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Look who's talking now: Implications of AV's explanations on driver's trust, AV preference, anxiety and mental workload

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Designing autonomous vehicle incentive program with uncertain vehicle purchase price



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ABSTRACT

This study proposes an autonomous vehicle incentive program design (AV-IPD) problem for a local government, which aims to promote the adoption of autonomous vehicles (AVs) by deploying AV lanes and subsidizing the purchase of AVs with the objective of system optimum subject to a fixed budget. As it is difficult to anticipate the changing AV market conditions in the future, we take into account the uncertainty in AV purchase price. For the AV-IPD problem, we firstly decide which regular lanes should be converted into exclusive AV lanes. Secondly, we determine the AV purchase subsidy for each realization of AV purchase price with a given AV lane deployment scheme. A binary logit model is applied to characterize the vehicle choice behavior of users. The AV-IPD problem is formulated as a two-stage stochastic programming model with equilibrium constraints. To solve the AV-IPD problem, we develop a solution method based on linear approximation techniques and duality theories. Numerical experiments are conducted to demonstrate the effectiveness of the proposed model and solution method.

1. Introduction

The last few years have witnessed an explosive development on autonomous vehicle (AV) technology. Compared to the conventional vehicles (CVs), AVs can achieve a wide range of benefits in terms of traffic capacity, safety and vehicular emission. A successful implementation of AVs depends on the deployment of transport infrastructure catered for AVs and sufficient AV adoption. To promote the market penetration of AVs, a local government could design an AV incentive program to deploy exclusive AV lanes and subsidize users to purchase AVs subject to a fixed budget.

The purpose of setting AV lanes is to separate AVs and CVs and exploit AV's advantage in improving road capacity. Current studies demonstrated that the road capacity could become up to triple in full AV environment by a shorter headway (Tientrakool et al., 2011). However, AVs become less beneficial under the mixed traffic with AVs and CVs. With a low AV ratio, the road capacity is possible to decrease due to speed variation and shock waves (Van Arem et al., 2006). To eliminate the issues arising from the mixed traffic, a local government can divide roads into regular lanes and AV lanes (Talebpour et al., 2017) so that AVs can travel with very short headway on AV lanes. The AV users have right of way on both regular and exclusive lanes while CV users can only travel on regular lanes. Different from conventional road construction projects, deploying AV lanes does not increase the number of lanes but converts some regular lanes into AV exclusive lanes. Some sensors are installed to facilitate the vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communications (Jia et al., 2016). Some roads may require a novel traffic signal system (Li and Zhou, 2017). Besides, the users' vehicle choice between AVs and CVs is another feature needed to consider for the deployment of AV lanes. This is

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because AV lanes save the travel time for AV users and thus stimulate the AV demand. To accurately predict the AV demand in planning horizon, we need to consider the vehicle choice behavior of users.

In addition to the support of road infrastructures, the purchase subsidy is recognized as another effective way to promote the ownership of emerging vehicles. For example, the governments of U.S. and China both implement the purchase subsidy for electric vehicles (Helveston et al., 2015). Now there is no purchase subsidy policy implemented for AVs since the full self-driving vehicles have not yet appeared on market. Currently, TESLA requires an extra \$8000 to equip its vehicles with level-4 auto driving capability (TESLA, 2019). This purchase cost already exceeds the average willingness-to-pay (\$7253) revealed in the study by Bansal et al. (2016). Although the subsidy is effective to remove the barrier of high purchase cost, the incurred financial burden on the local government could be very heavy. Therefore, more attention needs to be paid to the government's subsidy policies. In reality, we can determine the AV purchase subsidy and AV lane deployment in a holistic manner, which not only promotes the adoption rate of AVs, but also achieves a reasonable allocation of government resources.

At the design stage of the AV incentive program, the local government needs to predict the AV purchase price in the future. However, the forecast of AV purchase price could be difficult because the rapid development of AV technology makes the AV manufacturing cost unpredictable. Therefore, the AV purchase price is regarded as a discrete random variable in this study. To deal with the uncertainty, the AV incentive program is divided into two stages: the government first deploys AV lanes before the realization of the random AV price and then implements the purchase subsidy after the random AV price is realized. This two-stage flexible AV incentive program is motivated by different temporal natures of the lane and subsidy: the layout of AV lanes is not easily altered for a fixed period. The purchase subsidy, on the other hand, can adapt to the AV purchase price easily. For example, if the AV traffic flow volume on AV lanes is below a target, the government can encourage the adoption of AVs by means of the AV purchase subsidy.

In this study, we focus on the development of model and algorithm for the autonomous vehicle incentive program design (AV-IPD) problem, which aims to promote the adoption of AVs by deploying AV lanes and subsidizing the purchase of AVs with the objective of system optimum subject to a fixed budget. For the AV-IPD problem, we firstly decide which regular lanes should be converted into exclusive AV lanes. Secondly, we determine the AV purchase subsidy for each realization of the stochastic AV purchase price with a given AV lane deployment solution. When users choose AVs or CVs, we assume that they follow the binary logit model in which the utility includes the part of purchase cost, operational cost, travel time of commute trips and travel time savings by automated driving.

1.1. Literature review

There is an emerging trend of AVs related studies. Milakis et al. (2017) made a comprehensive review of the potential impacts of self-driving technology on society and policy making. We first review the traffic assignment studies with CVs and AVs. Levin and Boyles (2015) proposed a multi-class traffic assignment model with CVs and AVs by considering the impact of the percentage of AVs on road capacity. Noruzoliaee et al. (2018) considered both vehicle type and route choice behavior in determining the optimal AV purchase price from the perspective of an AV manufacturer. Bagloee et al. (2017) built a mixed equilibrium model in which CVs follow the user equilibrium (UE) principle and AVs follow the system optimal (SO) principle. The built model is capable of addressing capacity constraint, elastic demand as well as multiple user classes. Levin and Boyles (2016a) incorporated AVs into a dynamic traffic assignment (DTA) model to assess the impacts of AVs on capacity improvement. Also, Levin (2017) assumed that AVs can be shared to provide a dial-a-ride service to travelers. A SO-DTA model is formulated to seek the optimal routing solution. Melson et al. (2018) incorporated cooperative adaptive cruise control (CACC) into the framework of link transmission model (LTM) for dynamic network loading. It was found that a naïve CACC exclusive lane deployment could increase the total travel time over the network. Hence, a well-designed planning and operation strategy for AV based transport infrastructure is of crucial importance.

Recently, a few studies have been concerned with traffic management strategies by taking the advantages of AVs. Chen et al. (2016) proposed an AV lane location (AVLL) problem as an extension of the conventional time-dependent network design problem (Szeto and Lo, 2006). Talebpour et al. (2017) simulated the traffic flow dynamics under mandatory/optional AV lane use and limited/non-limited autonomous driving mode. Levin and Boyles (2016b) put forward a dynamic lane reversal scheme to address the flow imbalance in opposite directions. Chen et al. (2017) proposed a concept of the restricted area called AV zone. AVs entering the zone would be fully controlled to minimize traffic congestion. In addition to the lane management, the government also needs to determine the number of AVs under control. Zhang and Nie (2018) proposed a mechanism to obtain the optimal ratio of AVs under control for each origin-destination pair. Liu (2018) studied the optimal parking pricing scheme considering AV users' choice on departure time and parking location. Iacobucci et al. (2019) optimized the charging and relocation schedule for autonomous electric vehicles through model predictive control. However, these studies do not consider the impacts of uncertainty in AV traffic demand on the management strategies. Consequently, the fluctuation of future AV price and demand could cause some unexpected influence on the performance of transport infrastructures.

Another body of relevant studies is about the users' adoption behavior with respect to AVs. Gkartzonikas and Gkritza (2019) made a comprehensive review of the studies on the stated preference on AVs. Some researchers investigated on the empirical evidence of customers' willingness-to-pay (WTP) on AVs. For example, Bansal et al. (2016) collected the opinions of the residents of Austin towards different levels of automated driving. Daziano et al. (2017) made a similar survey on WTP but involved CVs as an alternative option. These two studies revealed a WTP much less than the current purchase price provided by the manufacturers, which means that the mass adoption still needs some time. Talebian and Mishra (2018) predicted the AV adoption through an agent-based approach where WTP could be changed by peer-to-peer communication. Several researchers used stated preference (SP) survey to

examine the vehicle choice of users between CVs and AVs. Yap et al. (2016) applied the mixed logit model to explore users' preference on AVs for last-mile trips. Haboucha et al. (2017) analyzed the market penetration rate of shared AV (SAV) under various costs and customers' attitudinal factors. Shabanpour et al. (2018) modeled the adoption behavior of AVs by combining SP survey and best-worst analysis. The results indicate that people are sensitive to the purchase price and incentive policies such as exclusive AV lanes. The insights from these empirical studies motivate us to characterize the vehicle choice behavior of users by considering both the travel cost and purchase cost.

We note that two recent works by Chen et al. (2016) and Noruzoliaee et al. (2018) are closely related to our study. Here we point out some key difference between our study with these two works. First, we put forward a novel AV incentive program design problem, which included both AV lane location and purchase subsidy optimization under the uncertainty. Chen et al. (2016) studied AV lane location problem by building a multi-period network design model with deterministic demand. The purchase subsidy optimization in our study differs from AV pricing problem (Noruzoliaee et al., 2018) in minimizing total travel time rather than seeking the maximum profits of the manufacturers. We also consider a government's budget constraint to avoid excessive expenses of AV incentive program. Finally, we express the traffic equilibrium by a convex programming model, rather than the complementarity slackness constraints as in Chen et al. (2016) and Noruzoliaee et al. (2018). Therefore, the traffic assignment model in our study can be reformulated as a mixed integer linear programming (MILP) without introducing binary variables, which reduces the burden of computation.

1.2. Objectives and contribution

The objective of this study is to solve the proposed AV-IPD problem by finding the optimal AV lane deployment and purchase subsidy solution with uncertain AV purchase price. To achieve this goal, we develop a two-stage stochastic programming model with equilibrium constraints. In the first stage, we determine the optimal locations and number of AV lanes. In the second stage, we optimize the purchase subsidy in response to each realization of AV price. The users' choice on vehicle type and route is formulated as a combined traffic assignment model. We propose a solution method to solve the AV-IPD problem by transforming the two-stage stochastic programming model with equilibrium constraints into a MILP model.

The contributions of our study are threefold. First, the proposed AV-IPD problem is a novel research issue, since it takes into account the AV lane deployment and purchase subsidy optimization with the uncertainty of AV purchase price. Second, a two-stage stochastic programming with equilibrium constraints is developed for the AV-IPD problem. The combined choice of users on vehicle types and routes is formulated as a convex programming model. Third, a solution method based on linear approximation techniques and duality theories is designed to solve the AV-IPD problem. Numerical experiments are conducted on different sizes of networks.

The remainder of this study is organized as follows. Section 2 gives the notations, assumptions and elaborates the AV-IPD problem. A two-stage stochastic programming model with equilibrium constraints is formulated in Section 3. A solution method based on linear approximation and duality theories are presented to solve the AV-IPD problem. Numerical experiments are given in Section 4 to illustrate the effectiveness of the proposed model and solution method. Section 5 draws the conclusion and points out the potential future work. Appendices A, B and C give technical details, detailed description of notations and network data in this study.

2. Notations, assumptions and problem statement

Consider a local government that plans to design an AV incentive program that converts some regular road lanes into exclusive AV lanes and subsidizes users to purchase AVs. At the design stage for the AV incentive program, we predetermine a set of candidate roads (links) with some lanes that can be converted into exclusive AV lanes. The AV purchase price is assumed to be a discrete random variable with limited outcomes. This assumption can reflect the uncertainty of the future AV market. Note that if the AV purchase price is a continuous variable, we can still use a discrete random variable to approximate it. The government first deploys exclusive AV lanes over the transportation network via converting some regular road lanes into AV lanes. The government then determines AV purchase subsidy for each realization of the random AV purchase price to promote the adoption of AVs. It is assumed that users maximize their individual utility (or minimize their generalized travel time) by following the logit model to choose either AVs or CVs. The route choice of users is governed by user equilibrium (UE) conditions. The primary goal of designing the AV incentive program is to minimize the expected total travel time by deploying AV lanes and determining AV purchase subsidy. A government's budget is considered in order to achieve a reasonable resource allocation between the AV lane deployment and purchase subsidy under all realizations of AV purchase price.

