51. IWK
Internationales Wissenschaftliches Kolloquium
International Scientific Colloquium

PROCEEDINGS

11-15 September 2006

# FACULTY OF ELECTRICAL ENGINEERING AND INFORMATION SCIENCE

# INFORMATION TECHNOLOGY AND ELECTRICAL ENGINEERING - DEVICES AND SYSTEMS, MATERIALS AND TECHNOLOGIES FOR THE FUTURE



TECHNISCHE UNIVERSITÄT
ILMENAU

**Impressum**

Herausgeber: Der Rektor der Technischen Universität Ilmenau
Univ.-Prof. Dr. rer. nat. habil. Peter Scharff

Redaktion: Referat Marketing und Studentische Angelegenheiten
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Fakultät für Elektrotechnik und Informationstechnik
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Redaktionsschluss: 07. Juli 2006

Technische Realisierung (CD-Rom-Ausgabe):
Institut für Medientechnik an der TU Ilmenau
Dipl.-Ing. Christian Weigel
Dipl.-Ing. Marco Albrecht
Dipl.-Ing. Helge Drumm

Technische Realisierung (Online-Ausgabe):
Universitätsbibliothek Ilmenau
ilmedia
Postfach 10 05 65
98684 Ilmenau

Verlag: isle
Verlag ISLE, Betriebsstätte des ISLE e.V.
Werner-von-Siemens-Str. 16
98693 Ilrnenau



ISBN (Druckausgabe): 3-938843-15-2
ISBN (CD-Rom-Ausgabe): 3-938843-16-0

Startseite / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=12391

51st Internationales Wissenschaftliches Kolloquium
Technische Universität Ilmenau
September 11 – 15, 2006

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# The effect of rotor eccentricity and unbalance on one – sided magnetic pull and centrifugal force

## ABSTRACT

The analysis and computer simulation allowed for determining the force of one-sided magnetic pull, the centrifugal force, total shaft deflection under the external forces, and rotor shift of an induction motor under the conditions of eccentric displacement of rotor with respect to stator axes and rotor unbalance with respect to its rotation axis. The cases of parallel eccentricity and static unbalance have been considered.

The problem has been solved for various values of the magnetic induction in the air gap of the motor, the initial assembly displacement and the distance between the centre of gravity of rotor and its axis of symmetry, considered as functions of the shaft rigidity coefficient. The analysis of the problem may be useful in studying possibilities of reducing the one-sided magnetic pull and the centrifugal force.

## INTRODUCTION

During operation of an electric machine its shaft is subject to the following external forces:

- the force of gravity G of the rotor core and the part of the shaft located under the rotor;
- the force F acting on the end shaft neck, converted to the point of application of the force G;
- the force $N_m$ of the magnetic pull;
- the centrifugal force $P_{centrr}$.

The external forces cause partial deflections of the shaft. It may be assumed that the process undergoes within the range of shaft elasticity. The external forces are then balanced by inter nal elastic force. The relationship is described by the equation [2] :

$$G + F + C(e_{in} + f_t) + m\omega^2(f_t + \eta) = Kf_t \quad (1)$$

with the following denotations: C – unitary magnetic pull; $e_{in}$ - initial eccentric shift of the rotor axis with respect to the stator axis; $f_t$ – total shaft deflection; m – rotor mass; $\eta$ – shift of the rotor centre of gravity with respect to the shaft axis; $\omega$ – angular velocity; K – shaft stiffness coefficient.

Shaft deflection caused by all the above mentioned forces [2]:

$$f_t = \frac{G + F + Ce_{in} + m\omega^2\eta}{K - C - m\omega^2} \quad (2)$$

with the denotations as above.

The shaft deflection is conducive to the rotor axis shift with respect to the stator axis, and, according to the feedback principle, to one-sided magnetic pull and the centrifugal force.

## ONE-SIDED MAGNETIC PULL

The one-sided magnetic pull arises in an electric machine in result of magnetic field asymmetry. It may take considerable values [1, 3, 4].

A certain stator symmetry and indeformability was assumed. The main reason of asymmetry of the air gap and the magnetic field is the shaft deflection and eccentric shift of the rotor axis with respect to stator axis [4].

The one-sided magnetic pull is described by the relationship [5]:

$$N_m = C(e_{in} + f_t) \quad (3)$$

The consideration makes allowance to the relationship between the one-sided magnetic pull and magnetic induction in the air gap, and initial shift of the rotor with regard to stator axis.

In the process of machine exploitation the basic eccentricity increases, as the unbalanced magnetic pull acts towards the smaller air gap. The eccentricity exceeding 20 per cent of rated thickness of the air gap between the stator and rotor should be considered in the calculation.

The one-sided magnetic pull results in growing power losses in the bearings and intensification of vibration-acoustic phenomena. This might be conducive to rotor shocks against the stator.

## CENTRIFUGAL FORCE

Rotor unbalance, apart from shaft deflection, is a reason of centrifugal force. It is described by the relationship [5]:

$$P_{centr} = m\omega^2 (f_t + \eta) \tag{4}$$

It may arise irrespective of previous accurate balancing of the rotor during machine exploitation.

The unbalance is a common reason of machine vibration. Its amplitude is proportional to the unbalance amount.

It was assumed that the rotor is located at a rigid shaft and rotates with angular force $\omega$. Total mass of the rotor and shaft amounts to $m$. An additional unbalance mass $m_n$ is located at the distance $r$ from the axis of rotation (in the plane of the center of gravity $S$). The unbalance may be determined based on equating the inertial forces.

The inertial force corresponding to the unbalanced mass $m$ is provided by the relationship:

$$B = m_n r\omega^2 \tag{5}$$

In case of unbalance the equation takes a form:

$$B = (m_n + m)\eta\omega^2 \tag{6}$$

As $m_n \ll m$, the actual unbalance is given by the relationship:

$$\eta \cong \frac{m_n r}{m} \tag{7}$$

Admissible unbalance is determined by maximal velocity of the center of gravity of the unbalanced shaft-rotor system. It amounts for small motors to 1 – 2.5 mm/s.

