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APPLICATION OF DISTRIBUTED OPTIMIZATION TECHNIQUE FOR LARGE-SCALE OPTIMAL POWER FLOW PROBLEM

by

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Abstract

Optimal Power Flow (OPF) is one the most basic problems in power system analysis. In the last decades many studies have been done to provide a robust and fast solution for OPF. The main goal of the Optimal Power Flow problem is determining an operating point that minimizes the power system objectives such as generation cost, emission or power loss.

The conventional optimization algorithms for Optimal Power Flow are centralized algorithms. These conventional centralized algorithms encounter two challenges. First, most times the generation units in a power network belong to different owners that do not want to share their confidential information with other power generation companies. Second, when the number of buses significantly increase the optimization problem will be very complicated. In these cases, finding the optimal solution takes time and in some cases even the solution does not converge.

In order to solve the large-scale Optimal Power Flow problem in power networks and deal with the problems brought by the system size, distributed parallel processing algorithms, which are known distributed optimization techniques, are sought. In distributed parallel algorithms, each processor tries to solve a sub-problem independently based on limited information communication.

This research discusses a consensus-based Alternating Direction Method of Multipliers (ADMM) approach for solving the OPF problem. In distributed optimization, the whole power system should be split into some partitions. Sub-problems which are related to partitions should be solved by their assigned local processors in parallel. The local processors have to be networked. In the proposed distributed optimization technique, the optimal point of the whole system can be obtained throughout the ADMM iterative process. In this thesis, ADMM

implementation on an OPF problem for some IEEE cases has been presented and the optimal solution obtained by ADMM and MATPOWER (a MATLAB base program) are compared.

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Dedication

This is dedicated to my parents and my supervisor.

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Chapter 1

Introduction to Thesis

1.1. Research Background

Every year the size of power networks will be larger and larger. As a result, the power system problems are getting more complicated. Importance of providing an optimization technique to deal with these sophisticated power system problems is increasing.

In the last decades many optimization techniques for Optimal Power Flow (OPF) have been presented. The main goal of all proposed techniques in the literature is finding a better solution efficiently. Many studies are dedicated to using metaheuristic optimization techniques including genetic algorithm, particle swarm optimization, simulated annealing, ant colony optimization, bacterial foraging technique, imperial competition algorithm, etc. for solving Optimal Power Flow.

One of the most interesting methods to solve OPF is using distributed optimization technique that many researchers have paid attention in the last few years. The main idea of using distributed optimization for OPF is to split the main power system into some sub-systems and to solve them independently by local computers and then integrate the results of the sub-systems to find the optimal solution of the main problem.

1.2. Research Objective

The main objectives of this thesis are listed as follows:

- To understand the analytical and stochastic optimization techniques and their differences and multi objective optimization concepts.
- To study the formulation of Economic Dispatch and Optimal Power Flow and their equality and inequality constraints.
- To study different objectives of Optimal Power Flow problem and their formulation.
- To review the required mathematics for distributed optimization such as Lagrangian function, Duality theory, linear and quadratic programming, Augmented Lagrangian function and Alternating Direction Method of Multipliers.
- To demonstrate how the power system can be split into some partitions to be suitable for distributed optimization technique.
- To implement proposed splitting concept on some IEEE cases.
- To reformulate ADMM to solve AC-OPF and DC-OPF in a distributed fashion.
- To create sub-problems and their constraints and assigning them to local processor in order to solve them in parallel.
- To present the steps for solving the AC-OPF and DC-OPF by consensus optimization along with typical 6-bus and 3-bus power system, respectively.
- To present the communication strategies between local processors in a distributed manner.
- To implement the proposed algorithm on some IEEE cases and compare the results with conventional techniques.

1.3. Organization of the thesis

This thesis has been organized such that introduction to optimization, Economic Dispatch (ED) and Optimal Power Flow (OPF) formulations are discussed in early chapters and distributed optimization techniques and its application to AC and DC Optimal Power Flow problem is discussed in later chapters. In Chapter 2, a short review of classical optimization techniques is presented. Also, the results of applying these techniques to Economic Dispatch (ED) are provided. In Chapter 3, the Optimal Power Flow problem and its constraints are discussed in detail. In Chapter 4, required background for distributed optimization and some mathematical approach for optimization such as linear and quadratic programming, Kuhn-Tucker, Lagrangian method of multipliers, and Alternating Direction Method of Multipliers (ADMM) have been presented. In Chapter 5, the ADMM is reformulated for the Optimal Power Flow problem. The approaches for partitioning the power system to be suitable for solving by ADMM is discussed in detail. Also, the suitable form of distributed optimization for AC-OPF and DC-OPF has been explained through some typical power systems and the communication strategies between local processors are presented.

In Chapter 6, the proposed algorithm presented in Chapter 5 has been applied on some IEEE case studies and results are compared with the conventional solutions. Chapter 7 concludes the thesis by highlighting the key contribution of this approach and presenting various suggestions for future works.

Chapter 2

Review of Classical Optimization Techniques

2.1. Introduction

In the real world, many problems exist that need to be optimized. The main goal of optimization is finding all feasible solutions which correspond to minimizing or maximizing all specified objectives while satisfying all constraints of the problem. Fundamentally, all optimization problems could be classified into two categories: Single-objective optimization methods that involve a single objective function which result in a single solution, and multi-objective optimization technique that considers several conflicting objectives simultaneously. In this case, there is not a certain single optimal solution, but a set of alternatives, with different trade-offs, which is called Pareto optimal solutions. Despite the existence of multiple Pareto optimal solutions, only one of these solutions is acceptable. In this chapter, the most important classical methods of optimization are presented.

2.2. Problem Formulation

In order to optimize a problem, the problem must first be modeled. Creating an appropriate mathematical model for an optimization problem is as important as the optimization method itself. This research is not about modeling of the problems thus modeling aspects have not been considered in this thesis.

Typically, a comprehensive model of optimization problem can be presented as follows:

Minimize or Maximize

$F_1(x_1, x_2, \dots, x_n)$

$$F_i(x_1, x_2, \dots, x_n) \qquad i = 1, 2, \dots, m \tag{2.1}$$

$F_m(x_1, x_2, \dots, x_n)$

Subject to

Equality Constraints $\quad G_j(x_1, x_2, \dots, x_n) = 0 \qquad j = 1, 2, \dots, p \qquad (2.2)$

Inequality Constraints $\quad H_k(x_1, x_2, \dots, x_n) \leq 0 \qquad k = 1, 2, \dots, q \qquad (2.3)$

Where: $X = \{x_1, x_2, \dots, x_n\}$ are n independent variables that will be obtained by applying optimization technique.

$F = \{ F_1, \dots, F_m\}$ are m objective functions that need be minimized or maximized.

$G = \{G_1, \dots, G_p\}$ are p equality constraints that should be satisfied.

$H = \{H_1, \dots, H_q\}$ are q inequality constraints that should be satisfied.

In the case that $(m = 1)$, the problem known as single objective optimization problem. In the case that $(m > 1)$, the problem known as a multi objective optimization problem.

In case, there are not any equality or inequality constraints the problem is known as Unconstrained Optimization Problem. If there is at least one constraint, the problem is classified as a Constrained Optimization Problem. Practically, in engineering problems, most cases are Constrained Multi-Objective Optimization Problems.

2.3. Optimization Techniques

Many optimization algorithms are available to engineers. However, many methods are appropriate only for certain types of problems. Thus, it is important to be able to recognize the characteristics of a problem in order to identify an appropriate solution technique. Within each class of problems, there are different minimization methods, varying in computational requirements, convergence properties, and so on. Optimization problems are classified according to the mathematical characteristics of the objective function, the constraints, and the control variables. The most important characteristic is the nature of the objective function [1]. These classifications are summarized in Table 2-1.

Table 2-1: Classification of Objective Functions

Characteristic	Property	Classification
Number of Variables	One	Univariate Optimization
	Two or more	Multivariate Optimization
Type of Independent Variables	Continuous	Continuous Optimization
	Integers or binary	Integer Optimization
Problem Function	Both Continuous and Integer	Mixed Integer Optimization
	Linear Function of Independent Variables	Linear Optimization
	Quadratic Function of Independent Variables	Quadratic Optimization
	Nonlinear Function of Independent Variable	Nonlinear Optimization
Problem Formulation	With Constraints	Constrained Optimization
	Without Constraints	Unconstrained Optimization

2.4. Classification of Optimization Techniques

Optimization Techniques can be classified into two specific categories: Analytical Methods and Metaheuristic Optimization Algorithms.

2.4.1. Analytical Methods

These methods are based on classical methods such as gradient method, line search technique, Lagrange multiplier method, Newton-Raphson optimization technique and Karush-Kahn-Tucker method. Most of the classic methods are based on derivation concepts and are applicable to convex functions. These algorithms usually present some inconveniences due to the danger of convergence, algorithmic complexity, long execution time and generation of a weak number of non-dominated solutions.

Figure 2-1 illustrates convex function versus non-convex function. In convex functions, there is just one optimal point that is the global solution, while non-convex functions have many local optimal points that the global solution is one of them.

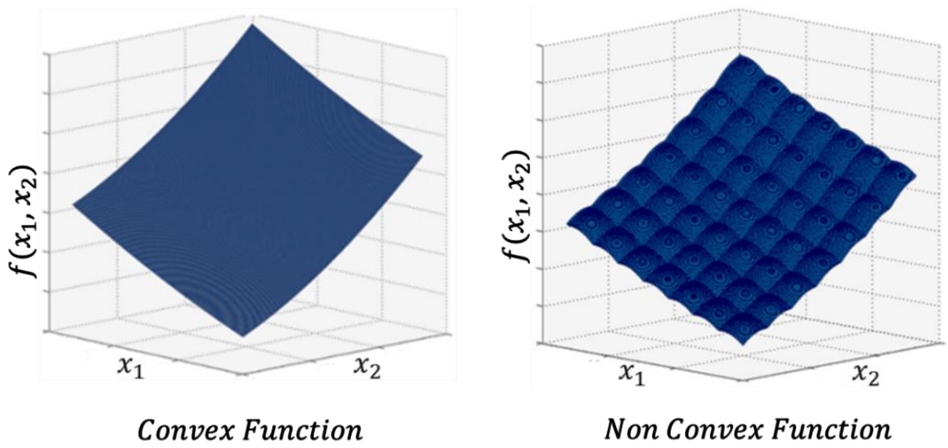

Figure 2-1: Comparison Between Convex and Non-convex Functions

2.5. Metaheuristic Optimization Algorithms

These algorithms are based on the evolutionary techniques such as the Genetic Algorithm (GA) [1], Particle Swarm Optimization (PSO) [2], Simulated Annealing (SA) [3], Imperial Competition Algorithm (ICA) [4], and Ant Colony Optimization (ACO) [3]. In these methods, an initial trial solution is selected, either using at random or common sense, and the objective function is evaluated. A move is made to a new point (second trial solution), and the objective function is evaluated again. If it is smaller than the value for the first trial solution, it is retained, and another move is made. The process is repeated until the minimum is found.

Evolutionary Techniques can be used in the following cases:

- The objective function is non-Convex.
- The number of variables and constraints is large.
- The problem functions (objective or constraint) are highly nonlinear.
- The problem functions (objective or constraint) are implicit in terms of the decision/control variables making the evaluation of derivative information difficult.

2.5.1. The Basic Concept of Particle Swarm Optimization

Particle Swarm Optimization (PSO) was developed by Kennedy and Eberhart in 1995. PSO is based on swarm behavior such as fish and bird schooling in nature. Many algorithms (such as ant colony algorithms and virtual ant algorithms) use the behavior of the so-called swarm intelligence. Though particle swarm optimization has many similarities with genetic algorithms and virtual ant algorithms, it is much simpler because it does not use mutation/crossover operators or pheromone. Instead, it uses the real-number randomness and the global communication among the swarm particles. In this sense, it is also easier to implement as there is no encoding or decoding of the parameters into binary strings as those in genetic algorithms.

This algorithm searches a space of an objective function by adjusting the trajectories of individual agents, called particles, as the piecewise path formed by positional vectors in a quasi-stochastic manner. The particle movement has two major components: a stochastic component and a deterministic component. The particle is attracted toward the position of the current global best while at the same time it has a tendency to move randomly. When a particle finds a location that is better than any previously found locations, then it updates it as the new current best for particle i. This is a current best for all n particles. The aim is to find the global best among all the current best until the objective no longer improves or after a certain number of iterations [3].

2.5.2. The Basic Concept of Simulated Annealing

Simulated Annealing (SA) is a random search technique for global optimization problems. It mimics the annealing process in material processing when a metal cools and freezes into a crystalline state which has minimum energy and larger crystal size so as to reduce the defects in metallic structures. The annealing process involves the careful control of temperature and cooling rate (often called annealing schedule). The application of simulated annealing into optimization problems is presented in [3]. Since then, there have been extensive studies. Unlike the gradient-based methods and other deterministic search methods which have the disadvantage of being trapped into local minima, the main advantage of the simulated annealing is its ability to avoid being trapped in local minima. In fact, it has been proved that the simulated annealing will converge to its global optimality if enough randomness is used in combination with very slow cooling.

Generally speaking, this is equivalent to dropping some bouncing balls over a landscape, and as the balls bounce and lose energy, they settle down to some local minima. If the balls are allowed to bounce enough times and loose energy slowly enough, some of the balls will eventually fall into the global [3].

2.5.3. The Basic Concept of Genetic Algorithms

Genetic Algorithms (GAs) are general-purpose search techniques based on principles inspired by the genetic and evolution mechanisms observed in natural systems and populations of living beings. Their basic principle is the maintenance of a population of solutions to a problem (genotypes) in the form of encoded individual information that evolves in time. A Genetic Algorithm for a particular problem must have the following five components:

- A genetic representation of a potential solution to the problem.
- A way to create an initial population of potential solutions.
- An evaluation function that plays the role of the environment, rating solutions in terms of their “fitness.”
- Genetic operators that alter the composition of children.
- Values for various parameters that the GA uses (population size, probabilities of applying genetic operators, etc.)

A genetic search starts with a randomly-generated initial population within which each individual is evaluated by means of a fitness function. Individuals in this and subsequent generations are duplicated or eliminated according to their fitness values. Further generations are created by applying GA operators. This eventually leads to a generation of high-performing individuals [1].

2.5.4. The Basic Concept of Imperial Competition Algorithm

Imperial Competition Algorithm (ICA) originally proposed by Atashpaz and Lucas [4], ICA is a population-based meta-heuristic search technique. In ICA, each member of the population is called a country and specified by a vector containing the problem variables. Some of the best countries are selected as imperialists, and the other countries are colonies of these imperialists. According to their power, all the colonies are distributed among the imperialists. An imperialist along with its colonies is called an empire.

Based on the assimilation policy, each colony moves towards the relevant imperialist by a deviation from the connecting line between the colony and its imperialist.

In order to escape local optima, ICA makes use of a revolution operator. This operator randomly selects some countries and replaces them with new random positions. As a colony moves towards an imperialist, there is the possibility that the colony will reach a position with better quality than that of the imperialist. In this case, the imperialist and the colony change their positions and the algorithm will be continued using this new country as the imperialist.

The most important process of ICA is the imperialistic competition in which all the empires attempt to take the possession of the colonies of the other empires and control them. Through the imperialistic competition the power of the weaker empires will decrease, and consequently, the power of the stronger ones will increase. This process is modeled by picking one of the weakest colonies of the weakest empires and creating a competition among all the empires to possess this colony. In this competition, based on its total power, each empire has the probability of taking the possession of the colony.

During the imperialistic competition, powerless empires collapse in the imperialistic competition and the corresponding colonies will be divided among the other empires. Moving colonies toward imperialists continuously and perform imperialistic competition during the search process. When the number of iterations reaches a pre-defined value, the search process stops [4].

2.6. Multi-Objective Optimization

Most of the optimization problems have only a single objective. Practically, multiple objectives need to optimize simultaneously. For instance, increasing the performance of a system while trying to minimize the cost at the same time. In this case, we are dealing with multi-objective optimization problems.

Multi-objective optimization problems, unlike single objective problems, do not necessarily have an optimal answer that minimizes all the objective functions simultaneously. In practice, different objectives may conflict with each other. This means that the optimal parameters of some objectives might make the optimality of other objectives worse. Many methods have been introduced for multi-objective optimization. The weighted sum method is one of them.

2.6.1. Weighted Sum Method

In the weighted sum method, among all objectives, a tradeoff or a certain balance of objectives need to be chosen. Finding a scalar-valued function that represents weighted combinations of all objectives is one of the most popular approaches. A simple way to construct this scalar function is to use the weighted sum formula as follows:

$$F = \sum_{i=1}^{n} \alpha_i f_i\,(\bar{x}) \tag{2.4}$$

Where

$\bar{x}$: is an independent variable vector.

F : is the obtained single objective problem.

α_i : are the coefficient of objective functions.

n : is the number of objective functions.

By using the weighted sum technique, a multi-objective problem including n objectives converts to a single objective problem.

2.7. Application of Optimization in Economic Dispatch Problem

2.7.1. Economic Dispatch Problem (EDP)

Economic Dispatch is a dynamic problem for minimizing fuel cost of thermal units while considering power system constraints which should iteratively be solved by changing the load demand in the network. Different methods have been proposed for solving this problem in the least possible time.

The fuel cost function for thermal units is a quadratic convex function. Considering valve loading effect, a sinusoidal term with limited range is added to this quadratic function which makes the fuel cost function non-convex.

The purpose of economic dispatch is calculating the power generated by thermal units for satisfying demand and network losses in order to minimize the total fuel cost of thermal units.

2.7.2. Cost Function of Thermal Units with Multiple Fuels

Each thermal unit has a unique cost function that shows the cost for any specific active power that is generated by unit. Equation (2.5) shows the thermal units' fuel cost model with multiple fuel types. Considering the loading effect adds a sinusoidal term to a quadratic function. In

Equation (2.5), $FC_i(P_i)$ is fuel a cost of ith thermal unit and $a_{i,k}, b_{i,k}, c_{i,k}, e_{i,k}, f_{i,k}$ are coefficients of kth fuel in the ith generator. In a multiple fuel model, certain fuel is used in each thermal unit depending on the produced power of each thermal unit [5].

$$FC_i(P_i) = \begin{cases} a_{i,1}p_i^2 + b_{i,1}p_i + c_{i,1} + \left|e_{i,1}\sin\left(f_{i,1}\left(p_i^{min} - p_i\right)\right)\right| \\ \quad p_i^{min} \le p_i \le p_{i,1} \qquad fuel\ type\ 1 \\ a_{i,2}p_i^2 + b_{i,2}p_i + c_{i,2} + \left|e_{i,2}\sin\left(f_{i,2}\left(p_i^{min} - p_i\right)\right)\right| \\ \quad p_{i,1} \le p_i \le p_{i,2} \qquad fuel\ type\ 2 \\ \quad \vdots \\ a_{i,k}p_i^2 + b_{i,k}p_i + c_{i,k} + \left|e_{i,k}\sin\left(f_{i,k}\left(p_i^{min} - p_i\right)\right)\right| \\ \quad p_{i,k-1} \le p_i \le p_i^{max} \qquad fuel\ type\ k \end{cases} \tag{2.5}$$

The purpose of EDP is minimizing thermal units costs based on Equation (2.6).

$$Min\ TC(P) = \sum_{i=1}^{N} FC_i(p_i) \tag{2.6}$$

$$P = [p_1, p_2, \dots, p_N] \tag{2.7}$$

Where:

$TC(P)$ is the total cost of thermal units.

N is the number of thermal units.

p_i is indicated power of ith generator that needs to be optimized.

2.7.3. Economic Dispatch Constraints

One of the EDP constraints is the equality of summation of load demand and power loss with total generated power of all thermal units. Equation (2.8) shows this constraint:

$$\sum_{i=1}^{N} p_i = P_D + P_L \tag{2.8}$$

In which P_D , P_L show consumed power and power line losses of network, respectively.

Another constraint is the power boundary of units which has been presented in Equation (2.9) in which $\left(p_i^{min}, p_i^{max}\right)$ is power bound of ith generator.

$$p_i^{min} \leq p_i \leq p_i^{max} \tag{2.9}$$

The total network loss is expressed by a quadratic function of generators' output power as Equation (2.10):

$$P_L = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} p_i B_{ij} p_j + \sum_{i=1}^{N_g} B_{0i} p_i + B_{00} \tag{2.10}$$

In which B_{ij} is ijth element of the B coefficient matrix.

2.7.4. Prohibited Operating Zones' Constraints

Errors in generators, pumps, boilers, and other equipment may create instability in a certain range of generator output power. Due to these errors, the generator is prohibited from power generation in that area. This matter results in a non-continuous fuel cost function. The prohibited operating zones (POZs) create constraint (2.11) for EDP.

$$p_j \in \begin{cases} p_j^{min} \leq p_j \leq p_j^{LB_1} \\ p_j^{UB_{K-1}} \leq p_j \leq p_j^{LB_K} & j = 1, .., N_g \\ p_j^{UB_K} \leq p_j \leq p_j^{max} \end{cases} \tag{2.11}$$

Figure 2-2 shows a typical fuel cost function considering valve effect, prohibited operating zone, and multiple fuel types.

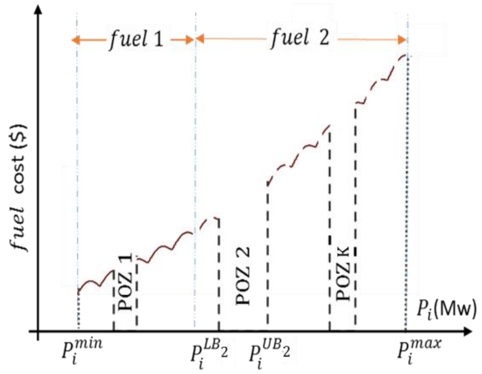

Figure 2-2: Fuel Cost Curve with Considering Multi-fuel and POZ

2.7.5. Case Study of Economic Dispatch (PowerWorld Simulator)

A 7-bus power system including 5 thermal units has been considered as a case study. Figure 2-3 demonstrates the power system in PowerWorld Simulator.

