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Optimal Routing of Wide Corridors

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Abstract

Techniques for tracing minimum-cost routes between nodes in a cost-weighted network have long been used in conjunction with geographic information systems (GIS) to allocate linear features such as roads, pipes, or cables. One of the problems with such techniques, however, is that they regard these features as truly linear in nature: having length but no appreciable width. When siting features such as rights-of-way, wildlife corridors, or greenways that clearly do encompass two significant dimensions, the whole area of each feature must be considered when attempting to minimize cost. Past approaches to the optimal routing of such wide corridors have generally been either ineffective or prohibitively complex. This paper proposes an alternative strategy that promises both efficiency and effectiveness and does so with both simplicity and flexibility. In fact, this technique requires no new software at all. It merely requires that the routing problem be seen from a different perspective: one that addresses path width not in terms of the path itself but in terms of the field of costs that the path must traverse.

1. Introduction

Greenways are often regarded as corridors: tracts of land associated with some sort of movement from one place to another. In some cases, this movement relates to directly to physical activities such as hiking, biking, or the migration of wildlife. In others, no actual motion is involved, but movement is nonetheless implied by an attempt to achieve physical connectivity among geographical features or conditions. In either case, the task of siting an effective and efficient greenway is sufficiently similar to the task of determining "the best way to get from here to there" that greenway planning often calls for the use of geographic information systems (GIS) to implement formal path-finding procedures.

These are procedures that have long been used in fields ranging from transportation planning to the design of electrical circuits, and they tend to be cast in terms of conventions relating to network theory. Here, a ("graph" or) "network" is a set of ("vertices" or) "nodes," any one of which may be connected to others by way of (an "arc," an "edge," or) a "link" whose traversal incurs a ("weight" or) "cost." An "optimal path" is one that connects one set of (one or more) "origin" nodes to another set of "destination" nodes by way of intervening links such that the accumulation of incremental costs associated with those links is as low as possible. And "optimal routing" is the process of identifying such a path.

Among the most often used algorithms for optimal routing is one (Dijkstra, 1959) that

- assigns a distance value of zero to all origin nodes and infinity to all others;
- places all nodes onto a distance-sorted list of available nodes;
- for the next available node listed, considers each of the other available nodes to which that listed node is directly linked and,
    - for each of those neighbors (unless a lower distance value

has already been assigned to that neighbor);
            - assigns a distance value equal to sum of the listed node's distance plus the cost of its link to that neighbor;
            - notes this was the link responsible for that neighbor's distance; and
            - repositions that neighbor on the distance-sorted list of available nodes; then
        - removes the listed node from the list of those still available; and
    - repeats the previous step until no more destination nodes are available.

The effect is analogous to the strategy that a hungry mouse might employ in making its way through a covered maze to a fragrant piece of cheese. How does that mouse know which way to turn? It is, of course, whichever direction yields the most intense scent. And why does this yield the shortest path? It's because of the way scents dissipate. As they waft through that maze, they all diminish at the same rate and are, in effect competing with one another to reach more and more distant locations. Thus, the first scent to reach any given location will always have done so by way of the shortest possible path from the cheese to that location.

Optimal routing techniques like this are often used to determine the shortest, quickest, or least-costly path from one place to another in a transportation network. If that network were to be a system of roads, for example, the nodes would be a set of road intersections or stops along the way; the links would be the stretches of road between those nodes; and the cost of each link would be expressed in terms of miles, minutes, or dollars. Routing capabilities for this sort of network have recently fallen into the hands of millions (quite literally) for whom mobile phones and global positioning systems (GPS) have made their use a routine part of everyday life.

Those with access to geographic information systems (GIS) routinely use optimal routing techniques to find minimum-cost paths not only through networks of well-defined alternatives, but also through landscapes in which there are no well-defined lines of transportation. In this case, "every square inch" of an area is characterized in terms of the incremental cost that would be incurred if that location were to be traversed by a path. This would be the case, for example, if one were to try to minimize the construction cost of a pipeline though a region in which there are no pre-existing rights-of-way, but there are variations topography, hydrology, geology, vegetation, ownership, and development that could all affect those costs.

In order to represent these variations across such a region, digital maps of the region are usually encoded as "raster"-based images (as opposed to "vector"-based drawings) in which "grid cells" act much like the pixels (picture elements) in a digital photograph. They are sample points arranged in a rectilinear grid pattern of equally spaced rows and columns that is projected onto a cartographic plane. Thus, each cell is uniquely associated with a particular point location on the ground. Numerical values are then used to record each cell's geographical conditions, and the set of values representing a particular condition (such as soil type, for example) over all cells within a given region constitute one of that region's cartographic "layers."