The unbalance may be conducive to rotor rubbing against the stator, bearing damage, resonance, etc.

## RESULTS OF THE CALCULATIONS

The shaft deflection, total shift of the rotor, one-sided magnetic pull, and the centrifugal force were determined.

Computer simulation of the above mentioned values was carried out with regard to magnetic induction, initial eccentric shift of the rotor with respect to stator axes, the shift of rotor gravity centre with respect to rotor axis, and with regard to coefficient of the shaft rigidity.

Value of magnetic induction in the air gap was assumed in the range from 0.1 to 0.8 T, excepting for 0.7 T for a four-pole motor. Initial assembly shift was equal from 0 to 0.5 $\delta$.

The shift $\eta$ of the rotor gravity centre with respect to the shaft axis varied from 0 to $\eta_{max}$, according to the rotor radius of a given motor and additional unbalanced mass amounting to 1 per cent of the rotor.

The shaft rigidity coefficient varied from 0.5 to 1.5 of rigidity of the shaft of an actual motor for the two-pole case and from 0.8 to 1.5 (due to the need of observing the limit of elasticity) of rigidity of the shaft of an actual motor for the four-pole case.

The simulation was carried out for selected single-phase two- and four-pole motors, meeting the condition of shaft elasticity.

Example calculations of the magnetic pull for two-pole motors are illustrated in Figs 1a, b, c, d, e, f, and in 2 a, b, c, d, e, f. Centrifugal forces for the two-pole and four-pole motors are illustrated in Fig. 3 a, b, c, d, e, f and in Fig. 4 a, b, c, d, e, f. Shaft deflections are illustrated in Fig. 5 a, b, c, d, e, f and in Fig. 6 a, b, c, d, e, f. Total shift of the rotor is illustrated in Fig. 7 a, b, c, d, e, f and in Fig. 7 a, b, c, d, e, f.

## CONCLUSIONS

The computer simulation allowed to make the following statement for two- and four-pole motors:

1. Increasing magnetic induction $B_\delta$ in the air gap is conducive to faster growing of the one-sided magnetic pull than it might be expected based on direct second-power relationship with the induction. This is due to the fact, that it additionally depends on induction in a more complex manner by the effect of shaft deflection.

   On the other hand, the centrifugal force increases insignificantly with growing $B_\delta$. It only depends on induction in second power through the shaft deflection $f_t$.

2. Absolute value of the magnetic pull depends to smaller degree on initial assembly displacement $e_{in}$ than it might be expected from common proportionality, as total rotor

