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M. Rančić, P. Rančić

EM Field of Vertical Hertz's Dipole (VHD) Placed above a Lossy Half-space: Approximate Expression for the Sommerfeld's Integral

INTRODUCTION

The electromagnetic field in the air in the surroundings of the vertical Hertz's dipole (VHD) placed above linear, isotropic and homogenous lossy half-space is determined in this paper. The influence of the electrical parameters of the lossy half-space on the EM field components is described by the Sommerfeld's integral kernel that, in numerical sense, presents a very complex problem. A very simple and accurate mathematical model, which is not limited by the values of electrical parameters of the lossy half-space or the VHD location, is proposed for solution of this numerically complex problem.

General theoretical solution for the EM field, which was proposed by Sommerfeld in 1909, [1], was formulated by a class of semi-infinite integrals, in literature often referred to as integrals of Sommerfeld's type, or Sommerfeld's integral kernel (SIK). Accurate numerical evaluation of integrals of this type demands very complex numerical procedure that depends on the values of the electrical parameters of the lossy half-space σ_1, $\varepsilon_1 = \varepsilon_0 \varepsilon_{r1}$ and $\mu_1 = \mu_0$ ($\sigma_1 \in [0, \infty)$ - conductivity, $\varepsilon_{r1} \in [1, 81]$ - relative permittivity), and the location of the VHD above a lossy half-space, $z'_k \in [0, \infty)$.

Thus, a great number of models for approximate evaluation of the SIK were proposed in order to solve this complex problem in a simpler way. However, the majority of these approximations were developed under certain assumptions, such as limited values of electrical parameters for which it is valid.

Authors of this paper have also proposed a number of simple models that are, besides accurate, general in a sense that they are not limited in any way by the values of the electrical parameters of the lossy half-space.

The SIK model that is presented in this paper is similar to the one proposed by the same authors in [3, 4] and the one used in [5] for evaluation of the input impedance of the vertical asymmetrical wire dipole antenna placed above a lossy half-space. In this paper, a SIK model, which represents arithmetical mean of the models from [3], [4] and [5] is proposed.

BRIEF THEORETICAL DESCRIPTION

Problem layout

The VHD is placed in the air ($\sigma_0 = 0, \varepsilon_0, \mu_0$) at height z'_k above a homogenous, isotropic lossy half-space of known electrical parameters (σ_1, $\varepsilon_1 = \varepsilon_{r1}\varepsilon_0$, $\mu_1 = \mu_0$). Following labels are also used: $\underline{\sigma}_i = \sigma_i + j\omega\varepsilon_i$, $i = 0,1$ - complex conductivity, $\underline{\gamma}_i = = \alpha_i + j\beta_i = (j\omega\mu_i\underline{\sigma}_i)^{1/2}$, $i = 0,1$ - complex propagation constant, $\underline{\varepsilon}_{r1} = \varepsilon_{r1} - j\varepsilon_{i1} = = \varepsilon_{r1} - j60\sigma_1\lambda_0$ - complex relative permittivity, $\underline{n}_{10} = (\underline{\varepsilon}_{r1})^{1/2}$ - complex refraction coefficient, λ_0 - wave length in the air, and $\omega = 2\pi f$ - angular frequency.

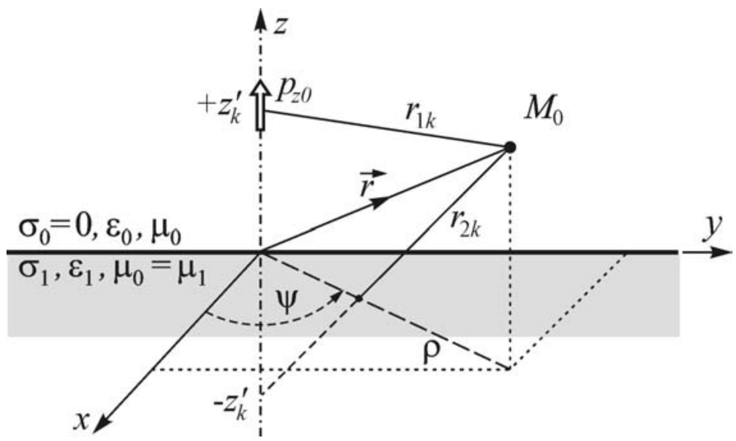

Fig.1. *Schematic illustration of the VHD placed above a lossy half-space.*

In the described case of the VHD, the Sommerfeld's integral kernel (SIK), which describes the field at point $M_0(x, y, z \geq 0)$ scattered from the lossy half-space, is given by the following expression:

