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Brief Paper

High-gain Adaptive λ-tracking for Nonlinear Systems*

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Key Words—Adaptive control; λ-tracking; nonlinear systems; tracking; robustness.

Abstract—It is shown that a simple modification (introducing a dead zone in the adaptation law) of the known adaptive high-gain control strategy $u(t) = -k(t)y(t)$, $\dot{k}(t) = \|y(t)\|^2$ yields λ-tracking in the presence of output corrupted noise for a large class of reference signals and a large class of multivariable *nonlinear* 'minimum-phase' systems of relative degree one. These results are applied to a realistic chemical reactor, showing the practical usefulness of these control laws. © 1997 Elsevier Science Ltd.

Nomenclature

$\mathbb{R}_+$ ($\mathbb{R}_-$)	the set of non-negative (non-positive) real numbers;
$\mathbb{C}_+$ ($\mathbb{C}_-$)	open right- (left-) half complex plane;
$\sigma(A)$	the spectrum of the matrix $A \in \mathbb{C}^{n\times n}$;
$\mu_{\min}(A)$, $\mu_{\max}(A)$	minimal and maximal singular values of the matrix $A \in \mathbb{C}^{n\times m}$;
$\|x\|_P =$	$\sqrt{\langle x, Px\rangle}$ for $x \in \mathbb{R}^n$, $P = P^T \in \mathbb{R}^{n\times n}$ positive-definite;
$\|x\| =$	$\|x\|_{I_n}$;
$\mathcal{B}_\lambda(0) =$	$\{x \in \mathbb{R}^m \mid \|x\| < \lambda\}$ for $m \in \mathbb{N}$, $\lambda > 0$;
$L_\infty(I)$	the vector space of measurable functions $f: I \to \mathbb{R}^n$, $I \subset \mathbb{R}$ an interval, n being defined by the context, such that $\|f(\cdot)\|_{L_\infty(I)} < \infty$, where $\|f(\cdot)\|_{L_\infty(I)} = \text{ess sup}_{s\in I}\|f(s)\|$;
$\mathcal{W}^{1,\infty}$	the Sobolev space of functions $f: [0,\infty) \to \mathbb{R}$ that are absolutely continuous on every compact interval and $f(\cdot), \dot{f}(\cdot) \in L_\infty(0,\infty)$.

1. *Introduction*

High-gain control is a well known and popular tool for robust stabilization of control systems that satisfy certain assumptions on the plant. In order to derive the necessary value of the high-gain parameter, certain properties of the system to be controlled, like the order of the system, the sign of its high-frequency gain and the size of the uncertainty, need to be known. In many cases these pieces of information are not available or the effort to obtain them cannot be justified. In order to circumvent this problem, the high-gain parameter can be determined in an adaptive way. Because no identification mechanism is employed, this adaptive control strategy is usually termed *non-identifier-based adaptive control* and is often of the simple form (or simple modifications of this form):

$$u(t) = -k(t)y(t)$$
$$\dot{k}(t) = \|y(t)\|^2, \quad k(0) = k_0 \in \mathbb{R}. \tag{1}$$

This approach was initiated by Mareels (1984), Mårtensson (1985), Morse (1983) and Willems and Byrnes (1984) for linear time-invariant systrems. If the nominal system is linear and satisfies the necessary assumptions, stability of the system thus controlled can be guaranteed for a large class of uncertainties. Furthermore, such control laws achieve an arbitrary desensitization of the feedback control system. (For a comprehensive bibliography, see the monograph by Ilchmann (1993).)

There are only few papers available where the nominal system is assumed to be nonlinear rather than with nonlinear disturbances. A first approach is due to Byrnes and Isidori (1984), who consider single-input/single-output systems of order one, relative degree one and known sign of the high-frequency gain. Khalil and Saberi (1987) and Saberi and Lin (1990) also consider the problem of high-gain adaptive stabilization with a high-gain parameter, which is increased on-line at discrete instants of time. Their approach is based on singular perturbation theory and allows the underlying multivariable system to have a strong relative degree of arbitrary, but known, finite order. However, the 'sign of the high-frequency gain' needs to be known. A simple modification of (1) has also been applied successfully to adaptively stabilize single-input/single-output, relative-degree-one systems with unknown order and positively homogeneous drift vector field (see Ryan, 1995).

In the present paper we consider multivariable, strong relative-degree-one, minimum-phase systems with unknown 'sign of the high-frequency gain'. The latter is more general than the assumptions made by Saberi and Lin (1990), whereas the former (relative degree one) is more restrictive. However, our control strategy is simpler and more transparent. The goal in the present paper is to apply the concept of λ-tracking to this class of nonlinear systems, thus making it attractive for many applications.

The concept of λ-tracking for linear systems has been studied by Miller and Davison (1991) and Ilchmann and Ryan (1994). λ-stabilization or λ-tracking refers to the specification that the output is no longer controlled to a setpoint but into a λ-neighbourhood of the setpoint (or the reference trajectory to be tracked), where $\lambda > 0$ is prespecified and may be arbitrarily small.

The main advantage of this control objective as opposed to the standard one is that a rather general class of nonlinear systems can be treated and that a serious robustness problem of previous formulations will be overcome. The necessary control law has a relatively simple structure that is a simple modification of (1) where a dead zone is introduced in the adaptation law, i.e. $k(t)$ stops increasing when $y(t)$ is within a λ-neighbourhood of the reference signal.

Certain assumptions on the plant to be controlled have to be satisfied. From an application point of view, these assumptions are mainly that the system be globally minimum-phase and have strong relative degree one. Clearly these are very restrictive assumptions, and many practical

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control problems will fail to meet these requirements. Also, the problems for which these assumptions hold could in many cases be termed as 'relatively simple control problems'. One further drawback of any high-gain control scheme, including ours, is its potential sensitivity to measurement noise. On the other hand, we have advantages. The control law is very simple, and can be implemented on almost any process-control equipment. The closed loop is guaranteed to be stable, even though no information about the system is needed other than that it satisfies the assumptions required. Thus we have a guarantee for robust stability for an unusually large class of uncertainties, including possibly parameter uncertainties, uncertainty of system order, uncertainty about the 'number of unstable modes', etc. In addition, the same structure of the feedback can be used for any plant, thus avoiding tedious controller calculations. There are a number of transparent design parameters that can be used to improve performance of the closed loop. But even if these parameters are chosen in an unfortunate manner, stability of the closed-loop will never be affected. We thus have a simple, reliable control scheme for a certain class of nonlinear systems that achieves stability and in many cases satisfying performance. From an application point of view, this control algorithm can thus certainly be considered as an attractive alternative to many other schemes.

This paper is organized as follows. Section 2 describes the system class along with a discussion of the assumptions. In Section 3 we state the main result, namely that adaptive λ-tracking is solved for nonlinear systems. In Section 4 we discuss the practicability of adaptive λ-tracking using a realistic process control problem. The performance and ease of design of the adaptive λ-tracker are compared with those of a nonlinear controller based on an I/O linearization of the system.