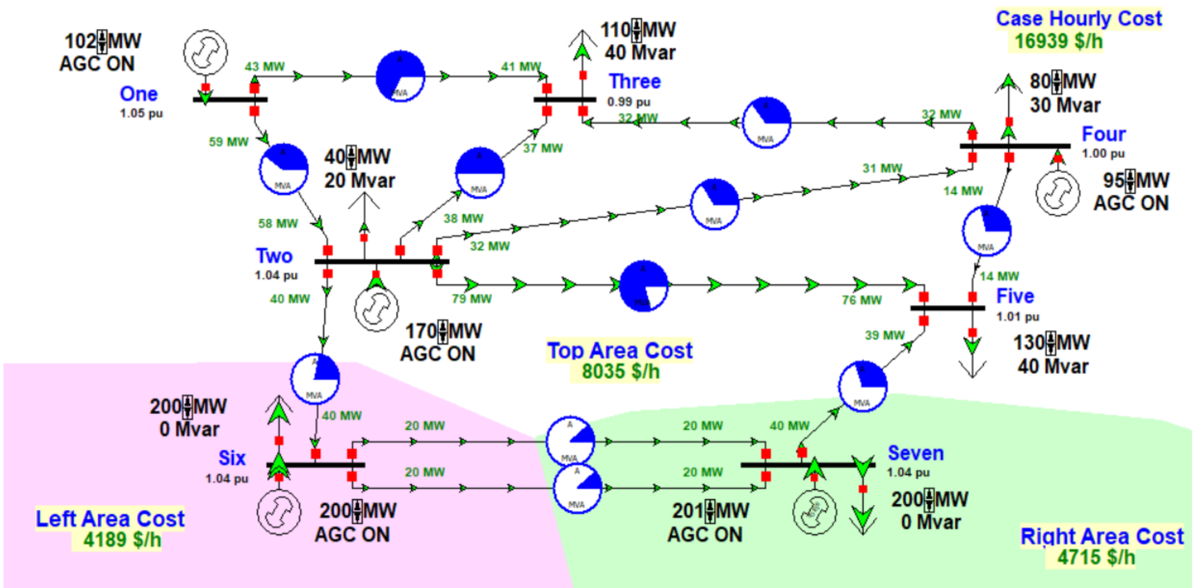

Figure 2-3: Seven-bus Power System in The PowerWorld Simulator

This case study has 5 generators that have to supply 760 MW. The total fuel cost function is provided by Equation (2.12) as follows:

$$total\ cost = \sum_{i=1}^{5} a_i p_i^2 + b_i p_i + c_i \tag{2.12}$$

Where

$a_1 = 0.0013$ / $a_2 = 0.00136$/ $a_3 = 0.00134$ / $a_4 = 0.00131$/ $a_5 = 0.00194$
$b_1 = 7.62$ / $b_2 = 7.52$ / $b_3 = 7.84$ / $b_4 = 7.57$ / $b_5 = 7.77$
$c_1 = 761.94$/ $c_2 = 831.84$ / $c_3 = 530.03$ / $c_4 = 831.92$ / $c_5 = 500.08$

The system equality and inequality constraints are as follows: (all numbers are in MW)

$$p_1 + p_2 + p_3 + p_4 + p_5 = 760 \tag{2.13}$$

$$100 < p_1 < 400 \tag{2.14}$$

$$150 < p_2 < 500 \tag{2.15}$$

$$50 < p_3 < 200 \tag{2.16}$$

$$150 < p_4 < 500 \tag{2.17}$$

$$0 < p_5 < 600 \tag{2.18}$$

In order to solve this case study, the MATLAB toolbox has been used. To compare the answers of the problem, two solutions “fmincon” and “genetic algorithm” have been applied. The obtained results for each optimization technique are as follows:

By applying “fmincon”, the total cost is C$9425.6, and active powers are as follows:

$p_1 = 179.77$MW, $p_2 = 208.60$MW, $p_3 = 92.31$MW, $p_4 = 197.48$MW, $p_5 = 81.80$MW

when “Genetic Algorithm (GA)” has been used, the obtained total cost is the same as “fmincon” while active powers are not same. GA results are as follows:

$p_1 = 181.38\text{MW}, \quad p_2 = 206.94\text{MW}, \quad p_3 = 94.60\text{MW}, \quad p_4 = 192.70\text{MW}, \quad p_5 = 84.35\text{MW}$

Due to the total cost of these techniques are equal both obtained active power can be used without any priority.

2.7.6. The Second Case Study for the Economic Dispatch Problem

A case study including 10 thermal units with 3 types of fuel and valve loading effect has been considered [5]. Table 2-2 represents the coefficients of multiple fuel cost functions. The total demanded power of the system is 2700MW. Optimal points for 10 units are presented in Table 2-3. These optimal points are obtained by solving the EDP with, the briefly presented methods, Particle Swarm Optimization (PSO) and Genetic Algorithm (GA). The optimal power and total cost of the thermal units are presented in Table 2-3 [5].

Table 2-2: Coefficients of Multiple Fuel Cost Functions [5]

Unit	Generation Min P_1 P_2 Max F1 F2 F3	Fuel type	Cost coefficients				
			a_i	b_i	c_i	e_i	f_i
1	100 196 250 1 2	1	.2697e2	-.3975e0	.2176e-2	.2697e-1	-.3975e1
		2	.2113e2	-.3059e0	.1861e-2	.2113e-1	-.3059e1
2	50 114 157 230 2 3 1	1	.1184e3	-.1269e1	.4194e-2	.1184e0	-.1269e2
		2	.1865e1	-.3988e-1	.1138e-2	.1865e-2	-.3988e0
		3	.1365e2	-.1980e0	.1620e-2	.1365e-1	-.1980e1
3	200 332 388 500 1 3 2	1	.3979e2	-.3116e0	.1457e-2	.3979e-1	-.3116e1
		2	-.5914e2	.4864e0	.1176e-4	-.5914e-1	.4864e1
		3	-.2875e1	.3389e-1	.8035e-3	-.2876e-2	.3389e0
4	99 138 200 265 1 2 3	1	.1983e1	-.3114e-1	.1049e-2	.1983e-2	-.3114e0
		2	.5285e2	-.6348e0	.2758e-2	.5285e-1	-.6348e1
		3	.2668e3	-.2338e1	.5935e-2	.2668e0	-.2338e2
5	190 338 407 490 1 2 3	1	.1392e2	-.8733e-1	.1066e-2	.1392e-1	-.8733e0
		2	.9976e2	-.5206e0	.1597e-2	.9976e-1	-.5206e1
		3	-.5399e2	.4462e0	.1498e-3	-.5399e-1	.4462e1
6	85 138 200 265 2 1 3	1	.5285e2	-.6348e0	.2758e-2	.5285e-1	-.6348e1
		2	.1983e1	-.3114e-1	.1049e-2	.1983e-2	-.3114e0
		3	.2668e3	-.2338e1	.5935e-2	.2668e0	-.2338e2
7	200 331 391 500 1 2 3	1	.1893e2	-.1325e0	.1107e-2	.1893e-1	-.1325e1
		2	.4377e2	-.2267e0	.1165e-2	.4377e-1	-.2267e1
		3	-.4335e2	.3559e0	.2454e-3	-.4335e-1	.3559e1
8	99 138 200 265 1 2 3	1	.1983e1	-.3114e-1	.1049e-2	.1983e-2	-.3114e0
		2	.5285e2	-.6348e0	.2758e-2	.5285e-1	-.6348e1
		3	.2668e3	-.2338e1	.5935e-2	.2668e0	-.2338e2
9	130 213 370 440 3 1 3	1	.8853e2	-.5675e0	.1554e-2	.8853e-1	-.5675e1
		2	.1530e2	-.4514e-1	.7033e-2	.1423e-1	-.1817e0
		3	.1423e2	-.1817e-1	.6121e-3	.1423e-1	-.1817e0
10	200 362 407 490 1 3 2	1	.1397e2	-.9938e-1	.1102e-2	.1397e-1	-.9938e0
		2	-.6113e2	.5084e0	.4164e-4	-.6113e-1	.5084e1
		3	.4671e2	-.2024e0	.1137e-2	.4671e-1	-.2024e1

In Table 2-2, the unit of all generation powers is MW. Also, the unit of coefficients a, b are $^{\$}/_{MW^2}$, $^{\$}/_{MW}$, respectively.

Table 2-3: Optimal Point Obtained by Some Metaheuristic Algorithms

Variables	PSO	GA
p_1 (MW)	225.573	219.996
p_2 (MW)	208.224	212.701
p_3 (MW)	278.806	283.739
p_4 (MW)	238.027	240.521
p_5 (MW)	282.414	282.313
p_6 (MW)	239.639	240.579
p_7 (MW)	285.427	293.085
p_8 (MW)	239.092	240.311
p_9 (MW)	425.586	406.980
p_{10} (MW)	277.212	279.775
Total Power (MW)	2700	2700
Total Cost ($)	624.304	624.963

2.7.7. The Third Case Study for Economic Dispatch

This power system includes forty power generation units with valve-point loading effects. The power loss is ignored in this case study. The demanded power is considered 10500 MW and the system information is obtainable from [6]. Because of the high dimension, this case study is challenging and obtaining the global optimum is time-consuming and difficult. In order to provide a bird's eye view on most of well-known metaheuristic techniques, the performance of the most familiar stochastic optimization techniques including ICA [4], HBMO [6], CHBMO [6], IHBMO [6], MPSO [8], DEC-SQP [8], PSO-SQP [8], FCASO [9], BBO [9], SOH-PSO [10], IFEP [11], MFEP [11], PAA [12], ESO [13], PSO-LRS [14], IGA [15], GA [10], PSO [10], NPSO [16], ST-HDE [17], HDE [8], EP-SQP [18], PSO-GM [19], TS [20], ACO [20], NPSO-LRS [14], APSO [14], SOH-PSO [17] and CSO [18] in terms of minimum, mean and maximum costs have been listed in Table 2-4.

The question that comes to mind is how the problem can be solved when the number of generating units is many times higher than 40. Clearly, results obtained by one processor in the case that there is a very large number of generating units have a low accuracy, and in some cases solution cannot even be converged. This matter shows the importance of using distributed optimization techniques that will be discussed in the next chapters.

Table 2-4: Comparison Between Stochastic Optimization Techniques

Algorithm	Minimum cost ($/h)	Mean cost ($/h)	Maximum cost ($/h)
MuICA	121430.45	121640.28	121846.04
ICA	122297.90	122625.46	123174.96
CASO	121865.63	122100.74	-
HBMO	121639.38	121851.77	121939.74
CHBMO	121639.38	121851.77	121939.74
IHBMO	121517.8	121589.18	121711.85
MPSO	122252.27	-	-
DEC-SQP	122174.16	122295.13	-
PSO-SQP	122094.67	122295.13	-
FCASO	121516.47	122082.59	-
BBO	121479.50	-	-
SOH-PSO	121501.14	-	-
IFEP	122624.35	123382.00	125740.63
MFEP	122647.57	123489.47	-
PAA	122243.18	122243.18	122243.18
ESO	122122.16	122558.45	123143.07
PSO-LRS	122035.79	122558.45	123461.67
IGA	121915.93	122811.41	123334.00
GA	121819.25	-	-
PSO	122513.91	122513.91	123467.40
NPSO	121704.73	122221.30	122995.09
ST-HDE	121698.51	122304.30	-
HDE	121698.51	122304.30	-
EP-SQP	122323.97	122379.63	-
PSO-GM	121845.98	122398.38	123219.22
TS	122288.38	122590.89	122424.81
ACO	121811.37	122048.06	121930.58
NPSO-LRS	121664.43	122209.31	122981.59
APSO	121663.52	122153.67	122912.39
SOH-PSO	121501.14	121853.57	122446.30
CSO	121461.67	121936.19	122844.53

2.8. Conclusion

This chapter has introduced classical optimization techniques for finding the optimal solution of a formulated problem. Modern optimization techniques based on metaheuristic search algorithms such as particle swarm optimization, simulated annealing, genetic algorithm and imperial competition algorithm have been described briefly. In addition, weighted sum, as one of the most basic techniques for Multi-objective optimization, has been presented. The formulation of the Economic Dispatch Problem, including the cost curve as the objective function and the prohibited operating zone as optimization constraints, has been explained.

Due to the main goal of this chapter was introducing classical optimization techniques for solving power system problems, the results of applying some classical techniques to 3 different case studies have been presented and compared.

Chapter 3

Optimal Power Flow Studies for Power Systems

3.1. Introduction to Power Flow

The flow of electrical power in any interconnected electrical system is termed load flow. A Load Flow Study is conducted to calculate the voltages at the various buses. The load flow study, one of the most basic and fundamental problems in the power system, concerns solving a set of static nonlinear equations. The load flow problem is formulated on the basis of Kirchhoff's laws in terms of voltages' amplitude and voltage phase at each node, and active and reactive power injections in the system.

Power systems typically have 3 different kinds of buses. Table 3-1 shows the types of buses and their associated known and unknown values.

Table 3-1: Power System Buses Classification

Type of Buses	Specified Quantities	Quantities to be Determined
Generation or P-V Bus	P, |V|	Q, δ
Load or P-Q Bus	P, Q	|V|, δ
Slack or Reference Bus	|V|, δ	P, Q

Where

P is the active power of the generator

Q is the reactive power of the generator

|V| is the amplitude of the voltage bus

δ is the phase angle of the voltage bus

3.1.1. Slack or Reference Bus

The slack bus in a power system emits or absorbs the active or reactive power. The magnitude and phase angle of the voltage are specified in the slack bus. The phase angle and magnitude of the voltage is usually set at zero and one per unit, respectively. The active and reactive power of this bus is usually determined through the solution of the load flow.

3.1.2. Generation Bus

The voltage magnitude and active power for the generation buses are specified, and the reactive power generation and voltage phase angle still have to be computed. The generation bus is also called the P-V bus, and in this kind of bus the voltage magnitude is maintained at a constant specified value by injection of reactive power.

3.1.3. Load Bus

In the load bus, also called the P-Q bus, the active and reactive power are specified, and the magnitude and phase angle of the voltage are to be computed.

To summarize, for a load flow problem, the active power and voltage magnitudes are specified values for load buses; the active and reactive power demands are also given. One generator bus is taken as the reference bus, with a specified voltage magnitude and phase angle. The problem is to find the reactive power generation and phase angles at the generating buses along with the voltage magnitudes and their angles at the load buses.

3.2. Power Flow Equations

Power flow studies are the backbone of power systems analysis. The principal information obtained from a power flow study is the magnitude and phase angle of the voltage at each bus and once all voltages and angles are available, the real and reactive power flow through transmission lines can be calculated. In any power system within N buses, the power injections at the buses can be defined by a set of $2N$ nonlinear equations.

$$P_G^k - P_D^k = V_k \sum_{i=1}^{N} \left[V_j \left[G_j^k \cos\left(\delta^k - \delta^j\right) + B_j^k \sin\left(\delta^k - \delta^j\right) \right] \right] \tag{3.1}$$

$$Q_G^k - Q_D^k = V_k \sum_{i=1}^{N} \left[V_j \left[G_j^k \sin\left(\delta^k - \delta^j\right) - B_j^k \cos\left(\delta^k - \delta^j\right) \right] \right] \tag{3.2}$$

Where $k = 1, 2, \ldots, N$.

V_k is the voltage magnitude at bus k^{th}.

δ^k is the voltage angle at bus k^{th}.

P_G^k is the active power generation at bus k^{th}.

P_D^k is the active power demand at bus k^{th}.

Q_G^k is the reactive power generation at bus k^{th}.

Q_D^k is the reactive power demand at bus k^{th}.

G_j^k is the real part of element (k, j) of the bus admittance matrix.

B_j^k is the imaginary part of element (k, j) of the bus admittance matrix.

Figure 3-1 shows a generic bus in a power system.

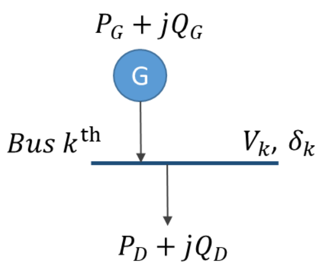

Figure 3-1: A Typical Bus in A Power System

3.3. Optimal Power Flow

For electric power systems with thermal generation units, an important economic operation function is Optimal Power Flow (OPF). In contrast to the load flow problem, where active power generations are specified, for Optimal Power Flow, the optimal generations are sought to optimize the OPF objectives, such as the operating cost of the power system.

The OPF problem seeks to find an optimal profile of active power along with voltage magnitudes. The aim is minimizing the total operating costs of an all-thermal electric power system while satisfying network security constraints. OPF is a more realistic formulation than the conventional economic dispatch, which does not account for the operating constraints of the network.

3.4. Optimal Power Flow Formulation

Based on the number of considered objectives, the Optimal Power Flow problem can be formulated mathematically as a nonlinear constrained optimization problem. The OPF problem has many control variables that need to be specified and subjected to a variety of equality and inequality constraints. The control variables are defined by the equation below in the form of a vector that is called $\bar{v}$. Optimal Power Flow state variables, which are called x, are defined as follows [1].

$$optimal\ power\ flow\ state\ variables: \quad \bar{x} = [V_L^t,\ \theta^t,\ P_{SG},\ Q_G^t] \tag{3.3}$$

$$optimal\ power\ flow\ control\ variables: \quad \bar{v} = [Q_c^t,\ TC^t,\ V_G^t,\ P_G^t] \tag{3.4}$$

Where

Q_c^t is the reactive power supplied by all shunt reactors.

TC^t is the transformer load tap changer magnitudes.

V_G^t is the voltage magnitude at the PV buses.

P_G^t is the active power generated at the PV buses.

N_L is the number of the load buses.

N_G is the number of the generator buses (PV buses).

V_L is the voltage magnitude at the PQ buses (load buses).

θ is the voltage angles of all buses, for the slack bus θ is considered zero.

P_{SG} is the active power of the slack bus.

Q_G is the reactive power of all generator units.

3.4.1. The Objective Functions

The Optimal Power Flow problem has typically six objective functions. These functions are fuel cost generation, active and/or reactive power transmission loss, reactive power reserve margin, security margin index, and emission index. As fuel cost is the main objective of the Optimal Power Flow problem, in this research, only the fuel cost has been considered as objective function. Some of these objective functions are conflicting in nature, which is what makes the OPF problem complicated. As such, a multi-objective optimization method needs to be applied to solve the OPF problem [1].

3.4.2. Minimization of Generation Fuel Cost (GFC)

The main objective function of the Optimal Power Flow problem is minimizing the Generation Fuel Cost of thermal units. The Generation Fuel Cost objective function can be expressed by a quadratic function as follows:

$$Minimize\ TFC = \sum_{k=1}^{N_G} FC_k(P_{Gk}) = \sum_{k=1}^{N_G} a_k P_{Gk}^2 + b_k P_{Gk} + c_k \tag{3.5}$$

Where:

TFC is the total fuel cost of the power system.

$FC_k(P_{Gk})$ is the fuel cost function of the k^{th} generator.

N_G is the number of generators including the slack generator.

a_k and b_k are the quadratic and linear cost coefficient of the k^{th} generator.

c_k is the basic cost coefficient of the k^{th} generator.

P_{Gk} is the real power output of the k^{th} generator.

3.4.3. Minimization of Active Power Transmission Loss (APTL)

Active Power Transmission Loss (APTL) is the second objective function for the Optimal Power Flow problem. The primary goal of this problem is to minimize power loss throughout the power network, an objective that can be represented by the following equations.

$$Minimize\ APTL = \sum_{k=1}^{N_G} P_{Gk} - \sum_{k=1}^{N_G} P_{Dk} \tag{3.6}$$

Where

P_{Gk} is the active generated power at the k^{th} bus.

P_{Dk} is the active demanded power at the k^{th} bus.

$APTL$ represents the total RI^2 loss in the transmission lines and transformers of the network.

3.4.4. Minimization of Reactive Power Transmission Loss (RPTL)

In the Optimal Power Flow problem, the Reactive Power Transmission Loss (RPTL) as an objective function needs to be minimized. The reactive power loss can be expressed by the difference between all generated reactive power and all demanded reactive power in the power system. It should not be forgotten that reactive power loss is not necessarily positive. The following equation represents the Reactive Power Transmission Loss.

$$Minimize\ RPTL = \sum_{k=1}^{N_G} Q_{Gk} - \sum_{k=1}^{N_G} Q_{Dk} \tag{3.7}$$

Where

$RPTL$ represents the total Reactive Power Transmission Loss in the power system.

Q_{Gk} is the reactive generated power at the k^{th} bus.

Q_{Dk} is the reactive demanded power at the k^{th} bus.

3.4.5. Maximization of Reactive Power Reserve Margin (RPRM)

Maximizing the Reactive Power Reserve Margins (RPRM) and seeking to distribute the reserve among the generators is one of the Optimal Power Flow objectives. This objective can be converted to a minimization problem and stated by the following function.

$$Minimize\ RPRM = \sum_{k=1}^{N_G} \left[\frac{Q_k^2}{Q_{k\ max}} \right] \tag{3.8}$$

Where

Q_k is the reactive power of the k^{th} generator.

$Q_{k\ max}$ is the maximum reactive power of the k^{th} generator.

3.4.6. Minimization of Emission Index (EI)

The Emission or environmental Index (EI) is taken as the index from the viewpoint of environmental conservation. Atmospheric pollutants like nitrogen oxides (NOx) and sulfur oxides (SOx) caused by fossil-fueled thermal units can be modeled separately. However, the OPF problem seeks to minimize the total (Ton/h) emission $E(P_G)$ of these pollutants, which can be stated by the following equations. As indicated, the amount of emissions is given as a function of the generators' active power output, which is the sum of the quadratic and exponential functions [1].

$$Minimize\ TE = \sum_{k=1}^{N_G} E_k(P_{Gk}) = \sum_{k=1}^{N_G} \alpha_k P_{Gk}^2 + \beta_k P_{Gk} + \gamma_k + \zeta_k e^{\lambda_k P_{Gk}} \tag{3.9}$$

Where

TE is the total emission of the power system.

$E_k(P_{Gk})$ is the emission function of the k^{th} generator.

$\alpha_k, \beta_k, \gamma_k, \zeta_k\ and\ \lambda_k$ are the coefficients of the k^{th} generator.

