player $\left(x_{1}<x_{2}\right)$, the high-endowment player's absolute contributions may still be higher (when $e_{1} x_{1}>e_{2} x_{2}$ ).

For our analysis, we only consider the first twenty rounds of the game because this was the minimum number all groups had in common. Moreover, for the results in the main text and in the following section, we have excluded groups in which at least one participant dropped out half-way through the game. However, in Section 5.3 we repeat our analysis using imputed values for the dropout groups instead. All our main results remain unchanged. Unless noted otherwise, all statistical tests have been performed with Stata SE 15.1.

### 5.2 Results

Average payoffs and contributions across treatments. In Fig. 4, we have depicted the players' average contributions across the five treatments, as well as the resulting surplus (i.e., the difference between the players' payoffs and their initial endowments, relative to the initial endowments).

When players are equally productive, we find that their total contributions make up $72.2 \%$ of their endowment in the full equality treatment, compared to $49.0 \%$ under endowment inequality (Mann-Whitney test, $Z=2.813, p=0.005$ ). As a direct consequence, the two treatments also differ in the generated surplus, which is $43.3 \%$ under full equality but only $29.4 \%$ under endowment inequality (Mann-Whitney test, $Z=2.813, p=0.005$ ).

For the case where players have unequal productivities, we find no significant differences in the players' contributions between productivity inequality and aligned inequality: players contribute $72.8 \%$ of their endowment under productivity inequality and $73.5 \%$ under aligned inequality (Mann-Whitney test, $Z=0.844, p=0.399$ ). However, both of these treatments differ significantly from the treatment with misaligned inequality, where players only contribute $45.5 \%$ of their endowment (Mann-Whitney tests, treatments (3) vs. (5): $Z=3.818, p<0.001$; (4) vs. (5): $Z=3.825, p<0.001$ ). With regard to the generated surplus all three treatments differ from one another. As predicted by our theoretical analysis, aligned inequality yields the highest surplus (the respective values are (3): $43.8 \%$, (4): $55.1 \%$, (5): $23.1 \%$; Mann-Whitney tests, (3) vs. (4): $Z=3.357, p<0.001$; (3) vs. (5): $Z=4.592, p<0.001$;
(4) vs. (5) $Z=4.802, p<0.001$ ).

Comparing the contributions of the two players. In Extended Data Fig. 7 we show the contributions and payoffs for each treatment, separately for the two players.

As one may expect, in the full equality treatment there are no differences in the players' contributions and payoffs. The average contribution of players 1 and 2 are $72.3 \%$ and $72.2 \%$, respectively, and their payoffs are 71.63 tokens and 71.69 tokens per round, respectively (Wilcoxon test, $Z=0.784, p=0.433$ ).

Under endowment inequality, the relative contributions of the high-endowment player 1 fall below the contributions of player 2, their average contributions being $47.2 \%$ and $54.4 \%$, respectively (Wilcoxon text, $Z=2.194, p=0.028$ ). As a consequence, the high-endowment player keeps more of their own units and thus earns a substantially higher payoff of 78.8 tokens, compared to the 50.6 tokens for the low-endowment player (Wilcoxon test, $Z=5.098, p<0.001$ ).

Under productivity inequality, we find that the relative contributions of the two players are similar, with the more productive player 1 contributing $73.4 \%$ of her endowment on average, and the less productive player 2 contributing $72.2 \%$ (Wilcoxon test, $Z=0.380, p=0.704$ ). As a result, also their payoffs are comparable, 71.6 tokens and 72.2 tokens, respectively (Wilcoxon test, $Z=0.366, p=0.714$ ).

We find a similar agreement in the players' contributions under aligned inequality. Here the highendowment and high-productivity player 1 contributed $73.4 \%$ of her endowment, whereas the lowendowment and low-productivity player 2 contributed $73.8 \%$ (Wilcoxon test, $Z=0.705, p=0.481$ ). However, since the players are not fully cooperative and player 1's initial endowment is higher, player 1 obtains a higher payoff on average, with 84.2 tokens compared to 70.8 tokens (Wilcoxon test, $Z=3.239$, $p=0.001$ ).

Finally, under misaligned inequality, the players differ in both their contributions and their eventual payoffs. Whereas player 1 contributes $62.9 \%$ of her endowment and earns a payoff of 43.5 tokens, player 2 contributes $39.7 \%$ of his endowment and earns 79.5 tokens (Wilcoxon tests, contributions: $Z=4.234, p<0.001$; payoffs: $Z=5.464, p<0.001$ ).

Dynamics of cooperation over time. In Extended Data Fig. 8 we show how much the two players in each treatment have contributed over time. To test whether there is any significant time trend within a treatment, we have compared the players' total contributions during the first five rounds with their total contributions during the last five rounds. For four of the five treatments we find no significant trends over time (Wilcoxon tests, $p>0.1$ for all four of these treatments); only under endowment inequality, we find that overall contributions slightly decrease, from $54.0 \%$ during the first five rounds to $43.9 \%$ during the last 5 rounds (Wilcoxon test, $Z=2.578, p=0.010$ ).

Reciprocal behavior across the five treatments. To gain some insights into the players' strategies, we have analyzed to which extent their behaviors are guided by reciprocity considerations. To this end, we say that a player's relative contribution is reciprocal if it matches or exceeds the co-player's relative
contribution in the previous round. That is, if one player contributes $60 \%$ of her endowment in one round, a reciprocal co-player would contribute at least $60 \%$ of his endowment in the following round. As illustrated in Extended Data Fig. 10, a majority of the players' contribution decisions can be classified as reciprocal, such that the frequency of reciprocal behavior is typically between $70 \%$ and $90 \%$.

The only exception is the treatment with misaligned inequality. Here, the low-endowment highproductivity player 1 reciprocates in $85.9 \%$ of all rounds, whereas the high-endowment low-productivity player 2 only reciprocates in $44.9 \%$ of all cases (Wilcoxon test, $Z=4.252, p<0.001$ ). To explore this mismatch in the players' reciprocity in more detail, Extended Data Fig. 9 shows the players' joint contribution decisions. For misaligned inequality, we observe that in a substantial fraction of rounds, the two players coordinate on a state where they both give the same absolute amount to the public good (for example, in $12.4 \%$ of the cases, both players contribute 25 tokens, which is the maximum contribution for player 1, but only a third of player 2's endowment). As a result, player 1's relative contributions systematically exceed the relative contributions of player 2 .

### 5.3 Robustness of the statistical results

As is usual in online experiments, some groups did not complete the study because players decided to drop out. These instances could potentially confound our analysis if there is a correlation between the game situation a participant experiences and his decision to leave the experiment. To quantify the effect of dropouts, we have performed two separate analyses. First, we have explored whether there are some systematic patterns in the groups that dropped out. Second, we have repeated all our statistical tests using imputed values for the groups that did not complete the study. Both analyses suggest that our statistical results are robust with respect to attrition.

Dropout analysis. Here, we investigate whether selection bias may have occurred on the basis of exogenous or endogenous variation (e.g. depending on the assigned treatment, or on the experienced contribution behavior during the game). In total, fifteen out of 218 groups ( $6.9 \%$ ) did not complete the game because at least one of the players dropped out. The following table shows how many groups completed the experiment in each treatment.

| Treatment | Completed game |  | Total |
| :--- | :---: | :---: | :---: |
|  | No | Yes |  |
| (1) Full equality | 2 | 42 | 44 |
| (2) Endowment inequality | 1 | 42 | 43 |
| (3) Productivity inequality | 3 | 40 | 43 |
| (4) Aligned inequality | 5 | 39 | 44 |
| (5) Misaligned inequality | 4 | 40 | 44 |
| Total | 15 | 203 | 218 |

Most dropouts occurred in the treatment with aligned inequality (4), whereas the fewest groups dropped out in the endowment inequality treatment (1). When we use the data of this table to test whether dropouts depend on the treatment, we find no significant association (Fisher's exact test, $p=0.531$ ).

Next we have explored whether dropouts are associated with other variables, such as the group's cooperation dynamics. To this end, we have explored several logistic regression models. Because dropouts are comparably rare, standard logistic regression can fail due to the problem of "separation" ${ }^{75}$. In that case, the estimate for the regression parameter diverges and the maximum likelihood estimate does not exist. We observe this divergence for our endowment inequality treatment, for which there is only a single group that dropped out. To address this problem, we use Firth's logistic regression ${ }^{76}$ instead, with standard errors clustered at the group level. We have verified that in those cases in which standard logistic regression produces finite parameter estimates, the results are qualitatively similar.

We have first explored whether less cooperative groups are more likely to drop out. As the independent variable we use the average sum of contributions across all rounds for each group. This variable takes values between 0 (if neither player ever contributed anything) and 100 (if both players always contributed everything). As the dependent variable we use an indicator variable with values 0 or 1 (dropped out or completed the game, respectively). When we pool across all conditions, there is no significant relationship between the dropout status of a group and the average contributions in a group ( $p=0.309$ ).