2. *System class*

Throughout this paper we consider nonlinear systems of the form

$$\left.\begin{aligned}\dot{y}(t) &= f(t, y(t), z(t)) + g(t, y(t), z(t))u(t),\\ \dot{z}(t) &= h(t, y(t), z(t)),\end{aligned}\right\} \tag{2}$$

where, for $n, m \in \mathbb{N}$ with $n > m$,

$$f: \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^{n-m} \to \mathbb{R}^m,$$
$$g: \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^{n-m} \to \mathbb{R}^{m \times m},$$
$$h: \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^{n-m} \to \mathbb{R}^{n-m}$$

are assumed to be *Carathéodory functions*† with equilibrium points $(y_e, z_e, u_e) \in \mathbb{R}^m \times \mathbb{R}^{n-m} \times \mathbb{R}^m$, i.e.

$$0 = f(t, y_e, z_e) + g(t, y_e, z_e)u_e \quad \text{and} \quad 0 = h(t, y_e, z_e) \quad \forall t \in \mathbb{R}.$$

As usual, $u(t)$ is considered as the manipulated variable and $y(t)$ is the output to be controlled. The state dimension n need not be known.

The system (2) is in the so-called Byrnes–Isidori normal form (see Byrnes and Isidori, 1984), where $\dot{z} = h(t, y_e, z)$ is the zero dynamics. The only assumption implied by starting out in this form is that the nonlinear system has a relative degree of one. As any nonlinear SISO system with relative degree one can be brought into the form (1) (see Isidori, 1989), this is certainly not restrictive for the SISO case.

In addition to requiring a relative degree of one, we have to require that the following assumptions on the nonlinear system will hold.

Assumption 2.1. f and g are globally Lipschitz at (y_e, z_e); i.e., for some unknown constants $M_f, M_g > 0$, independent of $t \in \mathbb{R}$, we have

$$\|f(t, y, z) - f(t, y_e, z_e)\| \le M_f \left\| \begin{matrix} y - y_e \\ z - z_e \end{matrix} \right\| \quad \forall (t, y, z) \in \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^{n-m}$$

and

$$\|g(t, y, z) - g(t, y_e, z_e)\| \le M_g \left\| \begin{matrix} y - y_e \\ z - z_e \end{matrix} \right\| \quad \forall (t, y, z) \in \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^{n-m}.$$

Assumption 2.2. h is continuously differentiable and globally Lipschitz at y_e; i.e. for some unknown constant $M_h > 0$, independent of (t, z),

$$\|h(t, y, z) - h(t, y_e, z)\| \le M_h \|y - y_e\| \quad \forall (t, y, z) \in \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^{n-m}.$$

Also, $\forall t \in \mathbb{R}$, $h(t, y_e, \cdot)$ is globally Lipschitz, i.e. for some unknown constant $\bar{M}_h > 0$, independent of $t \in \mathbb{R}$,

$$\|h(t, y_e, z) - h(t, y_e, \bar{z})\| \le \bar{M}_h \|z - \bar{z}\| \quad \forall (t, z, \bar{z}) \in \mathbb{R} \times \mathbb{R}^{n-m} \times \mathbb{R}^{n-m}.$$

Assumption 2.3. g is uniformly bounded and bounded away from 0; i.e., there exist positive-definite $P = P^T \in \mathbb{R}^{m \times m}$ and either $0 < \sigma_1 < \sigma_2$ or $\sigma_1 < \sigma_2 < 0$ such that

$$2\sigma_1 I_m \le Pg(t, y, z) + g(t, y, z)^T P \le 2\sigma_2 I_m$$

$\forall (t, y, z) \in \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^{n-m}$. P, σ_1, σ_2 are unknown; only existence is required.

Assumption 2.4. The zero dynamics is exponentially converging towards $\eta_e = z_e$ in the large; i.e., for any $\beta > 0$, there exist some (unknown) $M(\beta) > 0$, and $\varepsilon > 0$ such that the solution of

$$\dot{\eta}(t) = h(t, y_e, \eta(t)), \quad \eta(t_0) = \eta_0$$

satisfies

$$\|\eta(t) - \eta_e\| \le M(\beta) e^{-\varepsilon(t-t_0)} \|\eta_0\| \quad \forall t \ge t_0 \ge 0, \quad \|\eta_0 - \eta_e\| < \beta.$$

From an application point of view, Assumptions 2.1 and 2.2 can be considered as 'technical assumptions'. If the system is single-input/single-output, i.e. $m = 1$, then Assumption 2.3 simpifies to the assumption that $g(\cdot)$ is uniformly bounded away from zero and uniformly bounded from above, i.e. the following assumption.

Assumption 2.3'. There exist σ_1 and σ_2 with $0 < \sigma_1 < \sigma_2$ or $\sigma_1 < \sigma_2 < 0$ such that

$$\sigma_1 \le g(t, y, z) \le \sigma_2 \quad \forall (t, y, z) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}^{n-1}.$$

Assumption 2.3' means that g is either 'positive' or 'negative', but it is unknown which sign it has. It is merely required that g will not change sign anywhere. This is a rather strong assumption that essentially excludes the presence of any singular points in the whole state space. The strongest assumption, and the one that is probably the most difficult to show for an application, is Assumption 2.4, which requires the system to be globally minimum-phase. It is clear that these assumptions, together with the implied assumption that the relative degree is one, are not met by many practical control problems. On the other hand, many important practical control problems, for example many chemical reactors, will meet the requirements. In Section 4 we shall demonstrate results from a realistic chemical reactor for which the assumptions can be shown to hold.

In the following sections, we assume that the signals to be tracked belong to $\mathcal{W}^{1,\infty}$, i.e. absolutely continuous functions that are essentially bounded with essentially bounded derivatives.

3. *λ-tracking*

Before we prove the first result on our system class, the system (2) will be simplified by a straightforward coordinate transformation that converts the equilibrium into zero.

† $\alpha: \mathbb{R} \times \mathbb{R}^q \to \mathbb{R}$ is called a Carathéodory function if $\alpha(\cdot, x): t \mapsto \alpha(t, x)$ is measurable on $\mathbb{R}$ for each $x \in \mathbb{R}^q$, and $\alpha(t, \cdot): x \mapsto \alpha(t, x)$ is continuous on $\mathbb{R}^q$ for all $t \in \mathbb{R}$.

Remark 3.1. $(y(t), z(t))$ is a solution of (2) if and only if $(\bar{y}(t), \bar{z}(t)) := (y(t) - y_e, z(t) - z_e)$ is a solution of

$$\frac{d}{dt}\bar{y}(t) = \tilde{f}(t, \bar{y}(t), \bar{z}(t)) + \tilde{g}(t, \bar{y}(t), \bar{z}(t))v(u(t)),$$

$$\frac{d}{dt}\bar{z}(t) = \tilde{h}(t, \bar{z}(t)) + \bar{h}(t, \bar{y}(t), \bar{z}(t)),$$

where

$$\tilde{f}(t, \bar{y}, \bar{z}) := f(t, \bar{y} + y_e, \bar{z} + z_e) - f(t, y_e, z_e) + [g(t, \bar{y} + y_e, \bar{z} + z_e) - g(t, y_e, z_e)]u_e,$$

$$\tilde{g}(t, \bar{y}, \bar{z}) := g(t, \bar{y} + y_e, \bar{z} + z_e),$$

$$v(u) := u - u_e,$$

$$\tilde{h}(t, \bar{z}) := h(t, y_e, \bar{z} + z_e),$$

$$\bar{h}(t, \bar{y}, \bar{z}) := h(t, \bar{y} + y_e, \bar{z} + z_e) - h(t, y_e, \bar{z} + z_e).$$

This follows easily by rearranging (2) and using the fact that (y_e, z_e, u_e) is an equilibrium point. It follows from Assumptions 2.1, 2.2 and 2.4 that

$$\tilde{f}(t, 0, 0) = 0, \quad \tilde{g}(t, 0, 0) = g(t, y_e, z_e), \quad v(u_e) = 0,$$

whence $(0, 0, v(u_e))$ is an equilibrium point.