3.4.7. Maximization of Security Margin Index (SMI)

The last objective function is the Security Margin Index (SMI). The OPF problem seeks to operate all the transmission lines connected in a network to their maximum capability [1].

3.5. The Optimal Power Flow Constraints

For any nonlinear constrained optimization problem such as OPF, two types of constraints, equality and inequality, can typically be defined. For the Optimal Power Flow problem, the equality constraints are active and reactive power balances.

The inequality constraints of the optimal power flow problem are active and reactive power generation boundaries, the reactive power source capacity boundary, the transformer tap position range, and the line thermal limit for all transmission lines.

In order to represent a practical model of the Optimal Power Flow problem with the required accuracy, objective functions that have been subjected to some constraints should be considered.

3.5.1. Equality Constraints

Equality constraints of the Optimal Power Flow problem refer to the physics of the power system. These constraints indicate that the total injection of the active and reactive power at each bus is zero.

3.5.2. Active Power Constraints

Equation (3.10) states that the total active power in all power system buses is zero.

$$P_G^k - P_D^k = V_k \sum_{i=1}^{N} \left[V_j \left[G_j^k \cos\left(\delta^k - \delta^j\right) + B_j^k \sin\left(\delta^k - \delta^j\right) \right] \right] \qquad (3.10)$$

Where $k = 1, 2, \dots, N$.

V_k is the voltage magnitude at bus k^{th}.

δ^k is the voltage angle at bus k^{th}.

P_G^k is the active power generation at bus k^{th}.

P_D^k is the active power demand at bus k^{th}.

G_j^k is the real part of element (k, j) of the bus admittance matrix.

B_j^k is the imaginary part of element (k, j) of the bus admittance matrix.

3.5.3. Reactive Power Constraints

Equation (3.11) indicates that the total reactive power in all power system buses is zero.

$$Q_G^k - Q_D^k = V_k \sum_{i=1}^{N} \left[V_j \left[G_j^k \sin\left(\delta^k - \delta^j\right) - B_j^k \cos\left(\delta^k - \delta^j\right) \right] \right] \qquad (3.11)$$

Where $k = 1, 2, \dots, N$.

V_k is the voltage magnitude at bus k^{th}.

δ^k is the voltage angle at bus k^{th}.

Q_G^k and Q_D^k are the reactive power generation and power demand at bus k^{th}.

G_j^k is the real part of element (k, j) of the bus admittance matrix.

B_j^k is the imaginary part of element (k, j) of the bus admittance matrix.

3.5.4. Inequality Constraints

In order to present the limitations of the power system components, such as generators, transmission lines, and transformers, as well as system security, the inequality constraints of the OPF should be presented. The Optimal Power Flow inequality constraints are as follows:

3.5.5. Bus Voltage Magnitude Constraints

In the power system, the voltage of all buses, whether load buses or generations buses, is restricted by lower and upper limits as follows:

$$V_k^{Min} < V_k < V_k^{Max} \qquad (3.12)$$

Where

V_k is the voltage at the k^{th} bus.

V_k^{Min} is the minimum acceptable voltage at the k^{th} bus.

V_k^{Max} is the maximum acceptable voltage at the k^{th} bus.

3.5.6. Active and Reactive Power Generation Constraints for all Units

All power system generators have limitations for their generated power. Hence, the active and reactive power output of each generator in the power system is restricted by lower and upper limits. The below equations represent the boundaries of active and reactive power.

$$P_{Gk}^{Min} < P_{Gk} < P_{Gk}^{Max} \tag{3.13}$$

$$Q_{Gk}^{Min} < Q_{Gk} < Q_{Gk}^{Max} \tag{3.14}$$

Where:

P_{Gk} is the active power generated by the k^{th} generator.

Q_{Gk} is the reactive power generated by the k^{th} generator.

P_{Gk}^{Min} is the minimum active power generated by the k^{th} generator.

P_{Gk}^{Max} is the maximum active power generated by the k^{th} generator.

Q_{Gk}^{Min} is the minimum reactive power generated by the k^{th} generator.

Q_{Gk}^{Max} is the maximum reactive power generated by the k^{th} generator.

P_{Gi}^{Max} is the specified maximum MW generation by the i^{th} generator.

P_{Gi}^{Min} is the specified minimum MW generation by the i^{th} generator.

3.5.7. Reactive Power Source Capacity Constraints

In power systems, capacitors play the role of reactive power sources. They inject reactive power into the power network to maintain system stability. As a reactive power source, these capacitors are restricted by upper and lower reactive power limits.

3.5.8. Transformer Tap Position Constraints

In all transformers, the magnitude of the load tap changer is a discrete variable that can be changed by a specific increment. Load tap changing transformers also have a maximum and minimum tap ratio which can be adjusted.

3.5.9. Line Thermal Limit Constraints for All Transmission Lines

Considering the stability of the power system, the power flow over a transmission line must not exceed a specified maximum.

3.6. A Case Study for Optimal Power Flow

As an example, the IEEE 30-bus case study which its information extracted from [21] has been considered. This system includes 6 generators, 4 tap-changing transformers and 2 capacitor banks leading to 17 decision variables: 5 variables for the active power of the generators (the first generator considered as a slack generator), 6 variables for the voltage of the generation buses, 4 variables for tap changers, and finally 2 variables for capacitor banks. For this case study, the OPF problem is solved by Genetic Algorithm, Particle Swarm Optimization and Simulated Annealing by considering fuel cost as objective function and active power, reactive power, bus voltage, and phase angle boundaries as inequality constraints. Active and reactive power balances are considered as the equality constraints for this case study.

The cost function of each generator has been considered as Equation (3.15).

$$FC_i(P_{G_i}) = a_i {P_{G_i}}^2 + b_i P_{G_i} + c_i + \left| e_i \sin\left(f_i \left({P_{G_i}}^{min} - P_{G_i} \right) \right) \right| \tag{3.15}$$

The optimization vector for this case study is defined as follows:

$$x = \left[P_{G_2}, \dots, P_{G_6}, V_{G_1}, \dots, V_{G_6}, T_1, \dots, T_4, Q_1, Q_2\right] \quad (3.16)$$

Figure 3-2 shows the topology of the case study. The system information including the coefficients of generator, power limits, and prohibited operating zone are also listed in Table 3-2.

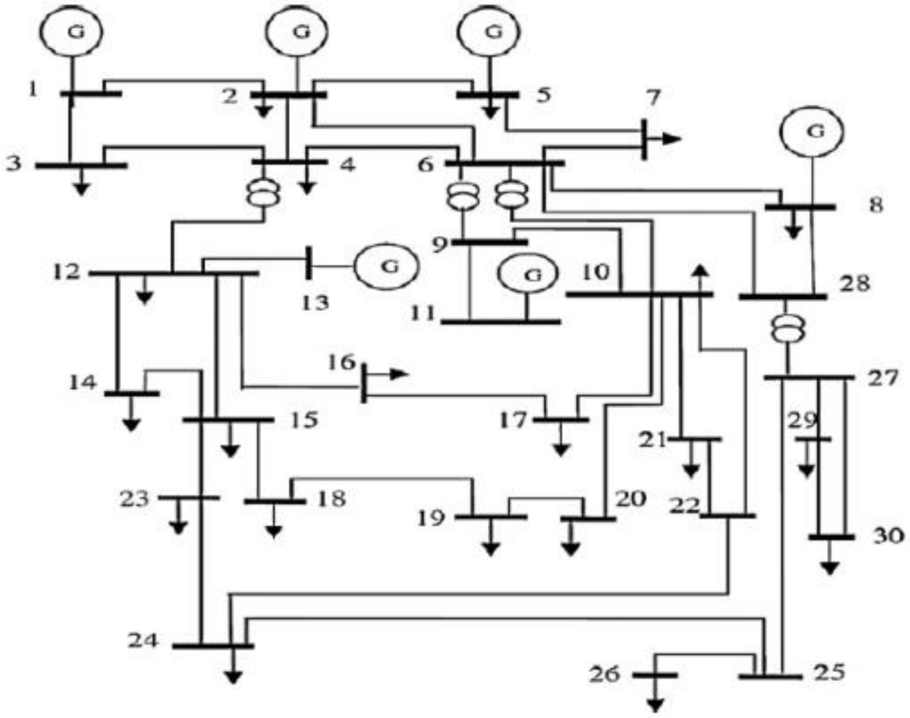

Figure 3-2: Schematic Diagram of IEEE 30-bus Power System

Table 3-2: Information for IEEE 30-bus Power System

Generator	1	2	3	4	5	6
a ($\$/MW^2$)	0.00375	0.0175	0.0625	0.0083	0.025	0.025
b($\$/MW$)	2	1.75	1	3.25	3	3
C	0	0	0	0	0	0
D	18	16	14	12	13	13.5
E	0.037	0.038	0.04	0.045	0.042	0.041
P_{min} (MW)	30	20	15	10	10	12
P_{max} (MW)	250	80	50	35	30	40
Prohibited	[55,66], [80-120]	[21-24], [45-55]	[30-36]	[25-30]	[25-28]	[24-30]
Bus Number	1	2	5	8	11	13

In this case study, bus 1 is considered as the slack bus and the limitations of bus voltage, transformer tap, and compensator reactive power are set to 0.95v< V <1.05v, 0.91< T <1.05, and 1 Mvar < QC < 20 Mvar, respectively.

Table 3-3 shows the minimum fuel cost of the system determined by 3 different well-known metaheuristic techniques including GA, PSO, SA.

Table 3-3: Fuel Cost of The System by 3 Different Solutions

Algorithms	Best
Genetic Algorithm (GA)	838.17 ($/h)
Particle Swarm Optimization	835.47 ($/h)
Simulated Annealing (SA)	836.53 ($/h)

Table 3-4 shows the final results of all optimization variables by Genetic Algorithm.

Table 3-4: The Results of The Case Study Solved by The Genetic Algorithm

PG1(MW)	PG2(MW)	PG5(MW)	PG8(MW)	PG11(MW)
219.7597	26.5523	15.7443	10.1166	10.0267
PG13(MW)	VG1(p.u.)	VG2(p.u.)	VG5(p.u.)	VG8(p.u.)
12.025	1.0199	1.0091	1.0223	1.0362
VG11(p.u.)	VG13(p.u.)	T6-9	T6-10	T4-12
1.0181	1.0450	0.96	0.94	1
T27-28	QC1(MVAR)	QC2(MVAR)	COST($/h)	
1.02	11	11	821.1647333	

3.7. Conclusion

In this chapter, first, a short review about power flow equations has been presented. Then, all buses of the power system have been explained. Also, the objective function and constraints of optimal power flow have been introduced. Finally, multi-objective techniques for solving the optimal power flow problem have been explained.

Also, in this chapter the results of solving the Optimal Power Flow problem on 30-bus IEEE power system by Genetic Algorithm, Particle Swarm Optimization, and Simulated Annealing has been presented and compared. This comparison shows that the results of all techniques are almost close thus depend on the flexibility of the algorithms and their running time any of them can be used.

Chapter 4

Distributed Optimization Techniques

4.1. Introduction

Conventional optimization algorithms are the most common methods for solving the economic dispatch and optimal power flow for power systems. Conventional algorithms can be solved by one computing center; in these techniques, all of the calculations should be solved by one processor. There are three challenges that conventional algorithms encounter. The first challenge is the size of the power system; when it becomes extremely large with more generation and loads, this results in high communication and computation costs because all data have to be transmitted to and processed by the central processor. Second, when the power system is large, there are many buses in the system, and this results in a bigger optimization problem that needs more processing time to be solved. Third, in some cases, the distribution network in a power system belongs to different owners. Due to security concerns, they might refuse to share some of their private data.

In order to solve power system optimization problems with large data sets and security concerns, distributed optimization algorithms are desired [22]. In these decentralized algorithms, the main optimization problem should be split into sub-problems. Each sub-problem is assigned to an agent to solve independently based on limited information communication. In distributed optimization, it has been assumed that each agent only knows its neighbors' information, and despite conventional optimization methods, communication errors and delays in multi-agent systems are not taken into account.

Distributed power system computing employed Alternating Direction Method of Multipliers (ADMM) as one of the well-known distributed optimization methods [23]. This chapter will discuss, the distributed optimization technique and methods of splitting problems into sub-problems.

4.2. Basic Definitions

4.2.1. Norms

For each vector $X = [x_1, x_2, \dots, x_n]$ the $\ell_1 - Norm$ and $\ell_2 - Norm$ can be defined as follows:

$$\ell_1 - Norm: \qquad |X| = \sum_{i=1}^{n} |x_i| \tag{4.1}$$

$$\ell_2 - Norm: \qquad \|X\| = \sqrt{\sum_{i=1}^{n} |x_i|^2} \tag{4.2}$$

4.2.2. Convex Functions

Typically, functions can be divided into two classes: convex and non-convex functions. Convex functions play a significant role in defining the cost or utility function and problem constraints.

A convex function is defined using the following definition.

A function $f: \mathbb{R}^n \Longrightarrow \mathbb{R}$ is convex if domain f is convex

Where

$x_i, x_j \epsilon$ domain f and $0 \leq \delta \leq 1$.

$$f\big(\delta x_i + (1-\delta)x_j\big) \leq \delta f(x_i) + (1-\delta) f(x_j) \tag{4.3}$$

Also, the function $f: \mathbb{R}^n \Longrightarrow \mathbb{R}$ is strictly convex if the inequality in (4.3) holds strictly for $0 < \delta < 1$. Figure 4-1 represents a typical convex and nonconvex function.

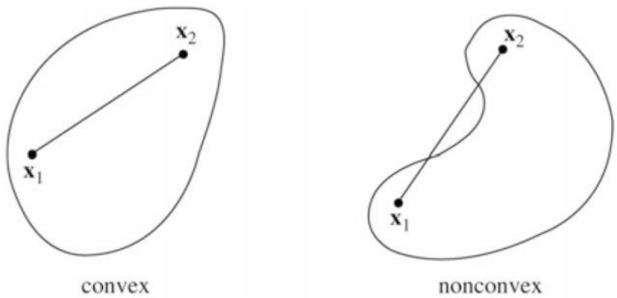

Figure 4-1: Convex vs Non-convex Function

4.3. Convex Optimization Problems

In order to better understand the convex optimization problem, consider the following minimizing optimization problem including equality and inequality constraints.

$$\min f(x) \tag{4.4}$$

$$\text{subject to} \quad \begin{array}{ll} g_i(x) \leq 0 & i = 1, \dots, q \\ h_i(x) = 0 & i = 1, \dots, p \end{array}$$

where

$f(x): \mathbb{R}^n \Longrightarrow \mathbb{R}$ is the objective function.

and, $g_i(x): \mathbb{R}^n \Longrightarrow \mathbb{R}$ for $i = 1, \dots, p$, and $h_i(x): \mathbb{R}^n \Longrightarrow \mathbb{R}$ for $i = 1, \dots, q$ are inequality and equality constraint functions, respectively.

The set defined by the presented constraints is referred to as the feasible set. The goal is to find a solution that minimizes the objective function while belonging to the feasible set. In order to do this, there must exist an x that satisfies all constraints. The obtained solution x is called a feasible solution.

The problem (4.2) is called a convex optimization problem if:

- the functions $f(x)$ and $g_i(x)$ for $i = 1, \dots, q$ are convex;
- the functions $h_i(x) = 0$ for $i = 1, \dots, p$ are affine.

In order to understand the meaning of affine functions as an equality constraint, a linear function from $\mathbb{R}^n \Longrightarrow \mathbb{R}^m$ space is presented in the form of $f(v) = Av$, while affine functions are expressed by $f(v) = Av + b$ where A is an arbitrary $m \times n$ matrix and b is an arbitrary $m \times 1$ matrix. Further, $\mathbb{R}$ can be replaced by any field.

The optimization problem can be classified into different categories, for instance, based on the convexity of the feasible set and the cost function, or according to the characteristics of the cost and inequality constraints.

To be more specific, convex optimization problems are categorized into several subcategories.

4.4. Linear Programming

An optimization problem is called linear if both the objective function and the equality and inequality constraints are all affine. These linear programs (LPs) can be written as follows:

$$\min \quad c^T x + d \tag{4.5}$$

$$\text{subject to} \quad \begin{aligned} Ax &\dot{\leq} a \\ Bx &= b \end{aligned}$$

where $c \in \mathbb{R}^n$, $d \in \mathbb{R}$, $A \in \mathbb{R}^{q\times n}$, $a \in \mathbb{R}^q$, $B \in \mathbb{R}^{p\times n}$ and $b \in \mathbb{R}^p$. Also, the notation $\dot{\leq}$ refers to element-wise inequality.

4.5. Quadratic Problems

Many engineering problems have a convex quadratic cost or utility function. Quadratic programming (QPs) refers to optimization problems with the discussed objective function and affined equality and inequality constraint functions. QPs can be presented as follows:

$$\min \quad \frac{1}{2} x^T P x + Q^T x + C \tag{4.6}$$

$$\text{subject to} \quad \begin{aligned} Ax &\dot{\leq} a \\ Bx &= b \end{aligned}$$

where $P \in \mathbb{S}_+^n$ which $\mathbb{S}_+^n$ is a set of $n \times n$ positive semidefinite matrices, $Q \in \mathbb{R}^n$, $c \in \mathbb{R}$, $A \in \mathbb{R}^{q\times n}$, $a \in \mathbb{R}^q$, $B \in \mathbb{R}^{p\times n}$ and $b \in \mathbb{R}^p$. The notation $\dot{\leq}$ refers to element-wise inequality.

Note that by considering $P = 0$, the quadratic programming problem can be converted to a Linear programming problem.

4.5.1. Quadratic Programming with Convex Quadratic Inequality Constraints

For the quadratic programming problem, simple inequality constraints can be presented in the form of convex quadratic inequality constraints. Equation (4.7) represents the quadratic programming problem with convex quadratic inequality constraints.

$$\min \ \frac{1}{2}x^T P x + Q^T x + c \tag{4.7}$$

$$\text{subject to} \quad \begin{aligned} \frac{1}{2}x^T P_c x + Q_c^T x + C_c \ &\leq 0 \\ Bx = b& \end{aligned}$$

where $P, P_c \in \mathbb{S}_+^n$, $Q_c^T \in \mathbb{R}^n$, $C_c \in \mathbb{R}$, $A \in \mathbb{R}^{q\times n}$, $B \in \mathbb{R}^{p\times n}$ and $b \in \mathbb{R}^p$.

This problem is referred to as a constrained quadratic programming.

4.6. Method of Lagrangian Multipliers

The Method of Lagrangian Multipliers (MLM) is a technique for finding the local maxima and minima of a function with equality constraints. The advantage of MLM is that it solves the optimization problem without explicit parameterization of the problem constraints.

The Method of Lagrangian Multipliers can be used to solve challenging differentiable constrained optimization problems.

The method is based on isolating any possible singular point of the solution set of the constraining equations in order to find all the stationary points of the Lagrange function; then it needs to be verified which of those stationary points and singular points are the global maxima of the objective function.

For the equality constrained optimization problem in the form of Equation (4.8):

$$\min F(x) \tag{4.8}$$

$$\text{subject to} \quad h_i(x) = 0 \quad i = 1, \dots, p$$

By defining λ as a vector of Lagrange multipliers as $\lambda = [\lambda_1, \lambda_2, \dots, \lambda_p]$, the Lagrangian function can be expressed by Equation (4.9).

$$L(x,\lambda) = F(x) + \sum_{i=1}^{p} \lambda_i \times h_i(x) \tag{4.9}$$

Solving $\nabla_{x,y,\lambda} L(x,,y,\lambda) = 0$ can result in all local minima or maxima of the objective function by taking into account all equality constraints.

In order to better understand, consider the example below:

$$Min \quad f(x,y) = x + y \quad s.t. \quad \mathrm{g(x,y)} = x^2 + y^2 - 1 \tag{4.10}$$

First, the Lagrangian function should be defined:

$$L(x,y,\lambda) = x + y - \lambda(x^2 + y^2 - 1) \tag{4.11}$$

The Gradient of $L(x,y,\lambda)$ can be defined as follows:

$$\nabla_{x,y,\lambda} L(x,,y,\lambda) = (\frac{\partial L}{\partial x}, \frac{\partial L}{\partial y}, \frac{\partial L}{\partial \lambda}) \tag{4.12}$$

By considering $\nabla_{x,y,\lambda} L(x,,y,\lambda)$ equal to zero, the following set of equations can be obtained.

$$\nabla_{x,y,\lambda} L(x,,y,\lambda) = 0 \Rightarrow (1 - 2\lambda x, 1 - 2\lambda y, x^2 + y^2 - 1) = 0 \tag{4.13}$$

$$\begin{cases} 1 - 2\lambda x = 0 \\ 1 - 2\lambda y = 0 \\ x^2 + y^2 - 1 = 0 \end{cases} \Longrightarrow x = \frac{\pm\sqrt{2}}{2} \quad , y = \frac{\pm\sqrt{2}}{2} \quad , \lambda = \frac{\pm 1}{\sqrt{2}} \tag{4.14}$$

Now, two local minima of the function have been found. By comparing the value of the function at these two stationary points, $f\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right) = -\sqrt{2}$ as the global minimum of the problem can be found.

4.7. Kuhn–Tucker Conditions

Most optimization problems have inequality constraints. In this case, Kuhn-Tucker conditions can be used in order to solve the problem by using the Lagrangian method. The optimal point is reached if the Kuhn–Tucker conditions are met. These can be stated as follows:

$$\min f(x) \qquad x = [x_1, \dots, x_n] \tag{4.15}$$

$$\text{subject to} \quad \begin{array}{ll} g_i(x) \leq 0 & i = 1, \dots, q \\ h_i(x) = 0 & i = 1, \dots, p \end{array}$$

The Lagrange function can be formed by Equation (4.16) as follows:

$$L(x, \lambda, \mu) = f(x) + \sum_{i=1}^{p} \lambda_i \times h_i(x) + \sum_{i=1}^{q} \mu_i \times g_i(x) \tag{4.16}$$

Consider $(\hat{x}, \hat{\lambda}, \hat{\mu})$ as an optimal point that should satisfy all Kuhn-Tucker conditions. These conditions have been presented as follows:

- $\frac{\partial L}{\partial x_i}(\hat{x}, \hat{\lambda}, \hat{\mu}) = 0 \quad for \quad i = 1,2, \dots, n$ (4.17)
- $h_i(\hat{x}) = 0 \quad for \quad i = 1,2, \dots, p$ (4.18)
- $g_i(\hat{x}) \leq 0 \quad for \quad i = 1,2, \dots, q$ (4.19)
- $\hat{\mu}_i g_i(\hat{x}) = 0 \ , \hat{\mu}_i \geq 0 \quad for \ = 1,2, \dots, q$ (4.20)

4.8. Slack Variable Method

Kuhn-Tucker conditions can be applied to problems that have inequality constraints. Using a slack variable is another technique which transforms the inequality constraints into equality constraints.