Next, we add a variable for the condition assignment to this logistic regression ( $1=$ full equality, $2=$ endowment inequality, $3=$ productivity inequality, $4=$ aligned inequality and $5=$ misaligned inequality). As before, average contributions do not predict dropout ( $p=0.409$ ). Furthermore, none of the condition indicators are significant ( $p$-value of (2) endowment inequality dummy: $p=0.754$; (3) productivity inequality: $p=0.655$; (4) aligned inequality: $p=0.269$; (5) misaligned inequality: $p=0.517$ ). These results suggest that there is no significant relationship across conditions between average contributions and whether or not the group completes the game.

We also consider each treatment separately. We use Firth's logistic regression with the same independent and dependent variables at the group level, but now for each treatment. The sum of contributions is not a significant predictor of group dropout in the full equality ( $p=0.579$ ), endowment inequality ( $p=0.313$ ), productivity inequality ( $p=0.736$ ), aligned inequality ( $p=0.271$ ) or misaligned inequality ( $p=0.319$ ) conditions.

Finally, we explore whether treatment assignment and contribution dynamics jointly affect dropouts. To this end, we pool across all conditions. As the independent variables, we use
(i) the sum of contributions (ranging between 0 and 100, abbreviated as SOC below),
(ii) the five condition indicators ( $1=$ full equality, $2=$ endowment inequality, $3=$ productivity inequality, 4 = aligned inequality, $5=$ misaligned inequality; full equality is also the baseline group for comparisons) and
(iii) the interaction term between these two variables.

The dependent variable is 0 if the group dropped out or 1 if the group completed the game. We find
that none of the interaction terms are significant predictors of dropout (SOC interacted with endowment inequality indicator: $p=0.646$; with productivity inequality indicator: $p=0.815$; with aligned inequality indicator: $p=0.330$; with misaligned inequality indicator: $p=0.881$ ). A post-hoc comparison of the interaction coefficients also does not reveal any significant differences (comparisons of interaction terms: SOC $\times(2)$ vs. $\operatorname{SOC} \times(3): p=0.487 ; \operatorname{SOC} \times(2)$ vs. $\mathrm{SOC} \times(4): p=0.185 ; \mathrm{SOC} \times(2)$ vs. $\mathrm{SOC} \times(5):$ $p=0.705 ; \mathrm{SOC} \times(3)$ vs. $\mathrm{SOC} \times(4): p=0.386 ; \mathrm{SOC} \times(3)$ vs. $\mathrm{SOC} \times(5): p=0.652 ; \mathrm{SOC} \times(4)$ vs. SOC $\times(5): p=0.152)$.
Overall, we find no indication that there is a systematic bias in the players' dropout decisions.

Robustness analysis with imputed missing values. For the experimental results reported in Section 5.2 we have only considered groups that have completed the game. In the following, we provide a complementary analysis in which we include missing data using a multiple imputation scheme.

Specifically, we apply the iterative, non-parametric imputation method introduced by Stekhoven and Bühlmann ${ }^{77}$. Their 'missForest' imputation method uses random forests (RF), which is a supervised machine learning method based on an iterative decision tree process. In our case, the generated trees split at nodes of variables that predict contributions on the basis of the existing data and makes predictions about the missing values. As is common with RF procedures, the final dataset of predicted values is then tested and validated against a hold-out sample.

As predictors we use treatment assignment, round, player type, endowment, productivity and nonmissing contributions. Using these variables, we predict out-of-sample the missing contributions of dropout groups. The parameters for the missForest function in R were as follows. We kept the standard number of trees supplied to the RF method (100), because our data is not high-dimensional. Decreasing or increasing the number of trees did not change the results. We set the maximum iteration parameter for the imputation procedure to 10 such that after 10 iterations, the procedure is forced to a hard-stop if it has not met its stopping criteria. The stopping criteria are such that after each iteration, the previous and the current imputed data matrix are compared. If the difference between them goes up for the first time, the most recent imputed dataset is returned. In practice, the stopping criteria were met after 4 iterations and the hard-stop was not enforced. The missForest function returns the normalized root mean squared error (NRMSE) to assess how the final imputed dataset fared in the out-of-sample prediction; the NRMSE was 0.000016 in our computations, suggesting a low error rate.

With the imputed dataset, we repeat the same analysis as in the previous Section 5.2. The results are summarized in Supplementary Table 1 (for the two treatments in which players have equal productivities) and in Supplementary Table 2 (for the three treatments in which players have different productivities). In each case, the left column ('main sample') displays our previous results based on only those groups who completed the study. The right column ('Imputed sample') shows the new results using the data of all groups, based on the imputation method described above. As before, we use nonparametric Mann-Whitney tests for between-treatment comparisons, and Wilcoxon signed-rank tests for within-treatment comparisons (two-tailed).

|  | Main sample | Imputed sample |
| :---: | :---: | :---: |
| Contributions | (1) $72.2 \%$ vs. (2) $49.0 \%, Z=2.813^{* *}$ | (1) $73.4 \%$ vs. (2) $50.2 \%, Z=2.847^{* *}$ |
| Surplus | (1) 43.3 vs. (2) $29.4, Z=2.813^{* *}$ | (1) 44.0 vs. (2) $30.1, Z=2.847^{* *}$ |
| Player 1 vs. Player 2 Contributions | (1) $72.3 \%$ vs. $72.2 \%, Z=0.765$ <br> (2) $47.2 \%$ vs. $54.4 \%, Z=2.194^{*}$ | (1) $73.5 \%$ vs. $73.3 \%, Z=0.524$ <br> (2) $48.5 \%$ vs. $55.4 \%, Z=2.234 *$ |
| Player 1 vs. Player 2 Payoffs | (1) 71.6 vs. $71.7, Z=0.784$ <br> (2) 78.8 vs. $50.6, Z=5.098^{* * *}$ | (1) 72.0 vs. $72.1, Z=0.548$ <br> (2) 78.8 vs. $51.3, Z=5.159^{* * *}$ |
| First versus last five rounds | (1) $71.1 \%$ vs. $72.8 \%, Z=0.350$ <br> (2) $54.0 \%$ vs. $43.9 \%, Z=2.578^{* *}$ | (1) $72.3 \%$ vs. $73.9 \%, Z=0.367$ <br> (2) $55.0 \%$ vs. $45.1 \%, Z=2.653^{* *}$ |
| Player 1 vs. Player 2 Reciprocity | (1) $90.0 \%$ vs. $88.5 \%, Z=0.804$ <br> (2) $71.6 \%$ vs. $80.1 \%, Z=1.449$ | (1) $90.3 \%$ vs. $88.5 \%, Z=0.552$ <br> (2) $72.1 \%$ vs. $80.5 \%, Z=1.533$ |

Supplementary Table 1: Results for main and imputed samples under equal productivites. Bold numbers in brackets refer to the respective treatment, (1) Full equality, (2) Unequal endowments. The number of data points is $N_{1}=N_{2}=42$ (left column) and $N_{1}=44, N_{2}=43$ (right column), respectively.

* $p<0.05,{ }^{* *} p<0.01, * * * p<0.001$, using either a Mann-Whitney test (for between-treatment comparisons) or a Wilcoxon signed-rank test (for within-treatment comparisons). All tests are two-tailed.

|  | Main sample | Imputed sample |
| :---: | :---: | :---: |
| Contributions | (3) $72.8 \%$ vs. (4) $73.5 \%, Z=0.844$ <br> (3) $72.8 \%$ vs. <br> (5) $45.5 \%, Z=3.818$ <br> (4) $73.5 \%$ vs. <br> (5) $45.5 \%, Z=3.825^{* * *}$ | (3) $73.5 \%$ vs. (4) $72.4 \%, Z=0.420$ <br> (3) $73.5 \%$ vs. <br> (5) $44.5 \%, Z=4.226$ <br> (4) $72.4 \%$ vs. <br> (5) $44.5 \%, Z=4.114^{* * *}$ |
| Surplus | (3) 43.8 vs . <br> (4) $55.1, Z=3.357^{*}$ <br> (3) 43.8 vs . <br> (5) $23.1, Z=4.592^{* * *}$ <br> (4) 55.1 vs . <br> (5) $23.1, Z=4.802^{* * *}$ | (3) 44.2 vs . <br> (4) $54.1, Z=3.014^{* *}$ <br> (3) 44.2 vs . <br> (5) $22.5, Z=4.986^{* * *}$ <br> (4) 54.1 vs . <br> (5) $22.5, Z=5.262^{* * *}$ |
| Player 1 vs. Player 2 Contributions | (3) $73.4 \%$ vs. $72.2 \%, Z=0.380$ <br> (4) $73.4 \%$ vs. $73.8 \%, Z=0.705$ <br> (5) $62.9 \%$ vs. $39.7 \%, Z=4.234^{* * *}$ | (3) $73.8 \%$ vs. $73.1 \%, Z=0.414$ <br> (4) $71.9 \%$ vs. $74.0 \%, Z=1.181$ <br> (5) $61.2 \%$ vs. $39.0 \%, Z=4.575^{* * *}$ |
| Player 1 vs. Player 2 <br> Payoffs | (3) 71.6 vs. $72.2, Z=0.366$ <br> (4) 84.2 vs. $70.8, Z=3.239^{* *}$ <br> (5) 43.5 vs. $79.5, Z=5.464^{* * *}$ | (3) 71.9 vs. $72.3, Z=0.401$ <br> (4) 84.3 vs. $69.8, Z=3.899^{* * *}$ <br> (5) 43.2 vs. $79.3, Z=5.736^{* * *}$ |
| First versus last five rounds | (3) $73.3 \%$ vs. $70.8 \%, Z=1.085$ <br> (4) $73.2 \%$ vs. $74.6 \%, Z=0.079$ <br> (5) $50.1 \%$ vs. $44.1 \%, Z=1.472$ | (3) $73.7 \%$ vs. $72.5 \%, Z=1.034$ <br> (4) $70.9 \%$ vs. $73.8 \%, Z=0.602$ <br> (5) $50.3 \%$ vs. $42.8 \%, Z=2.049 *$ |
| Player 1 vs. Player 2 Reciprocity | (3) $83.6 \%$ vs. $84.3 \%, Z=0.492$ <br> (4) $85.2 \%$ vs. $88.1 \%, Z=0.280$ <br> (5) $85.9 \%$ vs. $44.8 \%, Z=4.252^{* * *}$ | (3) $83.6 \%$ vs. $84.2 \%, Z=0.454$ <br> (4) $77.6 \%$ vs. $87.4 \%, Z=1.036$ <br> (5) $86.8 \%$ vs. $43.1 \%, Z=4.648^{* * *}$ |

