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THE INFLUENCE OF CONDUCTING BODY ON ELECTRIC FIELD HOMOGENEITY

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ABSTRACT

By analogy to Helmholtz coils that produce uniform magnetic field [1], it is possible to produce uniform electric field using two charged rings [2, 3]. Here, the system of thin coaxial toroidal electrodes is modeled in order to generate homogeneous electric field. The applied procedure for electrodes dimensions and positions determination enabled the largest area of homogeneity. In order to find the influence of an external body volume to the obtained field's homogeneity, the external object is modeled as a spherical electrode and placed in the centre of the system. Numerical results, obtained using the software Mathematica 7.0 and Femm, are compared and shown graphically.

Index Terms **-** Homogeneous electric field, toroidal electrodes, sphere external conducting body

1. INTRODUCTION

The problems of electric and magnetic field synthesis are of a great importance in many theoretical applications, where it is necessary to generate the field with a high accuracy and of required features. Perfect homogeneous electric field can be generated in a closed area like the eccentrical spherical hole inside the sphere or the eccentrical cylindrical hole inside the cylinder, having in both cases the uniform charge per unit volume. These areas are difficult to be reached, so various open systems for homogeneous electric fields generation are developed. One original analytical method for toroidal systems modeling for homogeneous electric field generation, is developed in [3].

The model consists of two thin toroidal electrodes, of absolutely equal but opposite potential values. These electrodes compose so called primary cell. The realized field homogeneity is as better as the used electrodes are thicker. But, in practical applications, a system of coaxial rings is used instead of two thick rings. Nevertheless, in that case, a few problems occur: how to find electrodes positions and dimensions in order to obtain the largest region of homogeneity, which exists in the central area of the system. For a purpose of high order homogeneity electric field generation, using thin toroidal electrodes, one quite general procedure is applied: a potential distribution along the system axis is expanded into a power series. In order to obtain the constant resulting field in the central area, it is necessary to annul as many dominant terms. Since the coefficients in the series are functions of positions and dimensions of toroidal electrodes, it is possible to choose system dimensions so that only linear term remains, while other series terms can be neglected. The larger the number of cells used, the larger the area of homogeneous electric field. In addition to the need to make a space of homogeneous electric field, very often there is a need for space in which there is no field. In that sense, specially generated electric fields, [3], can be used to neutralize some external field, for example the Earth electrostatic field.

A system for producing a highly homogeneous, variable, and directable magnetic field is described in [4]. That system is applied in gyromagnetic ratio experiments and directed against the Earth's magnetic field to obtain very nearly a free space field. In both cases, intensity of an external field, either electric or magnetic, is so decreased that its influence is practically equal to zero. It can be important because special physical conditions that differ a lot from the Earth's conditions and can be reached only in satellites, can be realized on the Earth. Therefore, the investigation of the ordinary life and many of technological processes in satellite conditions can be done on the Earth.

2. HOMOGENEOUS ELECTRIC FIELD GENERATION

In practice, very often there is a need for getting homogeneous electric fields, with required features and of high precision. Those fields are used widely in technique: they are applied in electrical machines for high field generation, NMR spectroscopy, MHD technique, nuclear and plasma physics, so the problem of electrostatic field is very actual from the standpoint of its realization as well as from the standpoint of its partial or complete elimination.

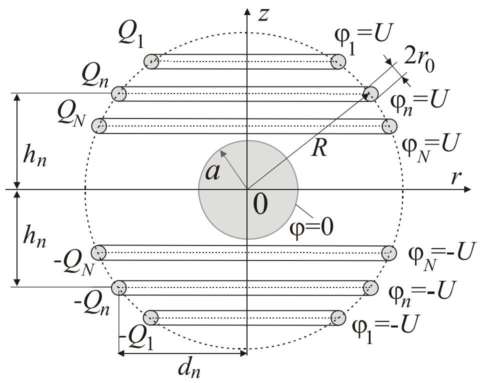


Figure 1 A system for homogeneous electric field generation

2.1. Toroidal electrodes system

Homogeneous electric field generation mechanism, formed by thin coaxial toroidal electrodes, is considered in this paper. As a starting element in modeling procedure, a so called primary cell is used. It consists of two thin coaxial toroidal electrodes, of absolutely equal potential values, but of opposite signs. Complex systems consist of a number of primary cells, Fig.1. Firstly, the system for homogeneous electric field generation without an external conducting body is observed.