$$S_{00}^{v}(r_{2k}) = \int\limits_{\alpha=0}^{\infty} \tilde{R}_{z10}(\alpha)\cdot\tilde{K}_0(\alpha, r_{2k})\mathrm{d}\alpha\,,\ \tilde{R}_{z10}(\alpha) = \frac{(\underline{n}_{10}^2 u_0 - u_1)}{(\underline{n}_{10}^2 u_0 + u_1)}, \tag{1}$$

where: $\tilde{R}_{z10}(\alpha) = \tilde{R}_{z10}(u_0)$ - spectral reflection coefficient (SRC), $u_i = \sqrt{\alpha^2 + \underline{\gamma}_i^2}$, i=0, 1, and $\tilde{K}_0(\alpha, r_{2k})$ - spectral form of the standard potential kernel from the image in the flat mirror, i.e.

$$K_0(r_{2k}) = \frac{\mathrm{e}^{-\underline{\gamma}_0 r_{2k}}}{r_{2k}} = \int\limits_{\alpha=0}^{\infty} \tilde{K}_0(\alpha, r_{2k})\mathrm{d}\alpha = \int\limits_{\alpha=0}^{\infty} \frac{\mathrm{e}^{-u_0(z+z'_k)}}{u_0}\alpha J_0(\alpha\rho)\mathrm{d}\alpha\,, \tag{2}$$

where $J_0(\alpha\rho)$ is the zero-th Bessel function of the first kind, $r_{2k} = \sqrt{\rho^2 + (z + z'_k)^2}$ - distance from the image in the flat mirror to the observed field point, and ρ - radial distance, $\rho = \sqrt{x^2 + y^2}$.

Approximate solution for the SIK

In the papers [3] and [5], authors have proposed two similar models for the approximation of the SIK. Firstly, the SRC is assumed in a form of a rational function with two unknown constants, which are determined by matching the SCR at characteristic points. Based on these models, the approximate form for the SRC proposed in this paper has the following form:

$$\tilde{R}_{z10}(u_0) \cong B + 0.5(A' + A'')\underline{\gamma}_0 / u_0 = B + A\underline{\gamma}_0 / u_0 \,, \tag{3}$$

where B, A' and A'' are unknown constants, obtained by matching $\tilde{R}_{z10}(u_0)$ at points $u_0 \to \infty$, $u_0 = \underline{\gamma}_0$ and $u_0 = \underline{\gamma}_0 / (\underline{n}_{10}^2 + 1)^{1/2}$, respectively. Their values are as follows:

$$B = (\underline{n}_{10}^2 - 1)/(\underline{n}_{10}^2 + 1)\,,\ A' = (\underline{n}_{10} - 1)/(\underline{n}_{10} + 1) - B\,, A'' = -B / \sqrt{\underline{n}_{10}^2 + 1}\,. \tag{4}$$

Substituting (4) into (1), the following approximate SIK model is obtained:

$$S_{00}^{V}(r_{2k}) \cong BK_0(r_{2k}) + 0.5(A' + A'')\underline{\gamma}_0 L(r_{2k}) = BK_0(r_{2k}) + A\underline{\gamma}_0 L(r_{2k})\,, \tag{5}$$

where $L(r_{2k})$ is the new integral kernel,

$$L(r_{2k}) = \int_{v=z+z'_k}^{\infty} K_0\left(r_{2kv}\right)\,\mathrm{d}v = -\int_{v=0}^{z+z'_k} K_0(r_{2kv})\mathrm{d}v - \frac{\pi}{2}[N_0(\beta_0\rho) + j\,J_0(\beta_0\rho)]\,. \tag{6a}$$

In some practical cases when $z + z'_k >> \rho$ (e.g. [9]), the new integral kernel $L(r_{2k})$ can be very accurately numerically solved using the Li integrals, $Li(X) = Ci(X) - jSi(X)$ ([10]). Approximate expression for these calculations is

$$L(r_{2k}) \cong \left(1 - j\frac{R^2}{4}\right) Li(R_{2k}) + \frac{R^2}{4R_{2k}}(1 - jR_{2k})\frac{e^{-jR_{2k}}}{R_{2k}}\,, \tag{6b}$$

where: $R = \beta_0\rho$, $R_{2k} = \beta_0 r_{2k}$ and when the condition $R << R_{2k}$ is satisfied.

