51. IWK
Internationales Wissenschaftliches Kolloquium
International Scientific Colloquium

PROCEEDINGS

11-15 September 2006

FACULTY OF ELECTRICAL ENGINEERING AND INFORMATION SCIENCE

INFORMATION TECHNOLOGY AND ELECTRICAL ENGINEERING - DEVICES AND SYSTEMS, MATERIALS AND TECHNOLOGIES FOR THE FUTURE

TECHNISCHE UNIVERSITÄT
ILMENAU

Impressum

Herausgeber: Der Rektor der Technischen Universität Ilmenau
Univ.-Prof. Dr. rer. nat. habil. Peter Scharff

Redaktion: Referat Marketing und Studentische Angelegenheiten
Andrea Schneider

Fakultät für Elektrotechnik und Informationstechnik
Susanne Jakob
Dipl.-Ing. Helge Drumm

Redaktionsschluss: 07. Juli 2006

Technische Realisierung (CD-Rom-Ausgabe):
Institut für Medientechnik an der TU Ilmenau
Dipl.-Ing. Christian Weigel
Dipl.-Ing. Marco Albrecht
Dipl.-Ing. Helge Drumm

Technische Realisierung (Online-Ausgabe):
Universitätsbibliothek Ilmenau
ilmedia
Postfach 10 05 65
98684 Ilmenau

Verlag: isle
Verlag ISLE, Betriebsstätte des ISLE e.V.
Werner-von-Siemens-Str. 16
98693 Ilrnenau

ISBN (Druckausgabe): 3-938843-15-2
ISBN (CD-Rom-Ausgabe): 3-938843-16-0

Startseite / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=12391

51. Internationales Wissenschaftliches Kolloquium
Technische Universität Ilmenau
11.-14. September 2006

Alexey Bulgakov

SPEED OPTIMIZATION OF BUILDING ROBOTS

Abstract: The paper presents the method of planning trajectories for building robots' motions according to the given points of support on the basis of the two-level interpolation. This method features the calculation of speeds in the intermediate points of the trajectory with due account of limitations for controlling moments. The combination of analytical and searching methods of calculation makes it possible to obtain a solution during the interval of control.

Keywords: Motion control; speed optimization; trajectory planning

1. INTRODUCTION

Building robots control requires solving the task of the movement trajectory planning. At the same time for the purpose of increasing operating speed and control stability the trajectory formation must be carried out with account of a variable speed setting and the possibility of its further optimization. Besides, the algorithms being planned should have the volume of calculations, which provides acceptance of controlling information for the degrees of movement during the control interval.

This makes it problematic to use conventional methods of planning based on application of approximating splines of power functions of the third and higher orders with their subsequent optimization by classical methods. To solve the stated problem we suggest to use a two-level interpolation as a basis for the movement planning algorithm, this enables to reduce the zone of speeds search with due account of limitations and to increase calculation speeds essentially.

2. CONSTRUCTION OF A THREE-DIMENSIONAL TRAJECTORY

Let us consider the ways of obtaining a trajectory of a building robot's manipulator movement as more complex control tasks can be reduced to solution of this problem. The main point of the suggested method of planning is that on the first level of interpolation we define $S_j(t)$ function describing a ***temporary*** law of a path change and on the second level a cubic splines approximation of generalized coordinates in the function of displacement is performed. With help of the obtained law of motion we can perform optimization of the first level parameters for constructing temporal splines taking into consideration limitations for generalized forces. To simplify optimization we require that approximating functions have no beating and limitations for controlling forces are reduced to limitation of accelerations in the supporting points as the forces in robots' drives don't essentially depend on speed.

Now we'll consider a mathematical description of the suggested method of planning. To avoid acceleration vibrations between supporting points we assume the dependency of speed on the distance having been passed in the *j*-th interval of the $V_j(s)$ path as a linear function:

$$V_j(s) = V_{\max}\left(\frac{p_j - p_{j-1}}{\overline{S}_j}s + p_{j-1}\right), \quad (1)$$
$$s = \left(0, \overline{S}_j\right),\ \ j = \overline{1, m},$$

where $V_{\max}$ – maximal movement speed; $p_j = V_j / V_{\max}$ – *dfs*; $\overline{S}_j$ – the path between supporting points in the *j*-th interval; *m* – number of approximating intervals.