Supplementary Table 2: Results for main and imputed samples under unequal productivities. Bold numbers in brackets refer to the respective treatment, (3) Unequal productivies, (4) Aligned inequality, (5) Misaligned inequality. The number of data points is $N_{3}=40, N_{4}=39, N_{5}=40$ (left column) and $N_{3}=43, N_{4}=44$ and $N_{5}=44$, respectively.

* $p<0.05,{ }^{* *} p<0.01,{ }^{* * *} p<0.001$, using either a Mann-Whitney test (for between-treatment comparisons) or a Wilcoxon signed-rank test (for within-treatment comparisons). All tests are two-tailed.

According to the two tables, the main sample and the imputed sample yield qualitatively and quantitatively similar results. In fact, across all tests, there is only one difference. For the treatment with misaligned inequality, we have reported in the previous section that there is no significant trend in the players' contributions over time. Players contribute $50.1 \%$ of their endowment during the first five rounds of the experiment, whereas they contribute $44.1 \%$ during the last five rounds ( $Z=1.472, p=0.141$ ). In contrast, for the imputed sample, the trend becomes significant. According to the imputed sample, contributions start at $50.3 \%$ during the first five rounds and drop to $42.8 \%$ during the last five rounds ( $Z=2.049, p=0.041$, see Supplementary Table 2).

Overall this additional analysis based on imputed data suggests that the major conclusions of our experiment remain unchanged. When players have equal productivities, equal endowments result in the highest surplus. In contrast, when players differ in their productivities, the highest surplus is achieved under aligned inequality, where the most productive player receives a larger share of the endowment.

Robustness with respect to multiple testing. In the previous analyses, we have tested multiple hypotheses, and we have reported individual test statistics. Running multiple tests on the same data set increases the risk that some null hypotheses are rejected by chance, even if all hypotheses were true ${ }^{78}$. A commonly employed guard against such false discoveries is the Bonferroni correction, which makes an adjustment to the relevant threshold for the $p$-value, based on the number of hypotheses tested ${ }^{79}$. More specifically, if $h$ is the number of tested hypotheses and $\alpha$ is the significance level of interest (e.g., $\alpha=0.05$ or $\alpha=0.01$ ), then the Bonferroni corrected significance level becomes $\alpha / h$.

In the following, we explore which of our key results reported in the main text remain valid if we use the Bonferroni method to correct for multiple testing. In Fig. 4, we explore the impact of inequality on two quantities, total contributions and generated surplus. To explore whether there are significant differences between treatments for these two quantities, we have performed eight statistical tests in total (see the first two rows in Supplementary Table 1 and Supplementary Table 2). As a consequence, the Bonferroni corrected significance levels become $0.05 / 8=0.00625$ ( 1 asterisk), $0.01 / 8=0.00125$ ( 2 asterisks), and $0.001 / 8=0.000125$ ( 3 asterisks).

Using these stricter significance levels, we find that all our main text conclusions remain valid after applying the Bonferroni correction. For example, for equal productivities we have found that unequal endowments lead to lower contributions and a reduced surplus. The corresponding $p$-value of $p=0.005$ remains significant even if we correct for multiple testing (although the relevant significance level is now $0.05 / 8$, i.e., one asterisk instead of two). Similarly, we also find for the treatments with unequal productivities that all reported differences in the players' contributions and their surplus remain significant (although the number of asterisks may again change).

In sum, this analysis suggests that our key findings reported in the main text remain unchanged if we account for multiple testing.

## 6 Discussion

In this last section, we close by critically reviewing some of our model assumptions, and by suggesting a few model extensions for future work.

On the rigidity of endowments and productivities. In the main text, we have discussed our framework by focusing on a linear public good game where players may differ in their endowments and their productivities. When comparing the players' contributions across different public good games, we have often tacitly assumed that a player's productivities are given, whereas their endowments can be varied. There are two reasons why we consider this assumption to be useful.

First, we believe that in many applications, the players' endowments are best interpreted as their monetary resources, whereas their productivities could reflect their skills. In such a scenario, it is plausible to assume that in the short run, endowments are more amenable to interventions than productivities.

The second reason is of a more conceptual nature. When comparing the outcome of different public good games, one needs to make sure that games are commensurable. In the case of endowments, there exists a natural linear budget constraint: after any endowment redistribution, the sum of total endowments needs to remain unchanged. This budget constraint allows us to compare the consequences of different endowment distributions (provided they all sum up to the same value). With respect to the players' productivities, it is somewhat less clear what the appropriate budget constraint would be. In some cases it may again be natural to assume that the sum over all players' productivities is constant. In some other cases, however, it may be more natural to assume that it becomes disproportionally difficult to further increase the productivity of individuals who already are highly productive. To allow for a clean comparison, we have thus decided to take the players' productivities as fixed, and to vary their endowments. However, our framework and our analytical results are general. Once a budget constraint for the players' productivities is specified, our theoretical results immediately apply.

Public good contributions and taxation. In our model we do not consider the process by which endowments could be redistributed among players. Instead, we take different distributions as given and explore their impact on cooperation and welfare. However, there seem to be natural parallels between our framework and the question of how a social planner should set the tax rate in a population. This social planner might collect some of the group members' endowments and redistribute them. Alternatively, the social planner might directly invest the collected endowments into the public good.

It is not clear how this latter mechanism could be mapped onto our framework. As a first approximation, one could assume that in addition to the players $1, \ldots, n$ there is another player 0 (the social planner). Player 0 has no own endowment. Instead Player 0 determines a tax rate $T$, which all other group members need to pay. Depending on the implementation of the tax, it may either be a player's total endowment that is subject to taxation, or only the proportion of the endowment the player decided to withhold. Given the implemented tax, all players $1, \ldots, n$ would then decide how much to contribute to
the public good. Again, these contributions would be multiplied by some individual factor $r_{i}$. In addition, the total amount of tax collected is multiplied by $r_{0}$. The resulting sum is then equally distributed among all $n$ group members (excluding Player 0 ).

In this extended framework, the problem of optimal taxation can be studied as a problem of mechanism design. How should the social planner set the tax rate $T$, and which implementation should be chosen, in order to maximize group payoffs? A particularly interesting case might arise if the social planner's own productivity $r_{0}$ is considerably below the maximum productivity $\max _{i} r_{i}$ of the group members.

However, we note that the above account of taxation abstracts away several important aspects. For example, it assumes that players and the social planner would routinely contribute to the same public good. In many applications it seems instead more natural that individuals can choose between different public goods (e.g., creating new jobs versus donating money to universities). This choice might in turn depend on which public good is most relevant to the respective individual. Due to these intricacies, we have refrained from modeling taxation explicitly, and leave it for future research instead. A related question is which kind of taxation a population considers as fair, which hinges heavily on perceptions of income inequality in a society ${ }^{80}$.

Central institutions and corruption. Herein, we have explored how cooperation in asymmetric groups can emerge when individuals interact repeatedly. In our model, individuals cooperate because there is a shadow of the future. If subjects were to withhold their contributions, they risk that also other group members reduce their contributions in subsequent rounds.

Instead, there is also a rich theoretical and experimental literature that explores the effects of centralized or decentralized punishment institutions on cooperation ${ }^{81-86}$. A few of these experimental studies explicitly take into account that players may be heterogeneous ${ }^{29,36,39}$. However, it seems to us that a full theoretical understanding of how punishment institutions act in asymmetric groups is still missing.

In this context, it may also interesting to study how the efficiency of central punishment institutions is undermined by bribery and corruption ${ }^{87}$. In the context of homogeneous groups, corruption possibilities can lead to a large decrease in public good provisioning, with the exact welfare losses depending on the cultural background of a society, the implemented transparency rules, and the extent to which leaders themselves are invested in the public good ${ }^{88}$. In the context of heterogeneous groups, corruption may have further detrimental implications. For example, when players with a higher endowment are better capable of bribing public sanctioning institutions, any efficiency advantages of unequal endowments may be dampened or reversed. While our present framework does not address these issues, we believe these questions are worthwhile of further study.