The Hertz's vector and electrical scalar potential

The Hertz's vector potential, $\vec{\Pi}_0 = \Pi_{z0}\,\hat{z}$, in the case of the VHD placed above a lossy medium is given, considering (5), by the following expression:

$$\Pi_{z0} = \frac{p_{z0}}{4\pi\underline{\sigma}_0}\left[K_0(r_{1k}) + S_{00}^{v}(r_{2k})\right] \cong \frac{p_{z0}}{4\pi\underline{\sigma}_0}\left[K_0(r_{1k}) + BK_0(r_{2k}) + A\underline{\gamma}_0 L(r_{2k})\right], \quad (7)$$

where: p_{z0} - moment of the Hertz's dipole, and $r_{1k} = [\rho^2 + (z - z'_k)^2]^{1/2}$ - distance from the VHD to the observed field point.

The electric scalar potential in the air in the vicinity of the VHD is as follows:

$$\varphi_0 = -\mathrm{div}\vec{\Pi}_0 = -\frac{\partial \Pi_{z0}}{\partial z} = -\frac{p_{z0}}{4\pi\underline{\sigma}_0}\left\{\frac{\partial}{\partial z}\left[K_0(r_{1k}) + BK_0(r_{2k})\right] - A\underline{\gamma}_0 K_0(r_{2k})\right\}. \quad (8)$$

EM field structure

The electric, $\vec{E}_0(\vec{r}) = -\mathrm{grad}\varphi_0 - \underline{\gamma}_0^2\vec{\Pi}_0$, and magnetic field, $\vec{H}_0(\vec{r}) = \underline{\sigma}_0\,\mathrm{rot}\vec{\Pi}_0$, in the vicinity of the VHD can be determined using the following definition expressions:

$$E_{x0} = \frac{\partial^2 \Pi_{z0}}{\partial x \partial z},\ E_{y0} = \frac{\partial^2 \Pi_{z0}}{\partial y \partial z},\ E_{z0} = \frac{\partial^2 \Pi_{z0}}{\partial z^2} - \underline{\gamma}_0^2\,\Pi_{z0}, \quad (9)$$

$$H_{x0} = \underline{\sigma}_0\frac{\partial \Pi_{z0}}{\partial y},\ H_{y0} = -\underline{\sigma}_0\frac{\partial \Pi_{z0}}{\partial x},\ H_{z0} = 0. \quad (10)$$

Complete form for the EM field components is obtained substituting the approximate solution for the Hertz's vector potential given by (7) into (9) and (10).

NUMERICAL RESULTS

Two groups of numerical results will be presented in this section:

- The first one corresponds to numerical experiments that illustrate the accuracy of the proposed model for SIK calculation; and
- The second one illustrates changes of EM field components on the ground surface in the air, versus *y*-, or ρ - coordinate for $x = 0$. The refraction coefficient $\underline{n}_{10}$ and position of the VHD z'_k are taken as parameters in these calculations.

Sommerfeld's integral kernel. The validity of the proposed SIK model was investigated through a number of numerical experiments, and the obtained results are compared to the accurate ones from [6].

Based on the approximate SIK model given by (5), modulus of the normalized SIK, $S_{00}^{v}(r_{2k})/\beta_0$, was determined for different values of the parameters on which it depends. Modulus of the normalized SIK versus normalized radial distance $Log(\beta_0\rho)$, is shown in Figs. 2a, b and c. Position of the VHD z_k', is taken as a parameter, relative permittivity is $\varepsilon_{r1} = 2$, and Figs. 2a, b and c correspond to the following values of the normalized conductivity $\sigma_1\lambda_0 = 10^{-4}\,S$, $\sigma_1\lambda_0 = 0.041667\,S$ and $\sigma_1\lambda_0 = 10\,S$, respectively. The results obtained applying the proposed SIK model (solid line) are compared to the ones obtained by accurate calculations from [6] (solid down triangle). For another group of electrical parameters, i.e. $\varepsilon_{r1} = 10$, and $\sigma_1\lambda_0 = 10^{-4}\,S$, $\sigma_1\lambda_0 = 0.175\,S$ and $\sigma_1\lambda_0 = 10\,S$, modulus of the normalized SIK is presented in Figs. 3a, b and c. The results obtained applying the proposed model when constant A takes value $A = 0$, are also given in the same figures (open up triangle).

As it can be seen from both Figs.2 and 3, the results obtained applying the proposed model for the SIK are in very good accordance with the ones of accurate calculations, regardless of the electrical parameters of the lossy half-space or the VHD position. In that sense, the proposed model can be, besides simple, characterized as general and accurate.