In this method, $\theta = [\theta_1, \theta_2, \dots, \theta_q]$ has been defined. The inequality constraints that need to be satisfied ($g_i(x) \leq 0 \quad i = 1, \dots, q$) can be modified to ($g_i(x) + \theta_i^2 = 0$).

Since $g_i(x) = -\theta_i^2 \leq 0$ the inequality constraints of the problem (4.4) are automatically satisfied.

By applying this method, all of the problem constraints are equality constraints. Thus, the Lagrangian method for equality constrained problems may now be applied:

$$L(x, \lambda, \mu, \theta) = f(x) + \sum_{i=1}^{p} \lambda_i \times h_i(x) + \sum_{i=1}^{q} \mu_i \times (g_i(x) + \theta_i^2) \qquad (4.21)$$

where λ_i and μ_i are respective Lagrange multipliers. The associated necessary conditions for a minimum at x are as follows:

$$\frac{\partial L}{\partial x_i} = \frac{\partial f(x)}{\partial x_i} + \sum_{i=1}^{p} \lambda_i \times \frac{\partial h_i(x)}{\partial x_i} + \sum_{i=1}^{q} \mu_i \times \frac{\partial g_i(x)}{\partial x_i} \qquad i = 1, 2, \dots, n \qquad (4.22)$$

$$\frac{\partial L}{\partial \theta_i} = 2\lambda_i \theta_i = 0 \qquad i = 1, 2, \dots, q \qquad (4.23)$$

$$\frac{\partial L}{\partial \lambda_i} = g_i(x) + \theta_i^2 = 0 \qquad i = 1, 2, \dots, p \qquad (4.24)$$

$$\frac{\partial L}{\partial \mu_i} = h_i(x) = 0 \qquad i = 1, 2, \dots, q \qquad (4.25)$$

The system expressed by Equations (4.22-25) represents a system of n+2q+p simultaneous non-linear equations. The answer obtained by solving these equations might be the global solution of the problem.

4.9. Duality Theory

The duality solution is a useful technique for some structural optimization problems. It is also employed in the development of the augmented Lagrange multiplier method to be discussed later.

The dual function of $L(x, \lambda)$ can be stated as follows:

$$h(\lambda) = \underset{x}{\text{minimize}}\ L(x, \lambda) \tag{4.26}$$

The minimization problem (4.18) can be converted to a maximization problem as follows:

$$\underset{\lambda}{\text{maximize}}\ h(\lambda) = \underset{\lambda}{\text{maximize}} \left\{ \underset{x}{\text{minimize}}\ L(x, \lambda) \right\} \tag{4.27}$$

Consider the problem of minimizing the generation costs of two power plants connected to a 10MW load as presented in Figure 4-2.

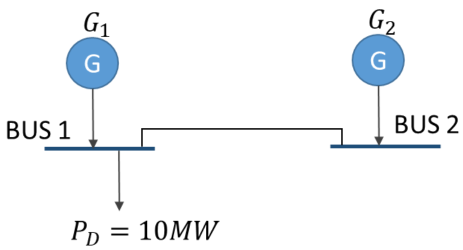

Figure 4-2: A Power Plant with 2 Generation Units

The total generation cost of power plants modeled by Equation (4.28):

$$TC(p_1, p_2) = C(p_1) + C(p_2) = p_1^2 + 2p_1 + 1 + p_2^2 + 4p_2 + 3 \tag{4.28}$$

$$\text{subject to:} \quad p_1 + p_2 = 10 \tag{4.29}$$

The Lagrangian function can be presented as follows.

$$L(p_1, p_2, \lambda) = p_1^2 + 2p_1 + 1 + p_2^2 + 4p_2 + 3 + \lambda(p_1 + p_2 - 10) \tag{4.30}$$

Thus,

$$\frac{\partial L}{\partial p_1} = 2p_1 + 2 + \lambda = 0 \rightarrow p_1 = \frac{-\lambda - 2}{2} \tag{4.31}$$

$$\frac{\partial L}{\partial p_2} = 2p_2 + 4 + \lambda = 0 \ \rightarrow p_2 = \frac{-\lambda - 2}{4} \tag{4.32}$$

$$\frac{\partial L}{\partial \lambda} = p_1 + p_2 - 10 = 0 \ \rightarrow p_1 + p_2 = 10 \tag{4.33}$$

By inserting (4.31) and (4.32) in (4.33), all parameters p_1 , p_2 , λ can be found as follows:

$$\lambda = -13 \ , \qquad p_1 = \frac{11}{2}mw, \quad p_2 = \frac{9}{2}mw \tag{4.34}$$

To solve the problem with the duality method, the optimal value of p_1 , p_2, which are presented by λ in Equations (4.31) and (4.32), should be inserted in the Lagrangian function to form the dual function that is called $h(\lambda)$:

$$h(\lambda) = (\frac{-\lambda - 2}{2})^2 + 2\left(\frac{-\lambda - 2}{2}\right) + 1 + (\frac{-\lambda - 4}{2})^2 + 4\left(\frac{-\lambda - 4}{2}\right) + 3$$

$$+ \lambda\left(\left(\frac{-\lambda - 2}{2}\right) + \left(\frac{-\lambda - 4}{2}\right) - 10\right) = -2\lambda^2 - 52\lambda - 4 = 0 \tag{4.35}$$

Then, by the definition of the duality theorem:

$$\underset{\lambda}{\text{maximize}}\, h(\lambda) = Maximize\ (-2\lambda^2 - 52\lambda - 4) \tag{4.36}$$

The necessary condition for maximizing $h(\lambda)$ is

$$\frac{\partial h(\lambda)}{\partial \lambda} = 0 \ \rightarrow \ \frac{\partial h(\lambda)}{\partial \lambda} = -4\lambda - 52 = 0 \ \rightarrow \lambda = -13 \tag{4.37}$$

Now, by taking $\lambda = -13$, obtained from (4.37), and inserting λ in (4.31) and (4.32) the value of p_1 , p_2 can be reached.

The optimal power values of the generation units are $p_1 = {}^{11}/_{2}\, mw, \quad p_2 = {}^{9}/_{2}\, mw.$

As expected, the results obtained by applying the Lagrangian method and using Duality Theory are the same.

4.10. Augmented Lagrangian Method of Multipliers

The augmented Lagrangian method is a practical solution to obtain the optimal point of constrained optimization problem. This method combines the classical Lagrangian method with the penalty function approach. In the multiplier methods, both of these approaches are combined to give an unconstrained problem which is not ill-conditioned.

As an introduction to the multiplier method, consider the equality constrained problem (4.4).

The augmented Lagrange function is introduced as follows:

$$L(x, \lambda, \rho) = f(x) + \sum_{i=1}^{q} \lambda_i \times h_i(x) + \sum_{i=1}^{q} \rho \times {h_i}^2(x) \tag{4.38}$$

Specifically, the unconstrained objective is the Lagrangian of the constrained problem, with an additional penalty term (the augmentation).

4.11. Alternating Direction Method of Multipliers

The Alternating Direction Method of Multipliers (ADMM) is a type of augmented Lagrangian algorithm which may be implemented in distributed computational environments.

In order to understand ADMM, consider the convex minimization model with linear constraints and an objective function which is the sum of two separable functions:

$$minimize\ f_1(x^1) + f_2(x^2) \tag{4.39}$$

Subject to $A_1x^1 + A_2x^2 = b$

Thus, the augmented Lagrangian function for (4.39) can be stated as follows:

$$\mathrm{L}_c(x^1, x^2, \lambda) = f_1(x^1) + f_2(x^2) + \lambda^T(A_1x^1 + A_2x^2 - b) + \frac{c}{2}\|A_1x^1 + A_2x^2 - b\|^2 \quad (4.40)$$

Note that the problem has at least one optimal solution.

Despite the method of multipliers, the alternating direction method of multipliers minimizes approximately $\mathrm{L}_c(x^1, x^2, \lambda)$ in an iterative order:

$$\begin{cases} x_{k+1}^1 := \arg\min\limits_{x^1} \mathrm{L}_c(x_k^1, x_k^2, \lambda_k) \\ x_{k+1}^2 := \arg\min\limits_{x^1} \mathrm{L}_c(x_{k+1}^1, x_k^2, \lambda_k) \\ \lambda_{k+1} = \lambda_k + c(A_1x_{k+1}^1 + A_2x_{k+1}^2 - b) \end{cases} \quad (4.41)$$

The main idea of ADMM is that each of the smaller minimization problems can be solved more efficiently. This technique is the base of distributed optimization.

4.12. Coupled Optimization Problems and Structure Exploitation

In distributed parallel algorithms, each agent is trying to solve a sub-problem independently based on limited communicated information. Alternating direction method of multipliers (ADMM) is a distributed optimization method that has been adopted in distributed power system computing. The main focus of this research is applying distributed optimization based on ADMM to optimal power flow.

Consider the optimization problem $f(x^1, x^2, \dots, x^n)$ that can be separated to the form of (4.42):

$$minimize\ f_1(x^1) + f_2(x^2) + \ldots + f_i(x^i) + \cdots + f_n(x^n) \quad (4.42)$$

Subject to $\quad x^i - z = 0$

where z is a common global variable for all x^i. This problem is called a global consensus problem. The augmented Lagrangian method of multiplier for conesus optimization can be expressed as (4.43).

$$\mathrm{L}_c(x^1, \dots, x^n, \lambda, c) = \sum_{i=1}^{n} f_i\left(x^i\right) + \sum_{i=1}^{n} \lambda_i^T\left(x^i - z\right) + \sum_{i=1}^{n} \frac{c}{2} \left\|x^i - z\right\|^2 \quad (4.43)$$

Some iterative steps based on k as following must be performed to solve this augmented Lagrangian function. These are as follows:

$$\begin{cases} x_{k+1}^i \coloneqq \arg\min_{x^i} f_i\left(x^i\right) + \lambda_i^{k^T}(x_i - z^k) + \frac{c}{2} \left\|x^i - z_k\right\|^2 \\ z_{k+1} \coloneqq \frac{1}{n}\sum_{i=1}^{n}\left(x_{k+1}^i - \frac{1}{c}\lambda_i^k\right) \\ \lambda_i^{k+1} = \lambda_i^k + c(x_{k+1}^i - z^{k+1}) \end{cases} \quad (4.44)$$

To clarify the method of multipliers, consider the following examples:

4.13. A Case Study for Method of Multipliers

Consider Figure 4-2, the total generation cost of power plants modeled by Equation (4.45).

$$TC(p_1, p_2) = C(p_1) + C(p_2) = p_1^2 + 2p_1 + 1 + p_2^2 + 4p_2 + 3 \quad (4.45)$$

subject to: $p_1 + p_2 = 10$ (4.46)

The augmented Lagrangian function can be presented as follows.

$$L(p_1, p_2, \lambda, c) = p_1^2 + 2p_1 + 1 + p_2^2 + 4p_2 + 3 + \lambda(p_1 + p_2 - 10)$$

$$+\frac{c}{2} \|p_1 + p_2 - 10\|^2 \quad (4.47)$$

Thus,

$$\frac{\partial L}{\partial p_1} = 2p_1 + 2 + \lambda + c(p_1 + p_2 - 10) = 0 \tag{4.48}$$

$$\frac{\partial L}{\partial p_2} = 2p_2 + 4 + \lambda + c(p_1 + p_2 - 10) = 0 \tag{4.49}$$

By utilizing (4.48) and (4.49), parameters p_1 , p_2 and λ can be found as follows:

$$p_1 = \frac{11c - \lambda - 2}{2 + 2c} \quad \& \quad p_2 = \frac{9c - \lambda - 4}{2 + 2c} \tag{4.50}$$

Also, λ can be updated by

$$\lambda^{k+1} = \lambda^k - c(p_1 + p_2 - 10) \tag{4.51}$$

By presenting the iterative formula, the below equations can be found.

$$p_1^{k+1} = \frac{11c - \lambda^k - 2}{2 + 2c} \tag{4.52}$$

$$p_2^{k+1} = \frac{9c - \lambda^k - 4}{2 + 2c} \tag{4.53}$$

$$\lambda^{k+1} = \lambda^k + c\left(p_1^k + p_2^k - 10\right) \tag{4.54}$$

Considering $k = 0$, $\lambda = 0$, $c = 1$ as initial values the iterative process can be started.

In this case in the first step by utilizing (4.52) and (4.53) the value of p_1 and p_2 can be obtained $1.25mw$ and $2.25mw$.

Utilizing (4.54), the value of λ will be -6.5.

In the second step, by inserting the new value of λ, the value of p_1 and p_2 will be $2.875mw$ and $3.875mw$. In iterative process when k reaches to 18, the best answer for p_1 and p_2 will be $4.5mw$ and $5.5mw$. After this iteration, the value of λ is fixed at -13. Figure 4-3 shows the optimal power at each iterations.

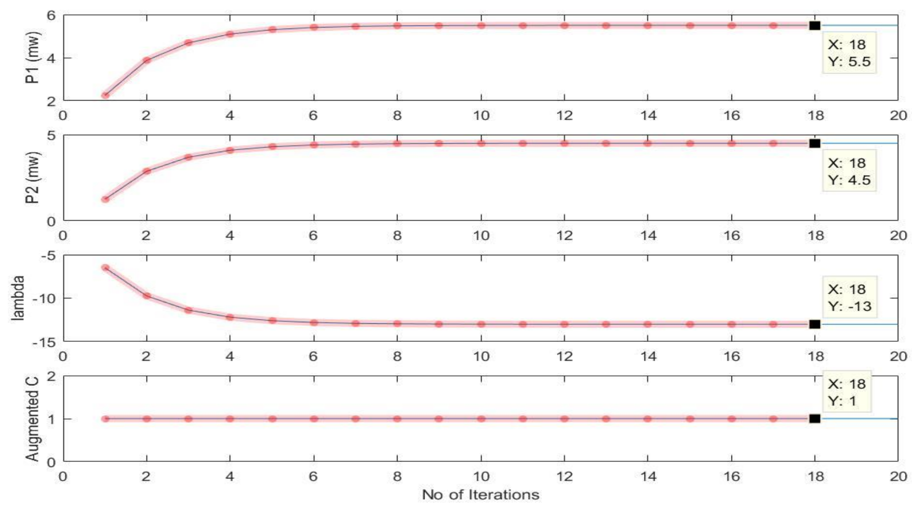

Figure 4-3: The Power of Thermal Units in ADMM

To be more efficient, in each iteration the value of c can be increased. This added step will reduce the number of iterations required for getting the best answer.

In this case, by applying $c^{k+1} = \alpha c^k$ and $\alpha = 2$, the optimal power can be determined after 6 iterations. Figure 4-4 shows this matter.

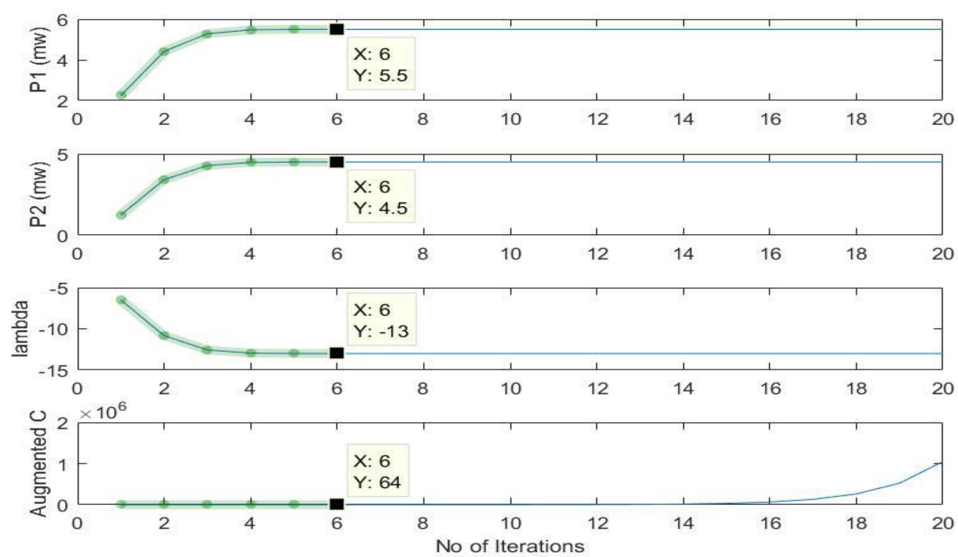

Figure 4-4: The Power of Units by Considering Updated c

4.14. A Case Study for ADMM

As a real example, take a 7-bus power system including 5 thermal units has been considered as a case study. The power network demand is $p_d\ mw$.

The total fuel cost function is provided by Equation (4.55) as follows:

$$total\ cost = \sum_{i=1}^{5} a_i p_i^2 + b_i p_i + c_i \tag{4.55}$$

Where

$a_1 = 0.0013$ / $a_2 = 0.00136$/ $a_3 = 0.00134$ / $a_4 = 0.00131$/ $a_5 = 0.00194$

$b_1 = 7.62$ / $b_2 = 7.52$ / $b_3 = 7.84$ / $b_4 = 7.57$ / $b_5 = 7.77$

$c_1 = 761.94$/ $c_2 = 831.84$ / $c_3 = 530.03$ / $c_4 = 831.92$ / $c_5 = 500.08$

$p_d = 760mw$

The load balance equality constraint is as follows:

$$\sum_{i=1}^{5} p_i = p_1 + p_2 + p_3 + p_4 + p_5 = p_d \tag{4.56}$$

In order to solve this case study, Augmented Lagrangian method of multipliers has been applied, and the Equation presented in (4.57) is obtained.

$$L(p_1, p_2, p_3, p_4, p_5, \lambda, c) = \sum_{i=1}^{5} f_i(p_i) = \sum_{i=1}^{5} a_i p_i^2 + b_i p_i + c_i + \lambda\left(\sum_{i=1}^{5} p_i - p_d\right)$$

$$+\frac{c}{2}\left\|\sum_{i=1}^{5} p_i - p_d\right\|^2 \tag{4.57}$$

The iterative relations for generated power by thermal units are as follows:

$$p_1^{k+1} \coloneqq \arg\min_{p_1} f_1(p_1) + \lambda\left(\sum_{j=1}^{5} p_j^k - p_d\right) + \frac{c}{2}\left\|\sum_{j=1}^{5} p_j - p_d\right\|^2 \tag{4.58}$$

$$p_2^{k+1} \coloneqq \arg\min_{p_2}\ f_2(p_2) + \lambda\left(\sum_{j=1}^{5} p_j^k - p_d\right) + \frac{c}{2}\left\|\sum_{j=1}^{5} p_j - p_d\right\|^2 \tag{4.59}$$

$$p_3^{k+1} \coloneqq \arg\min_{p_3} f_3(p_3) + \lambda\left(\sum_{j=1}^{5} p_j^k - p_d\right) + \frac{c}{2}\left\|\sum_{j=1}^{5} p_j - p_d\right\|^2 \tag{4.60}$$

$$p_4^{k+1} \coloneqq \arg\min_{p_4} f_4(p_4) + \lambda\left(\sum_{j=1}^{5} p_j^k - p_d\right) + \frac{c}{2}\left\|\sum_{j=1}^{5} p_j - p_d\right\|^2 \tag{4.61}$$

$$p_5^{k+1} \coloneqq \arg\min_{p_5} f_5(p_5) + \lambda\left(\sum_{j=1}^{5} p_j^k - p_d\right) + \frac{c}{2}\left\|\sum_{j=1}^{5} p_j - p_d\right\|^2 \tag{4.62}$$

$$\lambda^{k+1} = \lambda^k + c\left(\sum_{j=1}^{5} p_j^{k+1} - p_d\right) \tag{4.63}$$

By initializing $k = 0$, $p_{\mathrm{d}} = 760$ and $p_i^0,\ i = 0,1,\dots,5$, and utilizing Equations (4.58-63), the final result of the system can be obtained as follows:

$p_1 =\ 179.7\text{MW},\ \ p_2 =\ 208.6\text{MW},\ \ p_3 = 92.3\text{MW},\ \ p_4 = 197.4\text{MW},\ \ \ p_5 = 81.8\text{MW}$

4.15. Topologies of The ADMM Based Distributed Optimal Power Flow

Applicable topologies for optimal power flow can be categorized into distributed optimal power flow by the central controller, and, fully decentralized optimal power flow.

The conventional communication strategy of distributed ADMM employes a central controller. In this method, coupled variables of individual subsystems are transmitted to the central controller. The central controller computes the global variables and sends updated variables back to each subsystem. Local controllers in each subsystem can calculate multipliers by utilizing received parameters. In the next chapter, reformulation of optimal power flow in order to be suitable for solving by ADMM based distributed optimization will be discussed in details.

4.16. Convergence of Alternating Direction Method of Multipliers

In an optimization problem, the convergence of the solution in the iterative process to the optimal point is significant. The convergence of Alternating Direction Method of Multipliers (ADMM) can be specified regarding the primal residual.

$$\|r^{k+1}\|_2^2 = \|\lambda^{k+1} - \lambda^k\|_2^2 \leq \varepsilon \tag{4.64}$$

Utilizing Equation (4.41) and substituting in (4.64), the final form of stopping criteria can be found.

$$\left\|r^{k+1}\right\|_2^2 = \left\|\lambda^{k+1} - \lambda^k\right\|_2^2 = \left\|c\left(A_1 x_{k+1}^1 + A_2 x_{k+1}^2 - b\right)\right\|_2^2 \leq \varepsilon \tag{4.65}$$

where ε is the predefined threshold and can be defined by any small value depending on the problem and the required accuracy.

4.17. Conclusion

In this chapter, the augmented Lagrangian method and Alternating Direction Method of Multiplier algorithm for systems including two and more than two separable functions have been discussed and related iterative steps were presented. To understand the concept of these techniques more clearly, two power systems, 2-bus and 7-bus have been presented and solved by these methods.