2.2. Optimal system design

The axial potential distribution offers a convenient means to describe the electric field associated with used electrodes configuration and dimension. Keeping this in mind, the authors propose a technique for determining the electrode configuration required for developing a high homogeneous electric field in the largest area of interest. The procedure is based on the following: the potential function along the axis is expanded into a power series, which contains only odd powers of variable z. In order to obtain the high homogeneous electric field, it is necessary for resulting potential to be a linear function versus z from the central system point. For this requirement implementation, it is necessary to annul as many dominant terms. Primary cell dimensions of N^{th} order are obtained by setting the coefficient of z^{2N+1} to zero.

Along the system axis, the axial potential can be expanded in a power series which contains only odd powers of variable z:

$$\varphi(r=0,z)=\sum_{n=0}^{\infty}\varphi_{2n+1}z^{2n+1}\,, \qquad (1)$$

where

$$\varphi_{2n+1}=\frac{Q\,P_{2n+1}(\cos\alpha)}{2\pi\varepsilon R^{2n+2}}\,, \qquad (2)$$

$R=\sqrt{d^2+h^2}$ and $P_n(x)$ is Legendre polynomial of the first kind.

Since the coefficients in series (1) depend on electrodes positions and dimensions, it is possible to choose primary cell dimensions so that some series terms vanish and only one term dominates. This dominant term provides corresponding distribution of the electric field. So, in order to provide homogeneous electric field in the central system area, potential has to have the linear function versus the distance z from central system point. After setting the coefficient of z^{2N+1} to zero, where N defines primary cell order, the following equation is obtained: $\varphi_{2N+1}=0$. As this equation has N real positive roots, the primary cell of N-th order is determined. Each of these solutions defines one primary cell. The larger the number of cells used, the larger the area of homogeneous electric field.

3. MATHEMATICAL BACKGROUND

3.1. Calculation of primary cell dimensions

Let us first consider a system consisting of one primary cell, i.e. two thin toroidal electrodes, having the same absolute potential value, but of an opposite sign. The electrodes dimensions, necessary for high homogeneity electric field developing, are being obtained after assuming that coefficient of z^3, given by formula (2), is equal to zero, what yields: $P_3(\cos\alpha)=0$. This equation has only one root, $\cos\alpha=0.7745966692$, so the primary cell of the first kind has dimensions $h_1=0.7745966692R$ and $d_1=0.632455532R$.

By setting the coefficient of z^5 to zero, we get the equation that has two solutions, which define dimensions of the second order primary cell:

$d_1=0.422892524R$, $h_1=0.906179846R$,
$d_2=0.8426544122R$, $h_2=0.538469310R$.

In a general case, by setting the coefficient of z^{2N+1} to zero in (1), then the primary cell of the N^{th} order is obtained.

Table 1 presents the dimensions of primary cells of the first to fifth order.

Table 1. Optimal system dimensions for N^{th} order primary cell.

	d_5/R	d_4/R	d_3/R	d_2/R	d_1/R	
N	0.963	0.855	0.683	0.462	0.208	5
1	0.775	0.946	0.790	0.549	0.250	4
2	0.906	0.538	0.914	0.671	0.315	3
3	0.949	0.742	0.406	0.843	0.423	2
4	0.968	0.836	0.613	0.324	0.632	1
5	0.978	0.887	0.730	0.519	0.270	N
	h_1/R	h_2/R	h_3/R	h_4/R	h_5/R	

3.2. Charge calculation on the rings

In order to examine the influence on the realized field homogeneity with the smallest mathematical difficulties, the external body is modeled as a conducting sphere and placed into the central area, Figure 1. Its centre coincides with the coordinate origin. A potential distribution is expressed as:

$$\varphi=\frac{1}{2\pi^2\varepsilon}\sum_{n=1}^{\mathrm{N}}\sum_{m=1}^{4}Q_{nm}\left(\frac{\mathrm{K}(\pi/2,\mathrm{k}_{nm})}{R_{nm}}\right). \qquad (3)$$