EM field components. Based on the expressions for the electric field components (9), and (10) that describe the magnetic ones, a number of numerical calculations were performed, and the obtained results are presented further in this section.

The values for the normalized *z*- component of the electrical field, E_{z0}/β_0^2 and $p_{z0} = 1\text{Am}$, versus normalized radial distance $Log(\beta_0\rho)$, are shown in Figs.4a-c. Position of the VHD z_k' is taken as a parameter, relative permittivity is $\varepsilon_{r1} = 2$, and Figs.4a-c correspond to the values of the normalized conductivity $\sigma_1\lambda_0 = 10^{-4}\,S$, $\sigma_1\lambda_0 = 0.041667\,S$ and $\sigma_1\lambda_0 = 10\,S$, respectively.

Normalized *z*- component of the electric field versus normalized radial distance are also shown in Figs.5a-c, but for different value of the relative permittivity $\varepsilon_{r1} = 10$, taking the

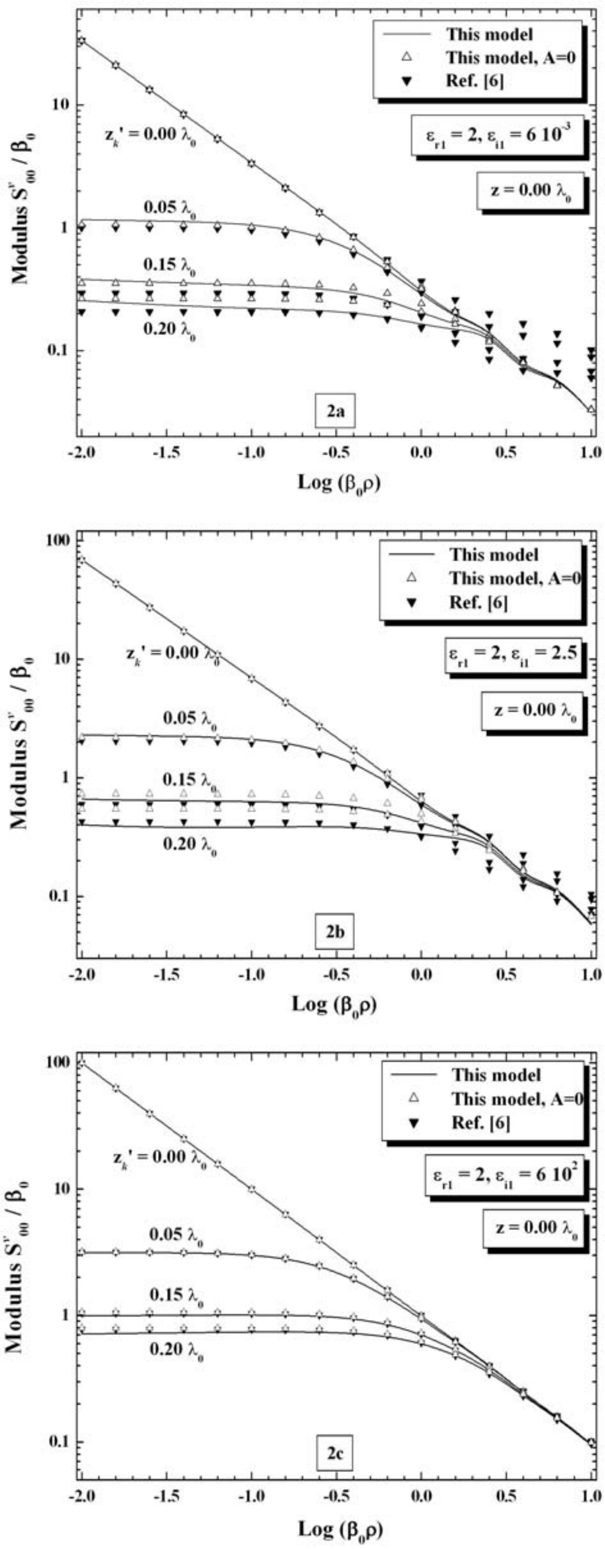

Fig.2. *Modulus of the normalized SIK versus normalized radial distance. Relative permittivity is* $\varepsilon_{r1} = 2$*, and VHD position* z'_k *and* $\varepsilon_{i1} = 60\sigma_1\lambda_0$ *are parameters.*