Most of the distributed optimization applications based on the Alternating Direction Method of Multipliers such as the optimal power flow problem include multiple separable objective functions with equality and inequality constraints. The presented ADMM based distributed optimization for solving multiple separable objective functions can be applied in these cases.

In the next chapter, ADMM based distributed optimization will be adjusted in the case of solving a real model of optimal power flow.

Chapter 5

Applying Distributed Optimization Technique on Optimal Power Flow

5.1. Introduction

As discussed in previous chapters, Optimal Power Flow (OPF) determines the minimum operation costs of power networks by dispatching generation resources to supply power demands. In its most realistic form, the OPF is a non-linear, non-convex problem, which consists of both binary and continuous variables. Fundamentally, implementation of realistic OPF includes thousands of variables and therefore it is difficult and in many cases will not converge because in its realistic form it is a complicated optimization problem. Typically, in large scale power systems, in order to reduce the complexity of the system, the simplified version of optimal power flow that is called DC-OPF can be used. DC-OPF is an approximation of AC-OPF for obtaining the optimal real power dispatch solution of the entire power system.

DC-OPF considers the power flows in a linearized form, the power flow limits of the lines, as well as active power. This chapter focuses on applying the distributed optimization technique to the optimal power flow problem.

In recent years, distributed optimization as an efficient technique for solving large scale problems has been applied [22]. This technique has many advantages in the solving of OPF in comparison with traditional centralized OPF. Because all detailed system information should be collected by the central computing station, the traditional centralized OPF needs high speed communication infrastructures, but in distributed OPF, the problem is wisely split to small scale sub-problems for local subsystems to solve in a distributed manner and only limited information should be transmitted to local computing stations during the optimization procedure.

On the other hand, in the distributed OPF, local subsystems are not required to disclose their confidential information to other local subsystems. Also, distributed OPF is more flexible and scalable with respect to system changes than centralized operations, especially in view of the fact that topologies of electric power and communication infrastructures are more dynamic in smart grids.

Distributed OPF is more robust than traditional OPF. In the centralized optimization, the performance of the entire system depends on the main computing station and the system will be disrupted when the main computer goes offline. Also, unlike traditional OPF, distributed OPF can be solved asynchronously via individual local computers. In this case, with the failure of certain local computers, other local controllers can continue their normal routines. In addition, accurate results will be finally achieved once the failed computers come back into service.

The chapter aims to show how the distributed optimization can be implemented on the AC-OPF and DC-OPF. Also, in this chapter, the communication strategies between local processors and the concept of splitting the power system into sub-systems has been presented.

5.2. A Short Review on Distributed Optimization for Optimal Power Flow

Over the last few years, power system operation studies have explored the application of distributed optimization to various power system problems, including distributed economic dispatch (ED) [24], distributed OPF [25], and distributed unit commitment [22]. Some fundamental methods are Lagrangian Relaxation (LR) [26], and Alternating Direction of Multipliers [27]. A dynamic multiplier-based Lagrangian Relaxation approach for solving ED in a fully decentralized manner has been presented in [28]. Alternating Direction Method of Multipliers (ADMM), as an appropriate and efficient method for Optimal Power Flow, is presented in [29].

The DC-OPF model, as a simplified version of OPF, is a convex problem with linear constraints. The ADMM approach can guarantee global convergence of DC-OPF [30].

Generally speaking, in some cases the convergence criterion may not hold. In these cases, by modification of the power system decomposition, the converging problem can be solved. Strategies for subsystem partitioning play a key role in convergence performance of the distributed power system problems. Practically, for applying distributed optimization to extremely large power systems, having information about the subsystem partitions will enhance the computational performance.

5.3. Reformulation of ADMM for Solving the OPF

In order to solve OPF by ADMM, the optimal power flow problem should be reformulated. For this purpose, a typical 6-bus power system, which is split into two areas, which are named A and B, has been considered. Each area only needs to consider its part and solve its constrained objective function. The system constraints can be classified into local and global constraints. Over an iterative process these areas update their decision variables in a parallel manner.

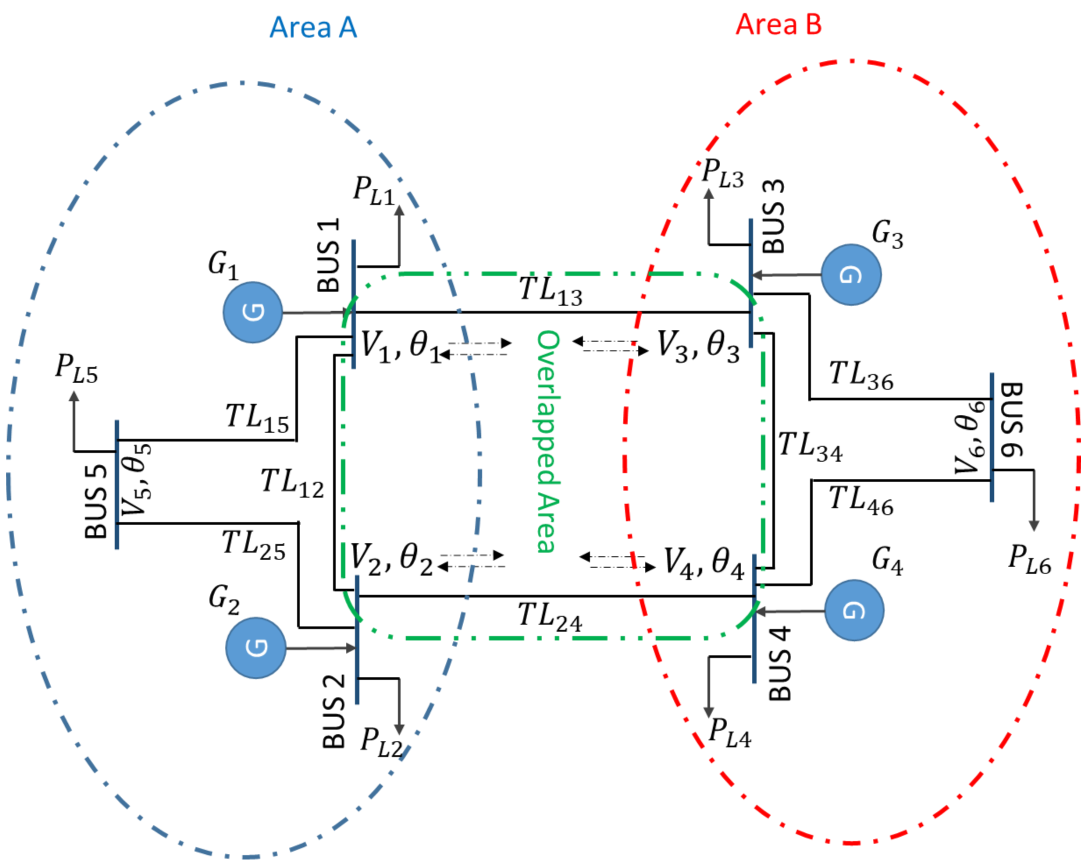

Figure 5-1: A Typical 6-bus Power System for ADMM

In this typical model buses 1-4 are connected to a generator and are generating buses, and buses 5 and 6 are considered load buses. Figure 5-1 demonstrates buses 1, 2 and 5, and buses 3, 4 and 6 in areas A and B, respectively. Also, as Figure 5-1 shows, the overlapped area includes transmission lines between buses 1 and 3, and buses 2 and 4, which are called TL_{13} , TL_{24} .

The system buses are indexed by i as bus number, where $i = 1, \dots, 6$.

Let Y be the branch admittance matrix, where $\overline{V_i}$ and $\overline{I_i}$ are voltage and current injection at bus i, respectively.

The net power injection equations at bus i can be stated as follows:

$$S_i = \overline{V_i} \times \overline{I_i}^* \tag{5.1}$$

$$P_i(V, \theta) = \sum_{j=1}^{N} V_i V_j \big(G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)\big) \tag{5.2}$$

$$Q_i(V, \theta) = \sum_{j=1}^{N} V_i V_j (G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)) \tag{5.3}$$

Also,

$$\overline{I_i} = \sum_{j=1}^{N} Y_{ij} \times \overline{V_j} \tag{5.4}$$

where

I_i as injected current to bus i can be derived by Y_{ij} and $\overline{V_j}$ as stated in Equation (5.4).

G is the conductance matrix, which is defined by $G_{ij} = Re(Y_{ij})$ as the real part of the admittance matrix.

B is the susceptance matrix, which is defined by $B_{ij} = Im(Y_{ij})$ as the imaginary part of the admittance matrix.

The Optimal Power Flow’s objective function can be formulated as follows:

$$\min_{x} \sum_{i=1}^{N} C(P_{gi}) \tag{5.5}$$

$$Subject\ to\!: \quad \underline{P_{gi}} < P_{gi} < \overline{P_{gi}}$$

$$\underline{Q_{gi}} < Q_{gi} < \overline{Q_{gi}}$$

$$\underline{V_i} < V_i < \overline{V_i}$$

$$\underline{\theta_i} < \theta_i < \overline{\theta_i}$$

$$P_{gi} - P_{di} = P_i(V, \theta)$$

$$Q_{gi} - Q_{di} = Q_i(V, \theta)$$

Where

$C(P_{gi})$ is the cost of i^{th} generator.

P_{gi} is the active power generated by i^{th} generator and $\overline{P_{gi}}$ and $\underline{P_{gi}}$ are upper and lower boundaries of active power, respectively.

Q_{gi} is the reactive power generated by i^{th} generator, and $\overline{Q_{gi}}$ and $\underline{Q_{gi}}$ are upper and lower boundaries of reactive power, respectively.

V_i is the voltage magnitude of i^{th} bus, and $\overline{V_i}$ and $\underline{V_i}$ are upper and lower boundaries of voltage magnitude, respectively.

θ_i is the voltage angle of i^{th} bus, and $\overline{\theta_i}$ and $\underline{\theta_i}$ are upper and lower boundaries of voltage angle, respectively.

P_{di} and Q_{di} are the active and reactive demanded power at i^{th} bus, respectively.

x is the optimization variable that is defined by Equation (5.6).

$$x = [\, \overline{P_g} \quad \overline{Q_g} \quad \bar{V} \quad \bar{\theta}] \tag{5.6}$$

where

$\overline{P_g}$ and $\overline{Q_g}$ are $N_g \times 1$ vectors which show active and reactive power generated by all generators, respectively. (N_g is the number of generators that for the aforementioned typical system is 4.)

$\bar{V}$ and $\bar{\theta}$ are $N \times 1$ vectors which represent voltage magnitude and angle of all buses, respectively. (In the presented system, because it is a 6-bus system, N is 6.)

The real OPF problem is a non-convex problem because of its nonlinear equality constraints.

5.3.1. Partitioning of The Proposed System

The described 6-bus network which is shown in Figure 5-1 is partitioned into 2 areas: Area A which is colored blue, and Area B which is colored red. The area enclosed by the green dotted line is the intersection area, which is called the overlapping area or consensus area.

Area A includes buses 1, 2 and 5, and all transmission lines inside this area as well as the branches in the overlapping area (TL_{13} , TL_{24}).

Area B includes buses 3, 4 and 6, and all transmission lines inside area B as well as the branches (TL_{13} , TL_{24}) in the consensus area.

Table 5-1: Information of Partitioned Areas

	Area A	Area B	Consensus Area
Own Buses	1, 2, 5	3, 4, 6	1, 2, 3, 4
Boundary Buses	1, 2	3, 4	-
Branches	$TL_{12}, TL_{15}, TL_{25}$	$TL_{34}, TL_{36}, TL_{46}$	TL_{13}, TL_{24}

Each area determines the active and reactive power of its generators, voltage magnitudes, and angles of all of its own buses.

Area A deals with the buses belonging to Area B in the consensus area as voltage sources and determines their voltage magnitudes and angles. In this case, the power injection equality of buses 3 and 4 can be neglected.

On the other side, Area B treats buses 1 and 2, which belong to area A but are connected to the overlapping area as the boundary buses. For its own buses, power injection equations will be imposed as equality constraints while considering buses 1 and 2 as two voltage sources.

Both areas will decide the voltage magnitudes and phase angles for the buses in the overlapping area. Thus, the voltage angles of buses 1, 2, 3, and 4 will be considered as local variables to achieve consensus.

5.3.2. Forming Local Sub-problems

The main OPF problem is partitioned into two areas. Thus, two sub-problems for solving by the local computing station can be formed. Consider x_A, x_B as optimization variables of partitions A and B, respectively.

$$x_A = [P_{g1_A}, P_{g2_A}, Q_{g1_A}, Q_{g2_A}, V_{1_A}, V_{2_A}, V_{5_A}, V_{3_A}, V_{4_A}, \theta_{1_A}, \theta_{2_A}, \theta_{5_A}, \theta_{3_A}, \theta_{4_A}] \tag{5.7}$$

$$x_B = [P_{g3_B}, P_{g4_B}, Q_{g3_B}, Q_{g4_B}, V_{1_B}, V_{2_B}, V_{6_B}, V_{3_B}, V_{4_B}, \theta_{1_B}, \theta_{2_B}, \theta_{6_B}, \theta_{3_B}, \theta_{4_B}] \tag{5.8}$$

Also, all variables which are related to the overlapping area are denoted by z and are defined as follows:

$$z = [V_1, V_2, V_3, V_4, \theta_1, \theta_2, \theta_3, \theta_4] \tag{5.9}$$

The default values of V and θ are considered as the initial value of global variable z.

The decision variable vectors λ for partitions A and B are defined as λ_A and λ_B, respectively.

The initial value of ρ is considered 20.

The vector z, which indicates global variable in overlapping area, is related to consensus local variables as follows:

$$x_{cA} = z \tag{5.10}$$

$$x_{cB} = z \tag{5.11}$$

Where

x_{cA} and x_{cB} are consensus local variables in partition A and B, respectively.

5.3.3. The partitioned objective functions

To solve the main OPF problem, the sub-problems A and B should be created. As discussed earlier about ADMM, the objective function for partition A can be defined as follows:

$$L_{\rho A}(x_A, z^k, \lambda_A^k) = \sum_{i=1,2} f_{pi}(P_{gi}) + (\lambda_A^k)^T (x_{cA} - z^k) + \frac{\rho}{2} \|x_{cA} - z^k\|_2^2 \tag{5.12}$$

Where

k is number of iterations.

Inequality constraints for $L_{\rho A}(x_A, z^k, \lambda_A^k)$ are as follows:

$$\underline{P_{gi}} < P_{gi} < \overline{P_{gi}} \qquad i = 1,2 \tag{5.13}$$

$$\underline{Q_{gi}} < Q_{gi} < \overline{Q_{gi}} \qquad i = 1,2 \tag{5.14}$$

$$\underline{V_i} < V_i < \overline{V_i} \qquad i = 1,2,5,3,4 \tag{5.15}$$

$$\underline{\theta_i} < \theta_i < \overline{\theta_i} \qquad i = 1,2,5,3,4 \tag{5.16}$$

Equality constraints for sub-problem A are as follows:

$$P_{gi} - P_{di} = P_i(V,\theta) = \sum_{j=1}^{N} V_i V_j \big(G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\big) \quad i = 1,2,5 \tag{5.17}$$

$$Q_{gi} - Q_{di} = Q_i(V,\theta) = \sum_{j=1}^{N} V_i V_j \big(G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)\big) \quad i = 1,2,5 \tag{5.18}$$

Similarly, the objective function of partition B can be expressed as follows:

$$L_{\rho B}(x_B, z^k, \lambda_B^k) = \sum_{i=3,4} f_{pi}(P_{gi}) + (\lambda_B^k)^T (x_{cB} - z^k) + \frac{\rho}{2}\|x_{cB} - z^k\|_2^2 \tag{5.19}$$

Inequality constraints for $L_{\rho B}(x_B, z^k, \lambda_B^k)$ are as follows:

$$\underline{P_{gi}} < P_{gi} < \overline{P_{gi}} \qquad i = 3,4 \tag{5.20}$$

$$\underline{Q_{gi}} < Q_{gi} < \overline{Q_{gi}} \qquad i = 3,4 \tag{5.21}$$

$$\underline{V_i} < V_i < \overline{V_i} \qquad i = 3,4,6,1,2 \tag{5.22}$$

$$\underline{\theta_i} < \theta_i < \overline{\theta_i} \qquad i = 3,4,6,1,2 \tag{5.23}$$

Equality constraints for sub-problem B are as follows:

$$P_{gi} - P_{di} = P_i(V,\theta) = \textstyle\sum_{j=1}^{N} V_i V_j \big(G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\big) \quad i = 3,4,6 \tag{5.24}$$

$$Q_{gi} - Q_{di} = Q_i(V,\theta) = \sum_{j=1}^{N} V_i V_j \big(G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)\big) \quad i = 3,4,6 \tag{5.25}$$

In iterative process for Part A, at each step k, Partition A computes the new value for x_{cA}, which is called x_{cA}^{k+1}. Similarly, Partition B finds the optimal solution x_{cB}^{k+1}.

As previously discussed, the decision variables of ADMM can be updated by Equations (5.26-28).

$$z^{k+1} = \frac{x_{cA}^{k+1} + x_{cB}^{k+1}}{2} \tag{5.26}$$

$$\lambda_A^{k+1} = \lambda_A^k + \rho(\, x_{cA}^{k+1} - z^{k+1}) \tag{5.27}$$

$$\lambda_B^{k+1} = \lambda_B^k + \rho(\, x_{cB}^{k+1} - z^{k+1}) \tag{5.28}$$

By defining the small value of ε, when Equation (5.29), which is the primal residual, is satisfied the iterative process can be terminated.

$$\|\lambda^{k+1} - \lambda^k\|_2^2 \leq \varepsilon \tag{5.29}$$

5.3.4. Summarized ADMM Steps

To summarize, the steps of ADMM for a 6-bus case study, are stated as follows:

i. Set initial values for ε, ρ, x_A and x_B.

ii. Set initial value of global variable vector z and variables λ_A and λ_B at each partition.

iii. Create sub-problems A and B.

iv. Continue with the following steps while the error is higher than ε.

v. Calculate x_A by minimizing the objective function A.

vi. Calculate x_B by minimizing the objective function B.

vii. Update x_{cA}^{k+1} with the x_A obtained in the previous step.

viii. Update $x_{cB}^{k+1}$with the x_B obtained in the previous step.

ix. Update z^{k+1} with the x_{cA}^{k+1} and x_{cB}^{k+1}.

x. Update λ_A^{k+1} and λ_B^{k+1} by Equations (5.27) and (5.28), respectively.

xi. Change k to $k + 1$.

xii. Go to step (iv) to check the termination condition.

xiii. When the process is terminated the optimal solution will be reached.

5.4. Power System Partitioning

Power system partitioning plays a key role in solving OPF in a distributed manner. The global variable vector, called $\boldsymbol{z}$, should be transmitted between local computing stations. In order to increase the performance of the solution, the number of global variables should be reduced. Practically, a lower number of global variables results in reduced information exchange between sub-systems and causes smaller numbers of iterations.

In order to reduce the number of general variables, the system should be decomposed to sub-systems with a minimum of coupling nodes. This enhances the performance of the introduced distributed approach.

Figure 5-2 illustrates the 14-bus IEEE system [21]. This typical model is split into two partitions in two different ways, models 1 and 2. As Figure 5-2 shows, in model 1, the system is partitioned into A_1 and B_1, and just 5 buses (4, 5, 6, 7, 9) are marked by red circles as coupling nodes. While in model 2, the system is partitioned into A_2 and B_2, and there are 8 coupling buses (2, 3, 4, 5, 6, 11,12, 13) which are marked by red circles.

The vector of global variables for partitioning models 1 and 2, denoted by $\boldsymbol{z_1}$ and $\boldsymbol{z_2}$, respectively, are as follows:

$$z_1 = [V_4, V_5, V_6, V_7, V_8, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8] \tag{5.30}$$

$$z_2 = [V_2, V_3, V_4, V_5, V_6, V_{11}, V_{12}, V_{13}, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_{11}, \theta_{12}, \theta_{13},] \tag{5.31}$$

Therefore, due to smaller global variable size, partitioning model 1 is more efficient and has fewer coupling nodes.

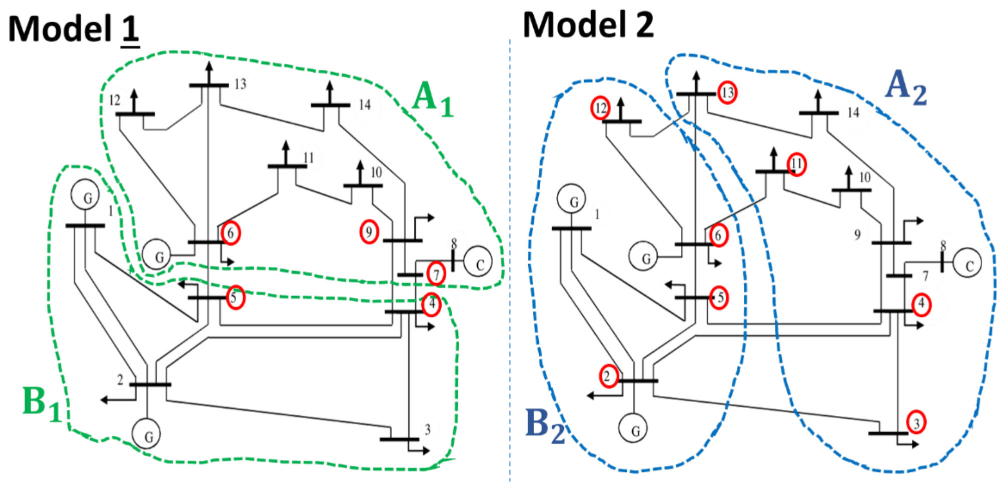

Figure 5-2: Comparison between Two Power System Partitioning

To deal with larger power systems, more partitions have to be considered. In these cases, a power system expert should partition the system by considering the structure of the power system. Figure 5-3 demonstrates a 30-bus IEEE system that is partitioned into three different sub systems A, B, and C.