Having integrated (1) with respect to the displacement we determine the average speed of the manipulator movement.

$$V_j(s)=V_{\max}\left(\frac{p_j-p_{j-1}}{2\overline{S}_j}s+p_{j-1}\right), \qquad (2)$$

Then the time of the end-effector displacement in the *j*-th interval will be

$$T_j=\frac{2\overline{S}_j}{V_{\max}\left(p_j+p_{j-1}\right)}. \qquad (3)$$

Time law of the path change in the interval of $S_j(t)$ can be defined by integrating (1) with respect to time. In order to change over to time dependence we assume that the average speed in the interval is constant and equal to the average speed $\overline{V}_j\left(\overline{S}_j\right)$ at the end of the interval, then $s=\tau\cdot\overline{V}_j\left(\overline{S}_j\right)$, where τ is an integration variable. In this case the path change with respect to time will be obtained in the following form

$$S_j(t)=\frac{\left(p_j^2-p_{j-1}^2\right)\cdot V_{\max}^2 t^2}{4\overline{S}_j}+p_{j-1}V_{\max}t, \qquad (4)$$
$$t\in\left(0,T_j\right).$$

Having differentiated (4) we can find the law of speed change with respect to time

$$V_j^*(t)=\frac{\left(p_j^2-p_{j-1}^2\right)\cdot V_{\max}^2 t}{2\overline{S}_j}+p_{j-1}V_{\max}, \qquad (5)$$

The obtained law of path change satisfies the requirement of beatings absence.
Using dependencies (4) and (5) let us go on to discussion of interpolation of the second level. We'll get the law of generalized coordinates change in the path function of $\mathbf{q}^{(j)}(s)$, $s=S_j(t)$ assuming that the movement trajectory is given by a set of generalized coordinates in supporting points of $\mathbf{Q}_j=\left\{Q_j^{(i)}\right\}_i$, $i=\overline{1,n}$, $j=\overline{0,m}$, where *n* is a number of generalized coordinates. To provide continuity of the manipulator position and speed in the supporting points we choose a cubic polynomial as an approximating function:

$$\mathbf{q}^{(j)}(s)=\sum_{k=0}^{3}\mathbf{a}_k^{(j)}s^k, \quad j=\overline{1,m}, \qquad (6)$$

where $\mathbf{a}_k^{(j)}$ are the vectors $[1\times n]$ of the polynomial coefficients defined on the basis of boundary conditions: in terms of position

$$\mathbf{q}^{(j)}(0)=\mathbf{Q}_{j-1},\ \mathbf{q}^{(j)}\left(\overline{S_j}\right)=\mathbf{Q}_j,$$

and in terms of speed:

$$d\mathbf{q}^{(j)}/ds\Big|_{s=0}=0{,}5\left(\overline{\mathbf{Q}}_j-\overline{\mathbf{Q}}_{j-1}\right),\ j=\overline{2,m},$$
$$d\mathbf{q}^{j}/ds\Big|_{s=\overline{S}_j}=0{,}5\left(\overline{\mathbf{Q}}_{j+1}-\overline{\mathbf{Q}}_j\right),\ j=\overline{1,m-1},$$
$$\overline{\mathbf{Q}}_j=0{,}5\left(\mathbf{Q}_j-\mathbf{Q}_{j-1}\right)/\overline{S}_j.$$

For boundary conditions with respect to derivatives we use generalized speeds averaged estimation of the path having been passed in the current and the near by intervals. Such boundary conditions promote the smoother kind of a spline. As a result we obtain an expression for determining interpolation coefficients

$$\mathbf{a}_3^{(j)}=\left(y\cdot\overline{S}_j-2\left(\mathbf{Q}_j-\mathbf{Q}_{j-1}-\mathbf{a}_1^{(j)}s\right)\right)/\overline{S}_j^2,$$
$$\mathbf{a}_2^{(j)}=\left(y-3\mathbf{a}_3^{(j)}\overline{S}_j^2\right)/\overline{S}_j,$$

where $y=\left(\mathbf{Q}_{j+1}-\mathbf{Q}_j\right)/\overline{S}_{j+1}-\mathbf{a}_1^{(j)}$.

Splines calculation in the first and last interpolation sections, if the speeds in the initial and final trajectory points are not given, are carried out without averaging the derivative with respect to movement:

$$d\mathbf{q}^{(1)} / ds\big|_{s=0} = \overline{\mathbf{Q}}_1 , \; d\mathbf{q}^{(m)} / ds\big|_{s=\bar{S}_j} = \overline{\mathbf{Q}}_m .$$

The derived law of interpolation (4), (6) makes it possible to amount the problem of optimization to calculation of relative speeds p_j in supporting points $j = \overline{1, m-1}$ with account of imposed restrictions on the generalized forces:

$$\mathbf{M}^{(1)}_{\lim} \le \mathbf{M}^{(j)}(t) \le \mathbf{M}^{(2)}_{\lim} , \qquad (7)$$
$$j = \overline{1, m} , \; t \in [0, T_j],$$

where $\mathbf{M}^{(j)}$ is a vector of generalized forces $[1 \times n_{\lim}]$; $n_{\lim}$ is number of coordinates having restrictions.