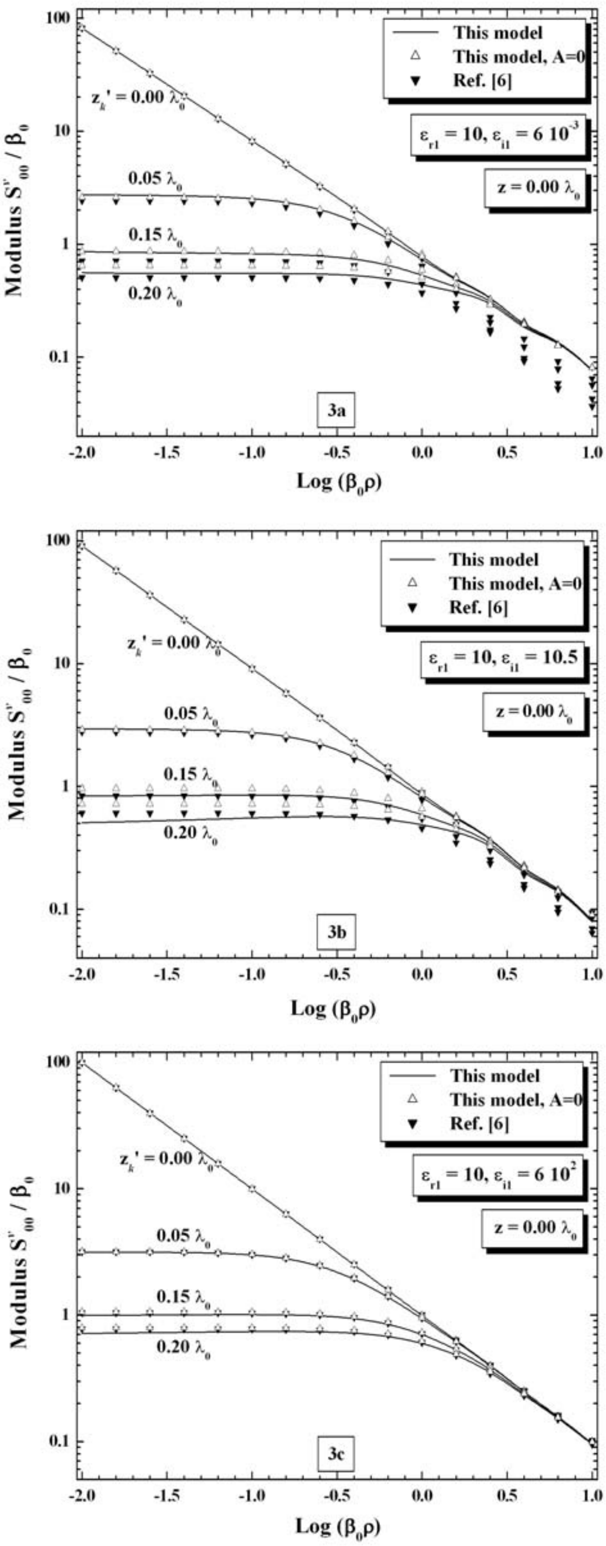

Fig.3. *Modulus of the normalized SIK versus normalized radial distance. Relative permittivity is* $\varepsilon_{r1} = 10$*, and VHD position* z'_k *and* $\varepsilon_{i1} = 60\sigma_1\lambda_0$ *are parameters.*