2.1. AV-CV network representation

Let $\mathcal{G}(\mathcal{N}, \mathcal{A})$ denote the transportation network managed by the local government, where \mathcal{N} is the set of nodes and \mathcal{A} is the set of directed links. Let $\mathcal{W} \subseteq \mathcal{N} \times \mathcal{N}$ be the set of origin–destination (OD) pairs. Define \bar{q}_w as the travel demand between OD pair $w \in \mathcal{W}$. Let $\bar{\mathcal{A}}_0 \subseteq \mathcal{A}$ be the set of given candidate links with some lanes that may be converted into exclusive AV lanes. Hence, $\mathcal{A}_0 = \mathcal{A} \setminus \bar{\mathcal{A}}_0$ is the set of links without candidate AV lanes. To represent the exclusive AV lanes converted from the regular lanes of a candidate link $b \in \bar{\mathcal{A}}_0$, we divide link b into one AV link denoted by b' and one regular link denoted by b'' , and (b', b'') is called an AV-regular link pair as both links connect the same pair of nodes. Let $\hat{\mathcal{A}}$ be the set of all these candidate AV links and $\bar{\mathcal{A}}$ be the set of regular links, namely, $\hat{\mathcal{A}} = \{b' \mid b \in \bar{\mathcal{A}}_0\}$ and $\bar{\mathcal{A}} = \{b'' \mid b \in \bar{\mathcal{A}}_0\} \cup \mathcal{A}_0$. Thus, we create an AV-CV network denoted by $\bar{\mathcal{G}}(\mathcal{N}, \hat{\mathcal{A}} \cup \bar{\mathcal{A}})$.

To facilitate real-time communication between AVs, V2I and V2V sensors should be installed on AV lanes. Hence, AV links could

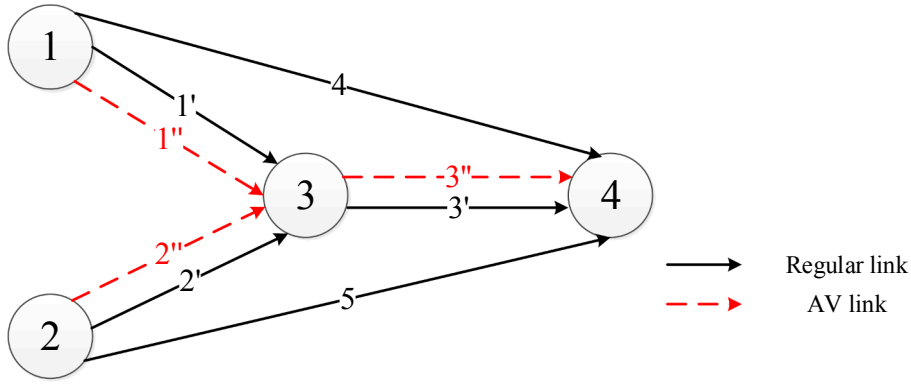


Fig. 1. Structure of illustrative network.

have very high capacity. Let CR be the ratio between AV lane capacity and regular lane capacity. Tientrakool et al. (2011) shows that CR could be nearly 3 under 100% cooperative driving. We assume that CVs do not have right of way on exclusive AV links. However, CVs and AVs are both allowed to travel on regular links. It is reasonable to assume that the capacity of a regular link will not change with respect to the variation of AV proportion, because AVs are not allowed to travel in very short headway on regular links to avoid the safety issues in mixed traffic.

For the sake of presentation, we index the link pairs $(b', b''), \forall b \in \bar{\mathcal{A}}_0$ from 1 to K . Let \mathcal{A}_k denote the set of one AV link and one regular link in the link pair $k \in \mathcal{K}$ where set $\mathcal{K} = \{1, 2, \dots, K\}$. We assume that the total number of lanes for link pair k is fixed, denoted by Y_k . It can be seen that the deployed AV lanes will occupy the space of the paired regular lanes, which is similar with the deployment of exclusive bus lane.

We now introduce the feasible paths for CVs and AVs over the AV-CV transportation network $\bar{\mathcal{G}}(\mathcal{N}, \widehat{\mathcal{A}} \cup \bar{\mathcal{A}})$. For OD pair w , let \mathcal{P}_w be the set of all paths and $\bar{\mathcal{P}}_w \subset \mathcal{P}_w$ be the set of paths only comprised by regular links. In other words, \mathcal{P}_w and $\bar{\mathcal{P}}_w$ are the sets of feasible paths for AVs and CVs, respectively.

We use a simple example to illustrate how to create an AV-CV transportation network. There are three links with candidate AV lanes, namely, links 1, 2 and 3 as shown in Fig. 1. Each of these three links is divided into one regular link and AV link, which results in three link pairs: $\mathcal{A}_1 = \{1', 1''\}$, $\mathcal{A}_2 = \{2', 2''\}$, and $\mathcal{A}_3 = \{3', 3''\}$. We also have an AV link set $\widehat{\mathcal{A}} = \{1'', 2'', 3''\}$ and regular link set $\bar{\mathcal{A}} = \{1', 2', 3', 4, 5\}$. For OD pair 1–4, the feasible paths for CV users include link 4 and link1'→link3', while AV users have three more paths to choose, namely, link1''→link3'', link1'→link3'' and link1''→link3'.

2.2. Random AV purchase price, purchase subsidy and vehicle choice behavior

To capture the uncertainty in future AV market, the AV purchase price is assumed to be a discrete random variable with limited outcomes. In other words, all possible values of AV prices and corresponding occurrence probabilities are given as the results of price prediction. We assume that random AV price has S realizations. Denote by $\tau_{AV}^s, \forall s \in S$ the realized price of AVs, where $S = \{1, 2, \dots, S\}$ is the index set of AV price realization. Let ϕ_s be the occurrence probability of τ_{AV}^s , and denote by τ_{CV}^s the purchase price of CVs. Note that CV price τ_{CV}^s is assumed to be a deterministic number under all AV price realizations since CV market is more mature and stable compared with AV market. Let \mathcal{R} be the set of candidate purchase subsidies for AVs. Given a subsidy $r \in \mathcal{R}$ and realization of AV purchase price τ_{AV}^s , the purchase cost of AVs can be expressed by $\hat{\tau}_{AV}^s = \tau_{AV}^s - r$, while the purchase cost for CVs is $\hat{\tau}_{CV}^s = \tau_{CV}^s$.

Define $\mathcal{J} = \{CV, AV\}$ as the set of vehicle types. We assume that users choose either AVs or CVs by following a binary logit model. For each realization of AV purchase price τ_{AV}^s , we denote $V_{w,j}^s$ as the utility of users between OD pair w for vehicle type j , including vehicle purchase cost $\hat{\tau}_j^s$, vehicle operational cost $OC_{w,j}$, UE travel time $TT_{w,j}^s$ and travel time savings by automated driving $TS_{w,j}$:

$$V_{w,j}^s = -\frac{b_3 \cdot \hat{\tau}_j^s}{L \cdot b_1 \cdot b_2} - \frac{OC_{w,j}}{b_2} - TT_{w,j}^s + TS_{w,j}, \quad \forall w \in \mathcal{W}, j \in \mathcal{J}, \tag{1}$$

where L is the lifespan of vehicle; b_1 is the number of commute trips in one year; b_2 is the value of time (VOT) for users. b_3 is the average percentage of commute trips among the yearly travel trips of users. The parameters b_1, b_2 and b_3 can be estimated through a household travel survey or historical data. The first term of Eq. (1) is the disutility measured by the generalized travel time of commute trips, which is converted from vehicle purchase cost. The second term is converted from trip-based vehicle operational cost (fuel and maintenance cost) $OC_{w,j}$, which is approximated by the unit operational cost (\$/mile) multiplying the distance of the shortest path between OD w . The third term is the UE travel time of vehicle type j between OD pair w after AV lanes are deployed, which will be discussed in Section 2.3. The fourth term is the approximated travel time savings by automated driving since users do not drive AVs directly during trips. We have $TS_{w,CV} = 0$ and $TS_{w,AV} = \varphi \cdot STT_w$, where STT_w is the UE travel time without AV lanes between OD pair w , and $\varphi \in [0, 1]$ indicates the proportion of travel time that drivers can save by dealing with their own things on AVs.

In the utility function defined by Eq. (1), it is reasonable to assume that a part of vehicle purchase cost is converted into the trip-based generalized travel time for commuting, while the other travel purposes may account for the remaining vehicle purchase cost. In reality, the utility function determines the modal split between AVs and CVs for commute trips, and it is comprised of trip based vehicle purchase cost, approximated operational cost and travel time between a given OD pair. Note that we consider the generalized travel time based on commuting because we assume that commute trip is the main purpose of purchasing private cars for road users. In addition, the commute trip has relatively fixed origin and destination and we can quantitatively measure the impact of AV lanes on vehicle choice behavior. It is common practice to consider the cost of commute trips to investigate vehicle ownership. For example, [Dissanayake and Morikawa \(2010\)](#) considered the distance of commuter trip for investigating the car ownership in Bangkok Metropolitan Region. [Haboucha et al. \(2017\)](#) used commute trip cost and purchase cost to investigate user preference on AVs. Those studies show that daily commute trips play a crucial role in users' vehicle choice behavior.

Based on the assumption that users follow the logit model to choose either AVs or CVs, the travel demand of vehicle type j for OD w can thus be calculated by

$$q_{w,j}^s = \bar{q}_w \frac{\exp\{\theta V_{w,j}^s\}}{\sum_{j \in \mathcal{J}} \exp\{\theta V_{w,j}^s\}}, \quad \forall w \in \mathcal{W}, j \in \mathcal{J}, \tag{2}$$

where θ is a positive scale parameter that can be estimated in stated preference (SP) survey in practice.

2.3. User equilibrium travel time pattern $TT_{w,j}^s$

In this section, we investigate the UE travel time involved in Eq. (1). For the realization of AV purchase price τ_{AV}^s , let $f_{w,j}^{p,s}$ denote the traffic flow on path $p \in \mathcal{P}_w$ connecting OD w . The corresponding path flow conservation equations can be expressed as follows:

$$\sum_{p \in \mathcal{P}_w} f_{w,j}^{p,s} = q_{w,j}^s, \quad \forall w \in \mathcal{W}, j \in \mathcal{J}, \tag{3}$$

$$f_{w,j}^{p,s} = 0, \quad \forall p \in \mathcal{P}_w \setminus \bar{\mathcal{P}}_w, w \in \mathcal{W}, j = CV, \tag{4}$$

$$f_{w,j}^{p,s} \geq 0, \quad \forall p \in \mathcal{P}_w, w \in \mathcal{W}, j \in \mathcal{J}, \tag{5}$$

$$\sum_{j \in \mathcal{J}} q_{w,j}^s = \bar{q}_w, \quad \forall w \in \mathcal{W}, \tag{6}$$

$$q_{w,j}^s \geq 0, \quad \forall w \in \mathcal{W}, j \in \mathcal{J}, \tag{7}$$

The traffic flow on link $a \in \widehat{\mathcal{A}} \cup \bar{\mathcal{A}}$ under the realization of AV purchase price τ_{AV}^s , denoted by v_a^s , can be calculated by

$$v_a^s = \sum_{w \in \mathcal{W}} \sum_{p \in \mathcal{P}_w} \sum_{j \in \mathcal{J}} \delta_{a,p}^w f_{w,j}^{p,s}, \quad \forall a \in \widehat{\mathcal{A}} \cup \bar{\mathcal{A}}. \tag{8}$$

Without loss of generality, we use the BPR travel time function for each link in the AV-CV transportation network, namely:

$$t_a = t_a^0 \left(1 + \bar{\alpha} \left(\frac{v_a^s}{\Lambda_a} \right)^{\bar{\beta}} \right), \quad \forall a \in \widehat{\mathcal{A}} \cup \bar{\mathcal{A}}, \tag{9}$$

where t_a is the travel time on link a ; $\bar{\alpha}$ and $\bar{\beta}$ are two coefficients in Bureau of Public Roads (BPR) function. t_a^0 is the free-flow travel time on link a ; Λ_a is the capacity of link a . When no AV lanes are deployed on a candidate AV link, i.e. $\Lambda_a = 0, \forall a \in \widehat{\mathcal{A}}$, we define $v/0 = \infty, v \in \mathbb{R}^+$ so that such AV link is inaccessible.