Furthermore, $\bar{h}(\cdot)$ is continuously differentiable,

$$\|\tilde{f}(t, \xi, \eta)\| \le (M_f + M_g \|u_e\|) \left\| \begin{matrix} \xi \\ \eta \end{matrix} \right\| \quad \forall (t, \xi, \eta) \in \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^{n-m}, \quad (3)$$

$$\|\bar{h}(t, \xi, \eta)\| \le M_h \|\xi\| \quad \forall (t, \xi, \eta) \in \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^{n-m}, \quad (4)$$

and

$$\dot{\eta}(t) = \tilde{h}(t, \eta(t)) = h(t, y_e, \eta(t) + z_e)$$

is exponentially stable in the large.

If only Assumption 2.3 is assumed then the correct sign in the feedback law has to be found adaptively. This can be achieved by Nussbaum functions, a concept due to Nussbaum (1983), or, more restrictively, by scaling-invariant Nussbaum functions, a concept due to Logemann and Owens (1988).

Definition 3.2. Let $k' \in \mathbb{R}$. A piecewise, right-continuous function $N(\cdot): [k', \infty) \to \mathbb{R}$ is called a *Nussbaum function* if it satisfies

$$\sup_{k > k_0} \frac{1}{k - k_0} \int_{k_0}^{k} N(\tau)\, d\tau = +\infty,$$
$$\inf_{k > k_0} \frac{1}{k - k_0} \int_{k_0}^{k} N(\tau)\, d\tau = -\infty \quad (5)$$

for some $k_0 \in (k', \infty)$. A Nussbaum function is called *scaling-invariant* if

$$\hat{N}(t) := \begin{cases} \alpha N(t) & \text{if } N(t) \ge 0, \\ \beta N(t) & \text{if } N(t) < 0 \end{cases}$$

is a Nussbaum function for all $\alpha, \beta > 0$.

It is easy to see that (5) holds for some $k_0 \in (k', \infty)$ if and only if it is valid for all $k_0 \in (k', \infty)$.

Logemann and Owens (1988) have proved that $N(k) = \cos \frac{1}{2}\pi k\, e^{k^2}$ is scaling-invariant. A list of further Nussbaum functions can be found in Ilchmann (1993).

We are now in a position to prove the first main result. For a prespecified $\lambda > 0$, let $y_{ref}(\cdot) \in \mathcal{W}^{1,\infty}$ denote a reference signal that should be tracked and $\rho(\cdot) \in \mathcal{W}^{1,\infty}$ some noise. Then the adaptive λ-tracker is supposed to force the noise-corrupted error $e(t) = y(t) - y_{ref}(t) + \rho(t)$ not necessarily to zero but towards a λ-neighbourhood of 0 as t tends to infinity.

Theorem 3.3. Suppose that $\gamma, \lambda > 0$, $\delta \in \mathbb{R}$, and $N(\cdot): \mathbb{R} \to \mathbb{R}$ is a scaling-invariant Nussbaum function. For any system (2) satisfying $m = 1$, Assumptions 2.1–2.4 and $|f(t, y_e, z_e)| \le f_e$ for all $t \in \mathbb{R}$, the application of the feedback

$$u(t) = -N(k(t))[y(t) + n(t)] + \delta$$
$$\dot{k}(t) = \begin{cases} \gamma(|y(t) + n(t)| - \lambda)\, |y(t) + n(t)| & \text{if } |y(t) + n(t)| \ge \lambda, \\ 0 & \text{if } |y(t) + n(t)| < \lambda, \end{cases} \quad (6)$$

where $n(\cdot) \in \mathcal{W}^{1,\infty}$ is arbitrary, permits, for arbitrary initial conditions $y_0 = y(0) \in \mathbb{R}$, $z(0) = z_0 \in \mathbb{R}^{n-1}$, a solution $(y(\cdot), z(\cdot), k(\cdot)): [0, \omega) \to \mathbb{R}^{n+1}$ to the closed-loop system, and any solution satisfies on its maximal interval of existence, $[0, \omega)$, $\omega \in (0, \infty]$, the properties

(i) $\omega = \infty$;

(ii) $\lim_{t \to \infty} k(t) = k_\infty$ exists and is finite;

(iii) $y(\cdot), z(\cdot) \in L_\infty(0, \infty)$;

(iv) $y(t) + n(t)$ approaches the interval $[-\lambda, \lambda]$ as $t \to \infty$.

Theorem 3.3 states that the desired closed-loop objective can be achieved with the simple control strategy (6). Note that not much information about the plant to be controlled is needed. The only knowledge required is that the plant satisfies the assumptions—which is, admittedly, not always easy to show, however. Also note that the structure of the control law (6) is independent of the plant to be controlled: the feedback (6) can be used to 'stabilize' (in the sense stated in the theorem) a chemical reactor as well as, for example, a ship, provided the assumptions hold.

Remark 3.4.

(i) Note that (6) becomes almost the Willems–Byrnes adaptive stabilizer (1) if $n(\cdot) \equiv 0$ (i.e. noise and reference signal are zero), $\lambda = 0$ (i.e. asymptotic stabilization) and $N(k) = k$ (which is possible if the sign of the high-frequency gain is known to be positive).

(ii) It can be proved that adaptive stabilization (not λ-stabilization) of the given class of nonlinear systems is possible by using

$$u(t) = -k(t)[y(t) - y_e] + u_e,$$
$$\dot{k}(t) = \gamma \|y(t) - y_e\|^p, \quad k(0) = k_0, \quad (7)$$

where $p \ge 1$. However, this requires a proof of its own; even for $p = 2$, it is not possible by just putting $\lambda = 0$ in the proof of Theorem 3.3.

(iii) There are three tuning parameters, γ, δ and λ, that can be used to 'customize' the feedback for the application at hand and to improve its performance. Usually at least an approximate value for the required steady-state input u_e is known. Then $\delta = u_e$ should be chosen, in order to reduce the necessary asymptotic gain k_∞ and thus reduce sensitivity to noise. A sensible choice of the parameter γ lies in the order of magnitude of the inverse of the dominant time constant of the plant. This adjusts the speed of adaptation to the speed of the plant. The parameter λ is clearly related to the desired performance. There is one more (hidden) tuning parameter, namely the initial value of the high-gain parameter $k(0)$. If, for example by previous experiments or by further knowledge about the plant model and the size of disturbances, a good guess is known for the 'size' of k, then this value should be chosen as initial value $k(0)$. Note, however, that the size of k should never be overestimated, since unnecessary large values only increase the sensitivity to measurement noise.

(iv) The function $n(\cdot)$ in Theorem 3.3 can be interpreted in different ways. For one, we may view $n(\cdot)$ in (6) as a reference signal, $n(\cdot) \equiv -y_{ref}(\cdot)$. In that case the control strategy (6) ensures that the output tends to a λ-strip of the reference signal. If the output is corrupted by noise then $n(\cdot) \equiv \rho(\cdot)$ may be viewed as that noise. Note that it only has to satisfy $\rho(\cdot) \in \mathcal{W}^{1,\infty}$; an upper bound of the essential supremum of $|\rho(t)|$ or $|\dot{\rho}(t)|$ need not be known. But if $\|\rho(\cdot)\|_{L_\infty(0,\infty)} \le \bar{\rho}$ is known, it makes sense to set $\lambda := \bar{\rho}$. If the noise $\rho(t)$ is actually larger than λ, the controller forces the true output to follow the noise.