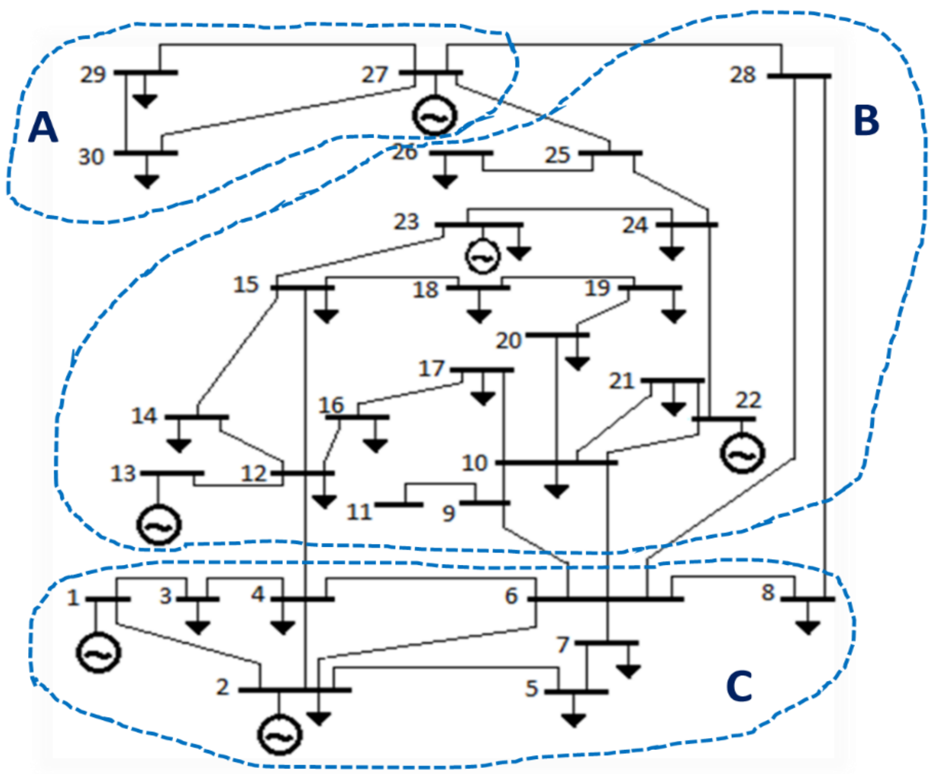

Figure 5-3: Partitioning of The 30-bus IEEE System into 3 Sub-systems

By considering the size of the power system, which is related to the number of buses and the number of processors, for solving the main problem by local processor related to each partition in parallel, the system can be partitioned into $\boldsymbol{N}$ sub-systems. Figure 5-4 demonstrates the 39-bus 10-generator IEEE power system which is partitioned into 4 subsystems A, B, C, D.

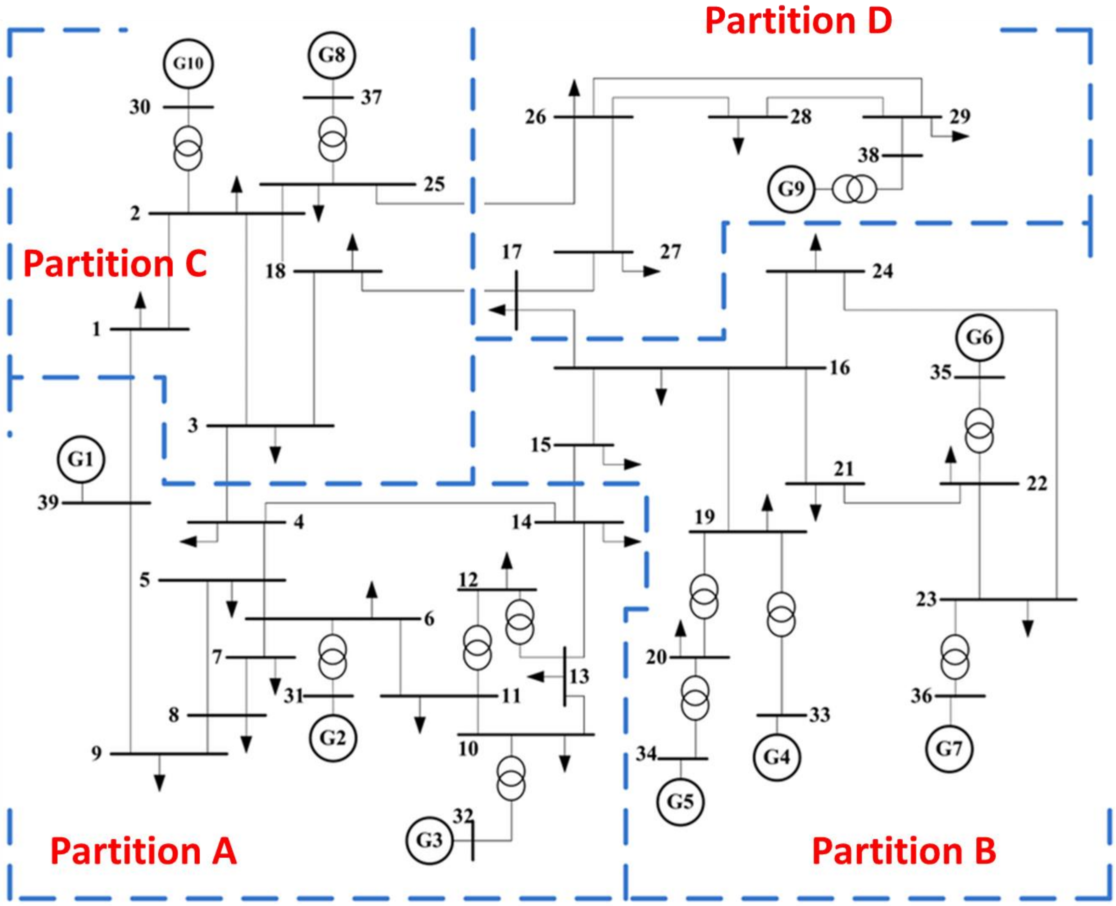

Figure 5-4: IEEE 10-generator 39-bus Power System

By considering the IEEE 39-bus power system with ten generators as one problem (see Figure 5-4), the obtained objective function includes the active power of generators 1 to 10. Thus, the optimization vector for this case can be as follows:

$$p = [p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}] \tag{5.32}$$

By splitting the power system into 4 partitions, the optimization vector of part $\boldsymbol{\xi}$, which is shown by p_ξ , where $\xi = A, B, C, D$, is as follows:

$$p_A = [p_{A1}, p_{A2}, p_{A3}] \quad (5.33)$$

$$p_B = [p_{B4}, p_{B5}, p_{B6}, p_{B7}] \quad (5.34)$$

$$p_C = [p_{C8}, p_{C10}] \quad (5.35)$$

$$p_D = [p_{D9}] \quad (5.36)$$

After splitting the main problem into four partitions, by assigning 4 processors (one for each sub-problem); which are connected in order to transmit the voltage and phase of the boundary buses; the problem can be solved in parallel.

5.5. Distributed DC-OPF Based on ADMM

5.5.1. DC-OPF

Because of the complexity, AC power flow algorithms are not fast but they have high calculation precision. In a real power system analysis, the calculation precision is not very important. The greatest concern is the calculation speed, especially for a large-scale power system.

The AC power flow model results in non-convex optimization problem that can be difficult to directly handle when using the distributed optimization technique. Therefore, many algorithms have focused on linear approximations and convex relaxations of AC power flow equations.

The most commonly used linear approximation is the DC power flow model, which is based on the following assumptions:

a. Reactive power flows can be neglected.

b. The lines are lossless and shunt elements can be neglected, thus (G ≈ 0)

c. The voltage magnitudes at all buses are approximately equal, so |Vi| ≈ 1 at all buses

d. Angle differences between connected buses are small such that

$sin(\theta_i - \theta_j) = \theta_i - \theta_j$, where i and j are system buses.

By applying these assumptions, Equations (5.37-38) can be transformed to the Equations (5.39) as demonstrated in the following equations.

AC Optimal Power Flow equality constraints are given by the following equations:

$$P_G^k - P_D^k = \underbrace{V_k}_{1} \sum_{i=1}^{N} \left[\underbrace{V_j}_{1} \left[\underbrace{G_j^k}_{0} \cos(\delta^k - \delta^j) + B_j^k \underbrace{\sin(\delta^k - \delta^j)}_{\delta^k - \delta^j} \right] \right] \tag{5.37}$$

$$\underbrace{Q_G^k - Q_D^k}_{0} = V_k \sum_{i=1}^{N} \left[V_j \left[G_j^k \, sin(\delta^k - \delta^j) - B_j^k \, cos(\delta^k - \delta^j) \right] \right] \tag{5.38}$$

DC Optimal Power Flow equality constraints:

$$P_G^k - P_D^k = B_j^k (\delta^k - \delta^j) \tag{5.39}$$

Also, the fuel cost as objective function of optimal power flow is presented in linear form through the following equation:

$$\sum_{k=1}^{N_G} a_k P_{Gk} + b_k \tag{5.40}$$

5.6. Fully Decentralized DC-OPF by ADMM

5.6.1. 3-partition Typical Power System

To simplify the explanation of distributed DC-OPF, a three-bus system (shown in Figure 5-5) is considered. ADMM can be applied to solve the DC-OPF in a distributed fashion. The illustrated system in Figure 5-5 is decomposed into three subsystems: P_1, P_2, and P_3.

Buses a, b and c are known boundary buses. Bus b is connected to buses a and c by transmission lines. θ_a, θ_b and θ_c are phase angles of boundary buses a, b and c, respectively.

Each subsystem has its variables and constraints which are called local variables and local constraints of that subsystem.

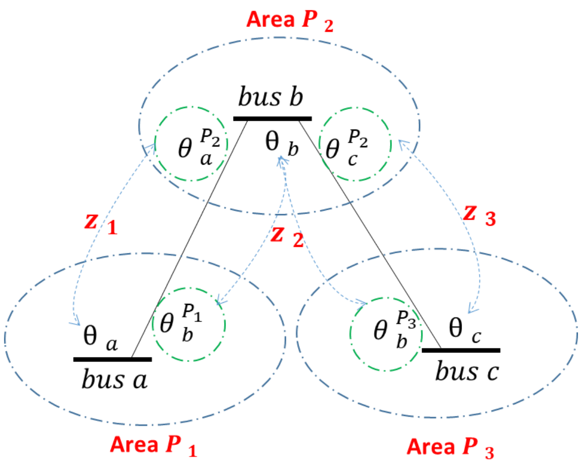

Figure 5-5: A typical 3-partition 3-bus Power System

The DC-OPF model presents an objective function that considers local variables and local constraints. The objective function for a subsystem is based solely on the information of local generators and demand. In DC-OPF, the inequality constraints are based on local variables. On the other hand, subsystems are coupled with each other via power flows on tie lines. Thus, constraints which are containing power flows of tie lines are global constraints. These constraints are not naturally separable.

To make the problem suitable for the Alternating Direction Method of Multipliers, in the proposed algorithm which is the basis of the distributed optimization, phase angles of boundary buses in each partition are duplicated in their adjacent subsystems.

As shown in Figure 5-5:

- θ_a in subsystem P_1 is duplicated in adjacent subsystem P_2 as $\theta_a^{P_2}$.
- θ_b in subsystem P_2 is duplicated in adjacent subsystems P_1 and P_3, as $\theta_b^{P_1}$ and $\theta_b^{P_3}$, respectively.
- θ_c in subsystem P_3 is duplicated in adjacent subsystem P_2 as $\theta_c^{P_2}$.

For each subsystem, the phase angle of boundary buses and the variables which are duplicated from other subsystems are called coupling variables.

Duplicating boundary bus angle variables, equality constraint of DC-OPF can be transformed into functions of local variables and coupling variables.

In order to keep the duplicated coupling variables in each subsystem equal, the vector of global variable z can be defined.

The global variables corresponding to subsystem n is presented by z_n.

For this 3-partition power system, z_1, z_2 and z_3 can be expressed as follows:

- z_1 is defined for guaranteeing coupling variables between partition P_1 and partition P_2 are equal.
- z_2 is defined for guaranteeing coupling variables between partition P_2 and partitions P_1 and P_3 are equal.
- z_3 is defined for guaranteeing coupling variables between partition P_3 and partition P_2 are equal.

$$\theta_a - z_1 = 0, \qquad \theta_a^{P_2} - z_1 = 0 \tag{5.41}$$

$$\theta_b - z_2 = 0, \qquad \theta_b^{P_1} - z_2 = 0, \qquad \theta_b^{P_3} - z_2 = 0 \tag{5.42}$$

$$\theta_c - z_3 = 0, \qquad \theta_c^{P_2} - z_3 = 0 \tag{5.43}$$

By defining these global variables, the DC-OPF model can be reformulated into a separable form which is suitable for the distributed optimization.

All variables of n^{th} subsystem are indicated by $\mathrm{x_n}$, which are classified into local variables $\overline{\mathrm{x_n}}$ and coupling variables $\overleftrightarrow{\mathrm{x_n}}$.

Local variables $\overline{\mathrm{x_n}}$ includes generation dispatches and bus angles in n^{th} subsystem, except boundary buses and coupling variables.

Coupling variables $\overleftrightarrow{\mathrm{x_n}}$ includes phase angle variables of boundary buses in $\mathrm{n^{th}}$ subsystem and duplicated phase angle variables of boundary buses in adjacent subsystems.

In the distributed DC-OPF problem, the objective function of the entire system can be represented as the summation of objectives for all individual subsystems.

$$\min_{\mathrm{x}} \sum_{\mathrm{n=1}}^{\mathrm{N}} \mathrm{C_n}(\mathrm{x_n}) \tag{5.44}$$

$$\text{subject to :} \qquad \mathrm{x_n} \in \chi_{\mathrm{n}} \quad \forall \mathrm{n} \in \mathrm{N} \tag{5.45}$$

$$\overleftrightarrow{\mathrm{x_n}} - \mathrm{z_n} = 0 \quad \forall \mathrm{n} \in \mathrm{N} \tag{5.46}$$

$\mathrm{x_n}$ satisfies constraints in $\mathrm{n^{th}}$ subsystem which is represented as $\mathrm{x_n} \in \chi_{\mathrm{n}}$, where:

N is the number of sub-systems.

$\mathrm{C_n}(\mathrm{x_n})$ represents the objective function of $\mathrm{n^{th}}$ subsystem.

χ_{n} is set of all constraints of $\mathrm{n^{th}}$ subsystem.

5.6.2. Augmented Lagrangian Function for DC-OPF

The augmented Lagrangian function for the presented 3-partition power system (Figure 5.3) can be expressed as follows:

$$\min_{x,z} L_{\rho}\,(x,z,\lambda) = \sum_{n=1}^{N} \left[\mathrm{C_n}(\mathrm{x_n}) + \lambda_n^T.\,(\overleftrightarrow{\mathrm{x_n}} - \mathrm{z_n}) + \left(\frac{\rho}{2}\right).\,\|\overleftrightarrow{\mathrm{x_n}} - \mathrm{z_n}\|_2^2\right] \tag{5.47}$$

$$x_n^{i+1} = \arg\min_{\mathrm{x_n} \in \chi_{\mathrm{n}}} (\mathrm{C_n}(\mathrm{x_n}) + \lambda_n^T.\,\mathrm{x_n} + \left(\frac{\rho}{2}\right).\left\|\overleftrightarrow{\mathrm{x_n}} - \mathrm{z_n}{}^{i}\right\|_2^2) \quad \forall \mathrm{n} \in \mathrm{N} \tag{5.48}$$

$$z_1 = \frac{(\theta_a \;+\; \theta_a^{P_2})}{2} \tag{5.49}$$

$$z_2 = \frac{(\theta_b \;+\; \theta_b^{P_1} + \theta_b^{P_3})}{3} \tag{5.50}$$

$$z_3 = \frac{(\theta_c + \theta_c^{P_2})}{2} \quad (5.51)$$

$$\lambda_n^{i+1} = \lambda_n^i + \rho.(\overleftrightarrow{\mathrm{x}_\mathrm{n}}^{i+1} - \mathrm{z_n}^{i+1}) \quad \forall \mathrm{n} \in \mathrm{N} \quad (5.52)$$

The variables of the preceding equations are defined as follows:

λ_n^i are Lagrangian multipliers at i^{th} iteration.

ρ is a predefined positive parameter.

i is the i^{th} iteration of the distributed optimization.

The i^{th} iteration of the distributed DC-OPF process includes steps of (5.48-52).

The equation (5.48) for individual subsystems can be solved in parallel in a distributed manner.

5.6.3. Termination Criteria

The algorithm should be terminated when primal and dual residuals in each subsystem n are less than pre-defined values ε_1 and ε_2, respectively.

$$\left\|{r_n}^{i+1}\right\|_2^2 = \left\|\lambda^{i+1} - \lambda^i\right\|_2^2 \leq \varepsilon_1 \quad (5.53)$$

$$\left\|{s_n}^{i+1}\right\|_2^2 = \left\|{z_n}^{i+1} - {z_n}^i\right\|_2^2 \leq \varepsilon_2 \quad (5.54)$$

Where

${r_n}^{i+1}$ is the primal residual at i^{th} iteration.

${s_n}^{i+1}$ is the dual residual at i^{th} iteration.

5.7. Communication Strategies for DC-OPF

Having an efficient communication strategy seem to be necessary for the presented iterative procedure. The communication strategies between local processors can be classified into two major classes: (1) Distributed DC-OPF with central processor and (2) Fully Decentralized DC-OPF.

5.7.1. Distributed DC-OPF with the Central Controller

The most conventional communication strategy of distributed Alternating Direction Method of Multipliers is based on considering the central processor. In this method, all coupling variables of individual subsystems should be sent to the central processor. The central processor calculates the global variable z and sends updated z^{i+1} back to each subsystem n. After receiving z^{i+1}, the local processor of each subsystem n can calculate λ_n. The communication strategy for the presented model is illustrated in Figure 5-6.

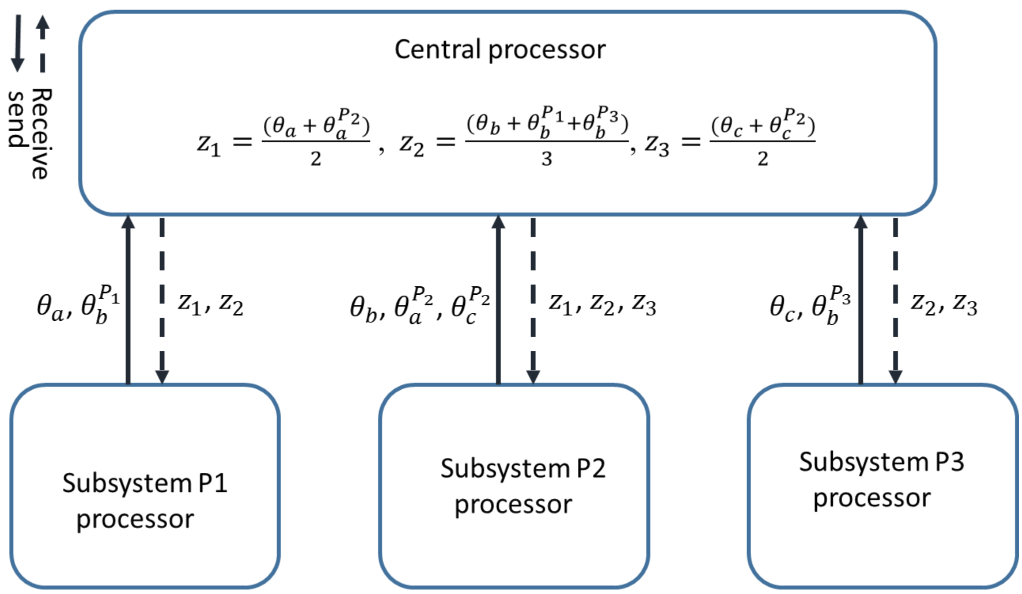

Figure 5-6: Communication Strategy Based on Central Processor

5.7.2 Fully Decentralized DC-OPF by ADMM

In order to have a fully decentralized DC-OPF approach, leading variables and subsystems should be defined. Leading variables are defined as the original phase angle variables (not the duplicated ones) and the leading subsystem is defined as the subsystem where the leading variables are located.

In this data exchange strategy, values of duplicated variables are sent to the leading subsystems. The local processor of the leading subsystem computes the global variable z and sends updated z^{i+1} back to corresponding subsystems. The communication procedure of the example is illustrated in Figure 5-7. The data exchange of the fully decentralized algorithm is much smaller than that of conventional communication strategy. Also, in this communication strategy the system is not dependent on the central controller. Figure 5-7 demonstrates the fully decentralized communication strategy.

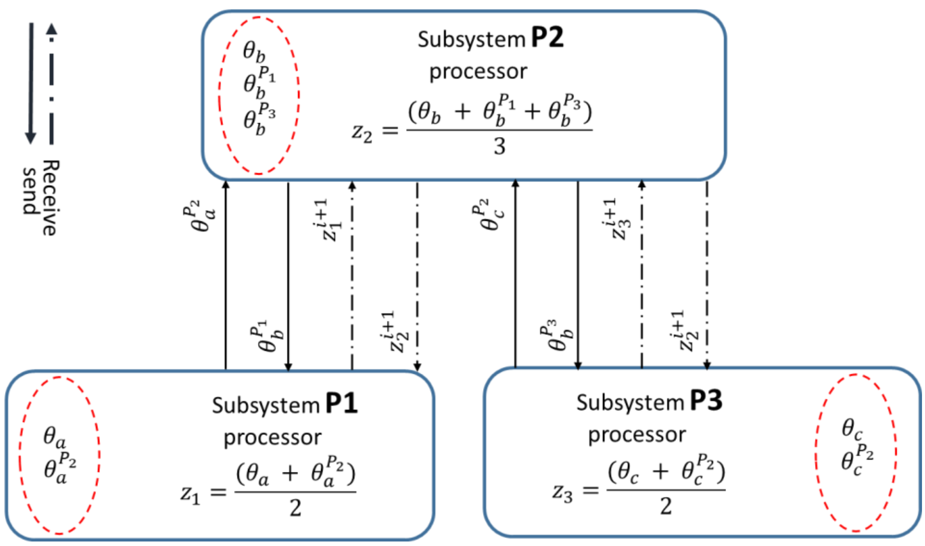

Figure 5-7: Fully Decentralized Communication Strategy For DC-OPF

5.8. Conclusion

In this chapter, the ADMM-based distributed optimization technique for solving the AC OPF problem is discussed using a 2-partition 4-bus power system. Also, the reformulation of optimal power flow, which is appropriate for solving by ADMM, has been presented. Power system wisely partitioning as a key factor in distributed optimization is explained.