Let $c_{w,j}^{p,s} = \sum_{a \in \widehat{\mathcal{A}} \cup \bar{\mathcal{A}}} t_a(v_a^s) \delta_{a,p}^w$ denote the travel time of path p between OD w for vehicle type j . At the UE state, neither CV nor AV users can reduce their travel time by unilaterally changing paths. The UE conditions for $TT_{w,j}^s$ can be described by

$$\text{If } f_{w,AV}^{p,s} = 0 \text{ then } c_{w,AV}^{p,s} - TT_{w,AV}^s \geq 0, \quad \forall p \in \mathcal{P}_w, w \in \mathcal{W}, \tag{10}$$

$$\text{If } f_{w,AV}^{p,s} > 0 \text{ then } c_{w,AV}^{p,s} - TT_{w,AV}^s = 0, \quad \forall p \in \mathcal{P}_w, w \in \mathcal{W}, \tag{11}$$

$$\text{If } f_{w,CV}^{p,s} = 0 \text{ then } c_{w,CV}^{p,s} - TT_{w,CV}^s \geq 0, \quad \forall p \in \bar{\mathcal{P}}_w, w \in \mathcal{W}, \tag{12}$$

$$\text{If } f_{w,CV}^{p,s} > 0 \text{ then } c_{w,CV}^{p,s} - TT_{w,CV}^s = 0, \quad \forall p \in \bar{\mathcal{P}}_w, w \in \mathcal{W}. \tag{13}$$

3. Two-stage stochastic programming model with equilibrium constraints

In this section, we build a two-stage stochastic programming model with equilibrium constraints for the proposed AV-IPD problem. The equilibrium constraints characterize users' vehicle choice and route choice, which can be described by a combined traffic assignment model. The proposed two-stage stochastic programming model can well capture the decision process of the government: the AV lane deployment is the first-stage decision before the realization of AV purchase price and difficult to be altered in practice.

The obtained AV lane deployment should be robust to the variation of AV price. The second-stage decision is implemented after the random AV purchase price is realized. Thus, the AV purchase subsidy can properly respond to each realization of uncertain AV price. For instance, once it is found that the AV purchase price is high and the AV adoption rate can be improved to reduce system cost, the government can implement AV purchase subsidy. The details of the model formulation are presented as below.

3.1. Two-stage stochastic programming model with equilibrium constraints

Let $I_k = \{0, 1, \dots, \bar{y}_k^{\max}\}$ be the set of feasible numbers of AV lanes to be deployed on link pair k , where \bar{y}_k^{\max} is the maximum number of AV lanes. Denote the first-stage binary decision variable $y_k^i = 1$ if i of AV lanes are deployed on link pair k , and 0 otherwise. Let \widehat{C}_a and \bar{C}_a be, respectively, the capacity of one AV lane and regular lane. The cost of AV lane, i.e. instalment of roadside sensors, is denoted as u_k^i if i of AV lanes are deployed on link pair k . For each AV purchase price realization τ_{AV}^s , the second-stage binary decision variable $z_r^s = 1$ if the purchase subsidy $r \in \mathcal{R}$ is implemented, and 0 otherwise. With the given decision variables \mathbf{y} , \mathbf{z}^s and AV price realization τ_{AV}^s , let $\Omega_{UE}^s(\mathbf{y}, \mathbf{z}^s, \tau_{AV}^s)$ be the set of demand \mathbf{q}^s and flow \mathbf{v}^s that satisfy the logit model and UE conditions respectively. Denote by BD the government’s budget on AV incentive program.

Before the realization of predicted AV purchase price, the government deploys AV lanes over the AV-CV network. The associated first-stage problem is formulated by

$$\min_{\mathbf{y}} \mathbb{E}[Q(\mathbf{y}, \tau_{AV}^s)] \tag{14}$$

$$\text{subject to } \Lambda_a = \widehat{C}_a \sum_{i \in I_k} i \cdot y_k^i, \quad \forall a \in \mathcal{A}_k \cap \widehat{\mathcal{A}}, k \in \mathcal{K}, \tag{15}$$

$$\Lambda_a = \bar{C}_a \left(Y_k - \sum_{i \in I_k} i \cdot y_k^i \right), \quad \forall a \in \mathcal{A}_k \cap \bar{\mathcal{A}}, k \in \mathcal{K}, \tag{16}$$

$$\sum_{i \in I_k} y_k^i = 1, \quad \forall k \in \mathcal{K}, \tag{17}$$

$$y_k^i \in \{0, 1\}, \quad \forall i \in I_k, k \in \mathcal{K}, \tag{18}$$

where objective function (14) minimizes the expected total travel time; $Q(\mathbf{y}, \tau_{AV}^s)$ is the optimal objective value of the second-stage model; Constraints (15) and (16) ensure that the total number of lanes in one link pair is unchanged; Constraints (17) enforce that only one AV lane deployment scheme can be deployed on each link pair; Constraints (18) define the binary decision variables for AV lane deployment.

For each AV price realization τ_{AV}^s , the government will choose one candidate AV purchase subsidy to further adjust AV adoption rate for the system optimum. Hence, the second-stage problem is formulated as follows:

$$Q(\mathbf{y}, \tau_{AV}^s) = \min_{\mathbf{v}^s, \mathbf{q}^s, \mathbf{z}^s} \sum_{a \in \widehat{\mathcal{A}} \cup \bar{\mathcal{A}}} t_a(v_a^s) v_a^s \tag{19}$$

$$\text{subject to } \sum_{k \in \mathcal{K}} \sum_{i \in I_k} y_k^i u_k^i + \sum_{w \in \mathcal{W}} \sum_{r \in \mathcal{R}} r \cdot z_r^s q_{w,AV}^s \leq BD, \tag{20}$$

$$(\mathbf{v}^s, \mathbf{q}^s) \in \Omega_{UE}^s(\mathbf{y}, \mathbf{z}^s, \tau_{AV}^s), \tag{21}$$

$$\sum_{r \in \mathcal{R}} z_r^s = 1, \tag{22}$$

$$z_r^s \in \{0, 1\}, \quad \forall r \in \mathcal{R}, \tag{23}$$

where constraints (20) guarantee that the expenses of incentive program cannot exceed the given budget, which connect the decision variables in the first-stage problem and second-stage problem; Constraints (21) impose the combined UE constraints, which are examined in the next sub-section; Constraints (22) enforce that only one AV purchase subsidy solution can be implemented; Constraints (23) require the binary second-stage decision variables.

As discussed in Section 2.2, we can estimate the expected total system travel time as: $\mathbb{E}[Q(\mathbf{y}, \tau_{AV}^s)] = \sum_{s \in \mathcal{S}} \phi_s Q(\mathbf{y}, \tau_{AV}^s)$. For the sake of presentation, we define a set $\Pi^s(\mathbf{y}, \tau_{AV}^s) = \{\mathbf{v}^s, \mathbf{q}^s, \mathbf{z}^s \mid \text{constraints (20)–(23)}\}$ for the second-stage problem. Now we formulate the AV-IPD problem as an integrated stochastic programming model with equilibrium constraints [SP-AV-IPD-EC] as:

$$[\text{SP} - \text{AV} - \text{IPD} - \text{EC}] \min_{\mathbf{y}, \mathbf{z}, \mathbf{v}, \mathbf{q}} \sum_{s \in \mathcal{S}} \sum_{a \in \widehat{\mathcal{A}} \cup \bar{\mathcal{A}}} \phi_s t_a(v_a^s) v_a^s \tag{24}$$

subject to Constraints (15)–(18),

$$(\mathbf{v}^s, \mathbf{q}^s, \mathbf{z}^s) \in \Pi^s(\mathbf{y}, \tau_{AV}^s), \quad \forall s \in \mathcal{S}, \tag{25}$$

We can know that model [SP-AV-IPD-EC] is equivalent to the two-stage stochastic programming model (14)–(23). By solving

model [SP-AV-IPD-EC], we can obtain the optimal AV lane deployment scheme and AV purchase subsidy for each realization τ_{AV}^s .

3.2. Convex programming formulation for combined traffic assignment

In this section, we use a combined traffic assignment model to express equilibrium constraints (21). The idea of combined traffic assignment is widely used in the multi-modal network design problems (Wang et al., 2015; Yu et al., 2015). In this study, with the realization of AV purchase price τ_{AV}^s and given solutions \mathbf{y} and \mathbf{z}^s , the logit-based vehicle choice model with travel time at UE, expressed by Eqs. (2) and (10)–(13), can be formulated as a combined traffic assignment model:

$$[\text{CTA-s}] \min_{(\mathbf{v}^s, \mathbf{q}^s) \in \bar{\Omega}_s} \sum_{a \in \bar{\mathcal{A}} \cup \bar{\mathcal{A}}} \int_0^{v_a^s} t_a(u) du + \sum_{w \in \mathcal{W}} \int_0^{q_{w,AV}^s} \left(\frac{1}{\theta} \ln \frac{u}{\bar{q}_w - u} + \psi_w^s - \sum_{r \in \mathcal{R}} \hat{r} \cdot z_r^s \right) du \quad (26)$$

where $\psi_w^s = \Delta\tau^s / (\Gamma \cdot b_1 \cdot b_2) + \Delta OC_w / b_2 - TS_{w,AV}$ is the difference of generalized travel time converted from the purchase price difference $\Delta\tau^s = \tau_{AV}^s - \tau_{CV}^s$, the operational cost difference $\Delta OC_w = OC_{w,AV} - OC_{w,CV}$ and the travel time saving by AVs $TS_{w,AV}$. $\hat{r} = r / (\Gamma \cdot b_1 \cdot b_2)$ is the trip-based generalized travel time based on purchase subsidy r . $\bar{\Omega}_s$ is the feasible set for link flow \mathbf{v}^s and demand \mathbf{q}^s , namely, $\bar{\Omega}_s = \{(\mathbf{v}^s, \mathbf{q}^s) | \text{constraints (3)–(8)}\}$.

In objective function (26), the difference of travel time, purchase cost and operational cost between AVs and CVs determines the AV modal split. With a fixed CV purchase price, we only need to consider $\Delta\tau^s$ for each realization of AV purchase price. Denote $\Gamma = \{\Delta\tau^s > 0 | \forall s \in S\}$ as the set of the difference of purchase price between AVs and CVs. Note that the purchase price of AVs is always greater than CVs due to a high manufacturing cost. It is straightforward to verify that the KKT conditions of model [CTA-s] are equivalent to the modal split shown in Eq. (2) and UE conditions (10)–(13). We define $\Omega_{UE}^s(\mathbf{y}, \mathbf{z}^s, \tau_{AV}^s)$ as the optimal solution of the model [CTA-s], namely:

$$\Omega_{UE}^s(\mathbf{y}, \mathbf{z}^s, \tau_{AV}^s) = \arg \min_{(\mathbf{v}^s, \mathbf{q}^s) \in \bar{\Omega}_s} \sum_{a \in \bar{\mathcal{A}} \cup \bar{\mathcal{A}}} \int_0^{v_a^s} t_a(u) du + \sum_{w \in \mathcal{W}} \int_0^{q_{w,AV}^s} \left(\frac{1}{\theta} \ln \frac{u}{\bar{q}_w - u} + \psi_w^s - \sum_{r \in \mathcal{R}} \hat{r} \cdot z_r^s \right) du$$

By examining the Hessian matrix of objective of model [CTA-s], it can be seen that the model [CTA-s] is a strictly convex with respect to the aggregated link flow v_a^s and the AV demand flow $q_{w,AV}^s$. In other words, v_a^s , t_a^s and $q_{w,AV}^s$ are uniquely determined by the model [CTA-s]. Note that link flows by specific vehicle types are not unique, but the total travel time over network is still unique since all used paths for an OD pair have the same travel time at equilibrium. This indicates that the proposed model [CTA-s] do not have the solution non-uniqueness issue of the multi-class user equilibrium problems.

