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ADAPTIVE REGULATION FOR BIO-PROCESSES*

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Abstract

Our aim is to explain the concept of adaptive λ-tracking, which is a simple proportional, time-varying output feedback control with nonlinear, output-driven gain adaptation. This control achieves asymptotic tracking of reference signals within a λ-neighbourhood of the signal. As a method of control it is applicable to various classes of systems, which are essentially minimum phase and have positive (or un-mixed) high-frequency gain. Nonlinear, as well as distributed parameter systems, are included. We develop the intuition behind the approach, present typical features of the control strategy in the simple case of first order systems and illustrate its applicability to industrial processes, including a Biogas Tower Reactor.

1 Introduction

The seminal contributions of Morse (1983), Mareels (1984) and Willems and Byrnes (1984) each tackled the problem of adaptive stabilization by using a *simple* feedback strategy, rather than invoking an identification mechanism. They proved that the proportional, time-varying output feedback

$$u(t) = -k(t)y(t), \tag{1.1}$$

with the gain adaptation

$$\dot{k}(t) = y(t)^2, \qquad k(0) \in \mathbb{R}, \tag{1.2}$$

when applied to

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t), \quad x(0) = x_0 \in \mathbb{R}^n \\ y(t) &= cx(t), \end{aligned} \tag{1.3}$$

yields

$$\begin{aligned} \lim_{t\to\infty} k(t) &= k_\infty \in \mathbb{R} \\ \lim_{t\to\infty} y(t) &= 0. \end{aligned} \tag{1.4}$$

This holds true under the weak structural assumptions that $A \in \mathbb{R}^{n\times n}$, b, $c^T \in \mathbb{R}^n$, (1.3) is minimum phase (i.e. the zeros of $c^T(sI-A)^{-1}b$ lie in the open left half plane) and the high-frequency gain is positive (i.e. $cb > 0$) .

To understand the intuition behind this adaptive control strategy assume for a moment that $k(\cdot) \equiv k$ is constant. Then it is well known from root locus theory that the poles of the closed-loop system (1.1), (1.3) tend to the zeros of $c^T(sI-A)^{-1}b$ (which are assumed to be stable) and the remaining pole tends to $-\infty$ since $cb > 0$. Hence there exists a $\bar{k}$ so that the closed-loop system (1.1), (1.3) is asymptotically stable for all $k \geq \bar{k}$. Of course, the frequency domain analysis does not hold true if the time-varying adative strategy (1.1), (1.2) is applied. However, by construction, $t \mapsto k(t)$ is monotonically nondecreasing until it becomes so large that the trajectory of (1.1), (1.3) decays exponentially which, in turn, leads to a converging $k(t) = k_0 + \int_0^t y(\tau)^2 d\tau$.

In the following decade, this simple approach was extended to numerous classes of systems. (See Ilchmann (1993) for a comprehensive bibliography.) However, these developments were still far from the applications, due to the following limitations:

- the controller is not robust with respect to noise corrupted output measurements,
- the class of reference signals which can be tracked by this approach consists only of finite sums of sinusoids including constant signals,
- for tracking an internal model is necessary which complicates the control,
- the approach is mainly suited for linear systems, although nonlinear perturbations could be incorporated.

A first step towards overcoming these limitations was the introduction of a dead-zone in the gain adaptation (1.2). For prespecified but arbitrarily small and positive

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λ replace (1.2) by

$$\dot{k}(t) = \begin{cases} (\|y(t)\| - \lambda)^2 & , \quad \|y(t)\| \geq \lambda \\ 0 & , \quad \|y(t)\| < \lambda. \end{cases}$$

In this case the gain increases so long as $y(t)$ is outside $\mathcal{B}_\lambda(0)$, the ball of radius λ centered around 0. When $k(t)$ is sufficiently large, then $y(t)$ tends to 0, so entering $\mathcal{B}_\lambda(0)$, the gain adaptation is switched off and $k(t)$ converges. Note that this control strategy weakens the control objective: Instead of asymptotic stabilization of the output, it is required that the output approaches or stays within $\mathcal{B}_\lambda(0)$. However, $\lambda > 0$ is prespecified and may be chosen as small as needed.