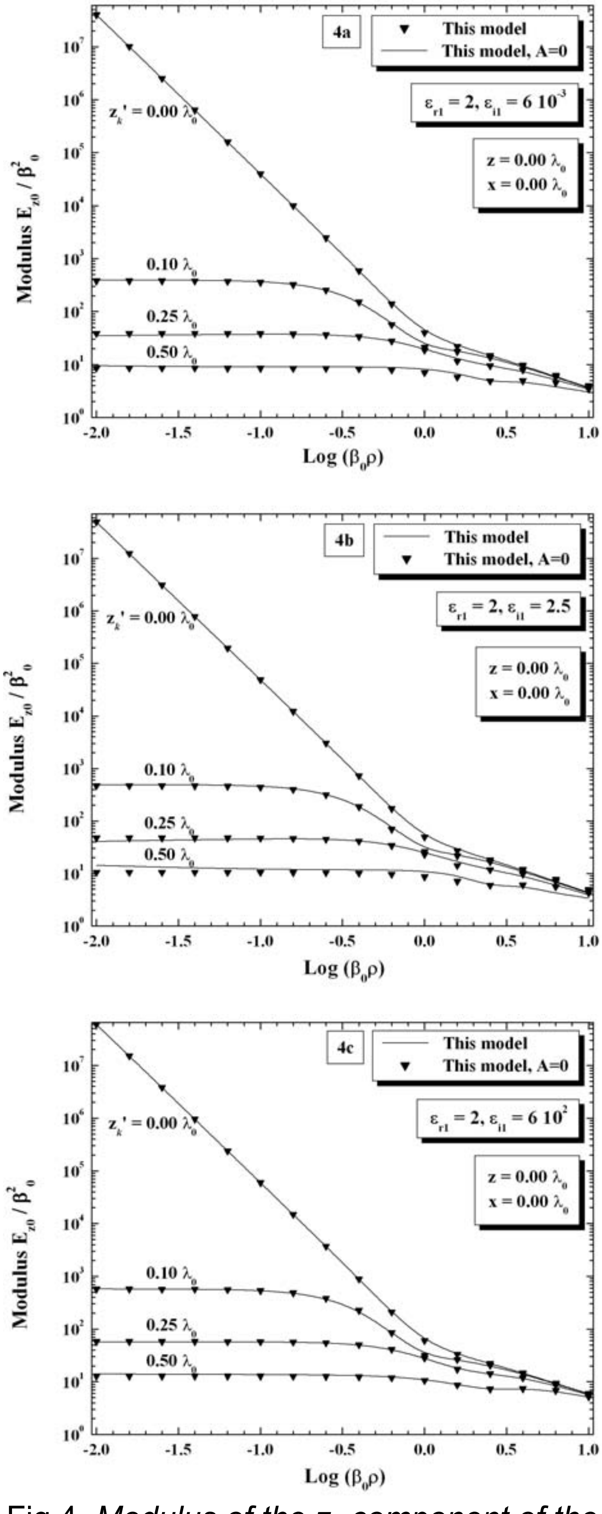

Fig.4. *Modulus of the z- component of the electrical field versus normalized radial distance. Relative permittivity is* $\varepsilon_{r1} = 2$*, and VHD position* z'_k *and* $\varepsilon_{i1} = 60\sigma_1\lambda_0$ *are parameters.*