There are some similarities between model [CTA-s] and the combined mode/traffic assignment model (Sheffi, 1985). The combined model integrates different dimensions of travel choice into a unified framework, and gives a steady system state characterized by the logit model and UE conditions. In reality, it takes several years for AV market penetration rate to reach the equilibrium point. Users can estimate AV and CV travel time at current AV market penetration level by means of V2I sensors and re-adjust their route choice every day. Increasing CV users would purchase AVs thanks to the benefits of AVs, while AV travel time would be gradually longer due to congestion in AV lanes. The rising trend of AV market penetration would continue until the AV modal split reaches the level governed by the logit model. For more information about the evolution of AV market share, interested readers can refer to Chen et al. (2016).

4. Mixed integer linear programming based solution method

The AV-IPD problem is difficult to solve because (i) the equilibrium constraints make the domain of feasible solutions nonconvex; (ii) the objective functions in both stochastic programming model and combined traffic assignment model are nonlinear. Hence, we propose a solution method based on linear approximation and duality theories. First, model [CTA-s] is approximated as a linear programming model [LP-CTA-s]. Based on the duality theories, the optimal conditions of model [LP-CTA-s] are further transformed as a set of linear constraints. Finally, the model [SP-AV-IPD-EC] is reformulated as a mixed integer linear programming (MILP) model that can be solved by the state-of-the-art solvers like CPLEX. The details of solution methods are presented as below.

4.1. Reformulation of the combined traffic assignment model

In the AV-CV network, we treat link $a \in \bar{\mathcal{A}} \setminus \mathcal{A}_0$ as $\bar{y}_k^{\max} + 1$ separated links representing all possible numbers of regular lanes under AV lane deployment scheme. Link $a \in \bar{\mathcal{A}}$ is divided into \bar{y}_k^{\max} separated links as we delete the AV link without any lane. Let \mathcal{L} be the set consisting of links in $\bar{\mathcal{A}}_0$ and $K + \sum_{k \in \mathcal{K}} 2\bar{y}_k^{\max}$ new links. Let $l(a, i, k) \in \mathcal{L}$ represent the link that corresponds to link a after deploying i AV lanes to link pair k . \mathcal{L}_n^+ is the set of links whose tail node is n ; \mathcal{L}_n^- is the set of links whose head node is n . We extract link pair 1 (links l' and l'') in the illustrative network in Fig. 1 as an example. Assume that total number of lanes $Y_1 = 4$ and the maximum number of AV lanes $\bar{y}_k^{\max} = 2$. The corresponding links are shown in Fig. 2. With one AV lane deployed, we have $l(l', 1, 1)$ as regular link and $l(l'', 1, 1)$ as AV link. The number of lanes for $l(l', 1, 1)$ and $l(l'', 1, 1)$ is 3 and 1, respectively.

Let \mathcal{N}_o and \mathcal{N}_d be the set of origins and destinations, respectively. Denote $h_{ij}^{o,s}$ as the flow of vehicle type j from origin $o \in \mathcal{N}_o$ on link l . Denote $o(w) \in \mathcal{N}_o$ as origin and $d(w) \in \mathcal{N}_d$ as destination for OD w . Let us consider three cases for parameter $H_{n,j}^{o,s}$. Case 1: $H_{n,j}^{o,s} = \sum_{w \in \mathcal{W} : o(w)=o} q_{w,j}^s$, if $n = o$; Case 2: $H_{n,j}^{o,s} = -q_{w,j}^s$ if $o = o(w)$, $n = d(w)$ and Case 3: $H_{n,j}^{o,s} = 0$, otherwise. Case 1 indicates that all

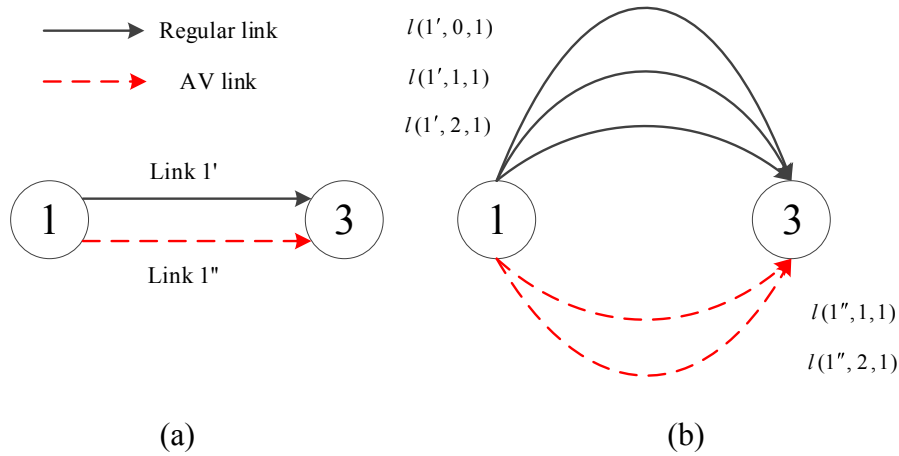


Fig. 2. Illustration of link pair 1 in Fig. 1.

flows with origin o depart from node n . Case 2 means that the flows between OD pair w arrive at node n if nodes o and n are respectively the origin and destination of OD pair w ; Case 3 implies that all flows with origin o pass through node n . Note that $q_{w,CV}^s$ can be replaced by $\bar{q}_w - q_{w,AV}^s$. The aggregated flow on link l is denoted as x_l^s . For each realization τ_{AV}^s and given solutions \mathbf{y} and \mathbf{z}^s , the original combined traffic assignment model can be reformulated as:

$$[\text{R-CTA} - s] \quad \min \sum_{l \in \mathcal{L}} \int_0^{x_l^s} t_l(u) du + \sum_{w \in \mathcal{W}} \int_0^{q_{w,AV}^s} \left(\frac{1}{\theta} \ln \frac{u}{\bar{q}_w - u} + \psi_w^s - \sum_{r \in \mathcal{R}} \hat{r} \cdot z_r^s \right) du \tag{27}$$

$$\text{subject to} \quad \sum_{l \in \mathcal{L}_n^+} h_{l,j}^{o,s} - \sum_{l \in \mathcal{L}_n^-} h_{l,j}^{o,s} = H_{n,j}^{o,s}, \quad \forall n \in \mathcal{N}, o \in \mathcal{N}_o, j \in \mathcal{J}, \tag{28}$$

$$\sum_{o \in \mathcal{N}_o} \sum_{j \in \mathcal{J}} h_{l,j}^{o,s} = x_l^s, \quad \forall l \in \mathcal{L}, \tag{29}$$

$$h_{l(a,i,k),CV}^{o,s} = 0, \quad \forall a \in \mathcal{A}_k \cap \widehat{\mathcal{A}}, i \in \mathcal{I}_k, k \in \mathcal{K}, o \in \mathcal{N}_o, \tag{30}$$

$$x_l^s \leq M_1 \cdot y_k^i, \quad \forall l(a, i, k): a \in \mathcal{A}_k, i \in \mathcal{I}_k, k \in \mathcal{K}, \tag{31}$$

$$h_{l,j}^{o,s} \geq 0, \quad \forall l \in \mathcal{L}, o \in \mathcal{N}_o, j \in \mathcal{J}, \tag{32}$$

$$q_{w,AV}^s \geq 0, \quad \forall w \in \mathcal{W}, \tag{33}$$

where M_1 is a sufficiently large number. Constraints (28) require the flow conservation. Constraints (29) define the total link flow. Constraints (30) indicate that CVs cannot travel on AV links. Constraints (31) state that when i of AV lanes are deployed on link pair k , only $l(a, i, k)$ for $a \in \mathcal{A}_k$ is considered in traffic assignment. Constraints (32) and (33) enforce nonnegative link flow and AV demand.

To take the advantage of the state-of-the-art solvers, the linear approximation techniques proposed by Nemhauser and Wolsey (1988) and Fontaine and Minner (2014) are applied to convert model [R-CTA-s] into a linear programming (LP) model. Besides, the nonlinear term in objective (24) can also be converted to a linear function. Hence, there are three nonlinear functions defined as: $B_l(x) = \int_0^x t_l(u) du$, $E_l(x) = -t_l(x)x$ and $U_w(x) = \int_0^x \frac{1}{\theta} \ln \frac{u}{\bar{q}_w - u} du$. Without loss of generality, we use $B_l(x)$ as an example to elaborate this method.

The principles of the linear approximation techniques are to create chord lines connecting adjacent breakpoints as approximated values. Within the feasible domain we set $M + 1$ breakpoints denoted as $\tilde{x}_{l,m}^s$, $m = 0, \dots, M$. The function value at breakpoints is denoted as $B_{l,m}^s = B_l(\tilde{x}_{l,m}^s)$. The feasible domain is divided into M identical intervals. Let slope $_m$ be the slope of m th chord line for interval $[\tilde{x}_{l,m}^s, \tilde{x}_{l,m+1}^s]$. All chord lines comprise a piecewise linear function denoted by $\bar{B}_l(\cdot)$. $\bar{B}_l(\cdot)$ provides an upper bound of original function $B_l(\cdot)$. The chord lines of $B_l(\cdot)$ are shown in Fig. 3.

With above linear approximation principles, the general mathematical expression of approximation function is given as below:

$$\bar{F}(e) = F_0 + \text{slope}_1 \cdot \tilde{g}_0 + \sum_{m=1}^{M-1} \tilde{g}_m (\text{slope}_{m+1} - \text{slope}_m), \tag{34}$$

$$\text{slope}_m = \frac{F_m - F_{m-1}}{\tilde{g}_m - \tilde{g}_{m-1}}, \quad \forall m = 1, 2, \dots, M, \tag{35}$$

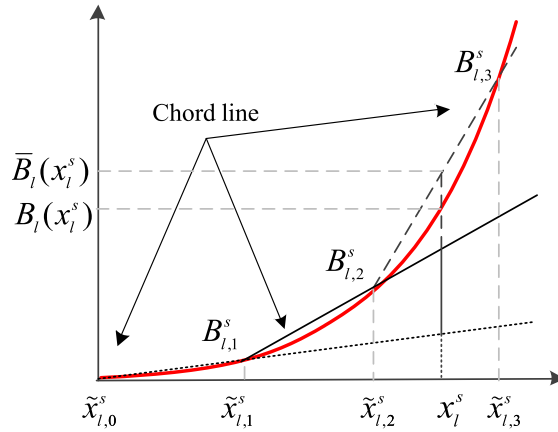


Fig. 3. Illustration of chord lines.

$$\tilde{g}_m \geq 0, \quad \forall m = 0, 1, \dots, M, \tag{36}$$

$$\tilde{g}_m \geq e - \tilde{g}_m, \quad \forall m = 0, 1, \dots, M, \tag{37}$$

$$\tilde{g}_m \in \tilde{G}_m = \{\tilde{x}_{l,m}^s, \tilde{q}_{w,AV,m}^s\}, \quad \forall l \in \mathcal{L}, w \in \mathcal{W}, m = 0, 1, \dots, M, \tag{38}$$

$$\tilde{g}_m \in \tilde{G}_m = \{\tilde{x}_{l,m}^s, \tilde{q}_{w,AV,m}^s\}, \quad \forall l \in \mathcal{L}, w \in \mathcal{W}, m = 0, 1, \dots, M, \tag{39}$$

$$F(e) \in \mathcal{F} = \{B_l(x_l^s), E_l(x_l^s), U_w(q_{w,AV}^s)\}, \quad \forall l \in \mathcal{L}, w \in \mathcal{W}, \tag{40}$$

$$\bar{F}(e) \in \bar{\mathcal{F}} = \{\bar{B}_l(x_l^s), \bar{E}_l(x_l^s), \bar{U}_w(q_{w,AV}^s)\}, \quad \forall l \in \mathcal{L}, w \in \mathcal{W}, \tag{41}$$

where \mathcal{F} is the set of nonlinear functions; $F_m = F(\tilde{g}_m)$ is the value of nonlinear function at breakpoints; $\bar{\mathcal{F}}$ is the set of approximation functions; e represents the given decision variable; G_m is the set of auxiliary variables involved in the approximation function; \tilde{G}_m is the set of m th break points in the feasible domain.