$K(\pi/2,k)$ is the complete elliptic integral of the first kind of moduli:

$$k_{n1}{}^2=\frac{4d_n r}{R_{n1}{}^2}\,,\quad k_{n2}{}^2=\frac{4d_n r}{R_{n2}{}^2}\,,\quad k_{n3}{}^2=\frac{4(a/R)^2 d_n r}{R_{n3}{}^2}\,,$$

$$k_{n4}{}^2 = \frac{4(a/R)^2 d_n r}{R_{n4}{}^2}$$, where

$$R_{n1} = \sqrt{(r+d_n)^2 + (z-h_n)^2} \ ,$$

$$R_{n2} = \sqrt{(r+d_n)^2 + (z+h_n)^2} \ ,$$

$$R_{n3} = \sqrt{\left(r+(a/R)^2 d_n\right)^2 + \left(z-(a/R)^2 h_n\right)^2} \ ,$$

$$R_{n4} = \sqrt{\left(r+(a/R)^2 d_n\right)^2 + \left(z+(a/R)^2 h_n\right)^2} \ .$$

$Q_{n1} = Q_n$ are the charges of ring electrodes of potential U, $Q_{n2} = -Q_n$ are the charges of ring electrodes of potential $-U$, $Q_{n3} = -Q_{n4} = -\frac{a}{R}Q_n$ are the charges of their images into the sphere mirror.

Using the condition that the toroidal electrodes are of equal potential U for $z > 0$, their unknown charges can be determined after matching the total potential to the potential U on them.

4. NUMERICAL RESULTS

In accordance with the analysis presented above, a general numerical program has been developed using the software Mathematica 7.0. The same problem is modeled using the software Femm, [5].

Let us observe a system consisting of one primary cell. Normalized value of the electric field strength along z axis, with and without the external sphere conducting body of normalized radius $a/R = 0.1$, is presented in Figure 2. Presented distributions show that external conducting body influences on field distribution only in the close vicinity of the body brought into the field.

Electric field distribution along the system axis for different number of primary cells, by application of both Mathematica and Femm, is presented in Figure 3. High homogeneous electric field is attained using primary cells of high order.

But, the electrostatic system that has to satisfy this electrostatic demand, has to be optimal in the view of simplicity and production economy, as well. Keeping this in mind, the recommended number of primary cells is $N = 3$.

The influence of the external body dimension on the achieved field homogeneity is shown in Figure 4. Increase of sphere size disturbs realized field homogeneity. Equipotential lines for $a/R = 0.2$, $N = 1$ and $a/R = 0.5$, $N = 3$, are shown in Figures 5(a) and 5(b), respectively.

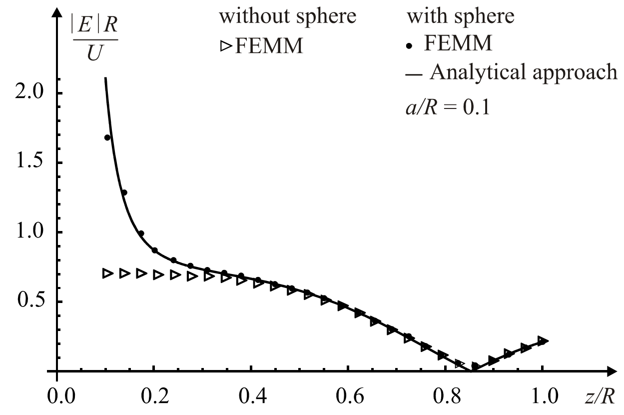


Figure 2 Normalized electric filed along z axis using one primary cell with and without external sphere of normalized radius s $a/R = 0.1$

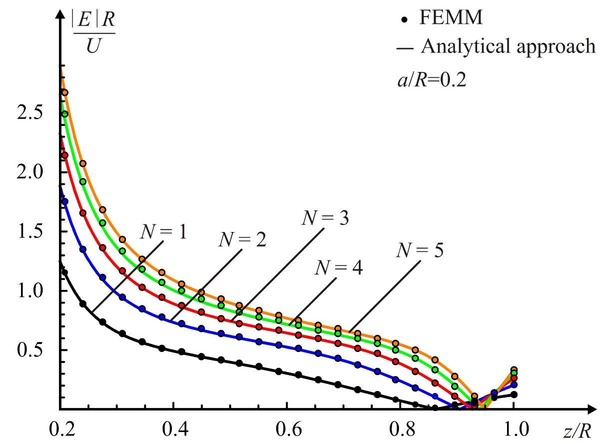


Figure 3 Normalized electric filed along z axis for different number of primary cells and $a/R = 0.2$