3. CALCULATION OF SPEEDS IN VIEW OF CONTROL RESTRICTIONS

During the trajectory optimization in the manipulator dynamics equation the gravitational forces can be omitted. Their values are used when generating restrictions in the supporting points. Taking into account that building robots’ speeds are rather low then centrifugal and Coriolis forces can be neglected. As a result, the manipulator dynamics equation has the following form

$$\mathbf{M}^{(j)}(t) = D\left(\mathbf{q}^{(j)}(t)\right) \cdot \ddot{\mathbf{q}}^{(j)}(t) , \qquad (8)$$

where $D(\cdot)$ is a submatrix $[n_{\lim} \times n]$ of the dynamics matrix.

The trajectory optimization task will be solved for p_j speeds on the basis of the equations (4) and (6) and with a view of imposed constraints (7). As a functional of the motion law optimization we use total time of the trajectory passing. The functional can be written on the basis of the equation (3) in the form of

$$\sum_{j=1}^{m} \frac{\bar{S}_j}{\left(p_j + p_{j-1}\right)} \to \min . \qquad (9)$$

The resulting functional doesn’t contain extremums for all $p_j \in (0, \infty)$, $j = \overline{1, m-1}$. Consequently, the functional limiting values are on the plane determined by inequalities (7). In order to simplify the process of optimization we assume that the limiting values of forces in each of the splines appear on its bounds in the trajectory supporting points:

$$t_{\max} = 0 , \; t_{\min} = T_j \qquad (10)$$
$$\text{or } t_{\max} = T_j , \; t_{\min} = 0 ,$$
$$\mathbf{M}^{(j)}(t_{\max}) = \max_{t \in [0, T_j]} \left(\mathbf{M}^{(j)}(t)\right),$$
$$\mathbf{M}^{(j)}(t_{\min}) = \min_{t \in [0, T_j]} \left(\mathbf{M}^{(j)}(t)\right).$$

This assumption is true as equation (4) has constant second derivative and the splines (6) calculation was carried on with regard to their smoothing. In this case the area of the extremum search is reduced to the vertices of the boundary surface, each of them being characterized by $m-1$ equations of the form

$$\sum_{k=1}^{n_{\lim}} D^{(i,k)}\left(\mathbf{q}^{(j)}(\tau)\right) \cdot \ddot{q}^{(k,j)}(\tau) = M^{(i,v)}_{\lim} \qquad (11)$$

Exhaustive search of all boundary surface points is performed by the method of parameters variation which characterize the points where the generalized coordinates have limiting values along the trajectory

$$i = \overline{1, n_{\lim}} , \; j = \overline{1, m-1} , \tau = 0, T_j , v = 1,2 , \qquad (12)$$

where *i*, *j* are the numbers of the generalized coordinate and the spline; ν is an index of the upper and the lower limits. The system of equations (11) solution should satisfy limitations on the spline bounds

$$\mathbf{M}_{\lim}^{(1)} \le \mathbf{M}^{(j)}(0) \le \mathbf{M}_{\lim}^{(2)}, \qquad (13)$$
$$\mathbf{M}_{\lim}^{(1)} \le \mathbf{M}^{(j)}(T_j) \le \mathbf{M}_{\lim}^{(2)}, \; j = \overline{1,m}$$

If in the process of optimization several solutions are found, then we choose the one providing minimum for the functional (9).

In order to carry out the task of bounding surfaces intersection points search we re-arrange the equation (8) into the form