## 7 Appendix

### 7.1 Proofs

## Proof of Proposition 1.

$(1) \Rightarrow$ (2) Suppose $\mathbf{e} \in E_{u}(\delta)$. Then there is a strategy for each player such that no player has an incentive to deviate by not cooperating in every round, for the given endowment distribution e. In the following, we refer to players who play their equilibrium strategy as 'residents', and to players who deviate as 'mutants'. If all players are residents, the equilibrium payoff is $\pi_{R}=u_{i}(\mathbf{e}, \mathbf{1})$.

Now suppose player $i$ considers deviating by using the strategy ALLD instead. Then her payoff in the first round is $u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)$. Due to Property (PE), her payoff in all subsequent rounds is at least $u_{i}(\mathbf{e}, \mathbf{0})$. Hence, the mutant payoff to a deviating ALLD player is at least

$$
\begin{equation*}
\pi_{M} \geq(1-\delta) u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)+\delta u_{i}(\mathbf{e}, \mathbf{0}) \tag{31}
\end{equation*}
$$

Since the strategies of the residents form a subgame perfect equilibrium, we can conclude that

$$
\begin{equation*}
(1-\delta) u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)+\delta u_{i}(\mathbf{e}, \mathbf{0}) \leq u_{i}(\mathbf{e}, \mathbf{1}), \tag{32}
\end{equation*}
$$

or equivalently,

$$
\begin{equation*}
\delta\left(u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{0})\right) \geq u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{1}) . \tag{33}
\end{equation*}
$$

$(2) \Rightarrow(3)$ To prove that Grim is a subgame perfect equilibrium, we can make use of the one-shot deviation principle ${ }^{44}$. That is, we only need to show that no mutant can get a higher payoff if she deviates in one round, and acts exactly as the other residents in all subsequent rounds. To this end, let us first assume that we are in the first round, or that all players have contributed their full endowment in all previous rounds. As the residents apply the strategy Grim, they continue to contribute their full endowment in the next round. A one-shot deviation consists in contributing an amount lower than the full endowment. In that case, Property (IF) implies that the maximum one-shot deviation payoff can be achieved by not contributing at all, in which case the continuation payoff becomes

$$
\begin{equation*}
\pi_{M}=(1-\delta) u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)+\delta u_{i}(\mathbf{e}, \mathbf{0}) \tag{34}
\end{equation*}
$$

Given the assumptions in (2), this continuation payoff is at most the resident payoff $u_{i}(\mathbf{e}, \mathbf{1})$.
Alternatively, let us now assume that players did not contribute their full endowment in all past rounds. In that case, a one-shot deviation requires player $i$ to contribute a positive amount in the next rounds, whereas all other players continue to contribute nothing. By Property (IF), such a deviation is unprofitable.
$(3) \Rightarrow$ (1) Since in a group of Grim players everyone contributes her full endowment in every round, and since Grim forms a subgame perfect equilibrium by assumption, full cooperation is feasible.

## Proof of Proposition 2.

1. Suppose $\mathbf{e} \in E_{u}(\delta)$. Because of Proposition 1, it follows that

$$
\delta\left(u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{0})\right) \geq u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{1}) .
$$

Since $\delta^{\prime}>\delta$ by assumption and $u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right) \geq u_{i}(\mathbf{e}, \mathbf{0})$ because of Property ( $\mathbf{P E}$ ), it follows that

$$
\delta^{\prime}\left(u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{0})\right) \geq \delta\left(u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{0})\right) \geq u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{1}) .
$$

Therefore, again by Proposition 1, $\mathbf{e} \in E_{u}\left(\delta^{\prime}\right)$.
2. Consider the map $\varphi: \Delta^{n} \rightarrow \Delta^{n}$, where $\Delta^{n}$ is the $n$-dimensional unit simplex, defined by

$$
\varphi(\mathbf{e})=\frac{u(\mathbf{e}, \mathbf{1})}{U(\mathbf{e}, \mathbf{1})} .
$$

Because $u(\mathbf{e}, \mathbf{x})$ is continuous in $\mathbf{e}$ due to property (C), and because $U(\mathbf{e}, \mathbf{1})>U(\mathbf{e}, \mathbf{0})=1$ due to property ( $\mathbf{O C}$ ), we conclude that $\varphi$ is continuous. Since $\Delta^{n}$ is compact and convex, it follows by Brouwer's fixed point theorem that there exists an endowment distribution $\mathbf{e}^{\prime}=\left(e_{1}^{\prime}, \ldots, e_{n}^{\prime}\right)$ such that $\varphi\left(\mathbf{e}^{\prime}\right)=\mathbf{e}^{\prime}$. As a consequence, we obtain that for all players $i$ the following relationship holds,

$$
\begin{equation*}
u_{i}\left(\mathbf{e}^{\prime}, \mathbf{1}\right)=U\left(\mathbf{e}^{\prime}, \mathbf{1}\right) \cdot e_{i}^{\prime}>e_{i}^{\prime}=u_{i}\left(\mathbf{e}^{\prime}, \mathbf{0}\right) . \tag{35}
\end{equation*}
$$

Let us now define $\delta^{\prime}$ by

$$
\begin{equation*}
\delta^{\prime}=\max _{i}\left\{\frac{u_{i}\left(\mathbf{e}^{\prime}, \mathbf{1}_{-i}\right)-u_{i}\left(\mathbf{e}^{\prime}, \mathbf{1}\right)}{u_{i}\left(\mathbf{e}^{\prime}, \mathbf{1}_{-i}\right)-u_{i}\left(\mathbf{e}^{\prime}, \mathbf{0}\right)}\right\} . \tag{36}
\end{equation*}
$$

By inequality (35), $\delta^{\prime}<1$. Moreover, $\mathbf{e}^{\prime} \in E_{u}\left(\delta^{\prime}\right)$ because the inequalities in condition (6) are satisfied by construction of $\delta^{\prime}$. The claim that $E_{u}(\delta) \neq \emptyset$ for all $\delta \geq \delta^{\prime}$ then follows from the first part of this proof.

Proof of Proposition 3. For a public good game with payoff function $u$ and continuation probability $\delta$, we define the following function with respect to player $i$,

$$
\begin{equation*}
F_{i}(\mathbf{e})=\delta\left(u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{0})\right)-\left(u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{1})\right) . \tag{37}
\end{equation*}
$$

According to Proposition 1, full cooperation is feasible for some given endowment distribution $\mathbf{e}$ if and only if $F_{i}(\mathbf{e}) \geq 0$ for all $i$. In the special case that player $i$ is given the whole endowment (that is, $\mathbf{e}$ is such that $e_{i}=1$ and $e_{j}=0$ for all $j \neq i$ ), it follows from Property $(\mathbf{P E})$ that $u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)=u_{i}(\mathbf{e}, \mathbf{0})$, whereas it
follows from Property (IF) that $u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)>u_{i}(\mathbf{e}, \mathbf{1})$. For that endowment distribution $\mathbf{e}$ we thus obtain $F_{i}(\mathbf{e})<0$. Since $u(\mathbf{e}, \mathbf{x})$ is continuous in $\mathbf{e}$ because of Property $(\mathbf{C}), F_{i}(\mathbf{e})$ is continuous as well. Hence there is an $\varepsilon_{i}>0$ such that $F_{i}\left(\mathbf{e}^{\prime}\right)<0$ for all $\mathbf{e}^{\prime}$ for which $e_{i}^{\prime}>1-\varepsilon_{i}$. That is, for all such endowment distributions $\mathbf{e}^{\prime}$, cooperation is infeasible.

The proofs of Propositions 4 and 5 is by example, provided in Sections 3.2 and 3.3. The proof of Proposition 6 is by iterated application of the following Lemma.

Lemma. Consider a standard public goods game with payoffs according to Eq. (10), and assume productivities satisfy $r_{1}<\ldots<r_{n}$. Let $\mathbf{e}^{1}$ be an endowment vector for which endowments are not ordered according to the players' productivities, such that there is an index $k$ such that $e_{k}^{1}>e_{k+1}^{1}$. Let $\mathbf{e}^{2}$ be the endowment vector for which $e_{k}^{2}=e_{k+1}^{1}, e_{k+1}^{2}=e_{k}$ and $e_{j}^{2}=e_{j}^{1}$ for all other $j$. If full cooperation is feasible for $\mathbf{e}^{1}$ then so it is for $\mathbf{e}^{2}$.