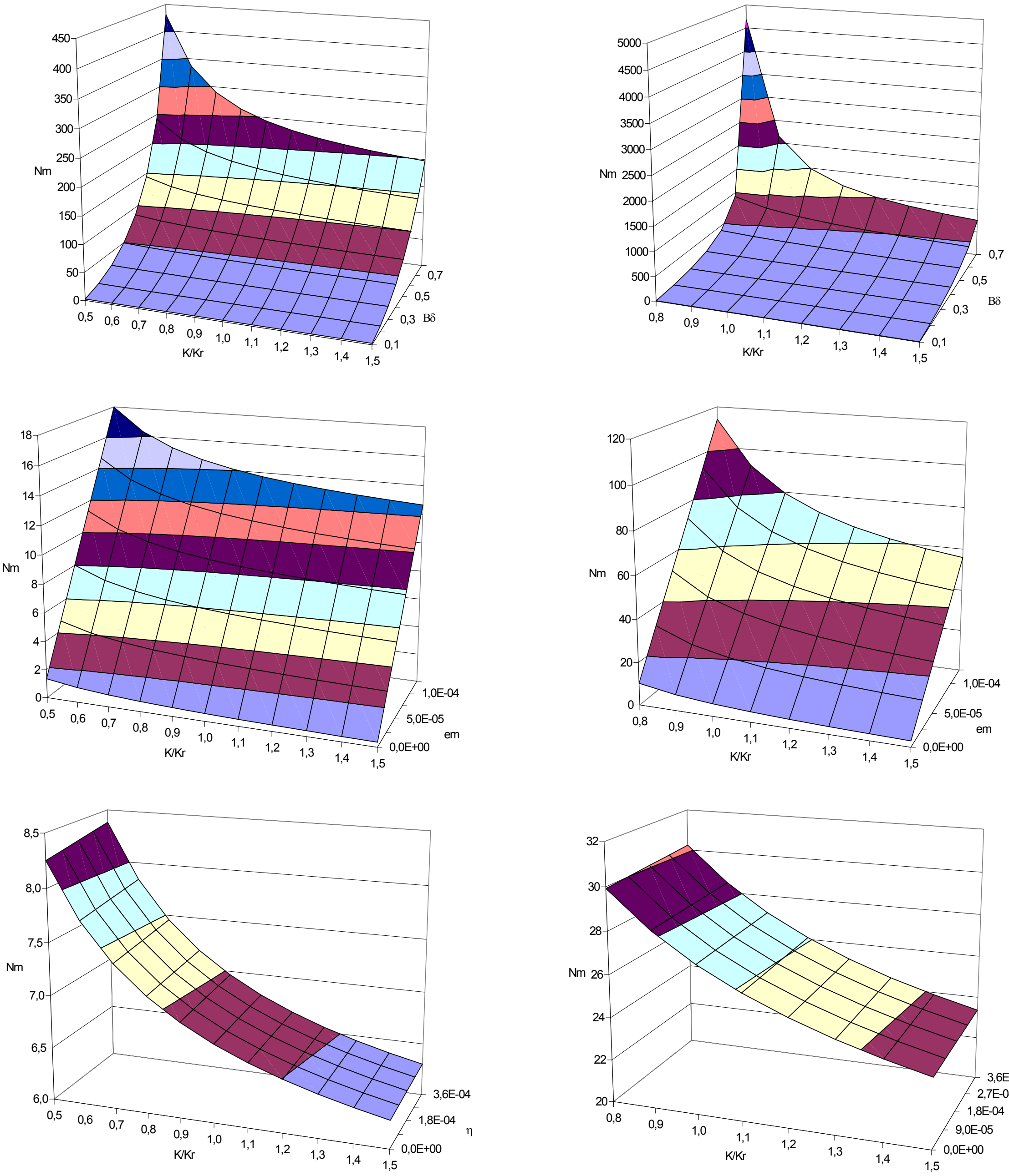


Fig.1. One-sided magnetic pull of a two-pole (a, c, e) and four-pole (b, d, f) motor (in Newtons); a, b.) $N_m = f(K, B_\delta)$, $e_{in} = 1 \times 10^{-4}$ m, $\eta = 0$; c, d.) $N_m = f(K, e_{in})$, $B_\delta = 0,6$ T, $\eta = 0$; e, f.) $N_m = f(K, \eta)$, $B_\delta = 0,3$ T, $e_{in} = 2,5 \times 10^{-5}$ m

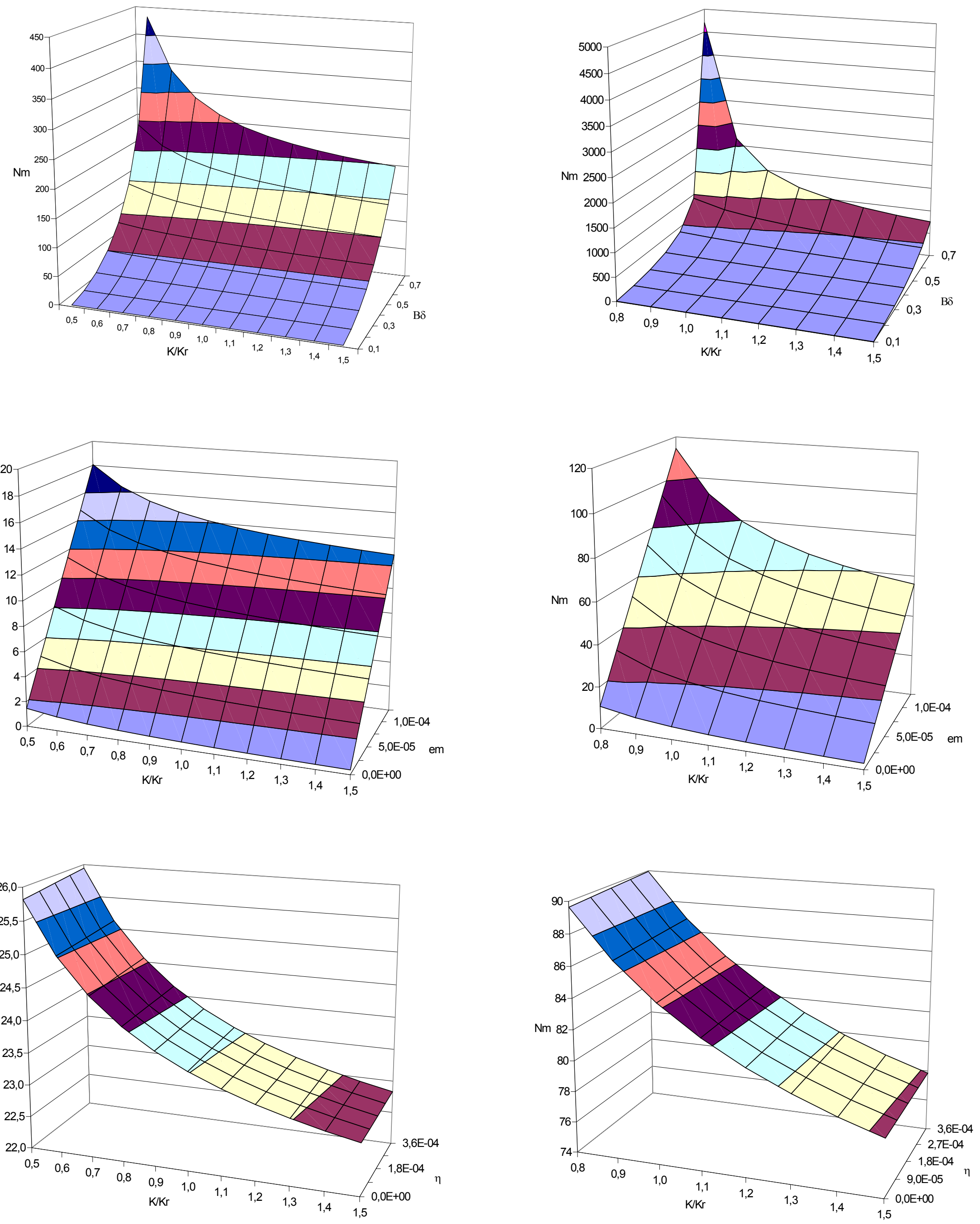


Fig. 2. One-sided magnetic pull of a two-pole (a, c, e) and four-pole (b, d, f) motor (in Newtons); a, b.) $N_m = f(K, B_\delta)$, $e_{in} = 1 \times 10^{-4}$ m, $\eta = 3{,}6 \times 10^{-4}$ m; c, d.) $N_m = f(K, e_{in})$, $B_\delta = 0{,}6$ T, $\eta = 3{,}6 \times 10^{-4}$ m; e, f.) $N_m = f(K, \eta)$, $B_\delta = 0{,}3$ T, $e_{in} = 1 \times 10^{-4}$ m