In practical applications, we shall usually have a mixture, and $n(t)$ must be seen as $n(\cdot) = \rho(\cdot) - y_{ref}(\cdot)$.

(v) In cases where the sign of the high-frequency gain is known, this information should be used. It is clear (see Ilchmann, 1993) that a choice of $N(k) = k$ instead of a 'switching' Nussbaum function usually leads to much better performance.

(vi) The adaptively stabilizing (not λ-stabilizing) feedback (7) has remarkable robustness properties, since no exact knowledge about the system structure and parameters is needed. There is, however, one drawback that applies equally to the results of Byrnes and Isidori (1986), Khalil and Saberi (1987) and Saberi and Lin (1990): these results have no robustness whatsoever to uncertainties that perturb the equilibrium, even for perturbations that are *arbitrarily* small. This can immediately be seen from the following argument. Assume that the nominal equilibrium lies at the origin, $(y_e, z_e, u_e) = (0, 0, 0)$, simplifying the control law to $u(t) = -k(t)y(t)$. We want to drive $y(t)$ to zero. Now assume that the real plant differs from the nominal plant in that its equilibrium is given by $(y_e, z_e, u_e) = (0, 0, \varepsilon)$, where $\varepsilon > 0$ is arbitrary. If $y(t)$ tends to zero then $u(t)$ has to tend to ε. For any ε (even arbitrarily small), $k(t)$ has to tend to infinity, and hence $k(t) \notin L_\infty$. This, of course, restricts the applicability of this result (and all other asymptotic stabilization strategies known from the literature and using $\dot{k}(t) = \|y(t)\|^p$) considerably, since, for example, even in the nominal, undisturbed case the equilibrium will never be known exactly, except for academic examples. The *nonlinear λ-tracker* overcomes this problem completely, and gives meaningful robustness guarantees for practical applications.

(vii) Unfortunately, even in the case of linear systems we were unable to prove that (6) can be extended to multi-input/multi-output systems (see Ilchmann and Ryan, 1994). Theorem 3.3 does, however, also hold for the MIMO case ($m \neq 1$) if the sign of the high-frequency gain is known to be positive and if the Nussbaum function $N(\cdot)$ is restricted to the special case $N(k) = k$. The proof can be found in Allgöwer and Ilchmann (1995).

Proof of Theorem 3.3. We proceed in several steps.

(a) The closed-loop system (2) and (6) possesses a solution $(y(\cdot), z(\cdot), k(\cdot))$: $[0, \omega) \to \mathbb{R}^{n+1}$ that is maximally extended over $[0, \omega)$ for some $\omega \in (0, \infty]$. This follows from the classical theory of ordinary differential equations.

(b) We shall prove the boundedness of $k(\cdot)$ on $[0, \omega)$. Set

$$w(t) := y(t) + n(t),$$

$$V_\lambda(w) := \begin{cases} \frac{1}{2}\gamma(|w| - \lambda)^2 & \text{if } |w| \ge \lambda, \\ 0 & \text{if } |w| < \lambda. \end{cases}$$

$$\theta_\lambda(w) := \begin{cases} \gamma \dfrac{|w| - \lambda}{|w|} w & \text{if } |w| \ge \lambda, \\ 0 & \text{if } |w| < \lambda. \end{cases}$$

Differentiation of the Lyapunov-like candidate $V_\lambda(\cdot)$ along the solution of

$$\dot{w}(t) = f(t, w(t) - n(t), z(t)) + \dot{n}(t) - N(k(t))g(t, w(t) - n(t), z(t))w(t) + g(t, w(t)) - n(t), z(t))\delta$$

yields, using Assumption 2.1, for all $t \in [0, \omega)$,

$$\begin{aligned} \frac{d}{dt} V_\lambda(w(t)) &= \theta_\lambda(w(t))\dot{w}(t) \\ &\le |\theta_\lambda(w(t))| \left[M_f \left\| \begin{matrix} w(t) - n(t) - y_e \\ z(t) - z_e \end{matrix} \right\| + f_e \right. \\ &\quad \left. + |\dot{n}(t)| + \max\{|\sigma_1|, |\sigma_2|\} |\delta| \right] \\ &\quad - N(k(t))g(t, w(t) - n(t), z(t))\theta_\lambda(w(t))w(t) \\ &\le M_1 |\theta_\lambda(w(t))| [|w(t)| + \|z(t)\| + 1] - \tilde{N}(k(t))\dot{k}(t), \end{aligned}$$

where

$$M_1 := M_f + M_f[\|y_e + n(\cdot)\|_{L_\infty(0,\infty)} + \|z_e\|] + \|\dot{n}(\cdot)\|_{L_\infty(0,\infty)} + f_e + \max\{|\sigma_1|, |\sigma_2|\} |\delta|,$$

$$\tilde{N}(k) := \begin{cases} \sigma_2 N(k) & \text{if } N(k) < 0, \\ \sigma_1 N(k) & \text{if } N(k) \ge 0, \end{cases}$$

and, without loss of generality, we have assumed $\sigma_1 > 0$ and $P = 1$ in Assumption 2.3; otherwise, we define $\tilde{N}(k)$ accordingly.

Since

$$|\theta_\lambda(w(t))| \, |w(t)| = \dot{k}(t) \quad \text{and} \\ |\theta_\lambda(w(t))| \le \lambda^{-1} |\theta_\lambda(w(t))| \, |w(t)|, \tag{8}$$

we conclude that

$$\frac{d}{dt} V_\lambda(w(t)) \le M_2 \dot{k}(t) + M_2 |\theta_\lambda(w(t))| \, \|z(t)\| - \tilde{N}(k(t))\dot{k}(t), \tag{9}$$

where

$$M_2 := M_1 + \frac{M_1}{\lambda}.$$

By Assumption 2.4 and 2.2, we have, for some ε, $M > 0$,

$$\|z(t)\| \le M e^{-\varepsilon t} \|z_0\| + M \int_0^t e^{-\varepsilon(t-s)} M_h \times [|\theta_\lambda(s)| + (|y(s)| - |\theta_\lambda(s)|) + |y_e|] \, ds,$$

and, since

$$|y| - |\theta_\lambda| \le |w| + |n| - (|w| - \lambda) \le |n| + \lambda,$$

we have

$$\|z(t)\| \le M_3 + M_3 \mathcal{L}(|\theta_\lambda(\cdot)|)(t), \tag{10}$$

where

$$M_3 := M \|z_0\| + M M_h [\|n(\cdot)\|_{L_\infty(0,\infty)} + \lambda + |y_e|] \times \sup_{t \ge 0} \int_0^t e^{-\varepsilon(t-s)} \, ds,$$

$$\mathcal{L}(|\theta_\lambda(\cdot)|)(t) := \int_0^t e^{-\varepsilon(t-s)} |\theta_\lambda(s)| \, ds.$$

