54. IWK

Internationales Wissenschaftliches Kolloquium
International Scientific Colloquium

Information Technology and Electrical Engineering - Devices and Systems, Materials and Technologies for the Future

Faculty of Electrical Engineering and Information Technology

Startseite / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=14089

TECHNISCHE UNIVERSITÄT ILMENAU

Impressum

Herausgeber:	Der Rektor der Technischen Universität Ilmenau Univ.-Prof. Dr. rer. nat. habil. Dr. h. c. Prof. h. c. Peter Scharff
Redaktion:	Referat Marketing Andrea Schneider Fakultät für Elektrotechnik und Informationstechnik Univ.-Prof. Dr.-Ing. Frank Berger
Redaktionsschluss:	17. August 2009

Technische Realisierung (USB-Flash-Ausgabe):
Institut für Medientechnik an der TU Ilmenau
Dipl.-Ing. Christian Weigel
Dipl.-Ing. Helge Drumm

Technische Realisierung (Online-Ausgabe):
Universitätsbibliothek Ilmenau
ilmedia
Postfach 10 05 65
98684 Ilmenau

Verlag:
isle
Verlag ISLE, Betriebsstätte des ISLE e.V.
Werner-von-Siemens-Str. 16
98693 Ilmenau

ISBN (USB-Flash-Ausgabe): 978-3-938843-45-1
ISBN (Druckausgabe der Kurzfassungen): 978-3-938843-44-4

Startseite / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=14089

APPROXIMATE CALCULATION OF CHARACTERISTIC PARAMETERS OF RECTANGULAR TRANSMISSION LINES

A. Milovanovic, B. Koprivica

Technical Faculty Čačak, Department of Electronics and Electrical Engineering

ABSTRACT

In this paper, we will present the application of Charge Simulation Method (CSM) for the calculation of the per unit length parameters of the transmission lines with rectangular cross section. We will present the results for different sizes, shapes and relative positions of conductors as well as the variation in the results for different insulating material between conductors. For these examples, the electric field disturbance and potential lines will be obtained by using Finite Element Method and Software package Femlab [1]. The results for transmission lines parameters obtained by CSM will be compared with those calculated with Femlab. We will provide the results confirming the agreement between these results.

In addition, the results for the capacitance per unit length of microstrip line using CSM will be also presented. Microstrip line model in Femlab has been made for this purpose and the results obtained by CSM will be compared both with the results obtained by using this model and those obtained by analytical expressions given in the literature.

Index Terms – Charge simulation method, Transmission lines, Rectangular cross section, Characteristic parameters, Microstrip line

1. INTRODUCTION

The transmission line model represents the transmission line as an infinite series of two-port elementary components, each representing an infinitesimally short segment of the transmission line:

- The distributed resistance R' of the conductors is represented by a series resistor (Ω/m).
- The distributed inductance L' is represented by a series inductor (H/m).
- The capacitance C' between the two conductors is represented by a shunt capacitor (F/m).
- The conductance G' of the dielectric material separating the two conductors is represented by a conductance shunted between the signal wire and the return wire (S/m).

Expressions for these parameters are functions of two sets of parameters: geometric parameters (sizes, shapes, relative positions of the two conductors) and electromagnetic constitutive parameters characteristic of the materials of which the conductors and the insulating material between them are made.

The exact analytical expression for the capacitance per unit length of the transmission lines with rectangular cross section does not exist. Calculation of the capacitance per unit length [2-9] can be done using different numerical methods and results obtained using some of them are presented in this paper. The basic method which is used is Charge Simulation Method [2].

2. CHARGE SIMULATION METHOD

Transmission line with rectangular cross section is presented in Fig.1.

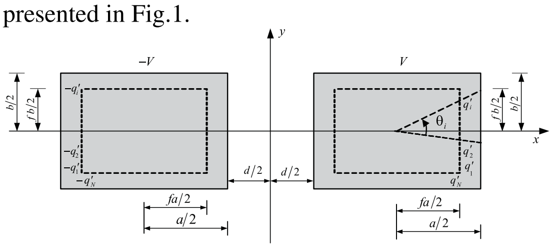

Fig. 1. Transmission line with rectangular cross section.

By applying CSM, real system of two electrodes with rectangular cross section is replaced with system of fictitious sources in form of line charges uniformly distributed inside the electrodes of the line, such is presented in Fig.1.