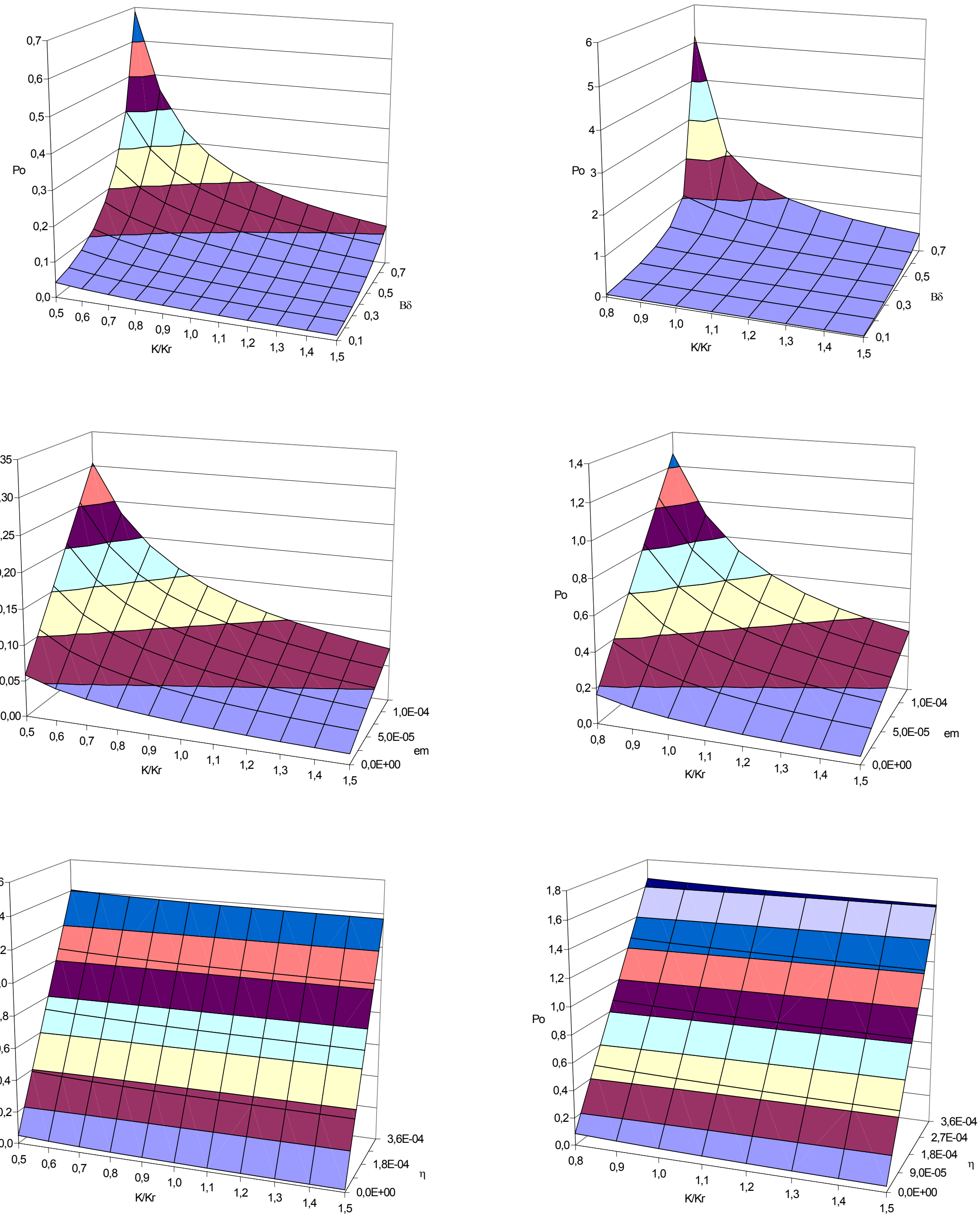


Fig.3. Centrifugal force of a two-pole (a,c,e) and four-pole (b, d, f) motor (in Newtons)

a, b.) $P_{centr} = f(K, B_{\delta})$, $e_{in} = 1 \times 10^{-4}$ m, $\eta = 0$; c, d.) $P_{centr} = f(K, e_{in})$, $B_{\delta} = 0,6$ T, $\eta = 0$;

e, f.) $P_{centr} = f(K, \eta)$, $B_{\delta} = 0,3$ T, $e_{in} = 2,5 \times 10^{-5}$ m