In Figure 1 is an example. Here, selected conditions within a typical landscape are depicted in shades of gray that indicate the relative cost of traversing each with a pipeline right-of-way. The darkest (most costly) conditions are ponds and areas in close proximity to buildings. Less dark (and less costly) are wetlands, then roads, then steep forests, then not-so-steep forests, and finally non-forested fields. Within each of these conditions, a subtle mottling of gray represents minor and random variation in these costs. Concentric black circles mark the two (arbitrarily selected) termini that are to be connected by this pipeline. Also in black is the pipeline path that makes this connection in a manner that minimizes the overall amount of darkness (cost) encountered. To generate that path, increments of cost are accumulated in waves that emanate from the left terminus as indicated by pale gray contour lines. The path of lowest total cost from any given location to that left terminus will always run perpendicular to those contours.

Fig. 1. A Path of Minimum Cost. The relative cost of traversing each location within a geographical area is indicated in gray, with darker shades corresponding to higher costs.
The linear path in black connects the two termini depicted as concentric black circles with the lowest possible sum of those incremental costs. Contour lines indicate how these costs accumulate with distance from the left terminus.

One of the fundamental limitations of this path-finding technique is that the path involved is truly linear in nature. Although it has length, it has no appreciable width. When siting features such as rights-of-way, wildlife corridors, or greenways that fully encompass a two dimensional area, that whole area must be considered when attempting to minimize cost.

2. Background

Surprisingly little has been published on the routing of such wide corridors. What is perhaps the most notable to date (Goncalves, 2011) cites only two other attempts. One (LaRue and Nelson,

2008) uses traditional methods to site what amount to the centerlines of paths that are then widened in order to consider costs in their vicinity. The problem with this approach is that the widening process encompasses areas that were never included in the routing procedure and whose costs may therefore result in sub-optimal routes. The second attempt (Majka, *et al.,* 2007) uses a well-known technique for the identification of N^{th}-best paths to delineate swaths whose widths vary according the manner in which those paths happen to meander. While this does allocate wider routes, it does not find the optimal corridor of a specified minimum width. The publication in which these attempts are cited (Goncalves, 2011) presents a new technique that is quite successful in routing optimal corridors of a specified width. It does so, however, by way of a procedure that appears to be much more complex than may be necessary.

3. Objective

This paper proposes an alternative strategy, a technique that promises both efficiency and effectiveness and does so with both remarkable simplicity and welcome flexibility. In fact, this technique requires no new software beyond that which is likely to be available in any general-purpose GIS. It merely requires that the routing problem be seen from a different perspective.

4. Methodology

The key to this strategy is suspiciously straightforward. Rather than attempting to route a wide corridor *per se*, that problem is cast in terms of routing the centerline for such a corridor. Since a centerline is a linear path with no appreciable width, conventional path-finding techniques can be used to find the centerline of minimum cost between two designated termini. And once an optimal centerline has been located, its corridor can be established by simply identifying all locations within a given distance (half the desired corridor width) of that line.

At first glance, this strategy may appear to be identical to that of LaRue and Nelson. Note, however, that the centerline of a wide corridor is not merely a linear path. Whereas an optimal path must thread its way through a field of obstructions in a manner that attempts to avoid them, a corridor's centerline must not only avoid those obstructions but also avoid coming close to them. How close? The minimum distance would be half of the corridor's width. Why? Because any obstruction whose proximity to a corridor's centerline is less than half of the corridor's width will lie within that corridor and therefore increase the corridor's cost.
Given this logic, the incremental cost of traversing any given grid cell with the centerline of a wide corridor must now reflect not only the local cost of that cell itself but also the cost of every other cell within a radius of half the corridor's width. How to do this? Start with the layer of incremental costs through which a corridor is to be routed. For each cell, determine the sum, the maximum, or some other function of those incremental-cost values for all cells that lie within its half-corridor-width vicinity. The result will be a revised layer of incremental costs that can now used in the same manner as the original.

In Figure 2 is an example. Here, the pipeline right-of way to be routed involves the same landscape, the same termini, and the same costs as were shown in Figure 1. In this case, however, those costs (still represented by shades of gray that get darker as costs increase) have been allowed to "blur" in order to reflect the fact that this right-of-way is to be wider. As a result, the centerline for that new right-of-way, again shown in black, is one that not only tries to avoid locations of higher cost but also tries to avoid areas in the vicinity of such locations.

Fig. 2. The Centerline of Minimum Cost. The relative cost of traversing each location within a geographical area is indicated in gray, with darker shades corresponding to higher costs. The linear path in black connects the two termini depicted as concentric black circles with the lowest possible sum of those incremental costs.

3. Results

To appreciate the effect of the routing technique presented in Figure 2, consider the three pipeline rights-of-way depicted in Figure 3. Here, the costs depicted in pale shades of gray are the same as those shown in Figure 1. So are the pipeline termini (concentric black circles) and the linear path of minimum cost (thin black line). In dark gray, however, is the set of all locations within a specified distance of the wider right-of-way centerline that was shown in Figure 2. Those locations comprise the right-of-way of lowest-possible total cost between the two termini at that width. And in a slightly lighter shade of dark gray is the optimal route for an even wider corridor.

Fig. 3. Minimum-cost Corridors of Different Widths. The relative cost of traversing each location within a geographical area is indicated in pale gray, with darker shades corresponding to higher costs. The linear corridor in black connects the two termini depicted as concentric black circles with the lowest possible sum of those incremental costs. The wider corridor in dark gray and the even-wider corridor in a lighter shade of dark gray do the same, given that each of the corridors must maintain a specified width.