Proof. Rewriting the condition (11), it follows that cooperation is feasible for a given endowment distribution $e$ if and only if

$$
\begin{equation*}
\left(n-(1-\delta) r_{i}\right) e_{i} \leq \delta \sum_{j=1}^{n} r_{j} e_{j} \tag{38}
\end{equation*}
$$

for all players $i$. Equivalently, if we define the two auxiliary functions $G(\mathbf{e}):=\max _{i}\left\{\left(n-(1-\delta) r_{i}\right) e_{i}\right\}$ and $H(\mathbf{e}):=\delta \sum_{j=1}^{n} r_{j} e_{j}$, cooperation is feasible if and only if $G(\mathbf{e}) \leq H(\mathbf{e})$. Now, using the properties of the two given endowment vectors $\mathbf{e}^{1}$ and $\mathbf{e}^{2}$, we have

$$
\begin{aligned}
\left(n-(1-\delta) r_{k}\right) e_{k}^{2} & =\left(n-(1-\delta) r_{k}\right) e_{k+1}^{1}<\left(n-(1-\delta) r_{k}\right) e_{k}^{1} \\
\left(n-(1-\delta) r_{k+1}\right) e_{k+1}^{2} & =\left(n-(1-\delta) r_{k+1}\right) e_{k}^{1} \leq\left(n-(1-\delta) r_{k}\right) e_{k}^{1},
\end{aligned}
$$

and

$$
\left(n-(1-\delta) r_{j}\right) e_{j}^{2}=\left(n-(1-\delta) r_{j}\right) e_{j}^{1}
$$

for all other $j \notin\{k, k+1\}$. Thus, we can conclude

$$
\begin{equation*}
G\left(\mathbf{e}^{2}\right) \leq G\left(\mathbf{e}^{1}\right) \tag{39}
\end{equation*}
$$

On the other hand, we have

$$
\begin{equation*}
H\left(\mathbf{e}^{1}\right)=\underset{j \notin\{k, k+1\}}{\delta} r_{j} e_{j}^{1}+\delta\left(r_{k} e_{k}^{1}+r_{k+1} e_{k+1}^{1}\right) \leq \underset{j \notin\{k, k+1\}}{\delta} \sum_{j} e_{j}^{1}+\delta\left(r_{k} e_{k+1}^{1}+r_{k+1} e_{k}^{1}\right)=H\left(\mathbf{e}^{2}\right) . \tag{40}
\end{equation*}
$$

Thus, if cooperation is feasible for $\mathbf{e}^{1}$, and hence $G\left(\mathbf{e}^{1}\right) \leq H\left(\mathbf{e}^{1}\right)$, it follows that

$$
\begin{equation*}
G\left(\mathbf{e}^{2}\right) \leq G\left(\mathbf{e}^{1}\right) \leq H\left(\mathbf{e}^{1}\right) \leq H\left(\mathbf{e}^{2}\right) . \tag{41}
\end{equation*}
$$

That is, cooperation is feasible for the endowment vector $\mathbf{e}^{2}$.

The proof of Propositions 7 is again by example, which is provided in Section 3.4. The proof of Proposition 8 is as follows.

Proof of Proposition 8. For arbitrary continuation probability $\delta$ and endowment distribution $\mathbf{e}=\left(e_{1}, e_{2}\right)$ it follows from Eq. (6) that $\mathbf{e} \in E_{u}(\delta)$ if and only if

$$
\begin{align*}
& \delta c_{12} e_{2} \geq-c_{11} e_{1}  \tag{42}\\
& \delta c_{21} e_{1} \geq-c_{22} e_{2} .
\end{align*}
$$

Dividing both inequalities by $e_{2}$ (which is positive due to Proposition 3), we obtain that $\mathbf{e} \in E_{u}(\delta)$ if and only if

$$
\begin{equation*}
\frac{-c_{22}}{\delta \cdot c_{21}} \leq \frac{e_{1}}{e_{2}} \leq \frac{\delta \cdot c_{12}}{-c_{11}} . \tag{43}
\end{equation*}
$$

In particular, an endowment distribution $\mathbf{e} \in E_{u}(\delta)$ exists if and only if $-c_{22} /\left(\delta c_{21}\right) \leq \delta c_{12} /\left(-c_{11}\right)$. This condition yields $\delta \geq \delta_{u}^{*}$, with $\delta_{u}^{*}$ as defined by the first equation in (21). Plugging this $\delta_{u}^{*}$ into either the left hand side or the right hand side of condition (43) shows that $e_{1} / e_{2}$ is given by the second equation in (21).

Proof of Proposition 9. The proof is similar to the proof of Proposition 1, by employing the one-shot deviation principle ${ }^{44}$. That is, we show that no mutant can get a higher payoff by deviating for one round, and by reverting to WSLS thereafter. We consider two cases, depending on whether the history is such that the $n-1$ residents would cooperate (contribute their full endowment) or defect (contribute nothing). Let us first consider the case where players are either in the first round, or they have all chosen the same relative contribution in the previous round. As the residents use WSLS, they cooperate. By sticking to the resident strategy the mutant $i$ obtains the usual resident payoff

$$
\begin{equation*}
\pi_{R}=u_{i}(\mathbf{e}, \mathbf{1}) \tag{44}
\end{equation*}
$$

If the mutant decides to deviate for one round instead, the best possible deviation is to contribute nothing at all. In that case the mutant's continuation payoff is

$$
\begin{equation*}
\pi_{M}=(1-\delta) u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)+\delta\left((1-\delta) u_{i}(\mathbf{e}, \mathbf{0})+\delta u_{i}(\mathbf{e}, \mathbf{1})\right) . \tag{45}
\end{equation*}
$$

For WSLS to be a subgame perfect equilibrium we require $\pi_{R} \geq \pi_{M}$, which yields the inequality (28).

As the second case, assume now that players differed in their last round's contributions. In that case, by adhering to the resident strategy WSLS player $i$ obtains the continuation payoff

$$
\begin{equation*}
\pi_{R}=(1-\delta) u_{i}(\mathbf{e}, \mathbf{0})+\delta u_{i}(\mathbf{e}, \mathbf{1}) . \tag{46}
\end{equation*}
$$

If the mutant instead contributes some positive amount in the next round, it follows from Property (IF) that her continuation payoff is at most

$$
\begin{equation*}
\pi_{M}=\left(1-\delta^{2}\right) u_{i}(\mathbf{e}, \mathbf{0})+\delta^{2} u_{i}(\mathbf{e}, \mathbf{1}) . \tag{47}
\end{equation*}
$$

In particular, since $u_{i}(\mathbf{e}, \mathbf{1}) \geq u_{i}(\mathbf{e}, \mathbf{0})$ due to Property $(\mathbf{O C})$, we can conclude that $\pi_{R} \geq \pi_{M}$.

## Proof of Proposition 10.

1. By Proposition 9, an endowment distribution e satisfies $\mathbf{e} \in E_{u}^{W}(\delta)$ if and only if

$$
\begin{equation*}
\delta \geq \frac{u_{i}\left(\mathbf{e}, \mathbf{1}_{-i}\right)-u_{i}(\mathbf{e}, \mathbf{1})}{u_{i}(\mathbf{e}, \mathbf{1})-u_{i}(\mathbf{e}, \mathbf{0})} \text { for all players } i \text { with } e_{i}>0 \tag{48}
\end{equation*}
$$

Because $\delta^{\prime}>\delta, \mathbf{e} \in E_{u}^{W}(\delta)$ thus implies $\mathbf{e} \in E_{u}^{W}\left(\delta^{\prime}\right)$.
2. For a linear and symmetric public goods game with payoff function (7), Proposition 9 implies that $E_{u}^{W}(\delta) \neq \emptyset$ if and only if there is some endowment vector $\mathbf{e}$ such that

$$
\begin{equation*}
\max e_{i} \leq \frac{\delta c_{1}}{\delta c_{1}+(1+\delta) c_{2}} \tag{49}
\end{equation*}
$$

Because $1 / n \leq \max \left(e_{i}\right)$, it follows that $(1 / n, \ldots, 1 / n) \in E_{u}^{W}(\delta)$.
3. As a simple counter-example, consider the asymmetric payoff function (10) where players have different productivities. Proposition 9 implies that $\mathbf{e} \in E_{u}^{W}(\delta)$ if and only if

$$
\begin{equation*}
\delta\left(\frac{1}{n} \sum_{j=1}^{n} r_{j} e_{j}-e_{i}\right) \geq\left(1-\frac{r_{i}}{n}\right) e_{i} \text { for all players } i \text { with } e_{i}>0 \tag{50}
\end{equation*}
$$

For the 3-player case with $r_{1}=1.1, r_{2}=1.5, r_{3}=2.9$ and $\delta=0.5$, a straightforward computation shows that condition (50) is violated for $\mathbf{e}=(1 / 3,1 / 3,1 / 3)$, but satisfied for $\mathbf{e}=(0,1 / 5,4 / 5)$.
4. For a fixed $\delta$ and $\mathbf{e}=\left(e_{1}, e_{2}\right)$, Proposition 9 implies that $\mathbf{e} \in E_{u}^{W}(\delta)$ if and only if the following two conditions hold,

$$
\begin{align*}
& \delta\left(c_{11} e_{1}+c_{12} e_{2}\right) \geq-c_{11} e_{1} \\
& \delta\left(c_{21} e_{1}+c_{22} e_{2}\right) \geq-c_{22} e_{2} . \tag{51}
\end{align*}
$$

Dividing both inequalities by $e_{2}>0$ shows that $\mathbf{e} \in E_{u}^{W}(\delta)$ if and only if

$$
\begin{equation*}
\frac{-(1+\delta) c_{22}}{\delta c_{21}} \leq \frac{e_{1}}{e_{2}} \leq \frac{\delta c_{12}}{-(1+\delta) c_{11}} \tag{52}
\end{equation*}
$$

This chain of inequalities can only be met if the left hand's side is less than the right hand's side, which yields the following implicit condition for the minimal continuation probability for which $E_{u}^{W}(\delta)$ is non-empty,

$$
\begin{equation*}
\frac{\delta}{1+\delta} \geq \sqrt{\frac{c_{11} c_{22}}{c_{21} c_{12}}} . \tag{53}
\end{equation*}
$$

By using this lower bound for $\delta /(1+\delta)$ and plugging it into (52), we obtain

$$
\begin{equation*}
\frac{e_{1}}{e_{2}}=\sqrt{\frac{c_{12} c_{22}}{c_{11} c_{21}}} \tag{54}
\end{equation*}
$$

That is, we have shown that for WSLS to be an equilibrium for the lowest possible continuation probability, endowments need to meet condition (54), which coincides with the condition for e to be maximally cooperative. The statement then follows from the first part of this Proposition.

