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DEVICES AND SYSTEMS,  
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FOR THE FUTURE**

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M. Tchobanou

## **Multidimensional Nonseparable Multirate Systems – a New Tool for Signal Processing**

### **MULTIMEDIA COMMUNICATIONS**

The growing demand for processing and compression of still two-dimensional images and video signals in telecommunications and multimedia technology motivates the fact that increasingly more attention is being paid to multi-dimensional (M-D) systems. An attempt to tackle the problem of nonseparable ("true") M-D multirate systems design is presented. It is supposed that nonseparable systems may have better compression and approximation results. But they are more difficult to be designed [1].

As usual the transformation part of the encoding system is performed by some multirate systems. Our main interest is connected with so-called true M-D systems, namely with nonseparable multirate systems. A good review of the results and open problems in the theory of M-D systems might be found in [2]. Multirate systems consist of M-D digital filters, decimators, interpolators and delay elements [3] – see Fig.1. They form analysis and synthesis filter banks (FB). The decimation matrix plays a very important role in it. Every known procedure for M-D multirate system design is based on a given decimation matrix [3, 4, 5, 6, 7]. Decimation matrix defines the filters in FBs and the shape of their pass-bands, it also heavily affects the implementation of such systems. The requirements that the decimation matrix should meet are known. But till nowadays the way to build these decimation matrices and a parameterization of them were still unknown. A method for generating all nonseparable admissible decimation matrices was developed for 2-D and 3-D cases. It allows to build any  $m$ -channel decimation matrix for which all requirements are met. The peculiarity of 4-D case was considered.

A wide variety of M-D filter banks of FIR and IIR types, having the property of perfect reconstruction or near perfect reconstruction, of orthogonal and biorthogonal types, currently receive much attention for use in different application areas. Compression of

images and video, computer tomography, texture characterization, high-definition television, three-dimensional TV (3D-TV) are the best known of them. Nonseparable decimation matrices and FBs are preferable because M-D signals by their nature are nonseparable. Nonseparable FBs have better characteristics than their separable counterparts (which consist of products of 1-D FBs along each dimension). The number of degrees of freedom is also much bigger for nonseparable FBs.

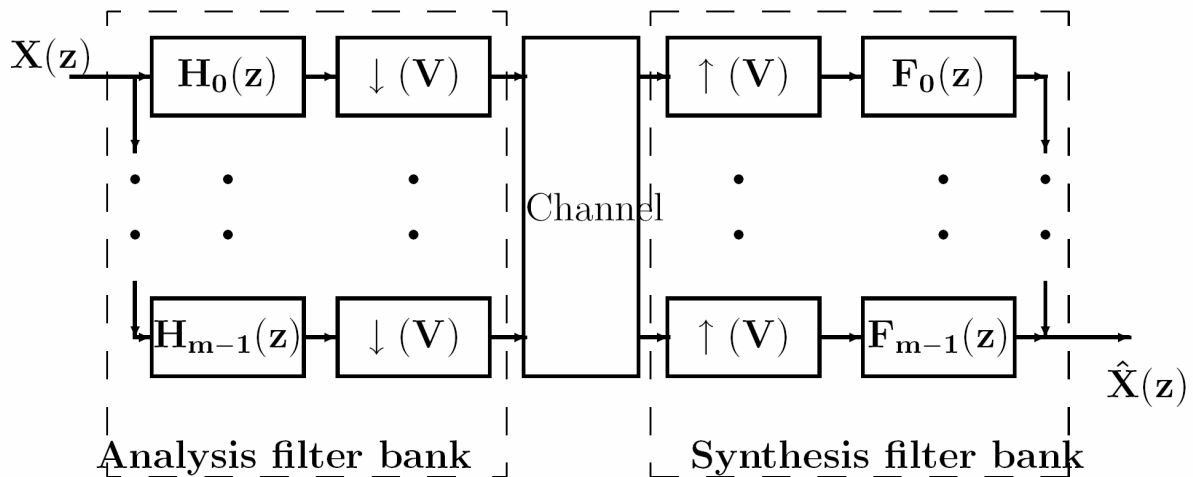


Fig 1. M-D multirate system

In tomography the 2-D separable wavelets impose a rectangular tiling of the frequency plane, which is not well suited to the radial band-limited assumption of the image [8]. The application of nonseparable multiresolution tomography to 2-D wavelets allows to respect the geometry of the system by tiling the frequency plane in a diamond-shaped fashion that is more respectful to the radial band-limited assumptions (see Fig. 2). Local tomography using these nonseparable bases shows an improvement in terms of PSNR. Another successful application of nonseparable wavelets in 3-D rotational angiography is given in [9]. In certain cases, it is desirable to use nonseparable subsampling to obtain useful 2-D wavelet representations. For example, nonseparable wavelet orthonormal bases can be used for texture discrimination and fractal analysis [10]. The selection of nonseparable decomposition filters has a significant influence on the result of texture characterization. In [11, 12] it is shown that nonseparable wavelets have features invariant to rotation of the texture image. That is why the classification and segmentation results are better if to use nonseparable symmetric wavelets.