By (8), (10) and the Cauchy–Schwarz inequality, we conclude that

$$\begin{aligned} &\int_0^t |\theta_\lambda(s)| \, \|z(s)\| \, ds \\ &\le M_3 \int_0^t |\theta_\lambda(s)| \, ds + M_3 \int_0^t |\theta_\lambda(s)| \, \mathcal{L}(|\theta_\lambda(\cdot)|)(s) \, ds \\ &\le \frac{M_3}{\lambda} \int_0^t |\theta_\lambda(s)| \, |w(s)| \, ds + M_3 \|\theta_\lambda(\cdot)\|_{L_2(0,t)} \|\mathcal{L}(\theta_\lambda)(\cdot)\|_{L_2(0,t)} \\ &\le \frac{M_3}{\lambda} \int_0^t \dot{k}(s) \, ds + M_3 \|\mathcal{L}\| \, \|\theta_\lambda(\cdot)\|^2_{L_2(0,t)} \\ &\le \frac{M_3}{\lambda} [k(t) - k(0)] + M_3 \|\mathcal{L}\| \int_0^t |\theta_\lambda(s)| \, |w(s)| \, ds \\ &= M_4 [k(t) - k(0)], \end{aligned} \tag{11}$$

where

$$M_4 := \frac{M_3}{\lambda} + M_3 \|\mathcal{L}\|.$$

Integration of (10) and insertion of (11) gives, for all $t \in [0, \omega)$,

$$V_\lambda(w(t)) \le V_\lambda(w(0)) + (M_2 + M_2M_4)[k(t) - k(0)] - \int_0^t \tilde{N}(k(\tau))\dot{k}(\tau)\,d\tau$$
$$\le V_\lambda(w(0)) + [k(t) - k(0)] \times \left[M_5 - \frac{1}{k(t) - k(0)}\int_{k(0)}^{k(t)} \tilde{N}(\mu)\,d\mu\right], \qquad (12)$$

where we have substituted $\mu = k(\tau)$ and set

$$M_5 := M_2 + M_2M_4.$$

Now $k(\cdot) \notin L_\infty(0, \omega)$ yields a contradiction, since $\tilde{N}(\cdot)$ is a Nussbaum function and hence the right-hand side of (12) takes negative values. This proves that $k(\cdot) \in L_\infty(0, \omega)$.

(c) Since $k(\cdot) \in L_\infty(0, \omega)$, (12) yields the boundedness of $V_\lambda(\cdot)$ and hence $w(\cdot)$, $y(\cdot) \in L_\infty(0, \omega)$. Since $\dot{\eta}(t) = h(t, y_e, \eta(t))$ is exponentially stable, a substitute of variation-of-constants (see Ilchmann, 1997) applied to

$$\dot{z}(t) = h(t, y_e, z(t)) + [h(t, y(t), z(t)) - h(t, y_e, z(t))],$$

and use of Assumption 2.2, yields, for some $M, \varepsilon > 0$,

$$\|z(t)\| \le Me^{-\varepsilon t}\|z(0)\| + M\int_0^t M_h e^{-\varepsilon(t-s)}\|y(s) - y_e\|\,ds,$$

and therefore $z(\cdot) \in L_\infty(0, \omega)$. It follows by maximality of ω that $\omega = \infty$. Hence (i)–(iii) are proved.

(d) It remains to show (iv). Note that

$$|\theta_\lambda(w(t))|\,\|z(t)\| \le \lambda^{-1}|w(t)|\,|\theta_\lambda(w(t))|\,\|z(t)\| \le \lambda^{-1}\|z(t)\|\,\dot{k}(t).$$

Let

$$M_6 := \|M_2 + M_2\lambda^{-1}\|\,z(\cdot)\,\| - \tilde{N}(k(\cdot))\|_{L_\infty(0,\infty)}.$$

Then, by (9),

$$\frac{d}{dt}V_\lambda(w(t)) \le -\dot{k}(t) + (M_6 + 1)\dot{k}(t).$$

Therefore the derivative of the sign-indefinite Lyapunov function

$$W(\cdot,\cdot): \mathbb{R}^{m+1} \to \mathbb{R}, \quad (w, k) \mapsto V_\lambda(w) - (M_6 + 1)k$$

along the solution components $w(t)$ and $k(t)$ of (2) and (6) is, for all $t \ge 0$,

$$\frac{d}{dt}W(w(t), k(t)) \le -\dot{k}(t) \le 0.$$

Now LaSalle's invariance principle for non-autonomous systems (see LaSalle, 1976) proves that the ω-limit set of the bounded solution $(w(\cdot), z(\cdot), k(\cdot))$ is contained in $\{(w, z, k) \in \mathbb{R}^{n+1} \mid |w| \le \lambda\}$. This proves (iv) and completes the proof. □

4. *λ-tracking control of a chemical reactor*

In this section we shall apply the feedback law (6) (adaptive λ-tracker) to a realistic process-control problem. The performance and ease of controller design are compared with those of a nonlinear controller based on exact I/O linearization.

The commercially relevant process we are studying is methanol synthesis in a polytropic, catalytic continuous stirred tank reactor (CSTR) on a solid-phase catalyst. The reversible exothermic reaction considered is given by a complex kinetic scheme following a four-step mechanism with realistic parameters. The model and data are taken from Berty *et al.* (1989) and Tátrai *et al.* (1992), and are based on the 'UCKRON-1' test problem for reaction-engineering modelling, which we extend to form a realistic control problem.

We consider two different models for this process: a rather simple one (which we shall call the *reduced model*) of order three on which the controller designs are based, and a detailed model of order nine, which represents the true dynamical behaviour of the real reactor in a very good way. This detailed model will be used to represent the real plant when testing the controllers in simulation.

For brevity, the equations and parameters of the detailed model are not given here. Instead, we refer to Berty *et al.* (1989) and Tátrai *et al.* (1992) for a detailed description of this model. The reduced model is given by the following equations:

$$\dot{\xi} = -q\xi + r(\xi, T_c)\frac{1}{\epsilon}, \qquad (13a)$$

$$\dot{T}_c = \frac{1}{(1-\epsilon)C_c\rho_c}[\Delta H_r r(\xi, T_c) + (T_g - T_c)U_cS_c], \qquad (13b)$$

$$\dot{T}_g = q(T_g^{feed} - T_g) + \frac{1}{\epsilon C_c\rho_c} \times [(T_w - T_g)U_wS_w + (T_c - T_g)U_cS_c], \qquad (13c)$$

where $r(\xi, T_c)$ is a nonlinear function of the reaction extent ξ and the catalyst temperature T_c (see Appendix). The first equation is an 'overall mass balance' for the reaction extend ξ, and the second and third equations are heat balances for the catalyst and gas phase. The variables T_g and T_c are the temperature of the gas phase and catalyst. T_w is the coolant temperature, which we use as the manipulated variable (input u). The temperature of the feed T_g^{feed}, the feedflow q and the feed concentrations (which appear explicitly only in the detailed model) are considered as disturbances. The reactor is very sensitive to these disturbances, where T_g^{feed} must be considered as having the main influence. The region of attraction of the stable operating point chosen (cf. Appendix) is very small. A change in T_w by as little as 1% results in a dramatic change in the gas temperature T_g (thermal runaway), and eventually to destruction of the catalyst (see Fig. 1). Thus the control objective is to avoid thermal runaway by maintaining the gas-phase temperature at a desired level (which assures proper production of methanol) and attenuating disturbances in the feed. Hence we have a SISO control problem with input T_w and output T_g (cf. Appendix).