More generally, and of greater relevance in many applications, we introduce the adaptive λ-tracker

$$\boxed{\begin{aligned} u(t) &= -k(t)e(t), \qquad e(t) = y(t) - y_{\text{ref}}(t) \\ \dot{k}(t) &= \begin{cases} (\|e(t)\| - \lambda)^2 & , \quad \|e(t)\| \geq \lambda \\ 0 & , \quad \|e(t)\| < \lambda. \end{cases} \end{aligned}} \tag{1.5}$$

If (1.5) is applied to a minimum phase system of the form (1.3) with $cb > 0$, then λ-tracking is achieved. This means that for arbitrary bounded $y_{\text{ref}}(\cdot)$, with essentially bounded derivative ($y_{\text{ref}}(\cdot)$ may have jumps), the feedback system (1.3), (1.5) has a unique solution $(x(\cdot), k(\cdot))$ on $[0, \infty)$, $k(\cdot)$ is bounded and

$$\lim_{t\to\infty} \text{dist}\,(\|y(t) - y_{\text{ref}}(t)\|, [0, \lambda]) \;=\; 0.$$

This concept was introduced by Ilchmann and Ryan (1994) and has been generalized to numerous classes of systems, such as multivariable nonlinear systems, distributed parameter systems and systems with sampled output. Furthermore, the λ-tracking approach has been proven as a suitable method for numerous industrial process control problems. Specific applications, respectively extensions, to biochemical processes, both in simulations and experimental studies, include:

- Set point control of different pH-values in a Biogas Tower Reactor. This includes an experimental implementation which was run over more than three months on an industrial pilot plant. In this application the system is MIMO with strong nonlinear couplings, so that the relative degree condition is not satisfied and the feedback law has to be modified, see Ilchmann and Pahl (1996).
- The anaerobic degradation of organic waste by microorganisms which is described by a five dimensional nonlinear SISO system. Here the input variable is the dilution rate and the control variable is the total organic concentration, see Ilchmann and Weirig (1996) and Ilchmann (1996).
- Methanol synthesis in a polytropic, catalytic continous stirred tank reactor on a solid phase catalyst was studied for a commercially relevant problem. In this case the nonlinear system is SISO but of order nine, see Allgöwer et al. (1997).
- The control of a binary distillation column by two input variables (the reflux stream and the vapour stream) in order to achieve set point regulation for one temperature in the rectifying section and another temperature in the stripping section, see Allgöwer and Ilchmann (1995).

In the remainder of this note we proceed as follows. In Section 2 we explain the λ-tracking concept in more detail but restrict our attention to first order systems. Many typical features of the adaptation strategy become already clear for this simple class of systems. In Section 3 we describe system classes to which the concept of λ-tracking has been applied. Finally, in Section 4 we discuss some of the industrial applications mentioned above.

2 λ-tracking for first order systems

In this section we focus our attention on the simple class of first order systems

$$\begin{aligned} \dot{x}(t) &= ax(t) + bu(t), \quad x(0) = x_0 \in \mathbb{R} \\ y(t) &= cx(t), \end{aligned} \tag{2.1}$$

with unknown system parameters $a, b, c \in \mathbb{R}$ and the only structural assumption imposed is that

$$cb > 0. \tag{2.2}$$

Proposition 2.1 *Let $\lambda > 0$ and assume that $y_{\text{ref}}(\cdot)$ is a bounded function, differentiable almost everywhere with essentially bounded derivative. If the λ-tracker (1.5) is applied to any system of the form (2.1), which satisfies (2.2), then the closed-loop system has a unique solution $(x(\cdot), k(\cdot))$ defined on $[0, \infty)$ and satisfies*

(i) $\lim_{t\to\infty} k(t) = k_\infty \in \mathbb{R}$,

(ii) $\lim_{t\to\infty} \text{dist}\,(|y(t) - y_{\text{ref}}(t)|, [0, \lambda]) \;=\; 0.$

A typical feature of all proofs in this high-gain context, even if the system classes are nonlinear and/or infinite dimensional, is that the most difficult part is to show boundedness of $k(\cdot)$. To this end an essential part of Proposition 2.1 relies on the following so called High-Gain Lemma.