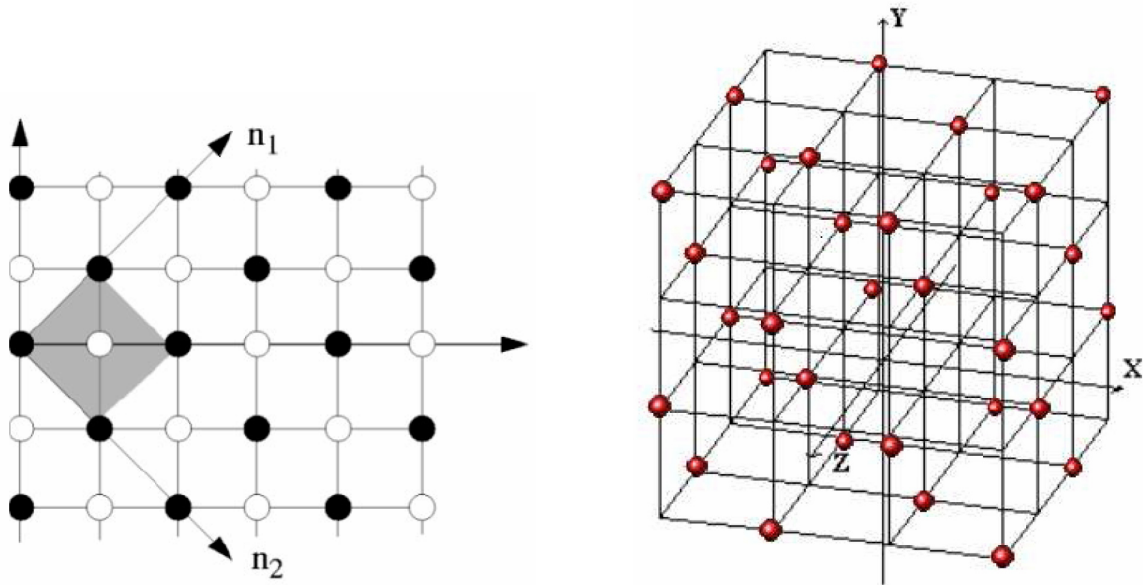


Fig. 2 Non-separable lattices (2D and 3D cases)

It is supposed that one of the main drawbacks of nonseparable filtration is its high computational complexity. In [13] it is shown that in general this is not true. The real complexity highly depends on many factors. Among some of them one should mention the size of the signal to be processed, the support of the wavelets (the size of FIR filter), the architecture of computing system and others. For example on multi-processor systems comparing convolution-based algorithms for different distributions, separable wavelet filters are more efficient for the block distribution for small values of kernel (filter) size  $L$ . In a block distribution, each processor holds a non-overlapping block of pixels of the image, and adjacent processors in the processor grid hold adjacent blocks of the input image. For nonseparable filters, the block distribution is more efficient if the image size  $n$  is large for a practical range of  $L$ . It is important to note that the results reported are specific to the parameters of the architecture. One can observe the high impact of architectural parameters (e.g., ratio of computation to communication speed) on the relative performance of the algorithms.

In high-definition television applications, in conversion between progressive and interlaced video, in motion parameter estimation, 3-D nonseparable filters are widely used [14]. The 3-D matched filtering approach results in nonseparable filters that produce the best SNR improvement among all linear filters, provided the object spatial signature is known. If to mention IIR filters, in [15] it is noted that one has to take advantage of well

known superior frequency behavior at a low computational cost obtainable from nonseparable IIR filters.

Beginning from the famous work [4] the task of M-D multirate system design was considered quite often during last 15 years. Different design procedures were given for the case when the decimation matrix is known. As it is clear the key building block in nonseparable M-D multirate systems is the decimation matrix. Till now under consideration there was only the case when this matrix is given. The most general task of M-D multirate system design requires to have a procedure for generating decimation matrices with given properties.

Our objective was to apply methods and techniques from different fields of mathematics (including approximation theory, computational commutative algebra and others) in the design of multi-dimensional (M-D) multirate systems. An important feature of the proposed design is that M-D non-separable case is under consideration (including the use of non-separable lattices and/or non-separable filter banks). Biorthogonal filter banks and wavelets are designed by application of polynomial approach. The extension of the results already obtained in our research group to arbitrary number of dimensions and channels (for arbitrary decimation matrix) was done. Another polynomial approach is connected with the M-D polynomial unimodular matrix completion method. It is based on Suslin and Quillen theorems. There exist several known algorithms, including ours, which rely on Grobner basis techniques.

Orthogonal M-D filter banks and wavelets are designed by application of polynomial and structural approaches. The method makes use of a structure of the filter that satisfies almost all necessary requirements (except the requirement of given regularity, which is answered by optimizing the parameters of that filter). An important issue is ensuring of given degree of regularity for wavelet function. The task was to build the wavelets which have maximum regularity for given support.