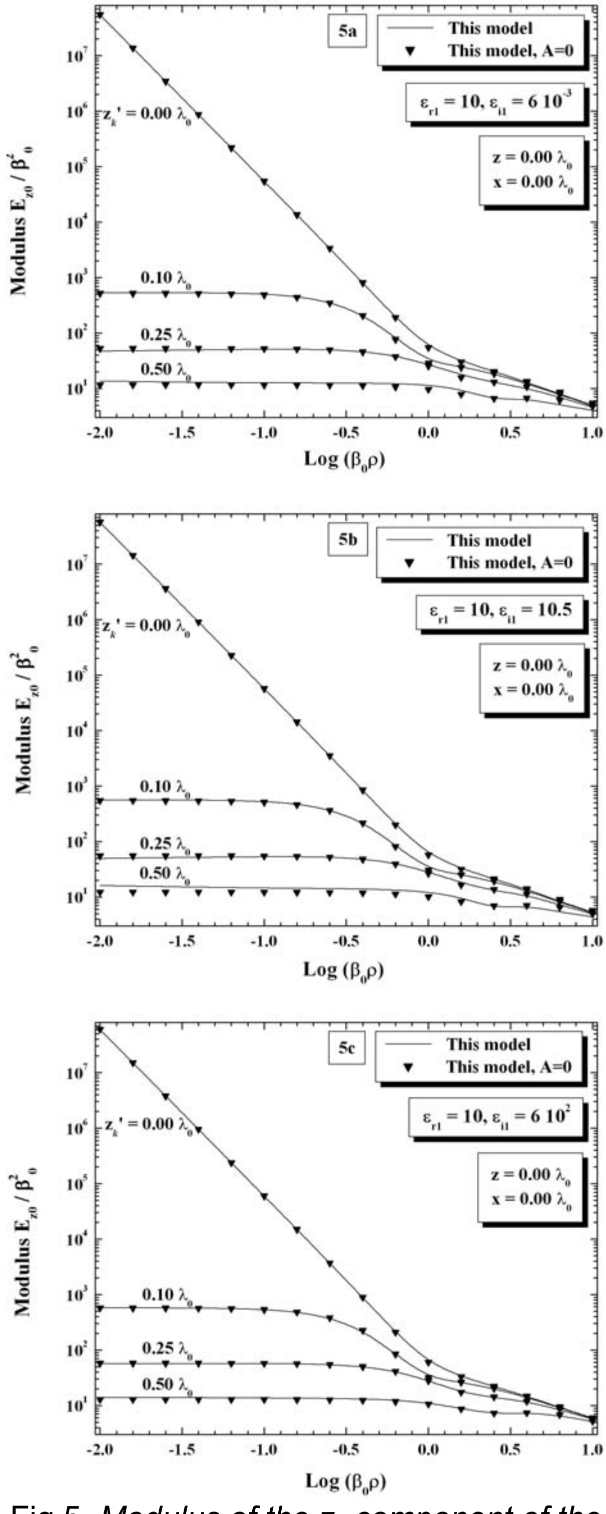

Fig.5. *Modulus of the z- component of the electrical field versus normalized radial distance. Relative permittivity is* $\varepsilon_{r1} = 10$*, and VHD position* z'_k *and* $\varepsilon_{i1} = 60\sigma_1\lambda_0$ *are parameters.*

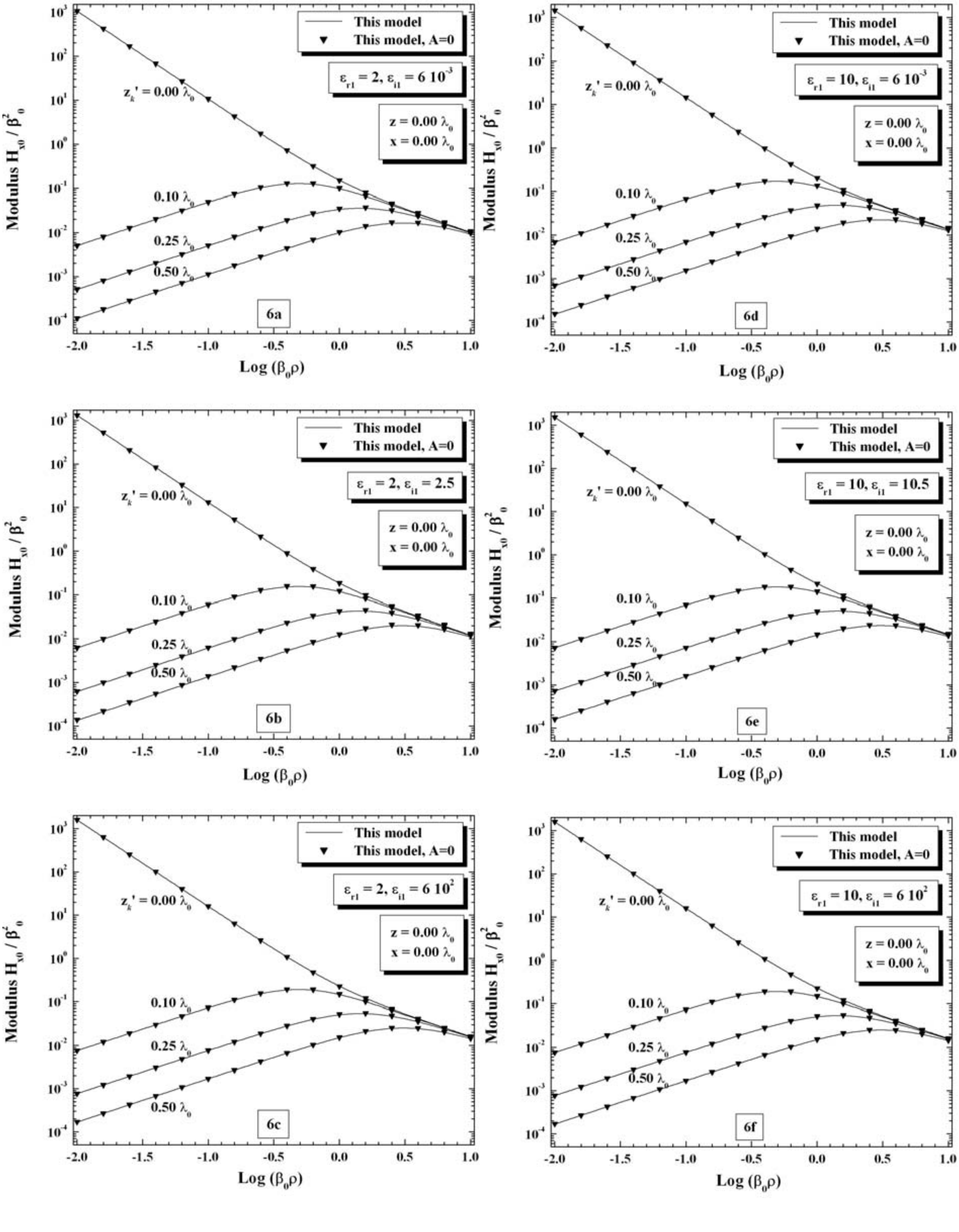

Fig.6. *Modulus of the x- component of the magnetic field versus normalized radial distance. Relative permittivity is* $\varepsilon_{r1} = 2$ (6a-c) and $\varepsilon_{r1} = 10$ (6d-f)*, and VHD position* z'_k *and* $\varepsilon_{i1} = 60\sigma_1\lambda_0$ *are parameters.*