Lemma 2.2 *Under the assumptions of Proposition 2.1 and if there exists a solution $(x(\cdot), k(\cdot))$ of the closed-loop system (2.1),(1.5) on $[0,\omega)$, for some $\omega \in (0,\infty]$, and if*

$$\lim_{t\to\omega} k(t) = \infty,$$

then

$$\lim_{t\to\omega} (y(t) - y_{\rm ref}(t)) = 0.$$

Proof: $e(t) = y(t) - y_{\rm ref}(t)$ satisfies

$$\dot{e}(t) = [a - k(t)cb]e(t) + \psi(t), \qquad (2.3)$$

where

$$\psi(t) := a y_{\rm ref}(t) - \dot{y}_{\rm ref}(t).$$

Hence, by the Variation-of-Parameters formula, for all $0 \le t_0 \le t < \omega$,

$$\begin{aligned} e(t) &= \exp\left\{\textstyle\int_{t_0}^t [a - k(\tau)cb]d\tau\right\} e(t_0) \\ &+ \textstyle\int_{t_0}^t \exp\left\{\int_s^t [a - k(\tau)cb]d\tau\right\} \psi(s)ds. \end{aligned}$$

Since, by construction (1.5), $t \mapsto k(t)$ is monotonically nondecreasing and $M := \sup_{t\ge 0} |\psi(t)|$ exists, a crude estimate yields

$$\begin{aligned} |e(t)| &\le \exp\{[a - k(t_0)cb](t - t_0)\}\, |e(t_0)| \\ &+ M \textstyle\int_{t_0}^t \exp\left\{\int_s^t [a - k(\tau)cb]d\tau\right\} ds. \end{aligned}$$

Now choose $t_0 \in (0,\omega)$ sufficiently large so that

$$[a - k(t)cb] < 0 \quad \text{for all} \quad t \ge t_0.$$

Then

$$|e(t)| \le exp\{[a - k(t_0)cb](t - t_0)\}\, |e(t_0)| + \frac{M}{-[a - k(t_0)cb]}.$$

The larger t_0 is chosen, the smaller the second term on the right hand side becomes and since the first summand tends to zero anyway when t tends to ω, the claim is proved. □

We are now in a position to prove Proposition 2.1.

Proof of Proposition 2.1: We proceed in five steps.

STEP 1: By the theory of ordinary differential equations there exists a unique solution $(x(\cdot), k(\cdot))$ on $[0,\omega)$, where $\omega \in (0,\infty]$ is maximal.

STEP 2: If $k(\cdot)$ is unbounded, then the High-Gain Lemma yields $e(t) \to 0$ and hence, by the dead zone incorporated in (1.5), $k(\cdot)$ is bounded. This contradiction establishes boundedness of $k(\cdot)$ on $[0,\omega)$.

STEP 3: If ω is finite, then by boundedness of $k(\cdot)$ and (2.3), $e(\cdot)$ cannot have a finite escape time. Therefore, $\omega = \infty$. This proves the statement (i), and it remains to prove (ii).

STEP 4: We prove boundedness of $e(\cdot)$ on $[0,\infty)$. To this end let

$$d_\lambda(e) := \begin{cases} (|e| - \lambda)^2 &, \ |e| \ge \lambda \\ 0 &, \ |e| < \lambda \end{cases}$$

and write (2.3) as

$$\dot{e}(t) = -e(t) + f_1(t) + f_2(t), \qquad (2.4)$$

where

$$\begin{aligned} f_1(t) &:= [1 - a - k(t)cb]\ [e(t) - d_\lambda(e(t))] + \psi(t), \\ f_2(t) &:= [1 - a - k(t)cb]\ d_\lambda(e(t)). \end{aligned}$$

Variation-of-Parameters applied to (2.4) yields

$$e(t) = e^{-t}e(0) + \int_0^t e^{-(t-s)}[f_1(s) + f_2(s)]ds,$$

and since $f_1(\cdot) \in L^\infty(0,\infty)$ and $f_2(\cdot) \in L^2(0,\infty)$ and the convolution of the exponential function with an L^∞-respectively an L^2-function is bounded (see Ilchmann and Logemann (1997), it follows from above that $e(\cdot)$ is bounded.