In order to apply the results of Section 3, we first have to check whether the applicability assumptions given in Section 2 are satisfied. This is, however, very difficult when using the detailed model. On the other hand, the reduced model is known to describe the structural properties of the reactor dynamics sufficiently well. Therefore we can check the assumptions using the reduced model. Note that the model (13) is given in the form (2), and thus has a relative degree of one. It is easy to check that Assumptions 2.1–2.3 are satisfied, while the exponential stability of the two-dimensional zero dynamics (Assumption 2.4) can be verified only by a numerical procedure. This means that all necessary assumptions are satisfied and that the control law (6) will guarantee λ-stabilization of the reactor.

The reduced model is also used to determine the desired

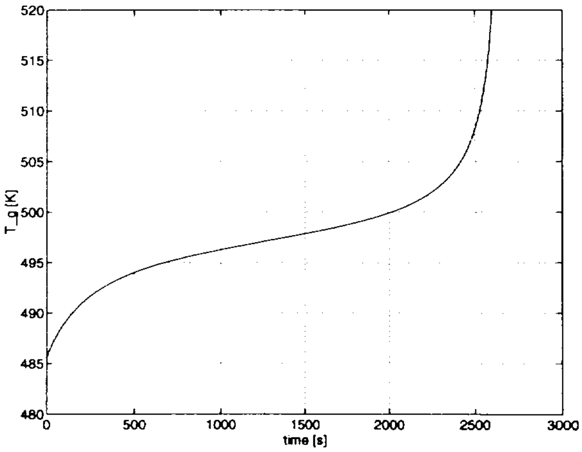

Fig. 1. Open-loop behaviour of gas temperature T_g after a change in the coolant temperature T_w by +1%.

steady-state output $y_e = T_{ge}$ and corresponding steady-state input $u_e = T_{we}$. Furthermore, the sign of the high-frequency gain, which is a positive constant $(U_w S_w / \epsilon C_g \rho_g)$ in this application, can be determined using the reduced model. Knowledge of the sign of the high-frequency gain is not required for applying the results derived in Section 3. If, as in the present example, the sign is known, however, this information will always lead to improved performance of the closed loop. Making use of our knowledge of the sign of the high-frequency gain, we choose the Nussbaum function $N(k(t))$ in the control law (6) to be $N(k(t)) = k(t)$. The design parameter λ, which represents the tolerated steady-state error, is fixed at $\lambda = 0.3$ K. γ is chosen to be $\gamma = 1\,\mathrm{K}^{-1}$, and δ is chosen as $\delta = u_e = 480$ K (i.e. the anticipated value of the necessary steady-state input needed to achieve the desired steady-state output $y_e = 487.4$ K. Note that for the detailed model a different steady-state input u_e is needed in order to achieve $y_e = 487.4$ K (cf. Fig. 2). This means that the equilibrium point is 'modelled' incorrectly. This will not affect the stability of the λ-tracker, but would cause the gain k in the control law (7) (or the gain in previously published high-gain adaptive control strategies) to grow without bound. The function $n(\cdot)$ in the feedback (6) is chosen as the (constant) desired output temperature y_e.

Figure 2 shows the gas-phase temperature T_g in the closed loop when the control law described above is applied to the *detailed* model of the reactor. During the first 200 s, no disturbances are applied. The initial conditions of the detailed model are chosen such that the reactor with input $u_e = 480$ K is at steady state ($T_g = 484.9\,\mathrm{K} \neq y_e$). It can be seen that within one minute the gas temperature reaches the desired level without overshoot. Figures 3 and 4 show the behaviour of the manipulated input T_w and the value of the high-gain parameter $k(\cdot)$ over time. At time $t = 200$ s a significant disturbance of +7% is applied to the feed temperature T_g^{feed} and taken back at time $t = 350$ s. Figure 2 shows that the controlled output only changes its value within the control tolerance of 0.3 K. Figure 3 displays the necessary input moves in T_w, and Fig. 4 shows that no further increase in the gain k is needed to attenuate this disturbance. The controller was also tested with other disturbances, measurement noise, additional parameter uncertainties, and for the task of tracking a reference trajectory. In all cases completely satisfying performance as for the disturbance described above could be observed. Figure 5 gives examples of the results for the same simulation experiment when band-limited white noise with sampling time 1 s and a variance of 0.1 K is added to the measured output y_e. Figure 6 shows the corresponding behaviour of the high-gain parameter k.

Remark 4.1. A possible redesign would use the information obtained by the closed-loop experiment described above. Choosing δ to have the proper steady-state input value for the detailed model ($\delta = 481.5$ K, which can be taken from Fig. 3) would then result in a lower value of the high-gain parameter k, which would further improve performance with respect to noise properties.

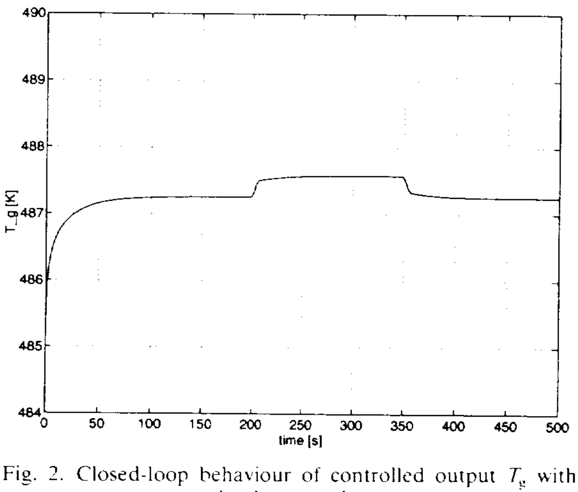

Fig. 2. Closed-loop behaviour of controlled output T_g with adaptive λ-tracker.

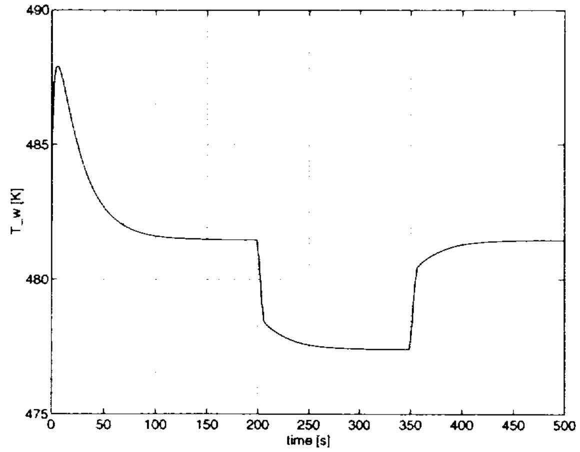

Fig. 3. Closed-loop behaviour of manipulated input T_w with adaptive λ-tracker.

In order to further assess the performance of the adaptive λ-tracker, we give, for comparison, results achieved with a different controller, based on an underlying exact linearization of the I/O behaviour (Isidori, 1989) of the reactor.