Since applying AC OPF on large-scale power systems makes the problem very complicated. A linearized form of optimal power flow, which is called DC-OPF, is introduced. Because DC-OPF is a convex problem with linear constraints, the proposed consensus-based ADMM algorithms can guarantee global convergence. The application of the proposed distributed consensus-based ADMM algorithms to DC-OPF is discussed along a 3-partition power system. Also, using central controller and fully decentralized optimization as possible communication strategies between local processors are introduced and compared.

Chapter 6

Case Studies and Results

6.1. Introduction

In this chapter, the proposed distributed optimization technique based on Alternating Direction Method of Multipliers is applied to some IEEE case studies.

The first case study is the 14-bus IEEE power system which has five generators. This IEEE case has been split into two partitions and optimized by two local processors in parallel. The obtained results of the proposed algorithm for this case study are compared with outputs of the conventional method. Also, the ADMM is applied to IEEE 30-bus 6-generator case study and the outputs for this case study is compared with MATPOWER results.

The 118-bus IEEE power system has been presented as a large-scale power system and it is split into 3 partitions with 17 coupling nodes to be suitable for distributed optimization by 3 local processors in parallel.

6.2. The First Case Study (IEEE 14-Bus System)

In order to demonstrate the results of the ADMM method, the 14-bus IEEE power system with 5 generator extracted from MATPOWER has been considered [30].

Figure 6-1 demonstrates the single line diagram of the case study. To make the OPF problem suitable for the proposed approach the case study is split into two partitions which are named A and B.

Area A includes buses 1 to 5 and the generators are connected to buses 1, 2, and 3.

Area B includes buses 6 to 14 and the generators are connected to buses 6 and 8.

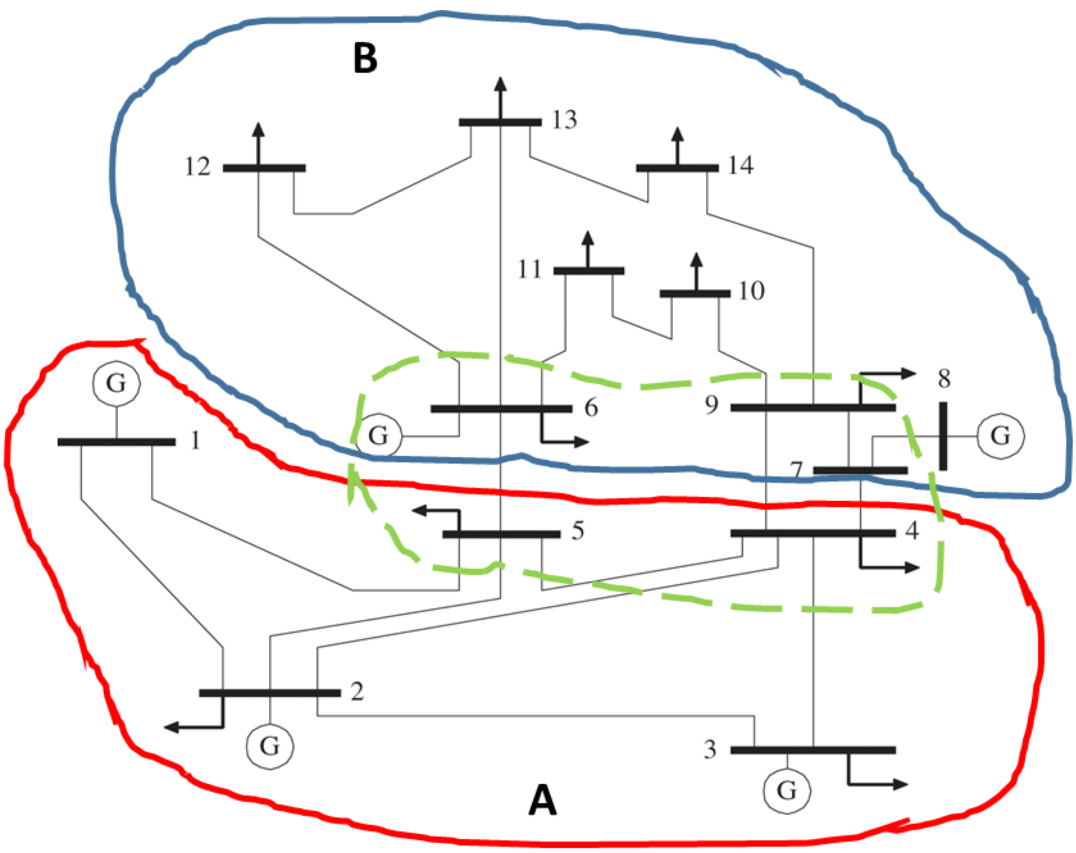

Figure 6-1: IEEE 14-bus Case Study Split into Two Partitions

Buses 4, 5, 6, 7 and 9 are boundary buses in the overlapping area between two partitions. The global variable vector which is related to voltage amplitude and the phase angle of the buses, which are located in the consensus area, is presented through the equation (6.1).

$$z = [V_4, V_5, V_6, V_7, V_9, \theta_4, \theta_5, \theta_6, \theta_7, \theta_9] \tag{6.1}$$

The global optimization vectors for partitions A and B which are called x_A and x_B are presented as follows:

$$x_A = [P_{g1_A}, P_{g2_A}, P_{g3_A}\ Q_{g1_A}, Q_{g2_A}, Q_{g3_A}, V_{1_A}, V_{2_A}, V_{3_A}, V_{4_A}, V_{5_A}, V_{6_A}, V_{7_A}, V_{9_A}, \ldots$$

$$\theta_{1_A}, \theta_{2_A}, \theta_{3_A}, \theta_{4_A}, \theta_{5_A}, \theta_{6_A}, \theta_{7_A}, \theta_{9_A}] \tag{6.2}$$

$$x_B = [P_{g6_B}, P_{g8_B}, Q_{g6_B}, Q_{g8_B}, V_{4_B}, V_{5_B}, V_{6_B}, V_{7_B}, V_{8_B}, V_{9_B}, V_{10_B}, V_{11_B}, V_{12_B}, V_{13_B}, V_{14_B}, \ldots$$

$$\theta_{4_B}, \theta_{5_B}, \theta_{6_B}, \theta_{7_B}, \theta_{8_B}, \theta_{9_B}, \theta_{10_B}, \theta_{11_B}, \theta_{12_B}, \theta_{13_B}, \theta_{14_B}] \tag{6.3}$$

Equations (6.4) and (6.5) define x_{cA} and x_{cB} which are consensus local variables in partitions A and B, respectively.

$$x_{cA} = [V_{4_A}, V_{5_A}, V_{6_A}, V_{7_A}, V_{9_A}, \theta_{4_A}, \theta_{5_A}, \theta_{6_A}, \theta_{7_A}, \theta_{9_A}] \tag{6.4}$$

$$x_{cB} = [V_{4_B}, V_{5_B}, V_{6_B}, V_{7_B}, V_{9_B}, \theta_{4_B}, \theta_{5_B}, \theta_{6_B}, \theta_{7_B}, \theta_{9_B}] \tag{6.5}$$

The sub-problem of sub-system partitions A and B of this case study are expressed as follows:

Problem A

$$\min_{x_A} L_{\rho A}(x_A, z^k, \lambda_A^k) = \sum_{i=1,2} f_{pi}(P_{gi}) + (\lambda_A^k)^T (x_{cA} - z^k) + \frac{\rho}{2} \|x_{cA} - z^k\|_2^2 \quad (6.6)$$

$subject\ to$:

$$P_{gi} - P_{di} = \sum_{j=1}^{N} V_i V_j \big(G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)\big) \quad i = 1, \dots, 5 \quad (6.7)$$

$$Q_{gi} - Q_{di} = \sum_{j=1}^{N} V_i V_j \big(G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)\big) \quad i = 1, \dots, 5 \quad (6.8)$$

$$\underline{P_{gi}} < P_{gi} < \overline{P_{gi}} \quad \& \quad \underline{Q_{gi}} < Q_{gi} < \overline{Q_{gi}} \quad i = 1,2,3 \quad (6.9)$$

$$\underline{V_i} < V_i < \overline{V_i} \quad \& \quad \underline{\theta_i} < \theta_i < \overline{\theta_i} \quad i = 1,2,3,4,5,6,7,9 \quad (6.10)$$

Problem B

$$\min_{x_B} L_{\rho B}(x_B, z^k, \lambda_B^k) = \sum_{i=3,4} f_{pi}(P_{gi}) + (\lambda_B^k)^T (x_{cB} - z^k) + \frac{\rho}{2} \|x_{cB} - z^k\|_2^2 \quad (6.11)$$

$subject\ to$:

$$P_{gi} - P_{di} = P_i(V, \theta) = \sum_{j=1}^{N} V_i V_j \big(G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)\big) \quad i = 6, \dots, 14 \quad (6.12)$$

$$Q_{gi} - Q_{di} = Q_i(V, \theta) = \sum_{j=1}^{N} V_i V_j \big(G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)\big) \quad i = 6, \dots, 14 \quad (6.13)$$

$$\underline{P_{gi}} < P_{gi} < \overline{P_{gi}} \quad \& \quad \underline{Q_{gi}} < Q_{gi} < \overline{Q_{gi}} \quad i = 6,8 \quad (6.14)$$

$$\underline{V_i} < V_i < \overline{V_i} \quad \& \quad \underline{\theta_i} < \theta_i < \overline{\theta_i} \quad i = 4,5,6,7,8,9,10,11,12,13,14 \quad (6.15)$$

In this step two local processors are assigned to sub-systems A and B.

The decision variables of the ADMM can be updated by the formula which are presented in Chapter 5. For updating these multipliers, we have made a communication network between two processors to transmit their consensus local variables. Figure 6-2 demonstrates the local processors and their problems.

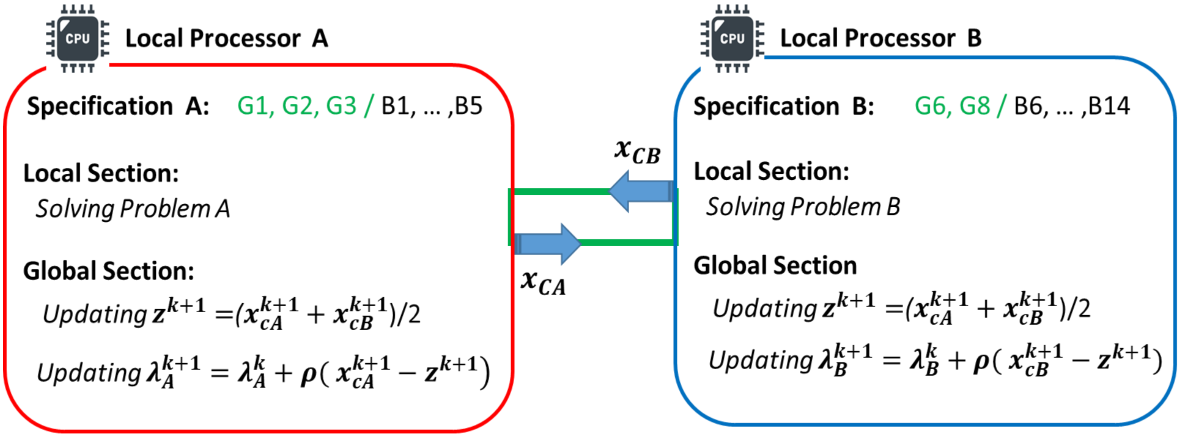

Figure 6-2: The Local Processors for IEEE 14-Bus Case Study

For this case study, by running the presented problems on both processors and updating global variables in a MATLAB code, the active and reactive powers of generators connected to buses in partition A and B have been found. Table 6-1 represents the obtained results of the processor A and B.

To verify the obtained results, this case study is solved by "*runopf* "command in MATPOWER. The results of MATPOWER are provided in Table 6-1.

The fuel cost for MATPOWER and the proposed algorithm are 8081 \$/h and 8080 \$/h, respectively. The results of the proposed method are close to the MATPOWER results.

Table 6-1: Obtained Results of Processor A for 14-bus IEEE Case

Part A	MATPOWER	Distributed	Part B	MATPOWER	Distributed
P Gen1 (KW)	194,330	194,300	P Gen6 (kw)	0	0
Q Gen1 (Kvar)	0	0	Q Gen6 (kvar)	11,550	11,490
P Gen2 (KW)	36,720	36,710	P Gen8 (kw)	8,490	8,470
Q Gen2 (Kvar)	23,690	23,520	Q Gen8 (kvar)	8,270	8,230
P Gen3 (KW)	28,740	28,740			
Q Gen3 (Kvar)	24,130	24,270			

6.3. The Second Case Study (IEEE 30-Bus System)

As the second case study, the 30-bus IEEE power system is considered. The system information of this case study has been extracted from MATPOWER [30].

Figure 6-3 illustrates the partitioned single line diagram of the IEEE 30-bus power system.

Area A includes buses 1 to 4 and 12 to 15 and generators which are connected to buses 1, 2, and 13.

Area B includes buses 5 to11 and 16 to 30 and generators which are connected to buses 5, 8, and 11.

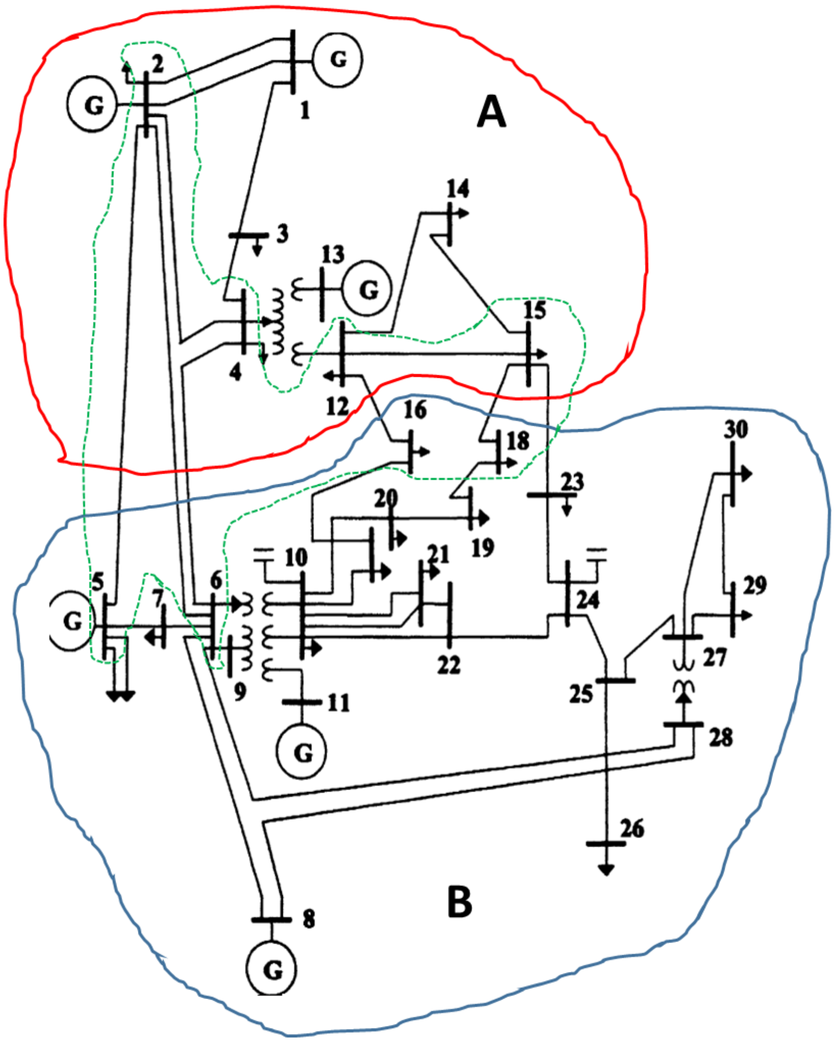

Figure 6-3: Single Line Diagram of 30-bus IEEE Power System

As Figure 6-3 shows, all buses which are located in the overlapping area that is indicated by the green dotted border are boundary buses. In this case study buses 2, 4, 5, 6, 12, 15, 16, and 18 are boundary buses.

Note that, the number of buses in consensus can be changed depending on splitting shape of the single line diagram. It is important to split the system to have minimum size of global variable vector.

For this case study as explained for the IEEE 14-bus case study, the local optimization vector of each area, the consensus local variables of each area and the global optimization variable can be formed.

By splitting the problem into two areas, two sub-problems can be extracted that should be individually solved by one local processor.

In order to review the sub-problems of each local processor, consider the following equations:

$$\min_{x_{Area}} \sum_{i=1,2} f_{pi}(P_{gi}) + (\lambda_{Area}^{k})^{T}(x_{cArea} - z^{k}) + \frac{\rho}{2}\|x_{cArea} - z^{k}\|_2^2 \tag{6.16}$$

$subject\ to$:

$$\begin{cases} P_{gi} - P_{di} = \sum_{j=1}^{N} V_i V_j \big(G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\big) \\ Q_{gi} - Q_{di} = \sum_{j=1}^{N} V_i V_j \big(G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)\big) \end{cases} \tag{6.17}$$

$$\begin{cases} \underline{P_{gi}} < P_{gi} < \overline{P_{gi}} \\ \underline{Q_{gi}} < Q_{gi} < \overline{Q_{gi}} \end{cases} \tag{6.18}$$

$$\begin{cases} \underline{V_i} < V_i < \overline{V_i} \\ \underline{\theta_i} < \theta_i < \overline{\theta_i} \end{cases} \tag{6.19}$$

To summarize the equations of each sub-problem, Tables 6-2 and Table 6-3 are presented.

For partition A, by substituting Area in Equation 6.16 with A, the main objective of sub-problem A can be found. Table 6-2 represents the i index in Equations (6.17-19) which are constraints of the sub-problem A for the 30-bus IEEE power system. Similarly, Table 6-3 demonstrates the i index in Equations (6.17-19) which are constraints of the sub-problem B for the case study.

Table 6-2: Information of Sub-problem A for 30-bus IEEE Power System

Index	Equation Number	Bus Number
i	6.17	1,2,3,4,12,13,14,15,
i	6.18	1,2,13
i	6.19	1,2,3,4,5,6,12,13,14,15,16,18

Table 6-3: Information of Sub-problem B for 30-bus IEEE Power System

Index	Equation Number	Bus Number
i	6.17	5,6,7,8,9,10,11,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30
i	6.18	5,8,11
i	6.19	2,4,5,6,7,8,9,10,11,12,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30

The global section of this case study which updates the global optimization vector and Lagrangian multipliers vector is the same as the first case study. The following equations show the iterative formula for updating vectors:

$$z^{k+1} = \frac{x_{cA}^{k+1} + x_{cB}^{k+1}}{2} \tag{6.20}$$

$$\lambda_A^{k+1} = \lambda_A^k + \rho(x_{cA}^{k+1} - z^{k+1}) \tag{6.21}$$

$$\lambda_B^{k+1} = \lambda_B^k + \rho(x_{cB}^{k+1} - z^{k+1}) \tag{6.22}$$

By running this case study on local processors, the obtained results of the processors which are assigned to partition A and B are presented in Table 6-4 and 6-5.

Table 6-4: 30-bus IEEE Power System Results Obtained by Processor A

Variable	ADMM	Variable	ADMM
P1 (MW)	178.28	Q1 (MVAR)	-2.983
P2 (MW)	48.301	Q2 (MVAR)	41.837
P13 (MW)	12.514	Q13 (MVAR)	33.118
V1 (p.u.)	1.06	θ1 (Deg)	0
V2 (p.u.)	1.045	θ2 (Deg)	-3.608
V13 (p.u.)	1.071	θ13 (Deg)	-10.409
Load (MW)	292.81	cost ($/h)	802

Table 6-5: 30-bus IEEE Power System Results Obtained by Processor B

Variable	ADMM	Variable	ADMM
P5 (MW)	20.924	Q5 (MVAR)	27.676
P8 (MW)	21.061	Q8 (MVAR)	22.467
P11 (MW)	11.736	Q11 (MVAR)	29.534
V5 (p.u.)	1.01	θ5 (Deg)	-10.301
V8 (p.u.)	1.025	θ8 (Deg)	-8.116
V11 (p.u.)	1.082	θ11 (Deg)	-8.783
Load (MW)	292.81	cost ($/h)	802

For implementing this case study, we should use two connected local computers but because of the hardware limitation, for this simulation, two different MATLAB programs; one for problem A as program of processor A and another one for problem B as the program of processor B; have been executed simultaneously on one computer.

By combining information from Tables 6-4 and 6-5 the active power and reactive power of all generators of the 30-bus IEEE case study are available.

To show the performance of the ADMM in solving this case study, this system is run by MATPOWER. The results of MATPOWER and results obtained by the proposed technique have been presented in Table 6-6.

Table 6-6: Comparison between the Results of ADMM and MATPOWER

Variable	MATPOWER	ADMM	Variable	MATPOWER	ADMM
P1 (MW)	176.28	178.28	Q1	-12.02	-2.983
P2 (MW)	48.79	48.301	Q2	30.63	41.837
P5 (MW)	21.48	20.924	Q5	29.48	27.676
P8 (MW)	22.07	21.061	Q8	46.89	22.467
P11 (MW)	12.19	11.736	Q11	5.41	29.534
P13 (MW)	12	12.514	Q13	2.8	33.118
V1 (p.u.)	1.06	1.06	θ1 (Deg)	0	0
V2 (p.u.)	1.047	1.045	θ2 (Deg)	-3.52	-3.608
V5 (p.u.)	1.02	1.01	θ5 (Deg)	-10.157	-10.301
V8 (p.u.)	1.029	1.025	θ8 (Deg)	-7.965	-8.116
V11 (p.u.)	1.06	1.082	θ11 (Deg)	-8.32	-8.783
V13 (p.u.)	1.06	1.071	θ13 (Deg)	-9.679	-10.409
Load	292.81	292.81	cost ($/h)	802.1	802

As Table 6-6 shows, for 292MW load, the hourly cost of both ADMM and MATPOWER are approximately the same at 802 ($/h).