STEP 5: Since

$$\frac{d}{dt} d_\lambda(e(t))^2 = 2d_\lambda(e(t))\frac{e(t)\dot{e}(t)}{e(t)^2} \le 2d_\lambda(e(t))|e(t)||\dot{e}(t)|,$$

and the right hand side of (2.3) is bounded, it follows that $\frac{d}{dt}d_\lambda(e(t))^2 \in L^\infty(0,\infty)$. This together with $d_\lambda(e(t))^2 \in L^1(0,\infty)$ yields $\lim_{t\to\infty} d_\lambda(e(t))^2 = 0$ (see, e.g., Ilchmann (1993), p.17), and hence statement (ii) is proved. □

3 Generalizations and extensions of the λ-tracking approach

The simple adaptive λ-tracker (1.5), which has been described in detail in the previous section for first order systems, has been successfully applied to many other classes of systems, such as linear distributed parameter systems and nonlinear systems of the form

$$\begin{aligned} \dot{y}(t) &= f(t, y(t), z(t)) + G(t, y(t), z(t))\, u(t) \\ \dot{z}(t) &= h(t, y(t), z(t)). \end{aligned} \qquad (3.1)$$

Here $f(t,y,z) \in \mathbb{R}^m$, $G(t,y,z) \in \mathbb{R}^{m\times m}$, $h(t,y,z) \in \mathbb{R}^p$ are smooth functions which satisfy certain upper bounds, but more crucially the system must have globally, exponentially stable zero dynamics and must satisfy a generalized relative degree one condition with G being "positive" and bounded away from zero. The following classes of systems have been investigated so far:

- multivariable linear systems subjected to nonlinear perturbations, and SISO systems with unknown sign of the high-frequency gain, see Ilchmann and Ryan (1994),
- linear SISO systems with nonlinear actuator characteristics and nonlinear perturbations, see Ryan (1994),
- nonlinear systems which are affine linearly bounded and SISO, see Allgöwer et al. (1997), and MIMO, see Allgöwer and Ilchmann (1995),
- larger classes of nonlinear systems (e.g. not in input affine form (3.1)) and improved transient behaviour, see Ilchmann (1996),
- linear MIMO systems with sampled output, see Ilchmann and Townley (1996),
- MIMO distributed parameter systems, see Ilchmann and Logemann (1997).

4 λ-tracking for industrial applications

We now show that the λ-tracker, or simple modifications of it, is suitable for industrial process control problems. Indeed the simple design and cheap implementation of this controller, together with its remarkable robustness properties, make it appealing in regulating those process control systems which are minimum phase and of relative degree one. Note that in many applications it is useful to modify (1.5) slightly by introducing additional off-line tuning parameters α, γ, r which can be used to "customize" the feedback for the specific application at hand:

$$\begin{array}{rcll} u(t) & = & -\alpha k(t)e(t) + \tilde{u}(t), & e(t) = y(t) - y_{ref}(t) \\ \dot{k}(t) & = & \gamma \begin{cases} (\|e(t)\| - \lambda)^r &, \ \|e(t)\| \geq \lambda \\ 0 &, \ \|e(t)\| < \lambda. \end{cases} & \end{array} \tag{4.1}$$

The parameters α, γ and r have transparent meaning and their choice is in general straightforward. The function $\tilde{u}(t)$ might be a bounded input function, e.g. a steady state input value which drives the system into a neighbourhood of the main operating point. A sensible choice for the parameter γ, which adjusts the speed of adaptation, is the order of magnitude of the inverse of the dominant time constant of the plant. The parameter $r \geq 1$ affects the speed of the gain adaptation. The larger r is, the faster that $k(t)$ grows. However, r should not be so large as to stiffen the closed-loop dynamics.

We now select some of the applications already listed at the end of Section 1 and describe them in more detail.