Standard exact I/O linearization can be applied if the system has a strong relative degree and stable zero dynamics. Both of these assumptions have already been checked in connection with the λ-tracker. We proceed again along the same lines as above. The feedback is designed based on the reduced model and then tested with the detailed model. Unlike the λ-tracker, exact linearizing feedback is a state-feedback control law. Since the number of states of the reduced and detailed model differ, one would expect that I/O linearization cannot be applied at all. Fortunately the heat balances (13b, c) of the detailed and reduced model coincide, and the other states do not enter the linearizing feedback law with new input v:

$$u = \frac{\epsilon C_g \rho_g}{U_w S_w}\left[-q(T_g^{\mathrm{feed}} - T_g) + \frac{U_w S_w}{\epsilon C_g \rho_g} T_g - (T_c - T_g)U_c S_c - (T_g - y_e) + v\right]. \quad (14)$$

Thus exact *I/O* linearization can be applied. Since all uncertainties, which are due to the difference between the reduced and detailed models, only appear in the zero dynamics, the effect of these uncertainties is decoupled from

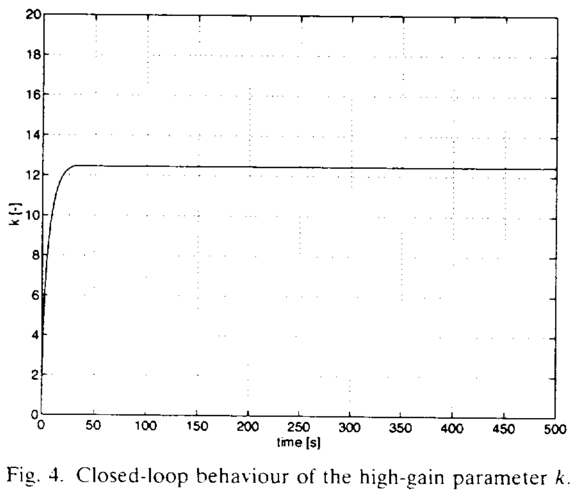

Fig. 4. Closed-loop behaviour of the high-gain parameter k.

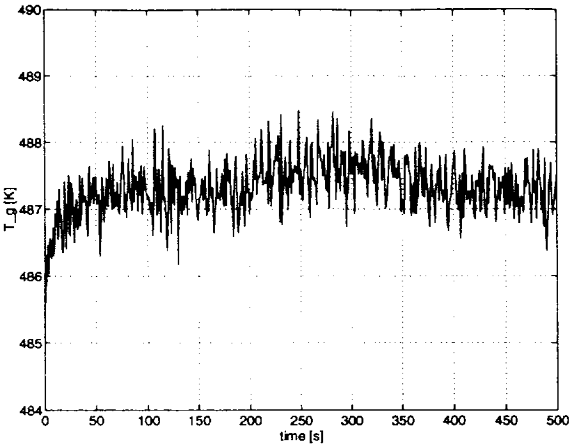

Fig. 5. Closed-loop behaviour of the controlled output T_g with adaptive λ-tracker in the presence of noise.

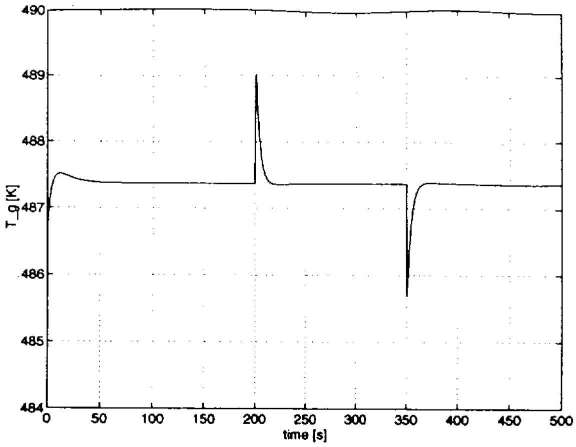

Fig. 7. Closed-loop behaviour of controlled output T_g with I/O-linearizing controller.

the output by the feedback (14). Hence exact I/O linearization is a well-suited nonlinear controller design scheme for this reactor control problem, and thus we expect this controller to behave very well for this example. The main disturbances T_g^{feed} and q have the same relative degree of one with respect to the output T_g as the manipulated input T_u. We assumed that both disturbances cannot be measured, and thus their effect on the closed-loop cannot be decoupled from the output. In order to attenuate their effect, an addition PI controler

$$v(s) = \left(k_p + \frac{k_I}{s}\right)[T_g(s) - y_e], \qquad (15)$$

with $k_p = 1500$ and $k_I = 750$, is incorporated to control the exactly I/O-linearized reactor. Figures 7 and 8 show the gas temperature T_g and the manipulated variable T_w of the closed-loop with the I/O-linearizing controller (including PI) applied. The same setup and disturbance as for the λ-tracker is used. Note that for this controller the catalyst temperature T_c needs to be measured in addition to the gas temperature T_g and that explicit knowledge of (at least part of) the model is required for controller design.

When comparing the two controllers, we can say that for this application both schemes lead to a similar level of performance for relevant disturbances, even though the closed-loop behaviour differs considerably. However, a major advantage of the λ-tracker is that the control law is very simple *and* very easy to design. Furthermore, a guarantee of stability can be given, no matter how bad the reduced model may be, as long as the assumptions are satisfied. The I/O-linearizing control scheme needs knowledge of a proper model and measurements of additional states; also, a PI-controller must be tuned. For this application, it is certainly questionable whether potential advantages with respect to closed-loop performance are worth the much greater effort needed to derive and implement the nonlinear controller based on I/O linearization.

5. *Conclusions*

We have introduced the concept of adaptive λ-tracking for nonlinear systems. This paper is based on previous results on non-identifier-based adaptive control of linear systems. By including a simple dead zone and a scaling-invariant Nussbaum function into the control law, it was possible to prove adaptive λ-tracking for the SISO case even if the sign of the high-frequency gain is not known. For the MIMO case, the proof requires that the sign of the high-frequency gain be known. Guaranteed achievement of the control objective has been proved for a large class of nonlinear systems satisfying certain assumptions, where the need for a globally exponentially stable zero dynamics and for a relative degree of one are the most restricting assumptions from an application point of view. The proposed adaptive control law overcomes serious robustness deficiencies with respect to uncertainties in the equilibrium that are shown by previously derived high-gain adaptive feedback laws for nonlinear systems. Using a realistic process control problem, we have demonstrated the practical applicability of this control law.

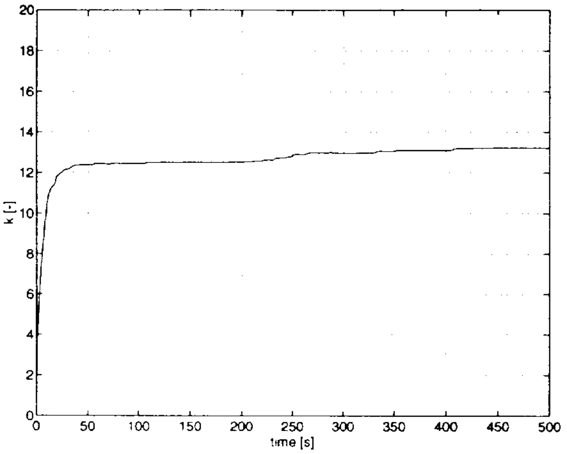

Fig. 6. Closed-loop behaviour of high-gain parameter k in the presence of noise.

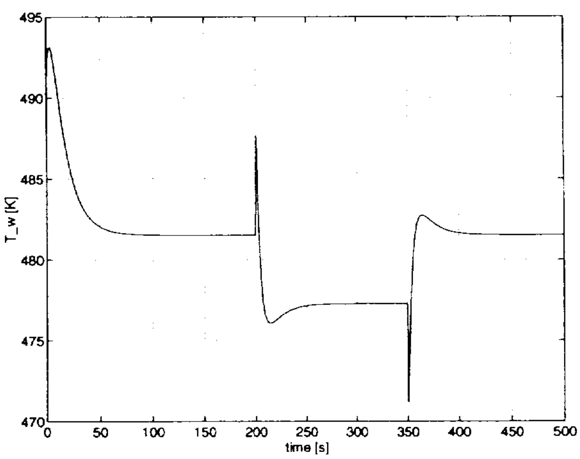

Fig. 8. Closed-loop behaviour of manipulated input T_w with I/O-linearizing controller.