Proof of Proposition 11. The proof is analogous to the proof of Proposition 9, with the only difference that now all one-shot deviations are required to yield a strictly lower payoff. As a consequence, the inequality (29) needs to be strict, and all players are required to have a positive endowment, in order to exclude neutral one-shot deviations. The result then follows from the argument given by Boyd ${ }^{62}$.

### 7.2 MATLAB code used for the evolutionary computations

In the following, we provide the MATLAB algorithms that we have used to $(i)$ calculate the payoffs of memory-1 players, (ii) simulate the introspection dynamics, and (iii) numerically compute the expected trajectories for introspection dynamics when players use pure strategies with two possible contributions, $X=\{0,1\}$. (iv) Moreover, we provide our code for games where players use finite state automata to represent their strategies. For simplicity, the following code considers groups of size two, which has been used to create Figs. 3-4 and Extended Data Figs. 1-6 . The respective code for $n$-player games is similar, and is available from the authors upon request.

Algorithm to calculate the payoffs in a game between memory-1 players

```
function [pi,coop,v,COS,M]=payoff(P1,P2,e1,r1,r2,m);
% Output:
% pi, coop, v .. payoffs, cooperation rates, and invariant distribution
% COS .. List of possible outcomes of each round
% M .. transition matrix of the Markov chain
% Input:
% P1 .. m^2xm matrix that represents player 1's strategy;
% First m&m matrix gives probability to choose contribution xl, etc.
% P2 .. m^2Xm matrix for player 2's strategy
% e1,r1,r2 .. players' endowments and their productivities
% m .. number of possible contribution levels, X={0,1/(m-1),...,1}
```

\% \% PARAMETERS AND PREPARATIONS \% \%
e2=1-e1;

$O S=z e r o s\left(m^{\wedge} 2,2\right) ; \operatorname{CoS}=z e r o s\left(m^{\wedge} 2,2\right) ; \operatorname{PayOS}=z \cos \left(\mathrm{~m}^{\wedge} 2,2\right)$;
\% Possible outcomes/contributions/payoffs of the one-shot game
for il=1:m \% Going through all possible contribution levels for player 1
for i2 $=1: m$ \% Same for player 2
$j=(i 1-1) * m+i 2$; $\%$ Storing i1, i2 in one variable that runs from 1 to m^2
OS (j,: ) = [i1, i2]; $\operatorname{COS}(j,:)=[(i 1-1) /(m-1),(i 2-1) /(m-1)]$;
$\operatorname{PayOS}(j,:)=(e 1 * r 1 * \operatorname{COS}(j, 1)+e 2 * r 2 * \operatorname{COS}(j, 2)) / 2+[(1-\operatorname{COS}(j, 1)) * e 1,(1-\operatorname{COS}(j, 2)) * e 2]$;
end
end
\% CONSTRUCTING THE MARKOV CHAIN \% \%
M=zeros (m^2, m^2); \% Initializing the transition matrix
for il=1:m, for i2=1:m \% Running over the players' previous actions
for $j 1=1: m$, for $j 2=1: m$ \% Running over players' next actions
$M((i 1-1) * m+i 2,(j 1-1) * m+j 2)=P 1(i 1+(j 1-1) * m, i 2) * P 2(i 1+(j 2-1) * m, i 2) ;$
end, end
end, end
 pi=PayOS'*v; coop=COS'*v;
\% Calculating cooperation rates and payoffs in the case of no discounting end

## Algorithm to simulate the introspection dynamics

```
function [AvCoop,AvPi,COS,Data]=SimulateIntrospection(e1,r1,r2,epsi,m,s,nGen);
% Output:
% AvCoop, AvPi, AvV .. average cooperation and payoff over course of simulation
% COS .. possible contributions in one-shot game
% Data .. Summary of all parameters used
% Input:
% e1,r1,r2 .. players' endowments and their productivities
% epsi .. error rate
% m .. number of possible contribution levels, X={0,1/(m-1),\ldots,1}
% s .. strength of selection
% nGen .. number of generations (number of elementary updating events)
%% PARAMETERS AND PREPARATIONS %%
C=clock; rng(C(5)*60+C(6)); % Setting up the random number generator
P1=zeros(m^2,m); P2=zeros(m^2,m); % Initially, both players use ALLD
[pi,coop]=payoff(P1,P2,e1,r1,r2,m); % Calculate initial payoffs and cooperation
AvPi=pi; AvCoop=coop; % Initialize variables for respective averages
Data=['e1=',num2str(e1), '; r1=',num2str(r1), '; r2=',num2str(r2),...
    '; eps=',num2str(epsi), '; m=',num2str(m), '; s=',num2str(s),...
    '; nGen=',num2str(nGen)];
%% SIMULATING THE EVOLUTIONARY PROCESS %%
for t=2:nGen
    iPop=randi(2); % Randomly choosing a player to update
    Mut=getrandomstrategy(m,epsi); % Creating a mutant strategy for that player
    % Case 1: Player 1 explores a new strategy
    if iPop==1
            [piM,coopM,vM,COS]=payoff(Mut,P2,e1,r1,r2,m); % Mutant strategy payoff
            rho=1/(1+exp(-s*(piM(1)-pi(1)))); % Player's updating probability
            if rand(1)<rho % If updating occurs
                Pl=Mut; pi=piM; coop=coopM; % Update respective variables
```

end

```
    % Case 2: Player 2 explores a new strategy
    else
        [piM, coopM, vM, COS]=payoff(P1,Mut,e1,r1,r2,m); % Analogous steps for player
2
    rho=1/(1+exp(-s*(piM(2)-pi (2))));
    if rand(1)<rho
            P2=Mut; pi=piM; coop=coopM;
        end
    end
    % Updating the averages
    AvPi=(t-1)/t*AvPi+1/t*pi;
    AvCoop=(t-1)/t*AvCoop+1/t*coop;
end
end
function Mut=getrandomstrategy(m,epsi);
% Auxiliary function that creates a sensible mutant strategy
Mut=zeros(m^2,m) ; % Initialize strategy
l=(0:m-1)*m; % Auxiliary vector
for il=1:m % Running through all previous actions of player 1
    for i2=1:m % Running through previous actions of player 2
        v=rand(1,m-1); v=[0, v, 1]; v=sort(v); v=diff(v); Mut(il+l,i2)=v;
        % Produces a random stochastic strategy by uniformly drawing a vector v
        % in the m-dimensional unit simplex
        % To produce a pure strategy, replace above line by following line
        % v=zeros(1,m); v(randi (m))=1; Mut (i1+1,i2)=v;
    end
end
Mut=(1-epsi)*Mut+epsi/(m-1)*(1-Mut); % Adding errors to the strategy
end
```