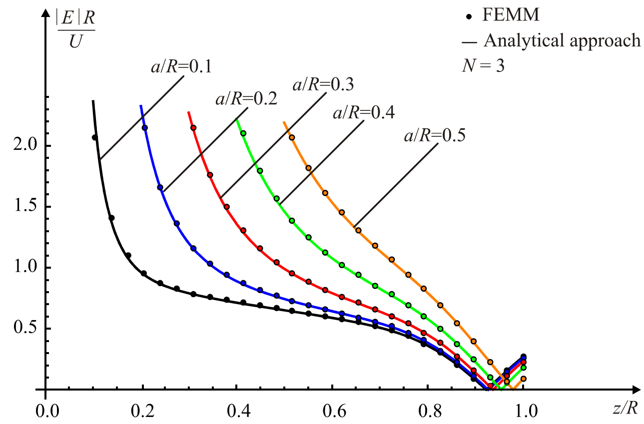
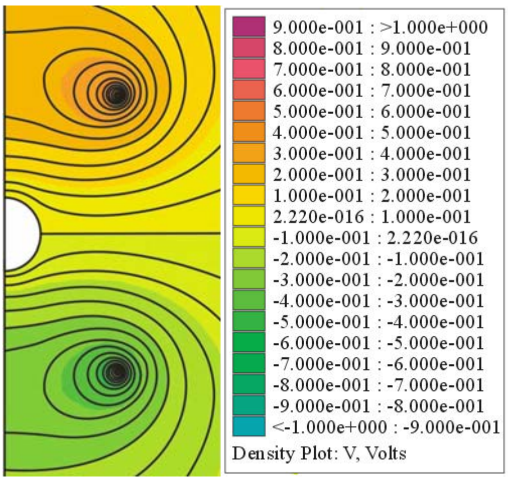
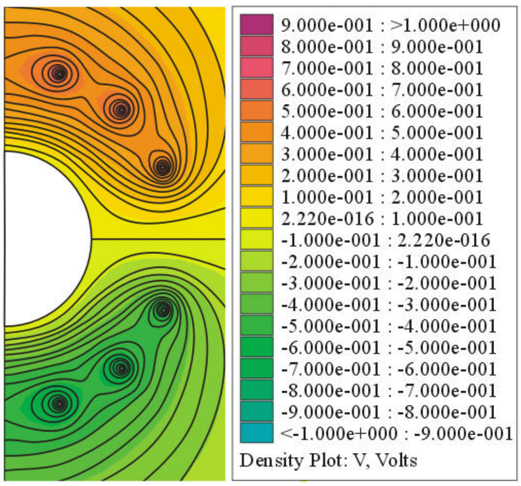


Figure 4 Influence of external sphere dimension on realized field homogeneity using $N = 3$ primary cells



(a)



(b)

Figure 5 Equipotential lines for $a/R = 0.2$, $N = 1$ *(a) and* $a/R = 0.5$, $N = 3$ *(b)*

5. CONCLUSION

On the basis on a very simple procedure for modeling electrostatic system for generating homogeneous electric field, [3], in this paper the authors investigate the influence of an external conducting body on the achieved field homogeneity. These systems are formed by thin coaxial toroidal electrodes, having the equal absolute potential values, but of the opposite signs. Along the system axis, an axial potential can be expanded into a power series. It is possible to choose primary cells dimensions so that only series linear term dominates. This condition implementation provides homogeneous electric field in the central system area. High homogeneity field is obtained with higher order primary cell: higher order cell provides the larger region of homogeneous electric field than the primary cell of the first order.

Based on the theoretical investigation, an affection of the external spherical body brought into the homogeneous electric field to the existing homogeneity in the central region of the system is observed. The homogeneity is disturbed mostly in the close vicinity of the body.

More primary cells will provide a larger region of homogeneous electric field. Decrease of the external body influence on realized field homogeneity is accomplished by increase of primary cells number.

External conducting sphere dimension has a strong influence on the achieved field homogeneity. The bigger the sphere, the smaller the area of the homogeneous electric field.

The further investigation will apply to different shaped external bodies brought into the system for uniform electric field generation. The authors will also put emphasis to the system of biconical electrodes, which also can successfully be used for high homogeneity field achievement.

When such toroidal electrodes are grounded and placed in the external electric field, then they can be used for the space protection against the overflow electric fields strengths. Existing fields for some applications need to be practically reduced or eliminated by specially generated electric and magnetic fields.

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