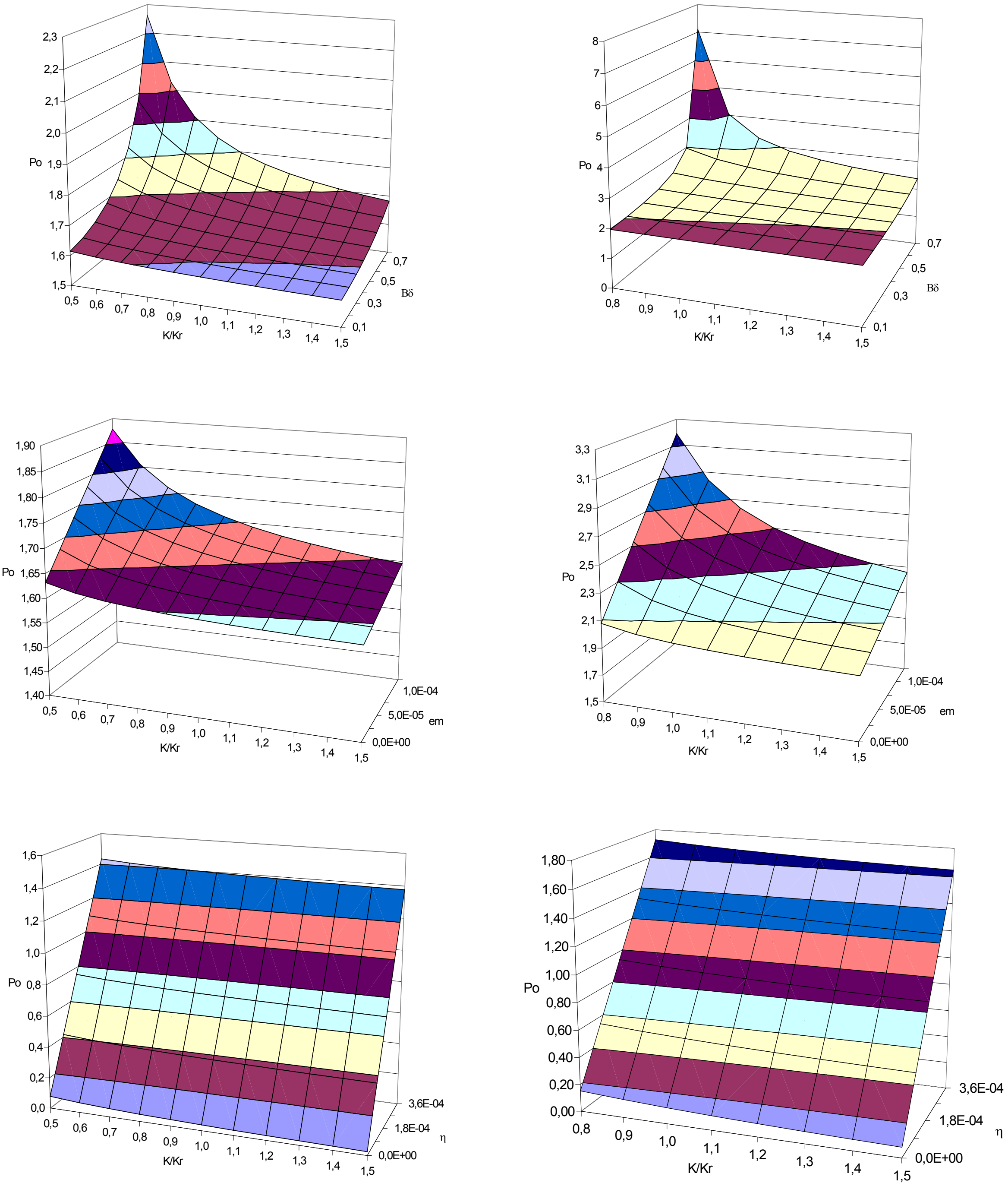


Fig.4. Centrifugal force of a two-pole (a,c,e) and four-pole (b,d,f) motor (in Newtons) a, b.) $P_{centr} = f(K, B_{\delta})$, $e_{in} = 1 \times 10^{-4}$ m, $\eta = 3,6 \times 10^{-4}$ m; c, d.) $P_{centr} = f(K, e_m)$, $B_{\delta} = 0,6$ T, $\eta = 3,6 \times 10^{-4}$ m; e, f.) $P_{centr} = f(K, \eta)$, $B_{\delta} = 0,3$ T, $e_{in} = 1 \times 10^{-4}$ m

displacement $e_c$ grows slower than $e_{in}$. The centrifugal force depends on the $e_{in}$ shift to still smaller degree, exclusively through $f_t$.

3. Absolute values of the magnetic pull are slightly higher for the shift $\eta>0$ of the rotor centre of gravity with respect to the shaft axis. The effect of $\eta$ is clearer for smaller values of the $e_{in}$ shift (predominating influence of $\eta$ through $f_t$). Absolute value of the centrifugal force grows as a function of the $\eta$ shift. Its higher growth is observed for lower induction (for the same coefficient K of shaft rigidity and the same $e_{in}$). Similarly, centrifugal force grows to higher degree for smaller $e_{in}$ shifts (for the same coefficient K).
4. Relative increase in the one-sided magnetic pull as a function of decreasing K coefficient is greater for smaller $e_{in}$ shifts (taking maximal value for $e_{in}=0$). This is met both in the case of existing and non-existing unbalance (the greatest effect of $f_t$ on $N_m$). Relative growth of centrifugal force as a function of decreasing K coefficient is subject only to insignificant variations for varying $e_{in}$ values. On the other hand, it changes significantly for various $\eta$ shifts, especially in the case of $\eta=0$.
5. Relative growth of the one-sided magnetic pull as a function of the $e_{in}$ shift for a constant K coefficient remains nearly unchanged for varying induction in the gap. Relative growth of centrifugal force as a function of the $e_{in}$ shift for a constant K coefficient is greater for smaller displacements $\eta$. The relative growth is larger for higher induction values. Maximal sensitivity of the growth occurs for $\eta=0$.
6. The magnetic pull growth as an $\eta$ shift function is insignificant. The growth is higher for smaller $e_{in}$ shift values, generally remaining independent on the induction value. On the other hand, the growth of the centrifugal force as a function of the $\eta$ displacement is significant. It is higher for smaller $e_{in}$ shift values (for unchanged induction values). In case of equal $e_{in}$ shift values the relative growth of centrifugal force, as a function of $\eta$, is greater for smaller induction values.
7. Relative growth of the magnetic pull as a function of induction is greater for smaller K coefficients. Relative growth of the centrifugal force considered as a function of induction is higher for smaller coefficients K.