4.1 pH-Control of a Biogas Tower Reactor

Adaptive λ-tracking has been applied to the control of a biogas tower reactor of pilot plant scale at Deutsche Hefewerke (DHW) in Hamburg, Germany. The reactor is used for anaerobic treatment of waste water which comes from a yeast production plant and contains harmful sulphuric acid and organic compounds. The waste water is fed into the biogas tower reactor where, because of micro-organisms, a mesophile anaerobic biochemical conversion of the organic influent compounds takes place.

The reactor, which is 20m high and has a diameter of 1m, consists of four identical modules, each similar in structure to an airlift loop reactor, which are arranged in a tower on top of each other. The waste water stream, with flow rate f_{feed}, is split up into four influent streams with flow rates $f_{feed,i}$, where the i-th stream is fed to the i-th module. These inflow rates are the four manipulated control variables u_i. The four variables y_i to be controlled are the pH-values in each module. These are measured on-line. The overall process is modelled as follows

$$\dot{y}(t) = Ay(t) + r(y(t)) - G(y(t))\,u(t)\,.$$

Here $y = (y_1, ..., y_4)^T \in \mathbb{R}^4$, $u = (u_1, ..., u_4)^T \in \mathbb{R}^4$ and the matrix

$$A = \begin{pmatrix} -a_1 & a_1 & 0 & 0 \\ a_1 & -(a_1+a_2) & a_2 & 0 \\ 0 & a_2 & -(a_2+a_3) & a_3 \\ 0 & 0 & a_3 & -a_3 \end{pmatrix},$$

with $a_1, a_2, a_3 > 0$, represents a compartmental model. The nonlinear function

$$r(y) = (r_1(y_1), \ldots, r_4(y_4))^T$$

is known to be affine linearly bounded and to have positive entries. Thus it reflects the increase of the pH-value if no waste water is fed into the reactor. Finally

$$G(y) = \begin{pmatrix} y_1 - y_{feed} & 0 & 0 & 0 \\ y_2 - y_1 & y_2 - y_{feed} & 0 & 0 \\ y_3 - y_2 & y_3 - y_2 & y_3 - y_{feed} & 0 \\ y_4 - y_3 & y_4 - y_3 & y_4 - y_3 & y_4 - y_{feed} \end{pmatrix}$$

with $y_{feed} > 0$, gives the waste water input at the different modules.

The reactor is open-loop unstable and the strong nonlinear couplings are obvious from the form of G. For details see Ilchmann and Pahl (1996).

In this case the minimum phase assumption is satisfied but the relative degree one assumption fails. For this reason we introduce a simple extension of the λ-tracker with the

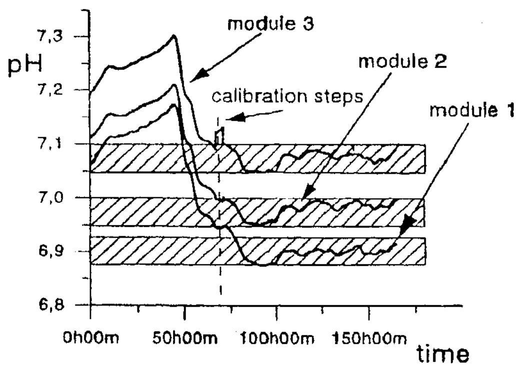

Figure 1: Stabilization of pH-values in the biogas tower reactor (experimental results).

following physically motivated nonlinear feedback law:

$$
\begin{aligned}
u_1(t) &:= \tfrac{k(t)}{y_1(t)-y_{feed}}(y_1(t)-y_{ref,1}),\\
u_2(t) &:= \tfrac{k(t)}{y_2(t)-y_{feed}}(y_2(t)-y_{ref,2}) - \tfrac{y_2(t)-y_1(t)}{y_2(t)-y_{feed}}\, u_1(t),\\
u_3(t) &:= \tfrac{k(t)}{y_3(t)-y_{feed}}(y_3(t)-y_{ref,3})\\
&\quad -\tfrac{y_3(t)-y_2(t)}{y_3(t)-y_{feed}}\,[u_1(t)+u_2(t)],\\
u_4(t) &:= \tfrac{k(t)}{y_4(t)-y_{feed}}(y_4(t)-y_{ref,4})\\
&\quad -\tfrac{y_4(t)-y_3(t)}{y_4(t)-y_{feed}}\,[u_1(t)+u_2(t)+u_3(t)].
\end{aligned}
$$

This feedback law can be viewed as a cascaded nonlinear compensator which is combined with the adaptation law for the gain $k(t)$ as in (4.1). It achieves λ-tracking and convergence of the gain adaptation.