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References

Allgöwer, F. and Ilchmann, A. (1995) Multivariable λ-tracking for nonlinear chemical processes. In *Proc. 3rd European Control Conf.*, Rome, pp. 1645–1651.

Berty, J. M., Lee, S., Szeifert, F. and Cropley, J. B. (1989) The 'UCKRON-1' test problem for reaction engineering modeling. *Chem. Engng Commun.* **76**, 9–33.

Byrnes, C. I. and Isidori, A. (1984) A frequency domain philosophy for nonlinear systems, with applications to stabilization and to adaptive control. In *Proc. 23rd IEEE Conf. on Decision and Control*, Las Vegas, NV, pp. 1569–1573.

Byrnes, C. I. and Isidori, A. (1986) Asymptotic expansions, root-loci and the global stability of nonlinear feedback systems. In *Algebraic and Geometric Methods in Nonlinear Control Theory*, ed. M. Fliess and M. Hazewinkel, Reidel, Dordrecht.

Ilchmann, A. (1993) *Non-identifier-based High-Gain Adaptive Control.* Lecture Notes in Control and Information Sciences, Vol. 89, Springer-Verlag, London.

Ilchmann, A. (1997) Universal adaptive stabilization of nonlinear systems. *Dyn. Control*, (to appear).

Ilchmann, A. and Ryan, E. P. (1994) Universal λ-tracking for nonlinearly-perturbed systems in the presence of noise. *Automatica* **30**, 337–346.

Isidori, I. (1989) *Nonlinear Control Systems*, 2nd ed. Springer-Verlag, Berlin.

Khalil, H. K. and Saberi, A. (1987) Adaptive stabilization of a class of nonlinear systems using high-gain feedback. *IEEE Trans. Autom. Control* **AC-32**, 1031–1035.

LaSalle, J. P. (1976) Stability of nonautonomous systems. *Nonlinar Anal. Theory, Meth. Applic.* **1**, 83–91.

Logemann, H. and Owens, D. H. (1988) input–output theory of high-gain adaptive stabilization of infinite dimensional systems with nonlinearities. *Int. J. Adaptive Control Sig. Process.* **2**, 193–216.

Mareels, I. (1984) A simple selftuning controller for stably invertible systems. *Syst. Control Lett.* **4**, 5–16.

Mårtensson, B. (1985) The order of any stabilizing regulator is sufficient a priori information for adaptive stabilization. *Syst. Control Lett.* **6**, 87–91.

Miller, D. E. and Davison, E. J. (1991) An adaptive controller which provides an arbitrarily good transient and steady-state response. *IEEE Trans. Autom. Control* **AC-36**, 68–81.

Morse, A. S. (1983) Recent problems in parameter adaptive control. In *Outils et Modèles Mathématiques pour l'Automatique, L'Analyse de Systèmes et le Traitment du Signal*, ed. I. D. Landau, pp. 733–740. Editions du CNRS 3, Paris.

Nussbaum, R. D. (1983) Some remarks on a conjecture in parameter adaptive control. *Syst. Control Lett.* **3**, 243–246.

Ryan, E. P. (1995) Universal stabilization of a class of nonlinear systems with homogeneous vector fields. *Syst. Control Lett.* **26**, 177–184.

Saberi, A. and Lin, Z. (1990) Adaptive high-gain stabilization of 'minimum-phase' nonlinear systems. In *Proc. 29th IEEE Conf. on Decision and Control*, Honolulu, HI, 3417–3422; identical version in *Control—Theory Adv. Technol.* **6**, 595–607 (1990).

Tátrai, F. Z., Varga, E. and Benedek, P. (1992) Dynamic simulation of catalytic reactors using the UCKRON-1 test problem as a kinetic model. *Ind. Engng Chem. Res.* **31**, 868–876.

Willems, J. C. and Byrnes, C. I. (1984) Global adaptive stabilization in the absence of information on the sign of the high frequency gain. In *Lecture Notes in Control and Information Sciences*, Vol. 62, pp. 49–57. Springer-Verlag, Berlin.

Appendix—Data for the methanol synthesis reactor

Reduced model

Manipulated input	$u = T_w$
Controlled output	$y = T_g$
Main disturbances	T_g^{feed}, q
Number of states	3
Operating point	$T_{we} = 280\,\text{K}$ $\xi_e = 61.8\,\text{mol m}^{-3}$, $T_{ce} = 489.4\,\text{K}$, $T_{ge} = 487.4\,\text{K}$ $q = 4.15\,\text{s}^{-1}$, $T_g^{\text{feed}} = 473\,\text{K}$ $c_{\text{ME}}^{\text{feed}} = 0$, $c_{\text{CO}}^{\text{feed}} = 772\,\text{mol m}^{-3}$, $c_{\text{H}_2}^{\text{feed}} = 1801\,\text{mol m}^{-3}$
Model parameters	$\epsilon = 0.4$, $C_c \rho_c = 8000\,\text{kJ m}^{-3}\,\text{K}^{-1}$, $\Delta H_1 = -97.97\,\text{kJ mol}^{-1}$ $U_c S_c = 288.46\,\text{kJ m}^{-3}\,\text{K}^{-1}\,\text{s}^{-1}$, $U_w S_w = 66.25\,\text{kJ m}^{-3}\,\text{K}^{-1}\,\text{s}^{-1}$, $P = 10\,120\,\text{kPa}$

The reaction rate $r(\xi, T_c)$ in (13) is given by the equation

$$r(\xi, T_c) = \frac{k\left(p_{\text{H}_2} - \sqrt{\frac{1}{K_p}\frac{p_{\text{ME}}}{p_{\text{CO}}}}\right)}{1 + b_{\text{H}_2} p_{\text{H}_2} + b_{\text{ME}} p_{\text{ME}}},$$

with

$$c_t = c_t^{\text{feed}} - \xi,$$

$$c_t^{\text{feed}} = c_{\text{H}_2}^{\text{feed}} + c_{\text{CO}}^{\text{feed}} + c_{\text{ME}}^{\text{feed}},$$

$$p_{\text{H}_2} = P\frac{c_{\text{H}_2}^{\text{feed}} - 2\xi}{c_t^{\text{feed}} - \xi},$$

$$p_{\text{CO}} = P\frac{c_{\text{CO}}^{\text{feed}} - \xi}{c_t^{\text{feed}} - \xi},$$

$$p_{\text{ME}} = P\frac{c_{\text{ME}}^{\text{feed}} + \xi}{c_t^{\text{feed}} - \xi},$$

and

$$K = 10^{3921/T_c - 7.971\log(T_c) + 2.499\times 10^{-3} T_c - 2.953\times 10^{-7} T_c^2 + 10.2}$$

$$K_p = \frac{K}{(101.325\,\text{kPa})^2},$$

$$k = 1.20\times 10^{-3}\exp\left[15\,181\left(\frac{1}{485} - \frac{1}{T_c}\right)\right],$$

$$b_{\text{H}_2} = 7.18\times 10^{-5}\exp\left[1545\left(\frac{1}{485} - \frac{1}{T_c}\right)\right],$$

$$b_{\text{ME}} = 9.88\times 10^{-4}\exp\left[-6628\left(\frac{1}{485} - \frac{1}{T_c}\right)\right].$$