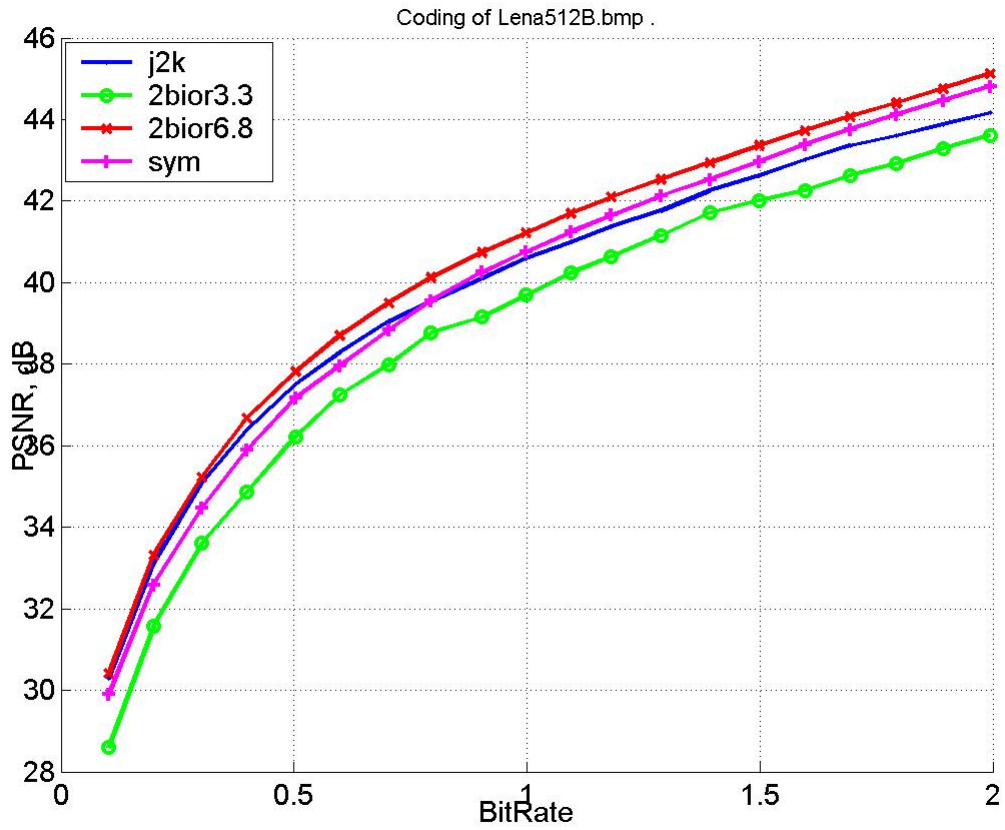
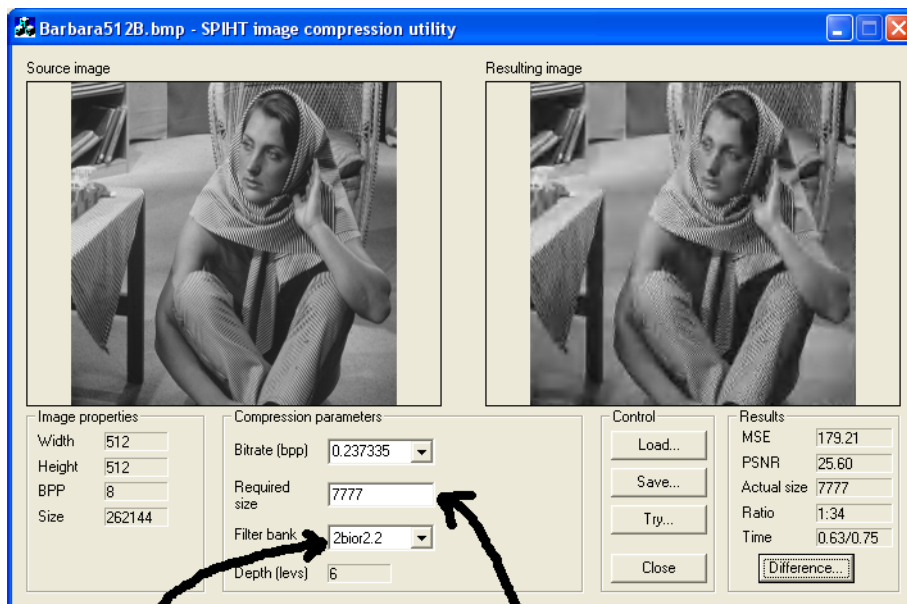


Fig 2. Rate distortion dependence  
(sym – one of our orthogonal non-separable FB)



характеристики  
результата

выбор банка  
фильтров

точный размер  
выходного файла

Fig.4 Program i\_comp

An encoding system for image and video compression was developed [16] (see Fig. 3 and 4). It is based on some known hierarchical type methods. The multirate system is based on separable/nonseparable filter banks. Progressive data transmission is a very important issue. In this case there were realized different types of scalability - resolution, quality and rate scalabilities.

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