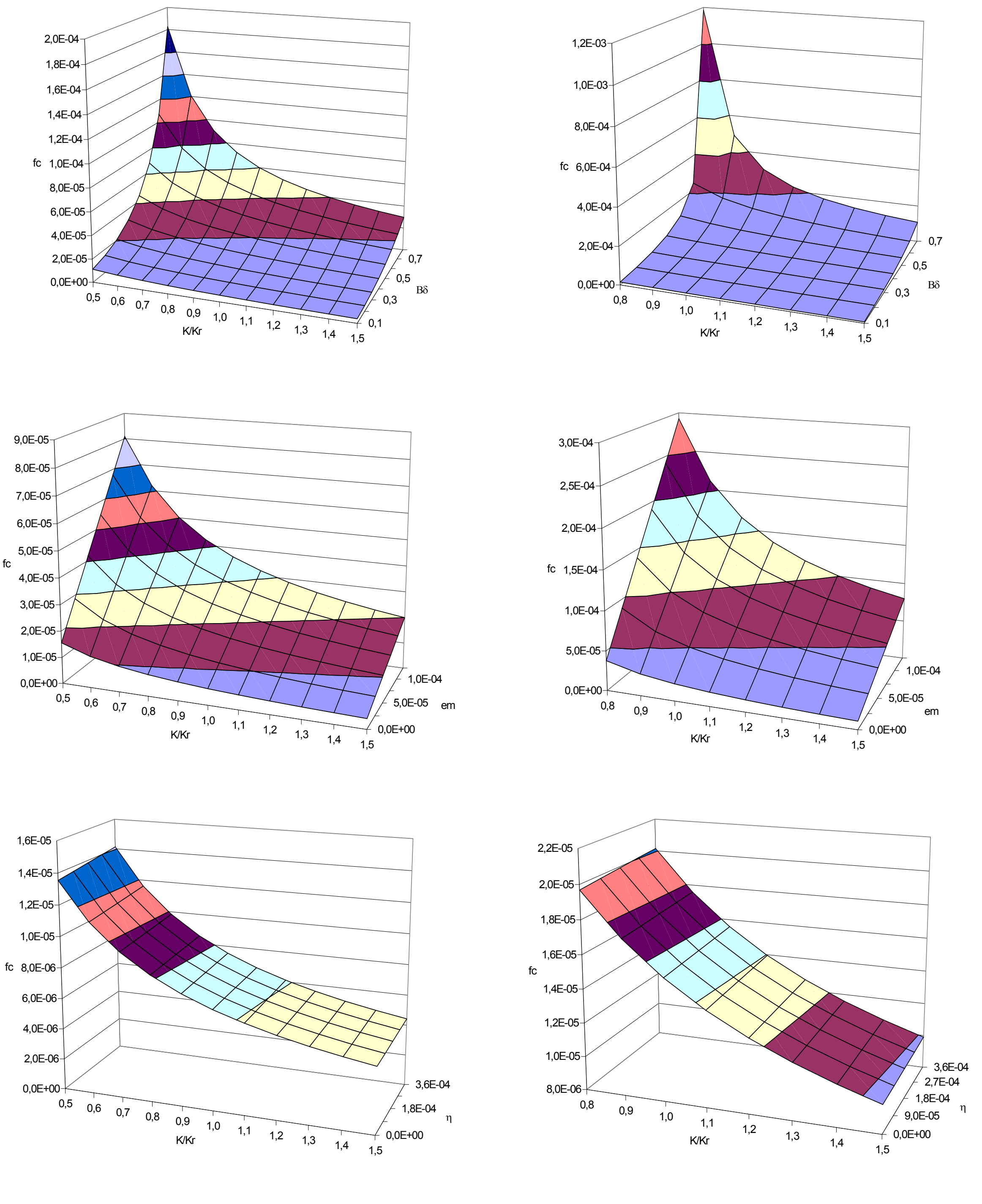


Fig.5. Shaft deflections (total) of a two-pole (a, c, e) and four-pole (b, d, f) motor (in m)

a, b.) $f_t = f(K, B_\delta)$, $e_{in} = 1\times10^{-4}$ m, $\eta = 0$; c, d.) $f_t = f(K, e_m)$, $B_\delta = 0{,}6$ T, $\eta = 0$;