The control law with parameters $\gamma = 0.002$, $\lambda = 0.05$, $r=2$, and $y_{ref} = [6.9, 6.975, 7.075]^T$ was implemented on a DCS using a discrete integration algorithm, with a sampling time of 6 minutes. For technical reasons, in the experiment, only the three modules 1-3 were considered and used.

Figure 1 shows experimental data for the pH-values y_i that were obtained at the pilot plant when the above "extended" λ-tracker was applied. The controller was switched on at time $t = 48h$. It can be seen that without the controller, the pH-values drift away from the required set-point. However, within only 24 hours of having switched on the λ-tracker, the pH-values are brought back to the desired λ-strips. These strips are depicted in Fig. 1 by the cross-hatched areas. In common with many other applications, the gain parameter k only rises to a modest level ($k=0.08$).

4.2 Chicken manure treatment

Many biotechnological processes can be described by

$$\dot{\xi}(t) \;=\; K\varphi(\xi(t)) - D(t)\,\xi(t) - q(\xi(t)) + f(t)\,, \quad (4.2)$$

where $\xi(t) \in \mathbb{R}^n$ is the state vector, $\varphi(\xi(t))$ contains the reaction rates, $f(t)$ the feed rates, $q(\xi(t))$ the gazeous outflow rates, $K \in \mathbb{R}^{n\times k}$ the stochiometric coefficients and $D(t)$ denotes the dilution rate. See Bastin and Dochain (1990) for details.

Many of these processes have strict relative degree one in certain practically relevant regions. Note that they are not in input affine form (3.1). However, many systems of the form (4.2) do have the advantage that certain practically relevant regions are invariants of their flows.

As an example we consider the anaerobic degradation of organic waste by micro-organisms, more precisely in the treatment of chicken manure. The five dimensional model (see Bastin and Dochain (1990), Stoyanov and Simeonov (1995) and Bastin and Van Impe (1995)) is based on the three-stage reaction scheme

$$
\begin{aligned}
S_0 &\hookrightarrow S_1\\
S_1 &\hookrightarrow X_1 + S_2\\
S_2 &\hookrightarrow X_2 + Q\,,
\end{aligned}
$$

where S_0 denotes the influent polluting organic concentration $[mg/l]$, S_1 denotes the substrate concentration for acidogenic bacteria $[mg/l]$, S_2 denotes the concentration of methanogenic bacteria $[mg/l]$, Q denotes the biogas production rate $[l/ld]$, and $S = S_0 + S_1 + S_2$ denotes the total organic concentration in the reactor.

The mathematical model is then described by:

$$
\begin{aligned}
\dot{S}_0(t) &= -bS_0(t)X_1(t) - [S_0(t) - y_pS_{in}]\cdot D(t)\\
\dot{X}_1(t) &= \mu_1(S_1(t))X_1(t) - k_1X_1(t) - X_1(t)\cdot D(t)\\
\dot{S}_1(t) &= bX_1(t)S_0(t) - \tfrac{\mu_1(S_1(t))X_1(t)}{y_1} - S_1(t)\cdot D(t)\\
\dot{X}_2(t) &= \mu_2(S_2(t))X_2(t) - k_2X_2(t) - X_2(t)\cdot D(t)\\
\dot{S}_2(t) &= y_b\mu_1(S_1(t))X_1(t) - \tfrac{\mu_2(S_2(t))X_2(t)}{y_2} - S_2(t)\cdot D(t)
\end{aligned}
\quad (4.3)
$$

where

$$\mu_i(S) \;=\; \frac{\bar{\mu}_{max,i}\cdot S}{S+K_{mi}}\,, \quad \text{for} \quad i=1,2\,,$$

denote the growth rates for bacteria by Michaelis-Menten. S_{in} denotes the organic concentration in the influent. The dilution rate $D(t)$ ($[1/d]$ at time $t[d]$) is the input variable $u(\cdot)$. The control objective is to force the output $y(\cdot) \equiv S(\cdot)$, the total organic concentration in the reactor, to track a (constant) reference signal.