The underlying principles of constraints (34)–(41) are elaborated as follows: suppose that we want to calculate $\bar{F}(e)$. Initially, the first chord line yields an approximated value. Since function $F(e)$ is convex, we have $\text{slope}_{m+1} - \text{slope}_m \geq 0$. From breakpoint (\tilde{g}_m, F_m) to $(\tilde{g}_{m+1}, F_{m+1})$, we can add $\tilde{g}_m(\text{slope}_{m+1} - \text{slope}_m)$, where $\tilde{g}_m = e - \tilde{g}_m$, to update the approximated value because of the minimization problem. This process continues until we reach the interval where the ending point \tilde{g}_m is larger than e . At this interval, we cannot subtract anything. Therefore, we have $\tilde{g}_m = \max\{0, e - \tilde{g}_m\}$ for $m = 0, 1, \dots, M$, corresponding to constraints (36) and (37) under a minimization problem.

The linear approximation method defined by constraints (34)–(41) can support the non-uniform distributed breakpoints by calculating \tilde{g}_m using a recursive algorithm (Noruzoliaee et al., 2018). As a result, the reformulated model requires a smaller number of breakpoints and the computational efficiency may improve. As the main objective of this study is to investigate the AV-IPD problem, the improvement of computational efficiency can be remained as a future research topic.

Based on above linear approximation principles, model [R-CTA-s] can be transformed into a LP model:

[LP-CTA-s]

$$\min \text{Obj}_s = \sum_{l \in \mathcal{L}} \bar{B}_l(x_l^s) + \sum_{w \in \mathcal{W}} \bar{U}_w(q_{w,AV}^s) + \sum_{w \in \mathcal{W}} \psi_w^s q_{w,AV}^s - \sum_{w \in \mathcal{W}} \sum_{r \in \mathcal{R}} \hat{r} z_r^s \cdot q_{w,AV}^s \tag{42}$$

subject to Constraints (28)–(41).

4.2. Transformation into a MILP

In this section, the equilibrium constraints of model [SP-AV-IPD-EC] are transformed as a set of linear inequalities which are equivalent to the optimal conditions of model [LP-CTA-s]. Recall that the objective function in model [SP-AV-IPD-EC] can also be approximated as a linear function through constraints (34)–(41). We can formulate a MILP model to utilize the state-of-the-art solvers like CPLEX.

Before giving the MILP model, we first linearize the term $z_r^s q_{w,AV}^s$ in constraint (20) and objective (42). As z_r^s is a binary variable, we introduce an auxiliary linear variable $\hat{z}_{r,w}^s$ to represent $z_r^s q_{w,AV}^s$ with following constraints:

$$\hat{z}_{r,w}^s \geq q_{w,AV}^s - (1 - z_r^s)M_2, \quad \forall r \in \mathcal{R}, w \in \mathcal{W}, s \in \mathcal{S}\Theta, \tag{43}$$

$$\hat{z}_{r,w}^s \leq q_{w,AV}^s, \quad \forall r \in \mathcal{R}, w \in \mathcal{W}, s \in \mathcal{S}\Theta, \tag{44}$$

$$\hat{z}_{r,w}^s \leq z_r^s M_2, \quad \forall r \in \mathcal{R}, w \in \mathcal{W}, s \in \mathcal{S}\Theta, \tag{45}$$

$$\hat{z}_{r,w}^s \geq 0, \quad \forall r \in \mathcal{R}, w \in \mathcal{W}, s \in \mathcal{S}\Theta, \tag{46}$$

where M_2 is sufficiently large value.

Based on the duality theories, we can represent the optimal primal and dual decision variables of model [LP-CTA- s] as the solutions of a set of linear inequalities. For the sake of presentation, we define a polyhedron $\Delta_s(\mathbf{y}, \mathbf{z}^s, \hat{\mathbf{z}}^s, \tau_{AV}^s)$ to involve these linear inequalities with the given AV price realization τ_{AV}^s , decision vectors \mathbf{y} , \mathbf{z}^s and $\hat{\mathbf{z}}^s$. The primal variables of model [LP-CTA- s] include \mathbf{x}^s , \mathbf{q}^s and $\bar{\mathbf{g}}^s$. The vectors of dual variable are set to be β^s , α^s , γ^s and ρ^s associated with constraints (28)–(31), respectively. Define vectors of dual variable η^s for $B_l(\cdot)$ and κ^s for $U_w(\cdot)$ in constraints (37). For more details about $\Delta_s(\mathbf{y}, \mathbf{z}^s, \hat{\mathbf{z}}^s, \tau_{AV}^s)$, readers can refer to constraints (A.2)–(A.20) in Appendix A. Since model [CTA- s] has a unique solution and the linear approximation does not violate the convexity of the objective function (Fontaine and Minner, 2014), $\Delta_s(\mathbf{y}, \mathbf{z}^s, \hat{\mathbf{z}}^s, \tau_{AV}^s)$ only involves one feasible solution. Thus, model [SP-AV-IPD-EC] can be transformed as a MILP model as:

$$[\text{MILP-AV-IPD}] \quad \min \sum_{s \in \mathcal{S}} \sum_{l \in \mathcal{L}} \phi_l \bar{E}_l(x_l^s) \tag{47}$$

$$\text{subject to } (\mathbf{x}^s, \mathbf{q}^s, \bar{\mathbf{g}}^s, \beta^s, \alpha^s, \gamma^s, \rho^s, \eta^s, \kappa^s) \in \Delta_s(\mathbf{y}, \mathbf{z}^s, \hat{\mathbf{z}}^s, \tau_{AV}^s), \quad \forall s \in \mathcal{S}, \tag{48}$$

$$\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}_k} y_k^i u_k^i + \sum_{w \in \mathcal{W}} \sum_{r \in \mathcal{R}} r \cdot \hat{z}_{r,w}^s \leq BD, \quad \forall s \in \mathcal{S}\Theta, \tag{49}$$

Constraints (15)–(18), (22)–(23), and (43)–(46),

where objective function (47) minimizes the approximation of expected total travel time; Constraints (48) approximate the equilibrium constraints (21). Constraints (49) define the government’s budget on AV incentive program.

By fixing the solution to model [MILP-AV-IPD], we can apply double-stage Frank Wolfe (F-W) algorithm (Sheffi, 1985) to calculate the demand and flow pattern at UE condition, and obtain the objective value of model [SP-AV-IPD-EC]. However, the optimal solution to model [MILP-AV-IPD] may not necessarily be optimal to model [SP-AV-IPD-EC]. In this paper, we use the percentage difference between the objective value of models [SP-AV-IPD-EC] and [MILP-AV-IPD] to measure the approximation error. If the number of breakpoints is sufficiently large, the approximation error could be very little but a long computation time is required. More tests about the impact of different numbers of breakpoints on the proposed model are conducted in Section 5.

5. Numerical experiments

In this section, the proposed model and solution method are applied to Nguyen-Dupuis network (Nguyen and Dupuis, 1984), and Sioux-Falls network. A personal computer with Intel Core (TM) i7-4790, 3.6 GHz CPU, 16 GB RAM, is used for all the tests. The solution approach is coded with C++, calling CPLEX 12.7 to solve the MILP models. In solving the traffic assignment, the relative optimality tolerance is set to be 10^{-5} to strike a balance between computation time and solution quality.

5.1. Nguyen-Dupuis network

The modified Nguyen-Dupuis network consists of 12 nodes, 26 links, 2 origins and 2 destinations. There are seven links with candidate AV lanes. The structure of AV-CV network is shown in Fig. 4. The link attribute is presented in Table 1, including the free flow travel time (unit: minute), the link capacity (unit: veh/hr), the link length (unit: km), and the number of lanes (#Lane). Here we assume that capacity ratio $CR = 2.5$. Table 2 gives the CV/AV links and the maximum number of AV lanes in each link pair. The total demand for OD pairs (1,12), (1,13), (3,12) and (3,13) is 4, 4, 3 and 3 (10^3 vehicles), respectively.

We first set the model parameters as below: number of AV price realizations $S = 4$; set of the purchase price difference between AVs and CVs $\Gamma = \{4, 6, 8, 10\}$ (unit: $\$10^3$); scale parameter of logit model $\theta = 1$; lifespan of vehicle $L = 10$ years; number of commute trips $b_1 = 500$; VOT $b_2 = 0.2\$/\text{min}$; set of candidate AV purchase subsidy $\mathcal{R}\Theta = \{0, 1, 2, 3\}$ (unit: $\$10^3$); proportion of commute trips $b_3 = 1$; number of breakpoints $M = 50$; unit cost of deploying AV lane is 10^5 dollars per kilometer; coefficient of travel time savings by automated driving $\varphi = 0.1$; unit operational cost for AVs and CVs are 0.284\$/km and 0.276\$/km (Noruzoliaee et al., 2018); All realizations of AV price have the same occurrence probability, namely, $\phi_s = 0.25, s \in \mathcal{S}$.

Table 3 reports the solutions and effects of the AV incentive program under different budgets. The column of BD represents the amount of budget (unit: $\$10^5$). The column of *Link pairs* indicates the link pairs where AV lanes are deployed, where the value in the parentheses is the number of deployed AV lanes. The column of %SC represents the percentage difference of average system total travel time with and without AV incentive program. The column of *Sub* refers to the purchase subsidy (unit: $\$10^3$) for each realization of AV prices. The column of MS_I is the modal split of AVs over network. It can be seen that the AV incentive program can reduce the total travel time in all cases. Meanwhile, the purchase subsidy increases with the AV price and improves the modal split of AVs. For example, the AV market share improves from 18.0% to 19.2% with $\Delta\tau^3 = 8$ and $\$1000$ subsidy. We notice that the optimal purchase subsidy is zero when the AV purchase price is not high ($\Delta\tau^1 = 4$ and $\Delta\tau^3 = 6$). If the government implements purchase subsidy in these cases, the budget constraint could be violated, and the resulting AV demand could cause congestion on AV lanes, which

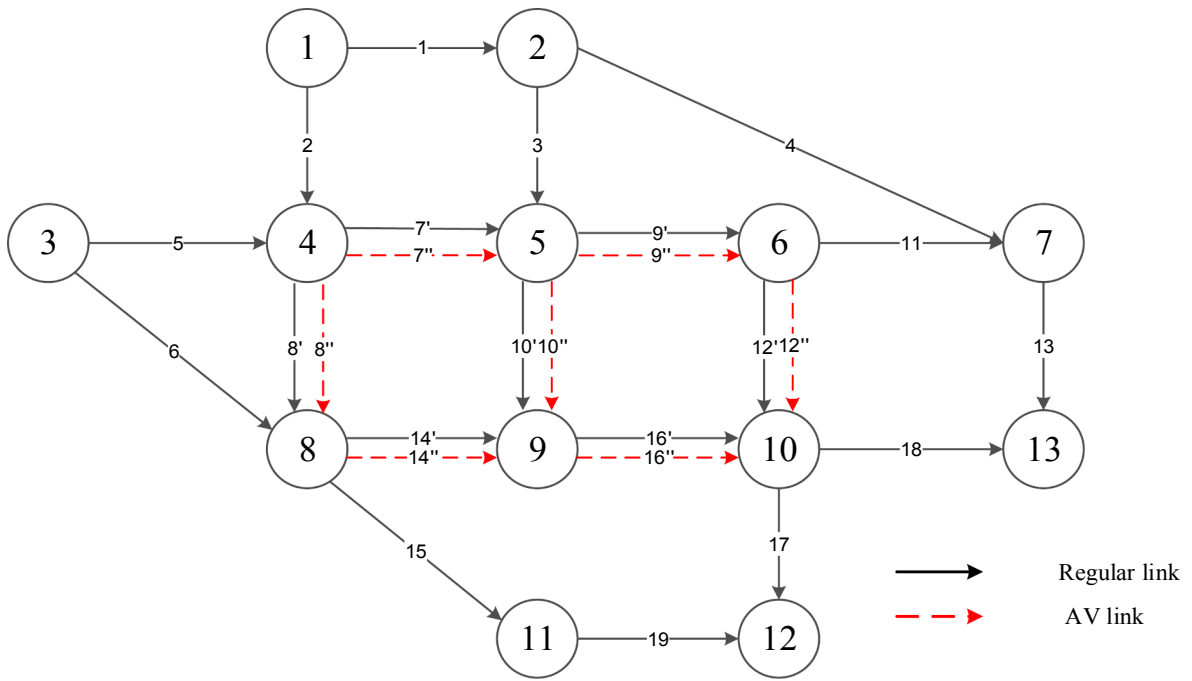


Fig. 4. Modified Nguyen-Dupius based AV-CV network.