position of the VHD as parameter. Figs. 5a-c correspond to the following values of the normalized conductivity $\sigma_1\lambda_0 = 10^{-4}\,S$, $\sigma_1\lambda_0 = 0.175\,S$ and $\sigma_1\lambda_0 = 10\,S$, respectively. The results obtained for the *x*- component of the normalized magnetic field, H_{x0}/β_0^2 and $p_{z0} = 1\text{Am}$, (for $x = 0$, $H_{y0} = 0$), versus normalized radial distance are presented in Figs.6a-f, taking the position of the of the VHD as parameter. Values shown in Figs.6a-c correspond to relative permittivity $\varepsilon_{r1} = 2$, and normalized conductivity $\sigma_1\lambda_0 = 10^{-4}\,S$, $\sigma_1\lambda_0 = 0.041667\,S$ and $\sigma_1\lambda_0 = 10\,S$, respectively. The results in Figs.6d-f correspond to the lossy half-space of relative permittivity $\varepsilon_{r1} = 10$ and normalized conductivity $\sigma_1\lambda_0 = 10^{-4}\,S$, $\sigma_1\lambda_0 = 0.175\,S$ and $\sigma_1\lambda_0 = 10\,S$, respectively.

Based on the presented results, it can be concluded that the greatest changes of the values of the EM field components (E_{z0} and H_{x0}) occur for the normalized radial distances up to $\beta_0\rho \cong 1$, where the proposed SIK model has performed most accurately. It should also be noted that E_{y0} - component of the electric field, so-called surface wave, exists and varies for different parameters of the lossy half-space, which is not the case when the ground is treated as an ideally conducting medium. Namely, based on the previous experience, the application of the proposed approximate SIK model as a function of normalized conductivity $\sigma_1\lambda_0$, i.e. $\varepsilon_{i1} = 60\sigma_1\lambda_0$, gives numerical results that correspond to the ones of the case of an ideally conducting ground, for all characteristics of the EM field already for $\sigma_1\lambda_0 > 10\,\text{S}$. Numerical values for the surface wave (E_{y0} component) are significant, i.e. it vanishes for much greater values of the normalized conductivity $\sigma_1\lambda_0 > 10^5\,\text{S}$. Therefore, the analysis of the surface wave in the frame of the proposed SIK model demands wider elaboration as well as much more space. Thus, the validity analysis of the proposed approximate models for the SIK will be the subject of further work.

CONCLUSION

Simple and satisfyingly accurate model for numerical calculation of the SIK (Sommerfeld's integral kernel, [1]) is proposed in this paper. This model is one of many simple ones proposed by signed authors ([2], [3], [4], [5], [8]).

Based on theoretical and numerical results presented in the paper, it can be concluded that the proposed model is reliable, simple, satisfyingly accurate and general, and it can be used for calculating potentials (Π_{z0} and φ_0) and EM field components ($\vec{E}_0$ and $\vec{H}_0$) in the surroundings of the VHD in the air.

It should be noted that these quantities are very accurately modelled at points on the ground surface. This is emphasized because the Hertz's vector in the ground, $\vec{\Pi}_1 = \Pi_{z1}\hat{z}$, $z \leq 0$, can also be approximately, but still very accurately, presented in a following form: $\Pi_{z1} \cong \Pi_{z0}(x, y, z=0) \cdot f_1(\underline{n}_{10}, z \leq 0)$, where Π_{z0} is evaluated according to (7), and f_1 is new exponential function that depends on electrical parameters of the ground and z- coordinate of the observed point in the ground. Thus, the EM field in the ground can be also modelled in a very simple and accurate manner.

The proposed model can be also very successfully used for modelling EM field structure of atmospheric discharge in frequency domain in the range $f \in (0, 10\,\mathrm{MHz}]$. This presents basis for evaluation of the Lightning EM field in time domain, at points in the surroundings of the Lightning channel using the FFT (Fast Fourier Transform). For this purpose, the model from [3] was successfully used in preliminary researches in [9].

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