## Algorithm to compute expected trajectories of introspection dynamics

```
function [Coop1,Coop2,XStr1,XStr2,PayMat1,PayMat2,Str,Data]=...
    ExpectedIntrospectionDynamics(e1,r1,r2,epsi,s,nGen);
% Output:
```

```
% Coop1, Coop2 .. The players' cooperation rates over time
% XStr1, XStr2 .. The players' strategies over time
% PayMat1, PayMat2 .. Payoff matrices for each pair of memory-1 strategies
% Str .. List of all memory-1 strategies in notation (pCC,pCD,pDC,pDD)
% Data .. List of the parameters used
% Input:
% e1,r1,r2 .. players' endowments and their productivities
% epsi .. error rate
% s .. strength of selection
% nGen .. number of generations (number of elementary updating events)
%% PARAMETERS AND PREPARATIONS %%
e2=1-e1;
POS1=[(r1*e1+r2*e2)/2,r1*e1/2,r2*e2/2+e1,e1]; % One-shot payoffs for player 1
POS2=[(r1*e1+r2*e2)/2,r1*e1/2+e2,r2*e2/2,e2]; % One-shot payoffs for player 2
Str=[0 0 0 0; 0 0 0 1; 0 0 1 0; 0 0 1 1; 0 1 0 0; 0 1 0 1; 0 1 1 0; 0 1 1 1;
    1 0 0 0; 1 0 0 1; 1 0 1 0; 1 0 1 1; 1 1 0 0; 1 1 0 1; 1 1 1 0; 1 1 1 1];
% List of all 16 memory-1 strategies with contributions either x=0 or x=1.
ns=size(Str,1); % number of strategies
Data=['e1=',num2str(e1), '; r1=',num2str(r1), '; r2=',num2str(r2),...
    '; eps=',num2str(epsi), '; s=',num2str(s), '; nGen=',num2str(nGen)];
%% CREATING PAIRWISE PAYOFF MATRICES FOR EACH PAIR OF MEMORY-1 STRATEGIES %%
PayMat1=zeros(ns,ns); PayMat2=zeros(ns,ns); % Initializing the payoff matrices
CopMat1=zeros(ns,ns); CopMat2=zeros(ns,ns); % Matrices for cooperation rates
for il=1:ns % Running through all strategies of player 1
    for i2=1:ns % Running through all strategies of player 2
        s1=Str(i1,:); s2=Str(i2,:);
        [pi1,pi2,coop1,coop2]=CalcPay(s1,s2,POS1,POS2,epsi);
        PayMat1(i1,i2)=pi1; PayMat2(i1,i2)=pi2;
        CopMat1(i1,i2)=coop1; CopMat2(i1,i2)=coop2;
    end
end
%% CREATING THE TRANSITION MATRIX W %%
W=zeros(ns^2, ns^2); % State space: all ns^2 combinations of strategies
for ilOld=1:ns % Previous strategy of player 1
    for i2Old=1:ns % Previous strategy of player 2
        pi1Old=PayMat1(i1Old,i2Old); pi2Old=PayMat2(i1Old,i2Old); % Previous payoffs
        % Case 1: Transitions where player 1 is chosen to update
        for ilNew=1:ns % Running through possible new strategies for player 1
        if i1New~=ilOld
```

```
            pi1New=PayMat1(i1New,i2Old); % New payoff
                rho=1/(1+exp(-s*(pilNew-pilOld))) ; % Update probability
                W(ns*(i1Old-1) +i2Old,ns*(i1New-1) +i2Old) =1/(2*(ns-1)) *rho;
            end
        end
        % Case 2: Transitions where player 2 is chosen to update
        for i2New=1:ns % Running through possible new strategies for player 2
        if i2New~}=i2Ol
            pi2New=PayMat2(i1Old,i2New);
            rho=1/(1+exp(-s*(pi2New-pi2Old)));
            W (ns*(ilOld-1) +i2Old,ns*(ilOld-1) +i2New) =1/(2*(ns-1))*rho;
        end
        end
    end
end
for k=1:ns^2
    W(k,k)=1-sum(W(k,:)); % Defining diagonal entries such that total sum is l
end
%% CALCULATING TRAJECTORIES OVER TIME %%
Coop1=zeros(1,nGen); Coop2=zeros(1,nGen); % Initializing the cooperation vectors
XStr1=zeros(ns,nGen); XStr2=zeros(ns,nGen); % List of the strategies over time
coop1v=zeros(ns^2,1); coop2v=zeros(ns^2,1);
ils=zeros(ns,ns); i2s=zeros(ns,ns); % Matrices that indicate which entries
    % of W correspond to player 1's strategy ils (row) and 2's strategy i2s (column)
for i=1:ns
    ils(i,:)=ns*(i-1)+1:ns*i;
    i2s(i,:)=(0:ns-1)*ns+i;
end
for i=1:ns % Transforming the possible cooperation rates into a vector
    coop1v(ns*(i-1)+1:ns*i,:)=CopMat1(i,:);
    coop2v(ns*(i-1) +1:ns*i,:)=CopMat2(i,:);
end
v=zeros(1,ns^2); v(1)=1; % Initially everyone plays ALLD
for t=1:nGen % Calculating the expected trajectory for each time step
    V}=\textrm{V}*W\mathrm{ ; % Iterate process for one time step
    Coop1(t)=v*coop1v; Coop2(t)=v*coop2v; % Expected cooperation rate at time t
    for i=1:ns
        XStr1(i,t)=sum(v(i1s(i,:))); XStr2(i,t)=sum(v(i2s(i,:)));
        % Expected frequency of each strategy at time t
    end
end
```

end

```
function [pi1,pi2,coop1,coop2]=CalcPay(p,q,POS1,POS2,epsi);
% Subroutine that calculates payoffs and cooperation rates for two players
% with memory-1 strategies p=(pCC,pCD,pDC,pDD) and q=(qCC,qCD,qDC,qDD)
p=p*(1-epsi)+(1-p)*epsi; q=q*(1-epsi) +(1-q)*epsi; % Adding noise
% Creating the transition matrix M
M=[p(1)*q(1), p(1)*(1-q(1)), (1-p(1))*q(1), (1-p(1))*(1-q(1));
    p(2)*q(3), p(2)*(1-q(3)), (1-p(2))*q(3), (1-p(2))*(1-q(3));
    p(3)*q(2), p(3)*(1-q(2)), (1-p(3))*q(2), (1-p(3))*(1-q(2));
    p(4)*q(4), p(4)*(1-q(4)), (1-p(4))*q(4), (1-p(4))*(1-q(4))];
v=null(M'-eye(4)); v=v/sum(v); % v .. normalized left eigenvector of M
pi1=POS1*v; pi2=POS2*v; coop1=v(1)+v(2); coop2=v(1)+v(3);
end
```


## Algorithm when strategies are represented by finite state automata

```
function [AvgC,AvgPay,Data]=EvolProcFSA(rvec,evec,nCL,s,nGen);
% Output:
% AvgC, AvgPay .. average cooperation and payoff over course of simulation
% Data .. string that stores the used parameter values
% Input:
% rvec=(r1,r2), evec=(e1,e2) .. productivity and endowment vectors
% nCL .. number of different contribution levels (e.g. 2, 3, or 4).
% s .. strength of selection, nGen .. number of updating events.
%% PARAMETERS AND PREPARATIONS
S1=ones(1,nCL+1); S2=S1; % Initially, both players use ALLD (choose first contribution
level, always go to state 1)
n1=1; n2=1; % current number of states of the two finite state automata (FSA)
[pi,coop]=CalcPayoff(S1,S2,rvec,evec,nCL);
AvgPay=pi; AvgC=coop;
Data=['e1=',num2str(evec(1)),'; r1=',num2str(rvec(1)),'; r2=',num2str(rvec(2)),...
    '; nCL=',num2str(nCL), '; s=',num2str(s), '; nGen=',num2str(nGen)];
%% EVOLUTIONARY PROCESS
for i=2:nGen
    % Randomly choosing a player
    iPop=randi(2);
```

```
    % Player 1 explores a new strategy
    if iPop==1
        Mut=getrandomstrategy(S1,nCL);
        [piM, coopM]=CalcPayoff(Mut,S2,rvec,evec,nCL);
    rho=1/(1+exp(-s*(piM(1)-pi(1))));
    if rand(1)<rho
        S1=Mut; pi=piM; coop=coopM;
    end
    % Player 2 explores a new strategy
    elseif iPop==2
        Mut=getrandomstrategy(S2,nCL);
        [piM,coopM]=CalcPayoff(S1,Mut,rvec,evec,nCL);
        rho=1/(1+exp(-s*(piM(2)-pi(2))));
        if rand(1)<rho
        S2=Mut; pi=piM; coop=coopM;
    end
    end
    % Updating the averages
    AvgPay=(i-1)/i*AvgPay+1/i*pi;
    AvgC=(i-1)/i*AvgC+1/i*coop;
end
end
function Mut=getrandomstrategy(S,nCL)
% Auxiliary program that creates a random mutant based on a given strategy S
% Strategies take the form of an ls x (nCL+1) matrix
% ls is the number of states of the FSA
% The first column defines which action to take in respective state
% The other nCL entries in each row specify the next state of the FSA,
% depending on which of the nCL possible actions the co-player has chosen
%% SETTING THE PARAMETERS
qAdd=0.3; % Probability to add a state
qRemove=0.35; % Probability to remove a state
qAction=0.175; % Probability to change an action
qArrow=0.175; % Probability to change an arrow
lS=size(S,1); % Number of states of the original FSA
Mut=S;
%% CREATING THE MUTANT
qRand=rand(1);
```

```
% Adding a state
if qRand<qAdd
    Mut(lS+1,:)=[randi(nCL),randi(lS+1,1,nCL)]; % Append new state to end of Mut
    Mut(randi(lS),1+randi(nCL))=lS+1; % Choose a random arrow to point to new state
end
```

```
% Deleting a state
```