Table 1
Link attributes of modified Nguyen-Dupius based AV-CV network.

Link	t_a^0	Λ_a	Length	#Lane	Link	t_a^0	Λ_a	Length	#Lane
1	9	0.8	5.4	3	10"	13	3.75	7.8	0
2	7	1	4.2	4	11	5	0.8	3	2
3	7	1	4.2	3	12'	9	0.8	5.4	4
4	14	1	8.4	2	12"	9	2	5.4	0
5	9	1	5.4	3	13	9	0.8	5.4	3
6	12	1.5	7.2	3	14'	10	1.5	6	4
7'	3	1.3	1.8	4	14"	10	3.75	6	0
7"	3	3.25	1.8	0	15	9	1.5	5.4	2
8'	9	1.3	5.4	3	16'	6	1	3.6	4
8"	9	3.25	5.4	0	16"	6	2.5	3.6	0
9'	5	0.8	3	3	17	5	1	3	3
9"	5	2	3	0	18	9	1.5	5.4	4
10'	13	1.5	7.8	3	19	11	1.5	6.6	3

Table 2
Link pair setting for Nguyen-Dupius based AV-CV network.

Link pair	Regular link	AV link	\bar{y}_k^{\max}	Link pair	Regular link	AV link	\bar{y}_k^{\max}
1	7'	7"	2	5	12'	12"	2
2	8'	8"	1	6	14'	14"	2
3	9'	9"	1	7	16'	16"	2
4	10'	10"	1				

consequently increases total travel time over the network. Therefore, Table 3 implies that the purchase subsidy is effective to adjust AV market share to the desired level that well matches the deployment scheme of AV lanes.

We further demonstrate the effects of AV incentive program. With $BD = 80$ and $\Delta\tau^3 = 8$, Table 4 gives the travel time for CVs and AVs in the column of CV time and AV time, and the modal split of AVs for a specified OD pair in the column of MS_2 . For the purpose of comparison, we also list the travel time and the modal split of AVs when no action is implemented. Without deploying AV lanes, the equilibrium travel time of AVs is the same with CVs between each OD pair, but AV modal splits could be different among OD pairs due to the travel time savings by automated driving. The results show that the AV incentive program reduces the travel time for both CV and AV users. This is because: (1) AV lane has a high road capacity and reduces the travel time of AVs; (2) some CV users shift to

Table 3
AV lane deployment and purchase subsidy with different budgets.

BD	Link pairs	%SC	$\Delta\tau^1 = 4$		$\Delta\tau^2 = 6$		$\Delta\tau^3 = 8$		$\Delta\tau^4 = 10$	
			Sub	MS ₁	Sub	MS ₁	Sub	MS ₁	Sub	MS ₁
110	1(1), 3(1), 5(2)	-3.83%	0	33.4%	0	21.4%	1	19.2%	3	19.2%
80	1(1), 3(1), 5(2)	-3.81%	0	33.4%	0	21.4%	1	19.2%	2	18.0%
50	1(1), 3(1), 5(2)	-3.74%	0	33.4%	0	21.4%	0	18.0%	1	16.4%
20	1(1), 3(1), 5(1)	-2.99%	0	30.9%	0	20.5%	0	16.9%	0	13.3%

Table 4
Effects of AV incentive program with $BD = 80$ and $\Delta\tau^3 = 8$.

OD pairs	AV incentive program			No actions	
	CV time	AV time	MS ₂	CV/AV time	MS ₂
1–12	41.4	38.5	27.9%	41.8	0.8%
1–13	42.6	40.2	22.7%	44.0	1.0%
3–12	36.5	36.5	1.6%	37.9	0.6%
3–13	40.7	38.2	20.6%	42.3	0.8%

AV users thus alleviate the congestion in regular lanes. We also notice that the modal split of AVs between OD pair 3-12 is much smaller than the other OD pairs. The reason is that no AV lanes are deployed on the shortest paths at UE state. The shortest path between OD 3-12 before and after deploying AV lanes is always 6-15-19 as shown in Fig. 4. Without the benefits from AV lanes, the impact of purchase subsidy on vehicle choice behavior is limited and most of users tend to choose CVs due to a high AV purchase price.

Next we conduct the sensitivity analysis of various parameters in the proposed model. Except the parameters tested, we keep the other model settings the same with the analysis above.

To investigate the impact of the approximation errors on the performance of solution method, numerical experiments are conducted with four different numbers of breakpoints in approximating the nonlinear terms: 20, 50, 80 and 200. In Fig. 5(a), the term “error” refers to the percentage difference between the objective value of models [MILP-AV-IPD] and [SP-AV-IPD-EC] with the given solution. Note that we obtain the objective value of model [SP-AV-IPD-EC] by calculating system total travel time through the double-stage F-W algorithm. We denote the objective value of model [SP-AV-IPD-EC] as “system cost” in Fig. 5(a). It can be seen that the error always decreases with the number of breakpoints, but the system cost does not necessarily decline. In our study, 50 of breakpoints already achieve a satisfactory result, while more breakpoints do not improve solutions but only increase the computation time.

The unit cost of AV lane deployment is subject to the technology used, such as V2V and V2I sensors. However, V2V and V2I communication has not been widely applied yet. Thus we explore the effects of various costs of AV lane deployment on system performance as shown in Fig. 5(b). The term “AV modal split” refers to the average AV market share over the network under all realizations of AV prices. It can be seen that, as the unit cost of AV lane deployment decreases, the AV market share increases and system cost declines monotonically. This is because cheap AV lanes allow the government to expand the coverage area of AV lanes, or increase the purchase subsidy. The AV market share can be maintained at a level that well matches the deployment of AV lanes, which reduces the total travel time over the network.

The capacity improving effects of AVs are crucial for AV lane deployment, so we test the proposed model with different ratios of road capacity (CR) between AV link and regular link. For example, $CR = 2$ means the capacity of AV link is two times of the paired regular link in the AV-CV network. In Fig. 5(c), as the capacity ratio rises, the total travel time decreases and AV market penetration increases. This is expected because the AV lane with a higher capacity will significantly decrease the travel time of AV users.

Fig. 5(d) depicts how AV market share and system cost changes with VOT. It can be observed that increasing VOT promotes the AV market penetration. This is because, with a higher VOT, the purchase cost would cause less trip-based disutility for AVs, which makes AVs more attractive. However, it should be further noticed that a high AV demand may not always be beneficial. Specifically, the system cost decreases when AV market share rises from 17% to 23%. Then, the system cost starts to grow after AV market share is greater than 23%. This is caused by the essence of UE: increasing number of AV users would minimize their individual travel time by making route choice without considering the effects of their choice on network performance. Consequently, AV lanes could become congested make system total travel time increase. We notice that Melson et al. (2018) also found that, after high-capacity and exclusive lanes are deployed, a growing market penetration of connected vehicles could increase the travel time for both connected and non-connected vehicles.

We next demonstrate the impact of travel time saving by automated driving on the system performance. Fig. 5(e) shows that a larger value of φ will increase AV market share. This is reasonable as automated driving enables AV drivers to be productive during commute trips. We can also observe that the system cost first decreases when φ rises from 0 to 0.1. However, the system cost then increases when φ further grows and the AV market penetration is more than 23%. The reason is that a relatively high AV market share

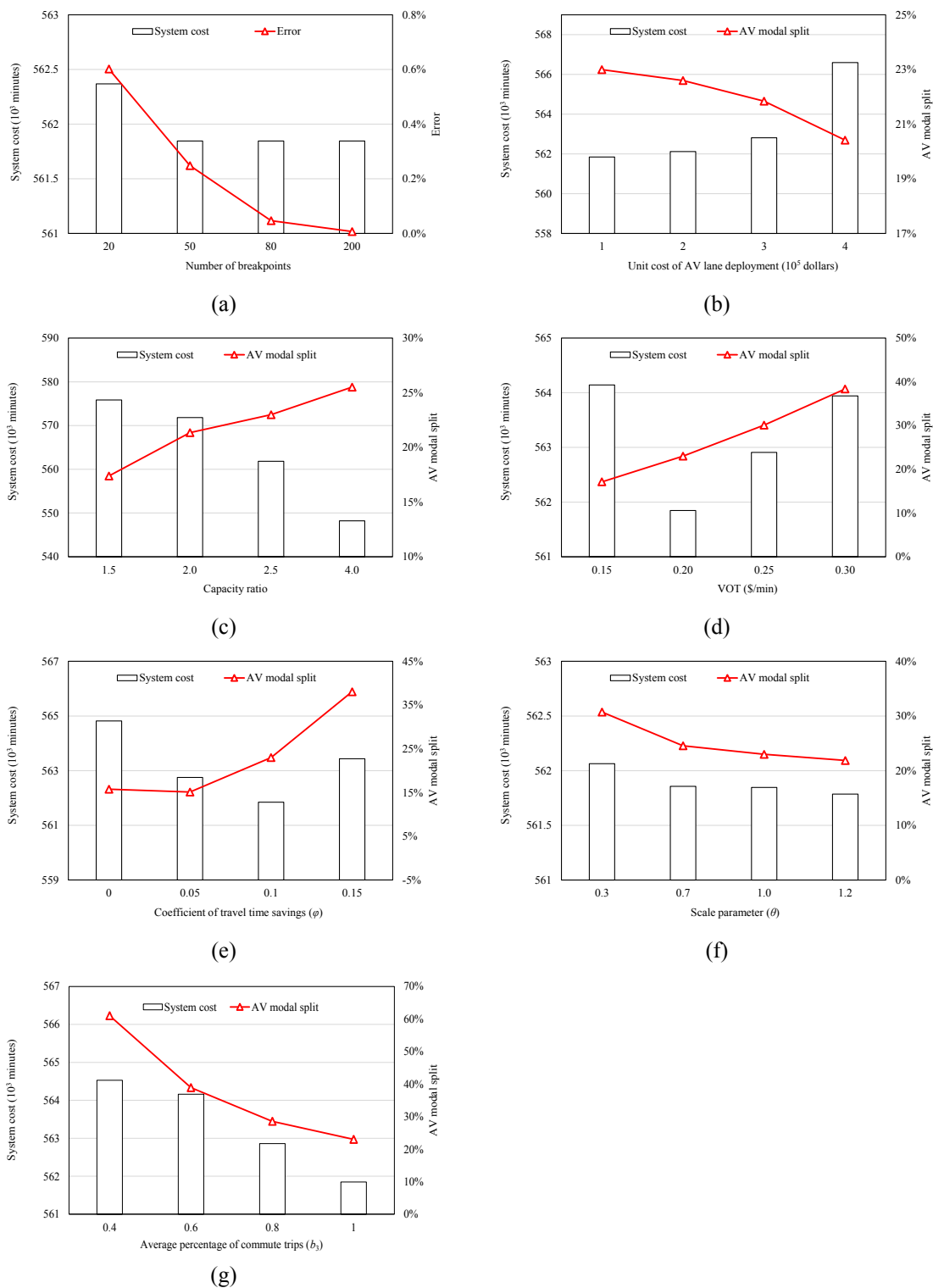


Fig. 5. The results of sensitivity analysis on: (a) number of breakpoints; (b) unit cost of AV lane deployment; (c) capacity ratio between AV lane to regular lane; (d) VOT; (e) coefficient of travel time savings by automated driving; (f) scale parameter in utility function; (g) average percentage of commute trips.