e, f.) $f_t = f(K, \eta)$, $B_\delta = 0{,}3$ T, $e_{in} = 2{,}5\times10^{-5}$ m

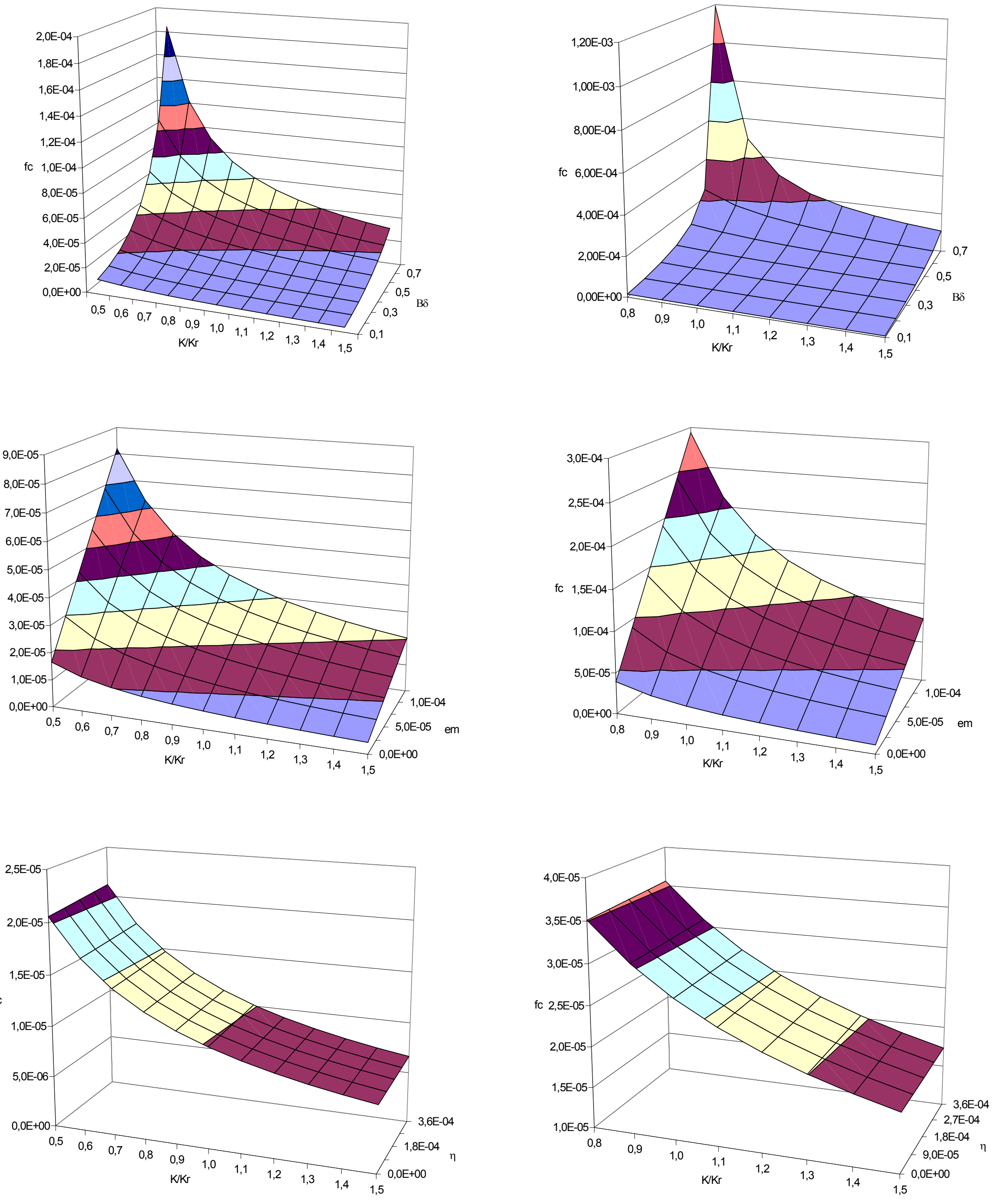


Fig.6. Shaft deflections (total) of a two-pole (a, c, e) and four-pole (b, d, f) motor (in m) a, b.) $f_t = f(K, B_\delta)$, $e_{in} = 1 \times 10^{-4}$ m, $\eta = 3,6 \times 10^{-4}$ m; c, d.) $f_t = f(K, e_{in})$, $B_\delta = 0,6$ T, $\eta = 3,6 \times 10^{-4}$ m; e, f.) $f_t = f(K, \eta)$, $B_\delta = 0,3$ T, $e_{in} = 1 \times 10^{-4}$ m

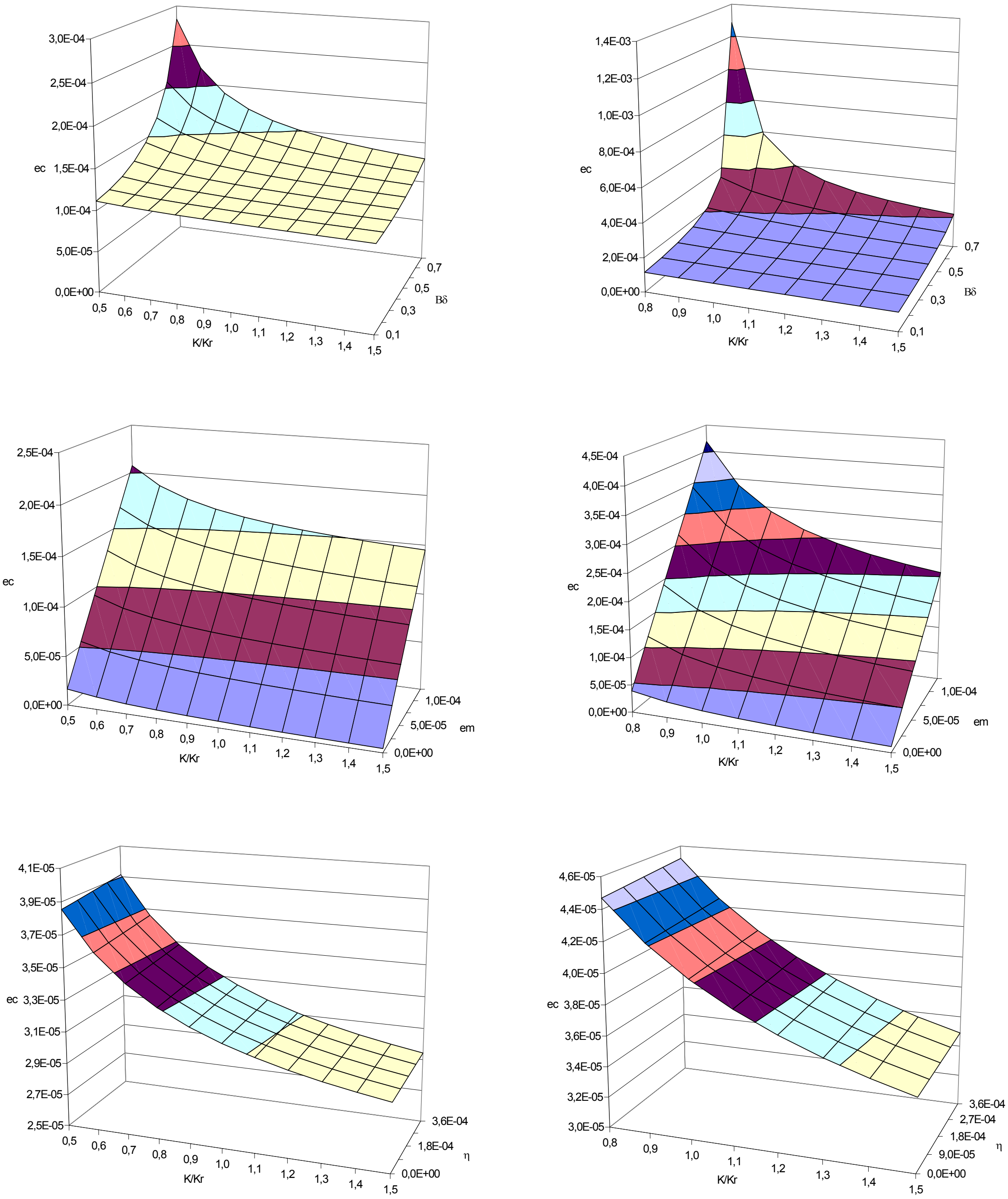


Fig.7. Total shift of the rotor of a two-pole (a, c, e) and four-pole (b ,d, f) motor (in m)