$$\tau = 0, \; \boldsymbol{\eta}_{1,1}^{(j)} p_j^2 + \boldsymbol{\eta}_{1,2}^{(j)} p_{j+1}^2 = \mathbf{M}_{\lim}^{(\nu)}, \qquad (15)$$
$$\tau = T_j, \; \boldsymbol{\eta}_{2,1}^{(j)} p_j^2 + \boldsymbol{\eta}_{2,2}^{(j)} p_{j+1}^2 = \mathbf{M}_{\lim}^{(\nu)},$$
$$\boldsymbol{\eta}_{1,1}^{(j)} = D_{tr}\left(\mathbf{q}_{tr}^{(j)}(0)\right)\left(\boldsymbol{\iota}_1^{(j)} - \boldsymbol{\iota}_2^{(j)}\right)\cdot V_{\max}^2,$$
$$\boldsymbol{\eta}_{1,2}^{(j)} = D_{tr}\left(\mathbf{q}_{tr}^{(j)}(0)\right)\cdot \boldsymbol{\iota}_2^{(j)} \cdot V_{\max}^2,$$
$$\boldsymbol{\eta}_{2,1}^{(j)} = D_{tr}\left(\mathbf{q}_{tr}^{(j)}(T_j)\right)\cdot \boldsymbol{\iota}_3^{(j)} V_{\max}^2,$$
$$\boldsymbol{\eta}_{2,2}^{(j)} = D_{tr}\left(\mathbf{q}_{tr}^{(j)}(T_j)\right)\left(\boldsymbol{\iota}_4^{(j)} - \boldsymbol{\iota}_3^{(j)}\right)\cdot V_{\max}^2,$$
$$\boldsymbol{\iota}_1^{(j)} = 2\mathbf{a}_2^{(j)}, \; \boldsymbol{\iota}_2^{(j)} = 0{,}5\mathbf{a}_1^{(j)} / \overline{S}_j,$$
$$\boldsymbol{\iota}_3^{(j)} = -1{,}5\mathbf{a}_3^{(j)} \overline{S}_j - \mathbf{a}_2^{(j)} - \boldsymbol{\iota}_2^{(j)},$$
$$\boldsymbol{\iota}_4^{(j)} = 6\mathbf{a}_3^{(j)} \overline{S}_j + \boldsymbol{\iota}_1^{(j)}.$$

The form of the resultant equations allows us to reduce the problem to solution of the system of linear equations for square values of speeds $\mathbf{p}_{sq} = \left\{p_j^2\right\}_j$: $\mathrm{H} \cdot \mathbf{p}_{sq} = \boldsymbol{\gamma}$, where H is a matrix $[(m-1)\times(m-1)]$ of coefficients $\eta_{r,k}^{(i,j)}$, $r = 1{,}2$, $k = 1{,}2$; *r* is an index of the spline boundary; γ is a vector of limitary forces $M_{\lim}^{(i,\nu)}$.

With the purpose of providing inequality with zero of the determinant $|\mathrm{H}|$ it is essential that the matrix main diagonal doesn't contain any zero values. To achieve this we assign each matrix line to the corresponding its index a *j*-th supporting point, and instead of the spline number variation $j = \overline{1, m-1}$ we perform an exhaustive search of the left and right splines. Therefore, an expression for non-zero elements of the *j*-th line of H matrix and γ vector will take the following form

$$\mathrm{H}^{(j,j-3+k+\upsilon)} = \eta_{r,k}^{(i,j)}, k = 1{,}2, \qquad (15)$$
$$\gamma^{(j)} = M_{spl\lim}^{(i,\nu)},$$

where $\upsilon = 1{,}2$ is a ***variate*** parameter indicating the left or right spline to the *j*-th supporting point.

Complete exhaustive search of possible combinations of bounding surfaces along M matrix lines can be achieved by ***varing*** parameters i_j, υ_j, r_j, ν_j, for all $j = \overline{1, m-1}$. As a result of exhaustive search relative speeds are determined which provide minimum time taking into account the chosen way of trajectory planning.

Correctness of assumptions concerning the absence of spline beatings can be substantiated by calculating the roots of the equation $\ddot{q}^{(i,j)}(t_r) = 0$ and checking their absence in the interval of $(0, T_j)$.

If only one root lies in the range of $t_r \in (0, T_j)$, then the value of the generalized force $M^{(i)}(t_r)$ should be checked. If it doesn't satisfy the condition (7), then one more supporting point is formed which is incorporated in exhaustive search during the next speeds optimization.

4. CONCLUSION

The presented method of movements planning for building robots makes it possible to form control actions for the manipulator drives in real time subject to limitations of forces. This allows to perform re-planning of the trajectory taking into account external loads and accumulated positioning error. The number of supporting points for the trajectory being planned is chosen depending on the trajectory length and form and the individual control strategy for different technological operations. The investigations have shown that the assumption of absence of forces beating proves to be correct in case when the movement trajectory has no sharp space bendings, and increased number of supporting points decreases smoothness of generalized forces function.

Autorenangabe:
Professor Alexej Bulgakow
132 Proswestschenija st.
346428 Nowotcherkassk / Russia
Tel: 007-86352 2 4568
Fax: 007-86352 2 4568
e-mail: a.bulgakow@gmx.de