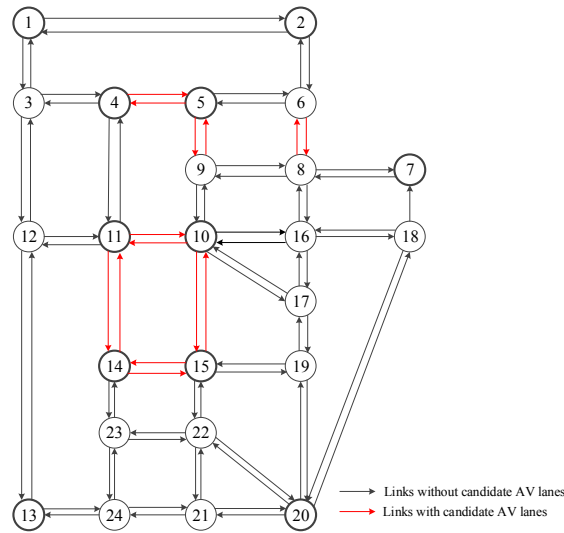


Fig. 6. The structure of Sioux Falls network.

makes AV lanes congested and results in the increase of total system travel time. This observation is similar with the results in Fig. 5(d).

The impact of various scale parameters is illustrated in Fig. 5(f). As it can be seen, an increasing θ will result in a lower AV modal split. Note that in our cases the market share of CVs is usually greater than AVs due to the limited coverage of AV lanes and high AV purchase cost. Therefore, a greater θ enlarges the utility gap between CVs and AVs, which makes users more likely to choose CVs. Furthermore, the results also show that the change of system cost is insignificant with the variation of θ .

Finally, we present average system costs and modal splits under various average percentages of commuting trips (b_3) in Fig. 5(g). The percentage of commute trips is set to be greater than 40%. The results show that a declining percentage of commute trips would result in a higher AV modal split since the purchase cost of AVs causes less generalized travel time per trip. However, the total travel time over network could increase with AV modal split due to the route choice of AV users.

5.2. Sioux Falls network

To demonstrate the computational performance of the solution method, we use a modified Sioux Falls network. The corresponding AV-CV network has 76 regular links, 14 AV links, and 24 nodes as shown in Fig. 6. The link pair settings are given in Table 5. We consider 11 nodes as the origin/destination nodes and there are 110 OD pairs in the network. The details of links and OD demands are shown in Appendix C.

Table 6 presents the computational performance of the proposed model and solution method on the modified Sioux Falls network. The number of breakpoints is set to be 20. The budget $BD = 1.5 \times 10^8$ dollars. The set of the difference of realized price between AVs and CVs is listed in the first column (unit: $\$10^3$). The second column gives the indexes of link pairs with AV lanes. Note that all specified link pairs in Table 6 are deployed with one AV lane and no link pair is deployed with two AV lanes. The column *Sub* shows the purchase subsidy (unit: $\$10^3$) corresponding to each realized AV price in the first column. The column of %SC represents the percentage difference of the system cost under AV incentive program compared with no-action plan. The last column shows the computation time (unit: minute). The results show that the proposed AV incentive program can significantly reduce the system cost. It can be seen that the computation time grows rapidly with the number of AV purchase price realizations. Therefore, it is worthwhile to develop some efficient heuristic algorithms to solve the large-scale problem.

Table 5
The link pairs for Sioux Falls based AV-CV network.

Link pair	Regular link	AV link	\bar{y}_k^{\max}	Link pair	Regular link	AV link	\bar{y}_k^{\max}
1	9	10	2	8	35	36	1
2	12	13	2	9	40	41	1
3	15	16	1	10	43	44	1
4	19	20	1	11	50	51	1
5	23	24	1	12	52	53	1
6	28	29	1	13	55	56	1
7	33	34	1	14	57	58	1

Table 6
The results AV incentive program for Sioux Falls network.

Γ	Link pairs	Sub	%SC	CPU time (min)
{5}	2, 6, 7, 8, 9, 12, 13, 14	{0}	−13.6%	95
{2, 8}	1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14	{1, 2}	−12.5%	954
{2, 5, 8}	1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14	{1, 1, 2}	−12.1%	8378

6. Conclusions

In this study, we propose a novel AV incentive program design problem with practical significance. The AV purchase price is regarded as a random variable to deal with the uncertainty in AV market. We build a two-stage stochastic programming model with equilibrium constraints for the proposed problem. The first stage involves the deployment of AV lanes, and the second stage determines the optimal purchase subsidy corresponding to each realization of the AV purchase price. The equilibrium constraints are formulated by a combined traffic assignment model which characterizes the vehicle choice and route choice behavior. To solve the proposed AV-IPD problem, we design a solution method based on the linear approximation and duality theories. Finally, various numerical experiments demonstrate the effectiveness of the proposed method. It is found that the proposed AV incentive program is beneficial to both AV and CV users. The AV purchase subsidy can be regarded as a flexible part of the incentive program to adjust the modal split for the system optimum.

There are some interesting issues needed to be further investigated. First, we can focus on the vehicle choice behavior study. For example, to capture the impact of all trip purposes on the vehicle choice behavior between CVs and AVs, we can apply activity-based traffic demand model. It is useful to incorporate annual mileage to enhance model accuracy. All users can be divided into several groups based on their annual mileage and each group’s preference on AVs needs to be estimated. A large-scale SP survey should be conducted to estimate the parameters in the logit model. The proposed model in this study is based on a critical assumption that commute travel time and purchase cost will jointly influence AV ownership, and this assumption can be further validated with more empirical studies. Finally, it is worthwhile to study the AV demand evolution with the construction of AV based infrastructures.

Besides, the proposed optimization model can be improved in following ways. The equity constraint can be added to guarantee the fairness of the autonomous vehicle incentive program among all OD pairs. A multi-class traffic assignment model can be used to incorporate the users who are heterogeneous in VOT and annual mileage. In addition, complementarity slackness constraint is another effective method for modelling vehicle choice behavior. This method also takes account of purchase cost, operational cost and OD based travel time, though an effective solution method should be designed to deal with the large number of binary variables in the reformulated model.

Finally, some efficient linear approximation strategies can be applied to improve the model efficiency, such as the non-uniform distributed breakpoints and domain reduction technique (Noruzoliaee et al., 2018). Some decomposition algorithms can be designed to solve the stochastic programming model, such as Benders’ decomposition and scenario decomposition. Finally, we can also apply some heuristic methods such as genetic algorithm (GA) or parallel computational techniques to solve the large-scale problems.

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Appendix A. Detailed description of $\Delta_s(\mathbf{y}, \mathbf{z}^s, \hat{\mathbf{z}}^s, \tau_{AV}^s)$

In this section, we describe the polyhedron that represents the optimal conditions of model [LP-CTA-s] in Section 4.2. Based on the duality theories of linear programming, the primal and dual formulation of model [LP-CTA-s] should have the same optimal objective value. The objective function of the dual formulation of model [LP-CTA-s] is presented as:

$$\bar{O}bj_s = \sum_{o \in \mathcal{N}_o} \sum_{n \in \mathcal{N}} \beta_{n,CV}^{o,s} \bar{H}_n^o + \sum_{a \in \mathcal{A}_k} \sum_{i \in \mathcal{I}_k} \sum_{k \in \mathcal{K}} M_i \cdot y_k^i \rho_{a,k}^{i,s} + \sum_{l \in \mathcal{L}} \sum_{m=0}^M \tilde{x}_{l,m}^s \eta_{l,m}^s + \sum_{w \in \mathcal{W}} \sum_{m=0}^M \tilde{q}_{w,AV,m}^s \kappa_{w,m}^s \tag{A.1}$$

where function $\bar{H}_n^o = \sum_{w \in \mathcal{W}: o(w)=0} \bar{q}_w$ if $n = o$, $\bar{H}_n^o = -\bar{q}_w$ if $o = o(w)$, $n = d(w)$ and $\bar{H}_n^o = 0$, otherwise; The first two terms are associated with constraints (28) and (31) respectively. The third and fourth terms correspond to the flow and demand variables in constraint (37).

Note that there is a nonlinear term $y_k^i \rho_{a,k}^{i,s}$ in (A.1). We here introduce an auxiliary variable $\zeta_{a,k}^{i,s}$ and following constraints for linearization:

$$\zeta_{a,k}^{i,s} \geq \rho_{a,k}^{i,s}, \quad \forall a \in \mathcal{A}_k, i \in \mathcal{I}_k, k \in \mathcal{K}, \tag{A.2}$$

$$\zeta_{a,k}^{i,s} \leq \rho_{a,k}^{i,s} + (1 - y_k^i) \cdot M_3, \quad \forall a \in \mathcal{A}_k, i \in \mathcal{I}_k, k \in \mathcal{K}, \tag{A.3}$$

$$\zeta_{a,k}^{i,s} \geq -M_3 \cdot y_k^i, \quad \forall a \in \mathcal{A}_k, i \in \mathcal{I}_k, k \in \mathcal{K}, \tag{A.4}$$

$$\zeta_{a,k}^{i,s} \leq 0, \quad \forall a \in \mathcal{A}_k, i \in \mathcal{I}_k, k \in \mathcal{K}, \tag{A.5}$$

where M_3 is a sufficiently large number. Thus, the objective of dual problem becomes:

$$Ob_j^s = \sum_{o \in \mathcal{N}_o} \sum_{n \in \mathcal{N}} \beta_{n,CV}^{o,s} \bar{H}_n^o + \sum_{a \in \mathcal{A}_k} \sum_{i \in \mathcal{I}_k} \sum_{k \in \mathcal{K}} M_1 \cdot \zeta_{a,k}^{i,s} + \sum_{l \in \mathcal{L}} \sum_{m=0}^M \tilde{x}_{l,m}^s \eta_{l,m}^s + \sum_{w \in \mathcal{W}} \sum_{m=0}^M \tilde{q}_{w,AV,m}^s \kappa_{w,m}^s. \tag{A.6}$$

Let $\bar{b}_{l,m}$ and $\bar{u}_{w,m}$ be the slope values of m th chord line segments for function $\bar{B}_l(\cdot)$ and $\bar{U}_w(\cdot)$, respectively. \mathcal{L}_0 represents the set of links without candidate AV lanes. With the given solutions \mathbf{y} , \mathbf{z}^s , $\hat{\mathbf{z}}^s$ and the realization of price τ_{AV}^s , $\Delta_s(\mathbf{y}, \mathbf{z}^s, \hat{\mathbf{z}}^s, \tau_{AV}^s)$ can be described by following linear constraints:

Constraints (28)–(41), and (A.2)–(A.6),

$$Ob_j^s \geq \sum_{l \in \mathcal{L}} \bar{B}_l(x_l^s) + \sum_{w \in \mathcal{W}} \bar{U}_w(q_{w,AV}^s) + \sum_{w \in \mathcal{W}} \psi_w^s q_{w,AV}^s - \sum_{w \in \mathcal{W}} \sum_{r \in \mathcal{R}} \hat{r} \cdot \hat{z}_{r,w}^s, \tag{A.7}$$

$$\alpha_l^s + \beta_{l(l),CV}^{o,s} - \beta_{h(l),CV}^{o,s} + \gamma_l^{o,s} \leq 0, \quad \forall o \in \mathcal{N}_o, l(a, i, k): a \in \mathcal{A}_k \cap \widehat{\mathcal{A}}, i \in \mathcal{I}_k, k \in \mathcal{K}, \tag{A.8}$$

$$\alpha_l^s + \beta_{l(l),CV}^{o,s} - \beta_{h(l),CV}^{o,s} \leq 0, \quad \forall o \in \mathcal{N}_o, l(a, i, k): a \in \mathcal{A}_k \cap \bar{\mathcal{A}}, i \in \mathcal{I}_k, k \in \mathcal{K} \text{ and } l \in \mathcal{L}_0, \tag{A.9}$$

$$\alpha_l^s + \beta_{l(l),AV}^{o,s} - \beta_{h(l),AV}^{o,s} \leq 0, \quad \forall o \in \mathcal{N}_o, l \in \mathcal{L}, \tag{A.10}$$

$$-\alpha_l^s + \sum_{m=0}^M \eta_{l,m}^s \leq \bar{b}_{l,1}, \quad \forall l \in \mathcal{L}_0, \tag{A.11}$$