4. Discussion

Since the key to this technique lies in the dispersal of incremental travel costs to neighboring locations, and since that can be done in a variety of ways, it is worthwhile to consider how this particular step affects routing results. The relationship is illustrated in Figure 4, where

- each of five typical locations is labeled **A**, **B**, **C**, **D**, or **E**;
- the degree to which each location's cost has been dispersed to neighboring locations is indicated by the gray circle surrounding it, with lighter shades of gray representing lower costs;
- a minimum-cost corridor is stippled; and

- the centerline of that corridor is dashed.

Here, the impact of each of those locations on that corridor is proportional to the total amount of its dispersed cost that is traversed by the corridor's centerline. Thus, the influence of any given location on a corridor that ultimately includes that location will decrease with (the cosine of) distance from the centerline of that corridor.

If each location's cost is dispersed such that less is distributed to more distance neighbors, as illustrated by Figure 4's location **D**, this will cause the influence of any given location to be even greater as the distance between that location and the centerline of a prospective corridor decreases.

Interestingly (and usefully), the converse is also true. If each location's cost is dispersed such that amounts increase with (the sine of) distance, as illustrated by Figure 4's location **E**, this can be used to nullify the inherently greater influence of costs that are located closer to a prospective corridor's centerline.

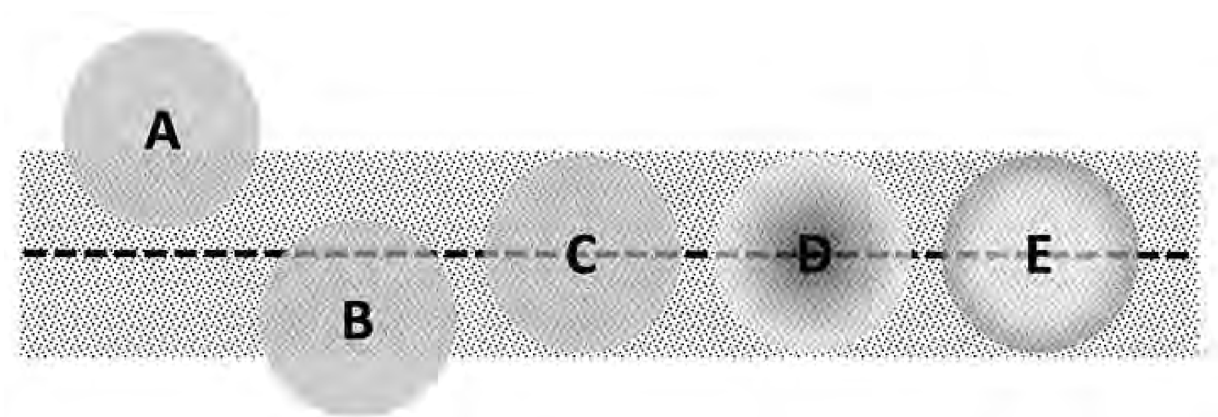

Fig. 4. The Local Effect of Cost Dispersal on Minimum-cost Routing. If the cost of traversing a given location (**A**, **B**, **C**, **D**, or **E**) is dispersed to all neighboring locations within a given distance (shaded in gray), any minimum-cost corridor (stippled) that is ultimately routed in that location's vicinity will be affected by the location's cost to the extent that is indicated by the amount and intensity of cost (grayness) crossed by that corridor's (dashed) centerline.

5. Conclusions

So what about greenways? Can the routing technique that so far been illustrated only by way of a pipeline-siting example be adapted for use in siting environmental corridors? Easily.

Perhaps most fundamental difference between pipeline- and greenway-siting problems is that the latter tend to be cast not only (or even) in terms of costs to be minimized but also (or instead) in terms of benefits to be maximized. To use the routing technique proposed requires that those benefits be expressed as costs that are lower than the norm (though, for technical reasons, are never allowed to be negative). This will cause prospective routes in the vicinity of such beneficial areas to be attracted toward them in order to enjoy their lower costs.

It is also quite easy to increase the geographical reach of this attractive force and thereby cause more distant routes to veer toward such beneficial areas. To do so requires only that proximity to those areas be included among the factors defining incremental costs.

If there are certain beneficial areas that must be included in a greenway to be sited, those areas need only be cast as additional termini. If more that one such additional terminus is introduced, however, this will require an efficient strategy to determine how best (i.e. with minimal cost) to connect those termini to one another, given what may well be a large number of alternatives. Fortunately, this is a familiar problem in network theory for which a number of solutions are available.

And just as fortunately, the wide-corridor routing technique introduced here is generally compatible with them all. This is because the routing component of that routing technique is really nothing new. The only new idea here is just as modest as it is effective: rather than change the optimization procedure, simply change the costs to be optimized. By doing so, well-known tools and techniques for the generation of optimal circuits, trees, networks, and so on can all be applied to problems in which those linear forms are not quite so linear after all.

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