a, b.) $e_t = f(K, B_\delta)$, $e_{in} = 1 \times 10^{-4}$ m, $\eta = 0$; c, d.) $e_t = f(K, e_{in})$, $B_\delta = 0{,}6$ T, $\eta = 0$;

e, f.) $e_t = f(K, \eta)$, $B_\delta = 0{,}3$ T, $e_{in} = 2{,}5 \times 10^{-5}$ m

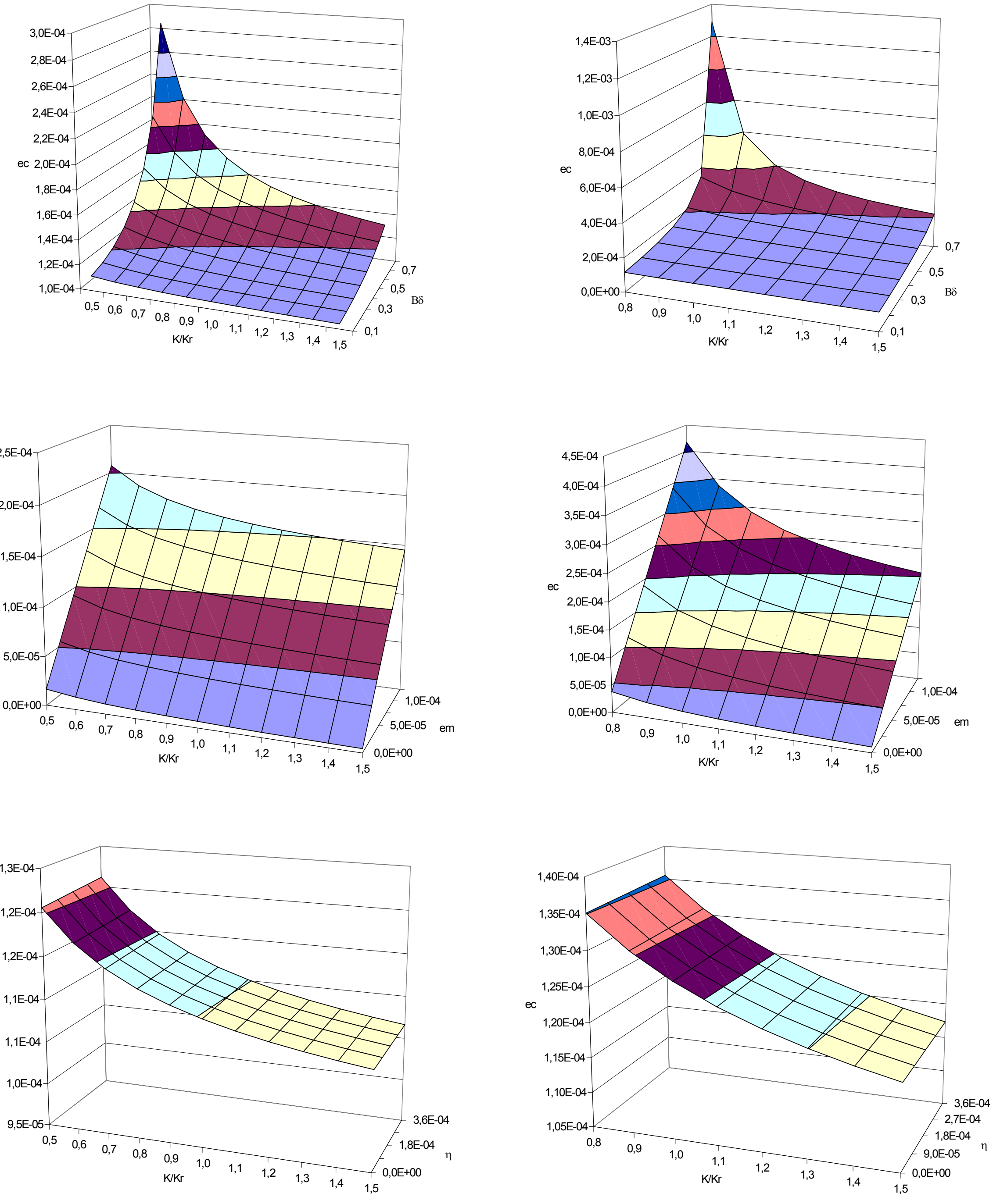


Fig.8. Total shift of the rotor of a two-pole (a, c, e) and four-pole (b, d, f) motor (in m) a, b.) $e_t = f(K, B_\delta)$, $e_{in} = 1 \times 10^{-4}$ m, $\eta = 3,6 \times 10^{-4}$ m; c, d.) $e_t = f(K, e_{in})$, $B_\delta = 0,6$ T, $\eta = 3,6 \times 10^{-4}$ m; e, f.) $e_t = f(K, \eta)$, $B_\delta = 0,3$ T, $e_{in} = 1 \times 10^{-4}$ m

8. Relative value of one-sided magnetic pull as a function of the $\eta$ shift is slightly higher for smaller coefficients K. Relative growth of centrifugal force considered as a function of the $\eta$ shift is considerably higher for greater coefficients K.
9. Absolute values of magnetic pull and centrifugal force as functions of the induction value in the air gap, and the $e_{in}$ and $\eta$ shifts, respectively, are higher for a four-pole than a two-pole motor.

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