6.4. The Third Case Study (IEEE 118-Bus System)

Due to the main application of the proposed algorithm in solving large scale optimal power flow for power systems, the 118-bus IEEE power system [21] has been considered as third case study.

To understand better, the single line diagram of the 118-bus IEEE power system has been provided. Figure 6-4 illustrates the 118-bus IEEE power system.

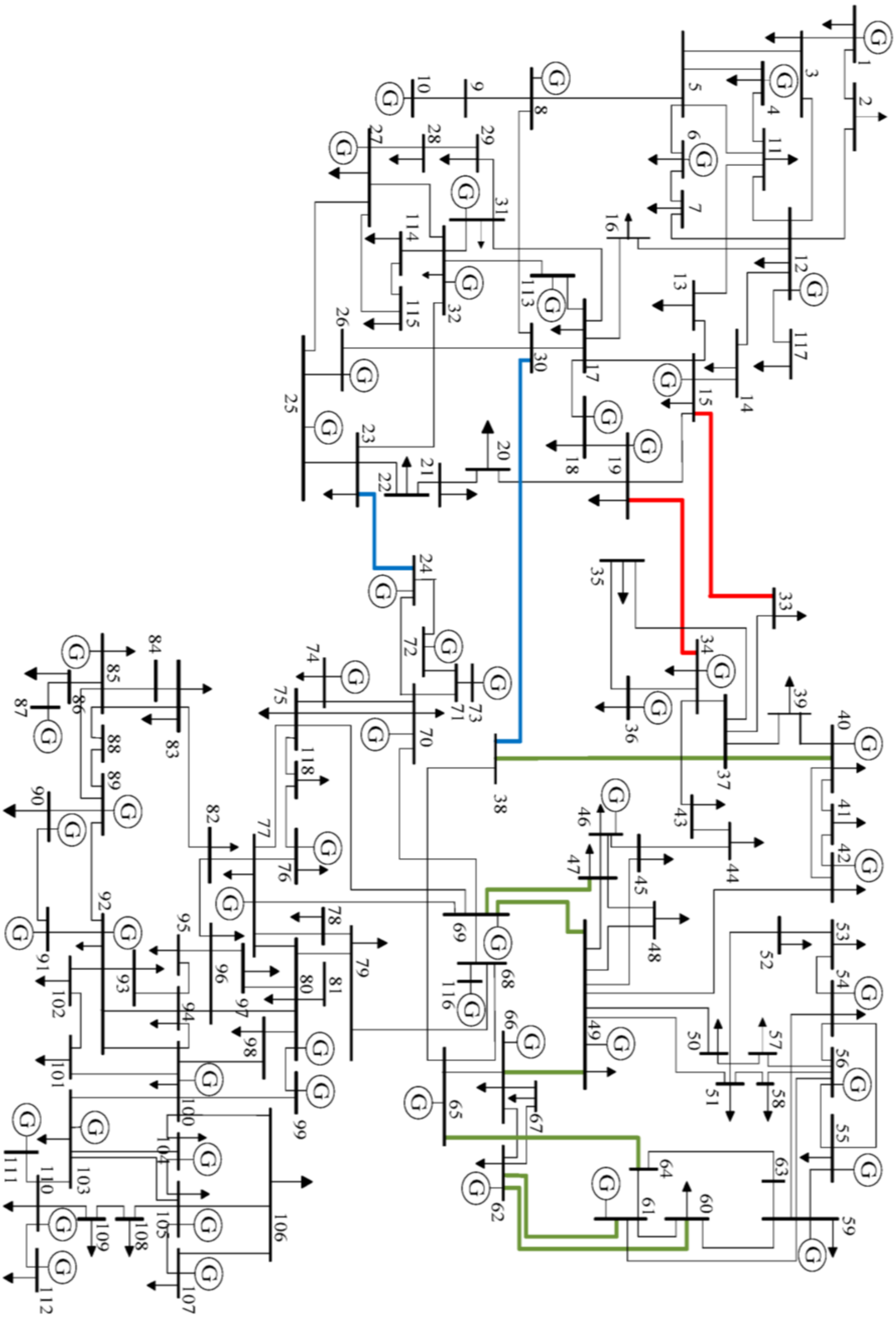

Figure 6-4: Single Line Diagram of The 118-bus IEEE Power System

In order to solve the third case study by the proposed distributed optimization technique, the 118-bus system has been split into 3 partitions which are named A, B, and C.

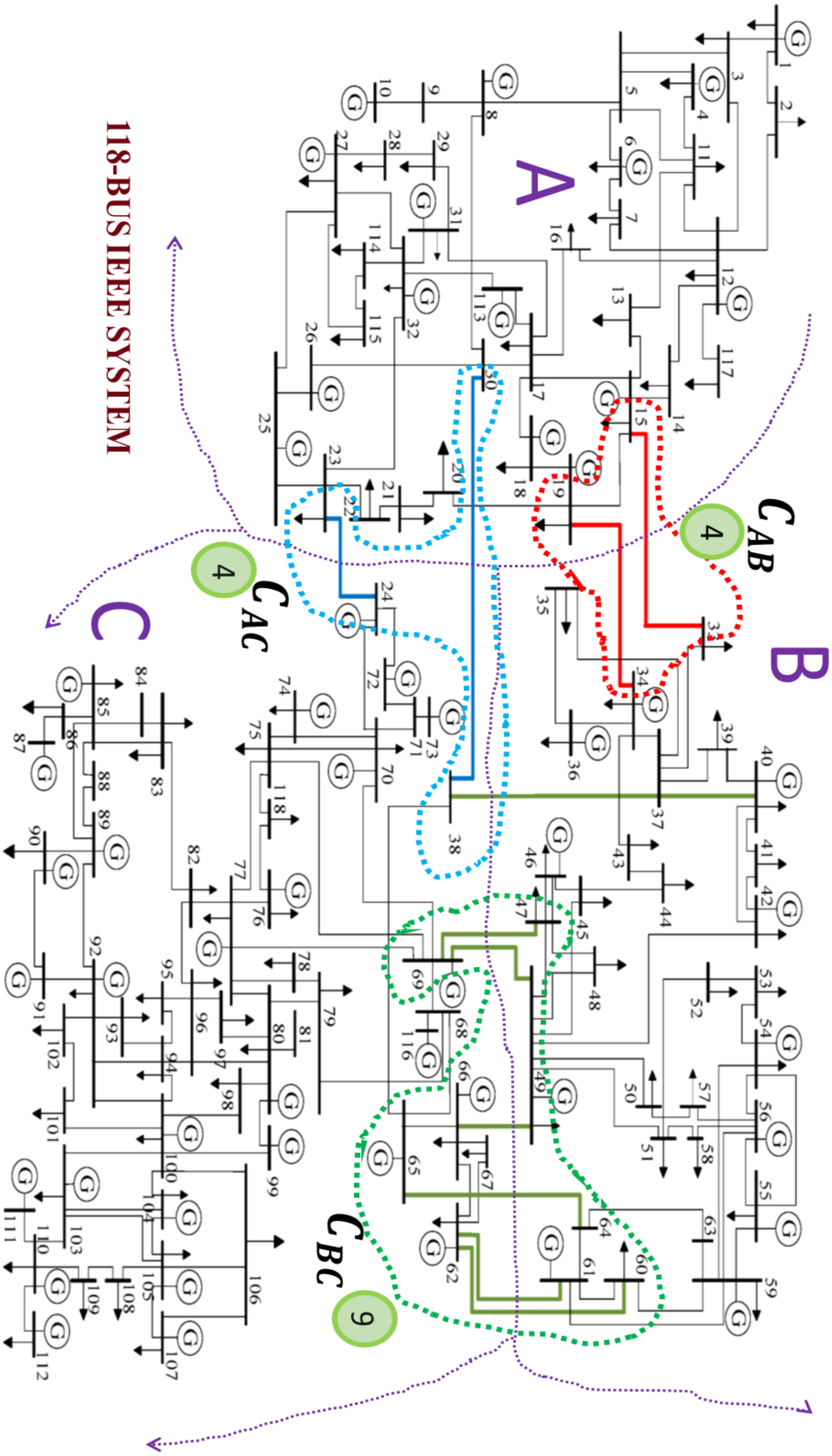

Figure 6-5: 3-partition 118-bus IEEE Power System

Figures 6-6, 6-7, and 6-8 demonstrate sub-systems A, B, and C, respectively.

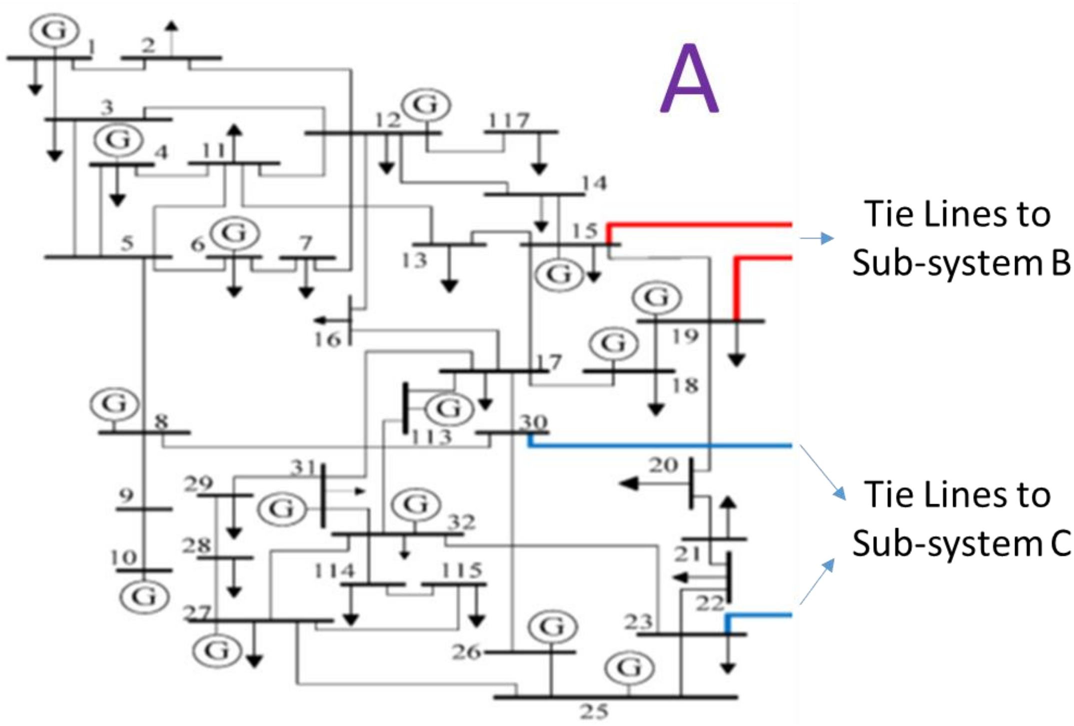

Figure 6-6: The First Partition of 118-bus IEEE (Sub-system A)

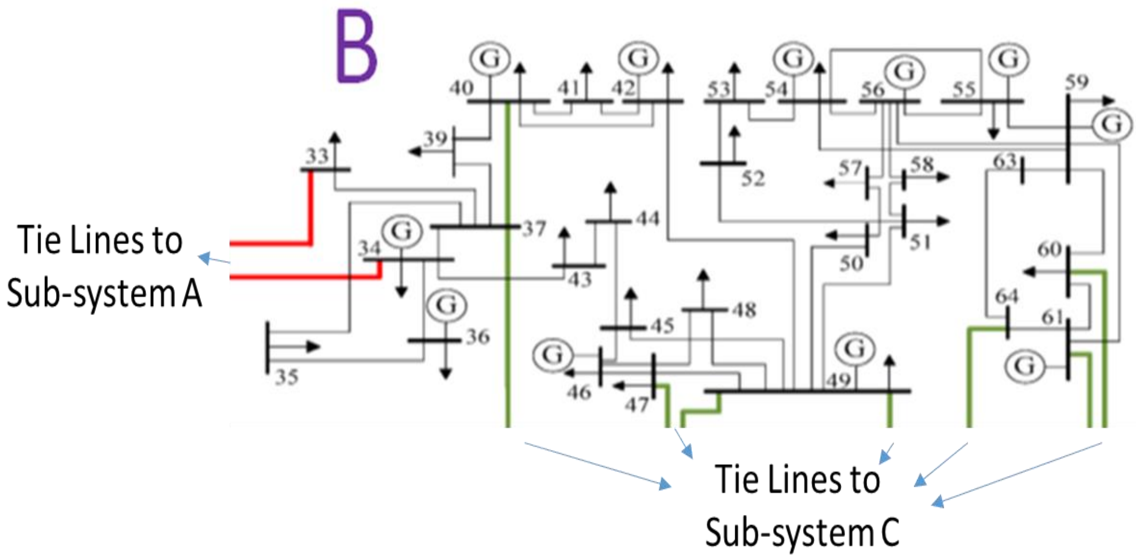

Figure 6-7:The Second Partition of 118-bus IEEE (Sub-system B)

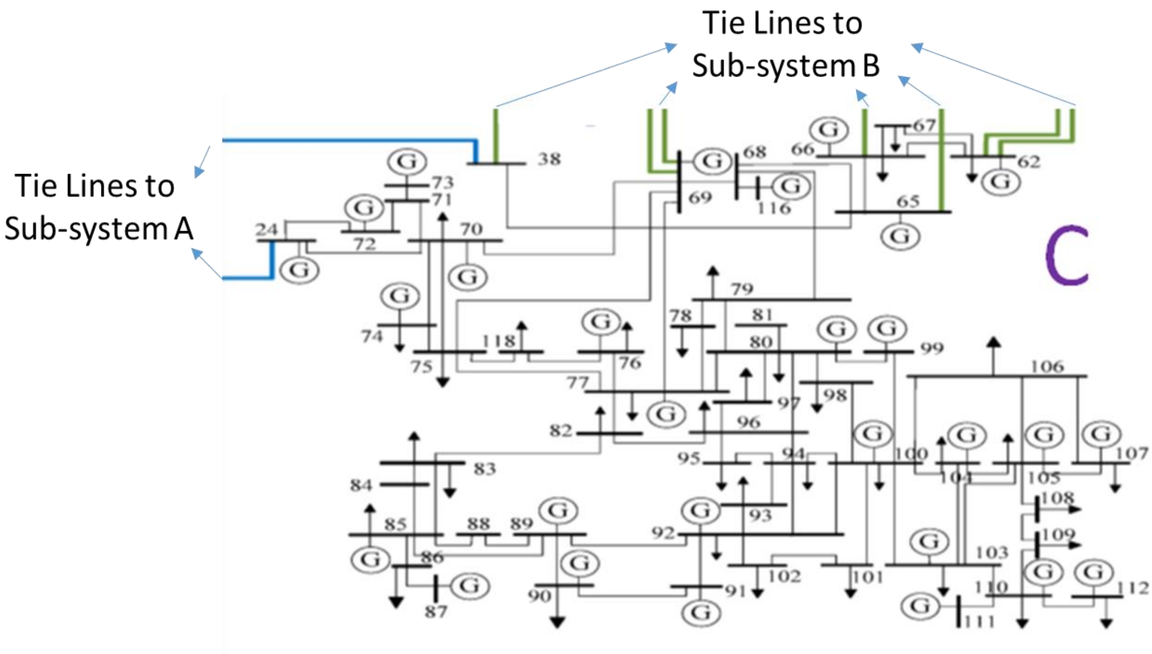

Figure 6-8: The Third Partition of 118-bus IEEE (Sub-system C)

By splitting the whole power system into 3 partitions, 3 different consensus area are created. The overlapping area between partitions A and B is named C_{AB}. This consensus area includes 4 buses (3, 15, 19, 34). The voltage amplitude and the phase angle of these boundary buses in both partitions A and B should be the same. Thus the global vector between these two partitions has 8 elements.

Similarly, the overlapping area between partitions A and C is named C_{AC}. This area has 4 buses (22, 24, 30, 38). The global vector between these two partitions has 8 elements.

The overlapping area between partitions B and C which is named C_{BC}. This area includes 9 buses (47, 49, 60, 61, 62, 64, 65, 66, 69). The voltage amplitude and the phase angle of these boundary buses in both partitions B and C should be the same. As a result, the global vector between these two partitions has 18 elements.

After forming the sub-problems for these three partitions, we have to consider three local processors. Figure 6-9 shows the diagram of the communication network between local processors.

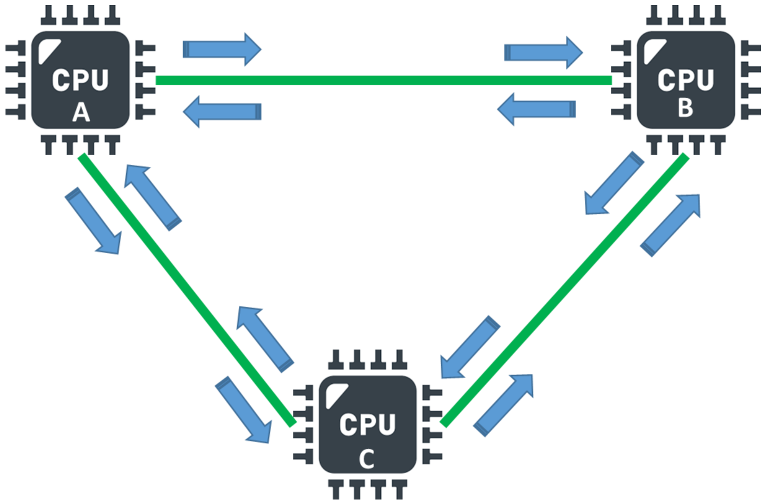

Figure 6-9: Communication Network between Sub-problems of 118-bus IEEE

Each processor should send its consensus local variable vector to the other processors through the communication platform.

By running all 3 local processors in parallel, the results of the system can be found. While previous case studies have 2 processors, this case study has 3 processors and it makes the problem complicated. In this research because of the complexity of the third case study which comes from having 3 processors that should work in parallel, we did not apply the proposed technique on the case study. For this case study, only the application of the presented technique to this problem has been described conceptually.

6.5. Conclusion

In this chapter, the proposed algorithm has been applied to the 14-bus IEEE power system and the 30-bus IEEE power system. For both cases the system was split into two partitions. Also, we used MATPOWER to solve both cases and the results of MATPOWER were compared with the proposed technique.

In the third case study, the 118-bus IEEE power system as a large-scale power system was considered. This case study was split into three partitions, and due to the complexity of the problem only the ADMM algorithm is described conceptually for this case study.

This chapter shows that for all case studies the obtained results by the proposed algorithm are approximately the same as the MATPOWER results. Because of the nature of the proposed algorithm, it can be used for any size power system while other conventional algorithms only can apply to limited-size power systems.

Chapter 7

Conclusion and Future Works

7.1. Conclusion

In this thesis, after providing a short review of the Optimal Power Flow and distributed optimization, the reformulated version of Alternating Direction Method of Multipliers as a basis of distributed optimization has been presented. The steps for applying the proposed algorithm to AC-OPF including splitting the whole power system into sub-systems, creating the OPF problem for each partition, providing the communication strategy between local processors and finally solving the main Optimal Power Flow problem in parallel have been discussed in detail.

Also, the ADMM technique has been applied to DC-OPF along with the 3-partition typical power system and all the steps ranging from forming the sub-problems to integrating the answers to get the final results have been presented.

In order to show the performance of the proposed algorithm, presented distributed optimization technique has been applied on some IEEE cases and the results are compared with the conventional optimization technique. The comparison between the solution of the MATPOWER and the proposed algorithm shows that the output of both methods are approximately equal. Ability of solving bigger power system problems by distributed optimization technique is the main advantage of the proposed algorithm. By applying the provided algorithm in this study to any OPF problem, regardless of its size the OPF problem can be solved by some local processors in parallel.

7.2. Contribution of the research

To summarize, in this research the distributed optimization technique has been used for solving optimal power flow problem. The research contributions are listed as follows:

- The concept of distributed optimization and the ADMM method for solving distributed optimization problems has been presented.
- The reformulated version of Alternating Direction Method of Multipliers which is suitable for Optimal Power Flow has been provided.
- In this research, two 6-bus and 3-bus power systems are considered as a typical model to show how the ADMM can be reformulated for AC and DC optimal power flow and determine the iterative steps of solving these systems.
- ADMM based distributed optimization technique for AC-OPF by considering fuel cost as objective functions; active and reactive power balance as equality constraints; and boundary of active power, reactive power, bus voltages, and phase angle of bus voltages as inequality constraints have been presented along with the 6-bus 2-partition typical power system.

- ADMM based distributed optimization technique for DC-OPF by considering fuel cost as objective functions and active power balance as equality constraints has been discussed through a 3-bus 3-partition typical power system.
- Two different communication strategies between local processors including using a central controller for communication between local processors and fully decentralized communication method have been explained.
- The power system partitioning for 14-bus, 30-bus, 118-bus IEEE power systems has been presented.
- The proposed approach has been applied on some IEEE cases and the obtained results are compared with conventional algorithms.

7.3. Future works

In this thesis, applying distributed optimization technique based on ADMM to AC and DC Optimal Power Flow has been effectively discussed. The following studies are possible future works related to this thesis topic:

- Application of ADMM algorithm to other power system problems, such as Economic Dispatch and Unit Commitment.
- Considering more objectives for OPF and apply distributed optimization to multi-objective functions.
- Apply ADMM to OPF by considering security constraints of the power network.
- Applying the proposed algorithm to dynamic Optimal Power Flow.
- Making the approach robust against communication networks failures.
- Developing the algorithm to increase its robustness for the case that one of the local processors goes offline.
- Developing the proposed algorithm for solving OPF problems with uncertainties.

- Reducing the size of the global variable in the consensus area.
- Analyzing the effectiveness of the algorithm by changing the number or the size of the partitions.
- Developing the communication strategies between local computers to make it smarter and more convenient.
- Analyzing the performance of the algorithm by varying the Lagrangian multiplies and studying the effect of getting different initial values to Lagrangian coefficients.
- Applying other optimization techniques for solving local sub-problems.
- Reducing the computational time of the individual sub-systems.

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