The simulations are based on experimental data given by Stoyanov and Simeonov (1995). The setpoint to be tracked is $y_{ref}(\cdot) \equiv S_{ref} = 45$ $[mg/l]$.
In the first simulation, we allow the system to converge to an equilibrium point, which takes 30 days, by setting $u(\cdot) \equiv \bar{u}(\cdot) \equiv 0.23$. At time $t \geq 30$, the system has settled and we switch on the λ-tracker (4.1) with design parameters

$$\gamma = 1, \quad \lambda = 0.5, \quad r = 2, \quad k(0) = 0, \quad \bar{u}(\cdot) \equiv 0.23.$$

Observe the nice transient behaviour shown in Fig. 2-4: Within a day, the output $S(t)$ is forced, without any oscillations, into the tolerance interval $[44.5, 45.5]$. The output stays in the tolerance interval and the gain converges to a finite value, which is less than 0.03. Note also the good performance of $u(t)$.

In order to illustrate that the controller can also cope with noise corrupted output we choose $\lambda = 0.3$ and corrupt the output by noise which is chosen as the first component of the three dimensional Lorenz equation. For the parameters chosen (see Ilchmann (1996) for details), Sparrow (1982) has shown that the noise is chaotic, bounded by 0.1 and has bounded derivative. In (4.1) we replace $e(t)$ by $e(t) = y(t) + n(t) - y_{ref}(t)$. We obtain the same qualitative results, see Figures 5–8, and we have added a cutting of the output dynamics around the point $t = 30$, where the regulator is switched on, see Fig. 6.

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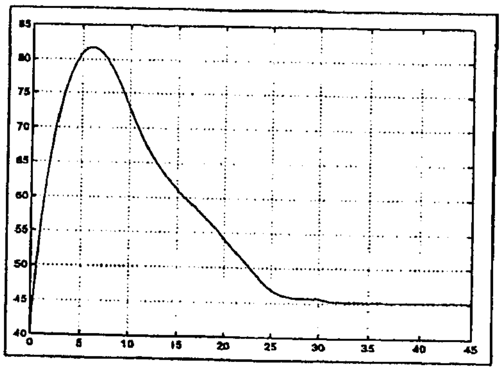

Figure 2: Output $t \mapsto S(t)$ for λ-tracker (4.1) with $\gamma = 1$, $r = 2$, $\lambda = 0.5$, $k(30) = 0$, $\bar{u}(\cdot) \equiv 0.23$ applied to (4.3).

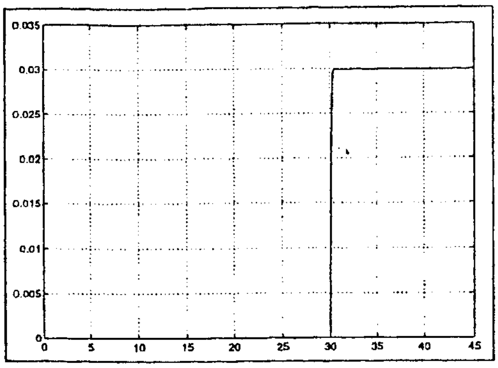

Figure 3: Gain $t \mapsto k(t)$ of λ-tracker (4.1) with $\gamma = 1$, $r = 2$, $\lambda = 0.5$, $k(30) = 0$, $\bar{u}(\cdot) \equiv 0.23$ applied to (4.3).

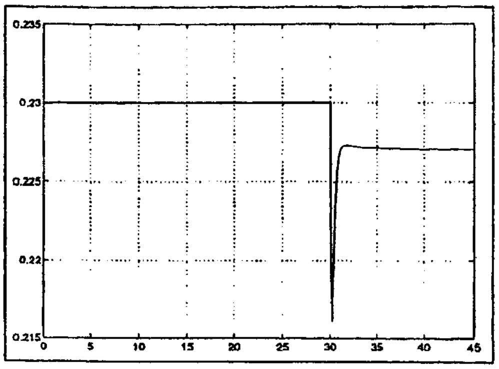

Figure 4: Input $t \mapsto u(t)$ for λ-tracker (4.1) with $\gamma = 1$, $r = 2$, $\lambda = 0.5$, $k(30) = 0$, $\bar{u}(\cdot) \equiv 0.23$ applied to (4.3).