% Deleting a state
if qRand>=qAdd \& qRand<qAdd+qRemove \& lS>1
if qRand>=qAdd \& qRand<qAdd+qRemove \& lS>1
iD=randi(lS); % Choosing a random state to be removed
iD=randi(lS); % Choosing a random state to be removed
Act=Mut(:,1); Trans=Mut(:,2:end); % Distinguishing between actions and transitions
Act=Mut(:,1); Trans=Mut(:,2:end); % Distinguishing between actions and transitions
[j1,j2]=find(Trans==iD); % Find all arrows that point to the state to be removed
[j1,j2]=find(Trans==iD); % Find all arrows that point to the state to be removed
for i=1:length(j1)
for i=1:length(j1)
Trans(j1(i),j2(i))=-1; % Overwriting all arrows that point to removed state
Trans(j1(i),j2(i))=-1; % Overwriting all arrows that point to removed state
end
end
for i=iD+1:lS
for i=iD+1:lS
[j1,j2]=find(Trans==i); % Find arrows that point to state with higher index
[j1,j2]=find(Trans==i); % Find arrows that point to state with higher index
for l=1:length(j1)
for l=1:length(j1)
Trans(j1(l),j2(l))=i-1; % Decrease index of those arrows by one
Trans(j1(l),j2(l))=i-1; % Decrease index of those arrows by one
end
end
end
end
[j1,j2]=find(Trans==-1);
[j1,j2]=find(Trans==-1);
for i=1:length(j1) % Assign new target states for arrows to removed state
for i=1:length(j1) % Assign new target states for arrows to removed state
Trans(j1(i),j2(i))=randi(lS-1);
Trans(j1(i),j2(i))=randi(lS-1);
end
end
Mut=[Act'; Trans']'; Mut(iD,:)=[]; % Removing the state with index iD
Mut=[Act'; Trans']'; Mut(iD,:)=[]; % Removing the state with index iD
end
end
% Changing an action
if qRand>=qAdd+qRemove \& qRand<qAdd+qRemove+qAction
rands=randi(lS); % Choosing a random state to change its action
as=S(rands,1);
while as==S(rands,1);
as=randi(nCL); % Assigning a new action to respective state
end
Mut (rands)=as;
end
% Changing an arrow
if qRand >= qAdd+qRemove+qAction \& lS>1
i=randi(lS); j=1+randi(nCL); % Choosing a random arrow to be changed
ar=Mut(i,j);
while ar==Mut(i,j)

```

Mut(i,j)=randi(lS); \% Assigning new target state to respective arrow end
end
end
function [pi,coop]=CalcPayoff(S1,S2,rvec, evec,nCL);
\% Auxiliary program that computes the payoffs and cooperation rates
\% when two players with FSA strategies S1 and S2 interact.
\% rvec=[r1,r2] and evec=[e1,e2] contain the players' productivities and endowments
\% nCL is the number of different contribution levels

\section*{응 PREPARATIONS}
ll=size (S1,1); l2=size(S2,1); \% number of states of the FSA of the two players
lContr=zeros(1,nCL); for \(i=1: n C L, ~ l C o n t r(i)=(i-1) /(n C L-1) ; ~ e n d\)
\% List of possible contributions (in percentage of the player's endowment
svec=zeros (l1*l2,2); cvec=svec; pivec=svec;
\% Lists that store the players' visited states, their contributions,
\% and their payoffs in the first \(11 * 12\) rounds
\% \% COMPUTING THE OUTPUT
stop=0; j=0; \% Initializing a stop variable and a counter cS1=1; cS2=1; \% Initial states of the two players
while stop==0
\[
j=j+1 ; \quad a 1=S 1(c S 1,1) ; ~ a 2=S 2(c S 2,1) ; ~ \% ~ P l a y e r ' s \text { actions in the respective round }
\]
svec(j,:)=[cS1,cS2]; \% Storing the player's current states
\(\operatorname{cvec}(j,:)=[l \operatorname{Contr}(a 1), l \operatorname{Contr}(\mathrm{a} 2)] ;\) o Storing their current contributions pivec (j, : ) = (rvec (1) *lContr (a1) *evec (1) +rvec (2) *lContr (a2) *evec (2)) / \(2+\ldots\)
\(+[(1-1 \operatorname{Contr}(\mathrm{a} 1)) * \operatorname{evec}(1),(1-1 \operatorname{Contr}(\mathrm{a} 2)) * \operatorname{evec}(2)]\);
\% Storing the players' current payoffs
\% Check whether current state combination has already occurred earlier: for \(k=1: j-1\)
if svec(k,:)==svec(j,:), stop=1; loopstart=k; end
end
\(\mathrm{cS} 1=\mathrm{S} 1(\mathrm{cS} 1,1+\mathrm{a} 2) ; \mathrm{CS} 2=\mathrm{S} 2(\mathrm{cS} 2,1+\mathrm{a} 1)\);
\% Compute the next states of the FSA of the two players
end
coop=mean(cvec(loopstart+1:j,:),1); pi=mean(pivec(loopstart+1:j,:),1);
\% Calculate average cooperation rates and payoffs by averaging over all
\% values in the loop that the two players will reach eventually
end

\subsection*{7.3 Game instructions of the behavioral experiment}

In the following, we provide screenshots of the 'aligned inequality treatment' from the perspective of player 1. The instructions for player 2 and for the other treatments are analogous. They are available from the authors upon request.

\section*{SUMMARY}
- You will be matched with another worker who is also taking this HIT at the same time.
- You will interact in real-time with this worker.
- There will be many interactive rounds with the same worker.
- The decisions you and the other worker make will affect your bonus payment.

Depending on the decisions you and the other worker make, you can earn a bonus between \(\$ 0.00\) and \(\$ 2.59\). However, you must stay on task at all times, otherwise you will automatically be disconnected and earn \(\$ 0.00\).

\section*{Detailed Instructions (1/3)}

In this HIT, you are matched with another worker who is also taking this HIT. You will interact with the same worker for the entire task. One of you will be Player 1 and the other will be Player 2. All interactions take place in real time.

\section*{Please be considerate of the worker's time!}

In this task, there are multiple rounds. At the beginning of every round, each player receives a certain number of units (income). The total number of units distributed in each round will be 100 units: Player 1 will receive 75 units in every round, while Player 2 will receive 25 units in every round.

There will be at least 20 rounds of the group task. After the 20 th round, there will be a \(50 \%\) chance that another round will follow. After that round, there will again be a \(50 \%\) chance that another round will follow, and so on until the group task ends. When the group task is over, you will see a brief questionnaire before you can submit the HIT.

Please answer the following questions:

What are the incomes of Players 1 and 2 in every round?

Players 1 and 2 each receive 50 units every round.
Player 1 receives 75 units per round and Player 2 receives 25 units per round.
Players 1 and 2 receive different units each round.

What is the chance that there will be another round after the 20th round?

0\%
50\%
100\%

What is the chance that there will be another round after the 21th round?

\section*{0\%}

50\%
100\%

\section*{Detailed Instructions (2/3)}

In every round, you have to decide how much of your income (in units) you want to contribute to the common group project and how much to keep for yourself.

All units that are contributed to the common group project are multiplied: every unit that Player 1 contributed is multiplied by a factor of 1.9 while every unit that Player 2 contributed is multiplied by a factor of 1.3.

Once multiplied, the total amount is split evenly among both players, regardless of whether they contributed. Thus for every unit players contribute, they personally get less than a unit back: so no matter what the other player contributes, you personally lose units on contributing, but contributing benefits the group as a whole.

The exchange rate is 800 units \(=\$ 1.00\).

Please answer the following questions:

By what value will Player 1's contribution be multiplied?
1.3
1.6
1.9

By what value will Player 2's contribution be multiplied?
1.3
1.6
1.9

What level of contribution earns the most money for the group as a whole?
None of your income.
Half of your income.
All of your income.

What level of contribution earns the most money for you personally?

None of your income.
Half of your income.
All of your income.

\section*{Detailed Instructions (3/3)}

These two examples illustrate how payoffs are calculated every round. Example A demonstrates that the highest payoff for the group is achieved if both contributed all their units to the common group project. Example B shows that a player will always earn more personally when contributing nothing.

\section*{EXAMPLE A:}


\section*{EXAMPLE B:}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow{6}{*}{Initial endowment} & \multicolumn{2}{|l|}{Individual players} & & & \multicolumn{2}{|l|}{Common group project} \\
\hline & Player 1 & Player 2 & & & Due to Player 1 & Due to Player 2 \\
\hline & 75 units & 25 units & & & & \\
\hline & -65 units & 0 units & \(\rightarrow\) & Contributions & 65 units & 0 units \\
\hline & & & & Multiplier & \(\times 1.9\) & \(\times 1.3\) \\
\hline & & & & Multiplied contributions & 123.5 units & 0 units \\
\hline Income from group project & 61.8 units & 61.8 units & \(\leftarrow\) & Total value (split evenly) & \multicolumn{2}{|c|}{\multirow[t]{2}{*}{123.5 units}} \\
\hline TOTAL EARNINGS & 71.8 units & 86.8 units & & & & \\
\hline
\end{tabular}

Please answer the following questions:

In Example A, what is Player 1's income from the common group project?

\section*{25.5 units}

70 units
87.5 units
\begin{tabular}{rll} 
& Player & Income \\
YOU \(\ggg\) & Multiplier \\
& Player 1 & 75 units \\
& 1.9 x \\
& Player 2 & 25 units
\end{tabular} 1.3 x.

Your income is 75 units.

Please decide now how many units you want to contribute to the common pool.

(Type a value between 0 and 75.)

Submit ...
\begin{tabular}{rllll} 
& Player & Income & Multiplier & Contribution
\end{tabular} Multiplied contribution

\section*{YOUR PAYOFF IN THIS ROUND:}
\begin{tabular}{|l|l|}
\hline Units you did not contribute: & \(\mathbf{2 5 . 0 0}\) units \\
\hline Units you earned from common pool: & \(\mathbf{6 0 . 5 0}\) units \\
\hline TOTAL THIS ROUND: & \(\mathbf{8 5 . 5 0}\) units \\
\hline
\end{tabular}

\section*{Continue ..}

\section*{Thank you for playing!}

You just finished playing the last round of the game.

In every game, there are at least 20 rounds. After that, another round is played with a \(50 \%\) probability. And after that, yet another round is plyed with a \(50 \%\) probability and so on. The random generator took this into account and determined a maximum length of 21 rounds for your group. You just finished with the 21th round and you can now claim your bonus.

Please finish the survey to receive your completion fee and your bonus. Your bonus is the number of units that you accumulated over all rounds.
```

Continue

```

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