$$-\alpha_l^s + \sum_{m=0}^M \eta_{l,m}^s + \rho_{a,k}^{i,s} \leq \bar{b}_{l,1}, \quad \forall l(a, i, k): a \in \mathcal{A}_k \cap \widehat{\mathcal{A}}, i \in \mathcal{I}_k, k \in \mathcal{K}, \tag{A.12}$$

$$-\eta_{l,m}^s \leq \bar{b}_{l,m+1} - \bar{b}_{l,m}, \quad \forall m = 1, 2, \dots, M-1, l \in \mathcal{L}, \tag{A.13}$$

$$\eta_{l,M}^s = 0, \quad \forall l \in \mathcal{L}, \tag{A.14}$$

$$\beta_{o(w),CV}^{o(w),s} - \beta_{d(w),CV}^{o(w),s} - \beta_{o(w),AV}^{o(w),s} + \beta_{d(w),AV}^{o(w),s} + \sum_{m=0}^M \kappa_{w,m}^s \leq \bar{u}_{w,1} + \psi_w^s - \sum_{r \in \mathcal{R}} \hat{r} \cdot z_r^s, \quad \forall w \in \mathcal{W}, \tag{A.15}$$

$$-\kappa_{w,m}^s \leq \bar{u}_{w,m+1} - \bar{u}_{w,m}, \quad \forall m = 1, 2, \dots, M-1, w \in \mathcal{W}, \tag{A.16}$$

$$\kappa_{w,M}^s = 0, \quad \forall w \in \mathcal{W}, \tag{A.17}$$

$$\rho_{a,k}^{i,s} \leq 0, \quad \forall a \in \mathcal{A}_k, i \in \mathcal{I}_k, k \in \mathcal{K}, \tag{A.18}$$

$$\eta_{l,m}^s \leq 0, \quad \forall m = 0, 1, \dots, M, l \in \mathcal{L}, \tag{A.19}$$

$$\kappa_{w,m}^s \leq 0, \quad \forall m = 0, 1, \dots, M, w \in \mathcal{W}, \tag{A.20}$$

where constraints (A.7) enforce the dual objective value to be equal to the primal objective value; constraints (A.8)–(A.10) are associated with variable $h_{ij}^{o,s}$; constraints (A.11) and (A.12) correspond to variable x_l^s ; constraints (A.13) and (A.14) are related to auxiliary variable $\tilde{x}_{l,m}^s$; Similarly, constraints (A.15)–(A.17) are defined for variables $q_{w,AV}^s$ and $\tilde{q}_{w,AV}^s$.

Appendix B. Summary of notations

Set	
\mathcal{N}	Set of nodes;
\mathcal{A}	Set of directed links in transportation network;
$\widehat{\mathcal{A}}$	Set of candidate AV links in AV-CV network;
$\bar{\mathcal{A}}$	Set of regular links in AV-CV network;
\mathcal{W}	Set of origin–destination (OD) pairs;
\mathcal{K}	Set of link pair index;
\mathcal{A}_k	Set of AV link and regular link in link pair $k \in \mathcal{K}$;
\mathcal{P}_w	Set of all paths connecting OD pair $w \in \mathcal{W}$;
$\bar{\mathcal{P}}_w$	Set of feasible paths connecting OD pair w for CVs;

S	Set of index of realization of AV purchase price;
\mathcal{R}	Set of candidate AV purchase subsidies;
\mathcal{J}	Set of vehicle types, $\mathcal{J} = \{CV, AV\}$;
\bar{I}_k	Set of feasible numbers of AV lanes to be deployed on link pair k ;
$\bar{\Omega}_s$	Set of feasible link flow and demand with AV price realization indexed by $s \in S$;
$\bar{\Omega}_{UE}^s$	Set of demand and link flow that satisfy logit model and UE conditions with AV price realization indexed by s ;
Γ	Set of the difference of purchase price between AVs and CVs;
Parameters	
τ_{CV}^s	Purchase price of CVs indexed by s ;
τ_{AV}^s	Realization of random purchase price of AVs indexed by s ;
Y_k	Total number of lanes for link pair k ;
\bar{y}_k^{\max}	Maximum number of AV lanes to be deployed for link pair k ;
L	Lifespan of vehicle;
θ	Scale parameter for logit model;
ϕ_s	Occurrence probability for AV price realization τ_{AV}^s ;
b_1	Number of commute trips in one year;
b_2	Value of time (VOT) for users;
b_3	Average percentage of commute trips among the yearly travel trips of users
STT_w	Equilibrium travel time for OD pair w before AV lanes are deployed;
φ	Coefficient of the travel time savings of automated driving;
$TS_{w,j}$	Approximated travel time savings by automated driving, $TS_{w,CV} = 0$ and $TS_{w,AV} = \varphi \cdot STT_w$ for OD pair w ;
$OC_{w,j}$	Approximated operational cost for OD pair w and vehicle type $j \in \mathcal{J}$;
\bar{q}_w	Travel demand between the OD pair w ;
\hat{c}_a	Capacity of a single AV lane for link $a \in \hat{\mathcal{A}}$;
\bar{c}_a	Capacity of a single regular lane for link $a \in \bar{\mathcal{A}}$;
u_k^i	Cost of deploying i of AV lanes at link pair $k \in \mathcal{K}$;
Decision variables	
$f_{w,j}^{p,s}$	Traffic flow on path $p \in \mathcal{P}_w$ connecting OD w under AV purchase price realization τ_{AV}^s ;
v_a^s	Traffic flow on link $a \in \hat{\mathcal{A}} \cup \bar{\mathcal{A}}$ for realization of AV purchase price τ_{AV}^s ;
$q_{w,j}^s$	Travel demand of vehicle type j for OD w with AV purchase price realization τ_{AV}^s ;
\mathbf{v}^s	Vector of link traffic flow with AV purchase price realization τ_{AV}^s , $\mathbf{v}^s = (v_a^s \forall a \in \hat{\mathcal{A}} \cup \bar{\mathcal{A}})$
\mathbf{q}^s	Vector of demands with AV purchase price realization τ_{AV}^s , $\mathbf{q}^s = \left(q_{w,j}^s \forall w \in \mathcal{W}, j \in \mathcal{J} \right)$
t_a	Travel time on link $a \in \hat{\mathcal{A}} \cup \bar{\mathcal{A}}$;
$c_{w,j}^{p,s}$	Travel time of path p between OD w for vehicle type j with AV price realization τ_{AV}^s ;
$TT_{w,j}^s$	User equilibrium travel time $TT_{w,j}^s$ for OD pair w and vehicle type j with AV price realization τ_{AV}^s ;
y_k^i	Binary variable, $y_k^i = 1$ if i of AV lanes are deployed on link pair k , and 0 otherwise;
z_r^s	Binary variable, $z_r^s = 1$ if purchase subsidy $r \in \mathcal{R}$ is implemented with AV price realization τ_{AV}^s , and 0 otherwise;
\mathbf{y}	Vector of binary variable of y_k^i , $\mathbf{y} = (y_k^i \forall i \in I, k \in \mathcal{K})$;
\mathbf{z}^s	Vector of binary variable of z_r^s , $\mathbf{z}^s = (z_r^s \forall r \in \mathcal{R})$;

Appendix C. Network data

See Tables C.1 and C.2.

Table C.1
Link attributes of modified Sioux Falls based AV-CV network.

ID	FN	TN	t_a^0	LT	ID	FN	TN	t_a^0	LT
1	1	2	6	0	39	11	4	6	0
2	1	3	4	0	40	11	10	5	0
3	2	1	6	0	41	11	10	5	1
4	2	6	5	0	42	11	12	6	0
5	3	1	4	0	43	11	14	4	0
6	3	4	4	0	44	11	14	4	1
7	3	12	4	0	45	12	3	4	0
8	4	3	4	0	46	12	11	6	0
9	4	5	2	0	47	12	13	3	0
10	4	5	2	1	48	13	12	3	0
11	4	11	6	0	49	13	24	4	0
12	5	4	2	0	50	14	11	4	0
13	5	4	2	1	51	14	11	4	1
14	5	6	4	0	52	14	15	5	0

(continued on next page)

Table C.1 (continued)

ID	FN	TN	t_a^0	LT	ID	FN	TN	t_a^0	LT
15	5	9	5	0	53	14	15	5	1
16	5	9	5	1	54	14	23	4	0
17	6	2	5	0	55	15	10	6	0
18	6	5	4	0	56	15	10	6	1
19	6	8	2	0	57	15	14	5	0
20	6	8	2	1	58	15	14	5	1
21	7	8	3	0	59	15	19	3	0
22	7	18	2	0	60	15	22	3	0
23	8	6	2	0	61	16	8	5	0
24	8	6	2	1	62	16	10	4	0
25	8	7	3	0	63	16	17	2	0
26	8	9	10	0	64	16	18	3	0
27	8	16	5	0	65	17	10	8	0
28	9	5	5	0	66	17	16	2	0
29	9	5	5	1	67	17	19	2	0
30	9	8	10	0	68	18	7	2	0
31	9	10	3	0	69	18	16	3	0
32	10	9	3	0	70	18	20	4	0
33	10	11	5	0	71	19	15	3	0
34	10	11	5	1	72	19	17	2	0
35	10	15	6	0	73	19	20	4	0
36	10	15	6	1	74	20	18	4	0
37	10	16	4	0	75	20	19	4	0
38	10	17	8	0	76	20	21	6	0
77	20	22	5	0	84	22	23	4	0
78	21	20	6	0	85	23	14	4	0
79	21	22	2	0	86	23	22	4	0
80	21	24	3	0	87	23	24	2	0
81	22	15	3	0	88	24	13	4	0
82	22	20	5	0	89	24	21	3	0
83	22	21	2	0	90	24	23	2	0

Note: FN-from node, TN-to node, and LT-link type, LT = 1 if link is an AV link and 0 otherwise.

Table C.2

OD demand in modified Sioux Falls network.

O	D	\bar{q}_w	O	D	\bar{q}_w	O	D	\bar{q}_w
1	2	0.35	5	14	0.35	13	7	1.4
1	4	1.75	5	15	0.7	13	10	6.65
1	5	0.7	5	20	0.35	13	11	3.5
1	7	1.75	7	1	1.75	13	14	2.1
1	10	4.55	7	2	0.7	13	15	2.45
1	11	1.75	7	4	1.4	13	20	2.1
1	13	1.75	7	5	0.7	14	1	1.05
1	14	1.05	7	10	6.65	14	2	0.35
1	15	1.75	7	11	1.75	14	4	1.75
1	20	1.05	7	13	1.4	14	5	0.35
2	1	0.35	7	14	0.7	14	7	0.7
2	4	0.7	7	15	1.75	14	10	7.35
2	5	0.35	7	20	1.75	14	11	5.6
2	7	0.7	10	1	4.55	14	13	2.1
2	10	2.1	10	2	2.1	14	15	4.55
2	11	0.7	10	4	4.2	14	20	1.75
2	13	1.05	10	5	3.5	15	1	1.75
2	14	0.35	10	7	6.65	15	2	0.35
2	15	0.35	10	11	14	15	4	1.75
2	20	0.35	10	13	6.65	15	5	0.7
4	1	1.75	10	14	7.35	15	7	1.75
4	2	0.7	10	15	14	15	10	14
4	5	1.75	10	20	8.75	15	11	4.9
4	7	1.4	11	1	1.75	15	13	2.45
4	10	4.2	11	2	0.7	15	14	4.55
4	11	4.9	11	4	5.25	15	20	3.85

(continued on next page)

Table C.2 (continued)

O	D	\bar{q}_w	O	D	\bar{q}_w	O	D	\bar{q}_w
4	13	2.1	11	5	1.75	20	1	1.05
4	14	1.75	11	7	1.75	20	2	0.35
4	15	1.75	11	10	13.65	20	4	1.05
4	20	1.05	11	13	3.5	20	5	0.35
5	1	0.7	11	14	5.6	20	7	1.75
5	2	0.35	11	15	4.9	20	10	8.75
5	4	1.75	11	20	2.1	20	11	2.1
5	7	0.7	13	1	1.75	20	13	2.1
5	10	3.5	13	2	1.05	20	14	1.75
5	11	1.75	13	4	2.1	20	15	3.85
5	13	0.7	13	5	0.7			

Note: O-origin, D-destination.

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