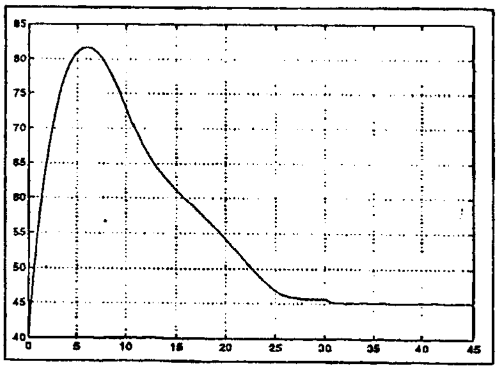

Figure 5: Output $t \mapsto S(t)$ for λ-tracker (4.1) with $\gamma = 1$, $r = 2$, $\lambda = 0.3$, $k(30) = 0$, $\bar{u}(\cdot) \equiv 0.23$ and chaotic noise applied to (4.3).

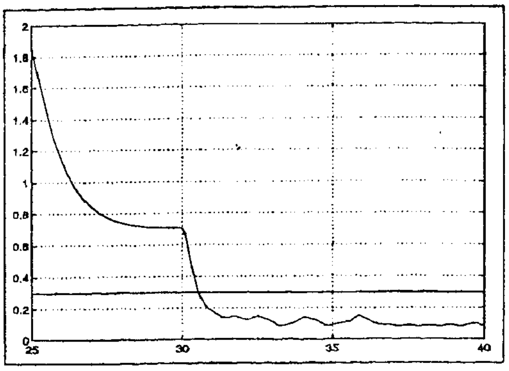

Figure 6: Error $t \mapsto dist(|S(t)| - y_{ref})$ on $[25, 40]$ for λ-tracker (4.1) with $\gamma = 1$, $r = 2$, $\lambda = 0.3$, $k(30) = 0$, $\bar{u}(\cdot) \equiv 0.23$ and chaotic noise applied to (4.3).

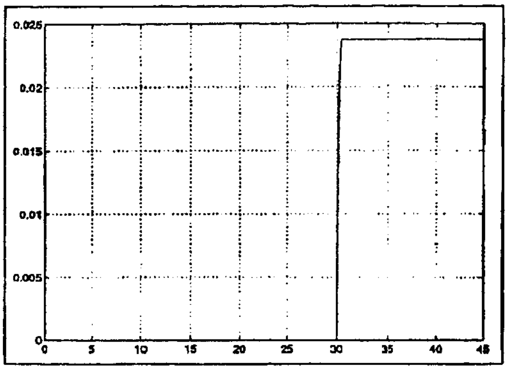

Figure 7: Gain $t \mapsto k(t)$ of λ-tracker (4.1) with $\gamma = 1$, $r = 2$, $\lambda = 0.3$, $k(30) = 0$, $\bar{u}(\cdot) \equiv 0.23$ and chaotic noise applied to (4.3).

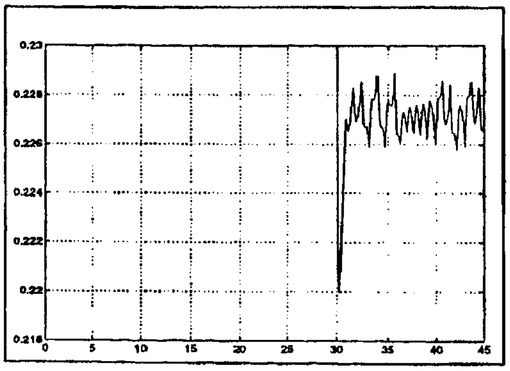

Figure 8: Input $t \mapsto u(t)$ for λ-tracker (4.1) with $\gamma = 1$, $r = 2$, $\lambda = 0.3$, $k(30) = 0$, $\bar{u}(\cdot) \equiv 0.23$ and chaotic noise applied to (4.3).