Using the superposition principle, potential in vicinity of the original electrode surface is expressed in terms of unknown weights of fictitious sources:

$$\varphi = \sum_{i=1}^{N} q_i' G_i\left(x, y, x_i, y_i\right), \tag{1}$$

where q_i' represents the (unknown) magnitude of the line charges and $G_i\left(x, y, x_i, y_i\right)$ is Green's function of potential of the FS including symmetric component. x_i and y_i define the position of the *i*-th FS.

The intensities of used FS can be determined after solving the linear equations satisfying the boundary condition in properly adopted points (matching points) uniformly distributed on the electrode's surface, $\varphi = V$, $k = 1, 2, \ldots, N$. By solving the system for unknown weight of FS, q_i' , $i = 1, 2, \ldots, N$, the capacitance per unit length, potential, electric field strength and other quantity of interest can be easily determined:

$$C' = \frac{\sum_{i=1}^{N} q_i'}{2V}\ ,\ Z_c = \frac{\sqrt{\varepsilon\mu}}{C'}\ ,\ E = -\operatorname{grad}\varphi\ . \quad (2)$$

3. NUMERICAL RESULTS

Based on the presented theoretical analysis a computer code is made and numerous calculations are performed.

3.1. Two wire line with rectangular cross section

The convergence of the results for $q'/4\pi\varepsilon$, with the number of fictitious sources, N, when $b/a = 0.6$, $d/a = 0.6$ and $f = 0{,}99$ are presented in Table 1.

TABLE 1
THE CONVERGENCE OF THE RESULTS FOR $q'/4\pi\varepsilon$.

N	$q'/4\pi\varepsilon$	N	$q'/4\pi\varepsilon$
4	0. 119 632	200	0. 237 050
20	0. 209 428	240	0. 237 292
32	0. 221 874	280	0. 237 437
40	0. 225 989	320	0. 237 526
60	0. 231 234	360	0. 237 582
80	0. 233 634	400	0. 237 616
100	0. 234 945	420	0. 237 628
160	0. 236 613	480	0. 237 647

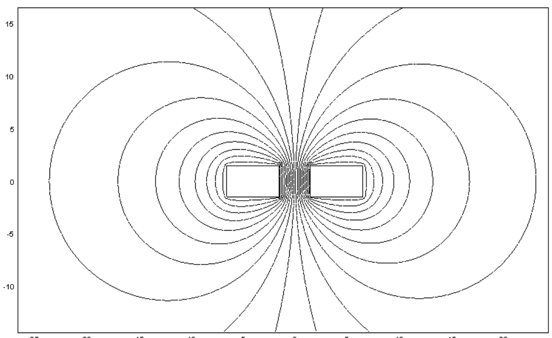

Fig. 2. Equipotential lines of two wire line with rectangular cross section (*a*, *b*), when $d/a = b/a = 0.6$.

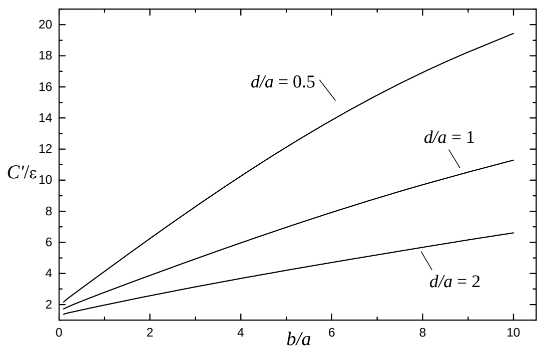

Fig. 3. Capacitance per unit length of the line, when $b/a = 0.6$.

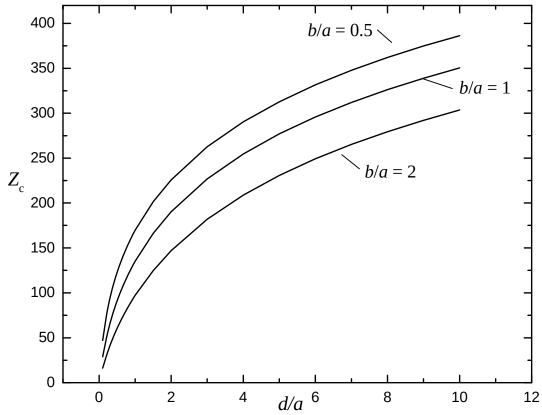

Fig. 4. Characteristic impedance of two wire line with rectangular cross section (*a*, *b*), when $d/a = 0.6$.

The comparison of the results for capacitance per unit length obtained by CSM and FEM is presented in Table 2.

TABLE 2
THE CAPACITANCE PER UNIT LENGTH, FOR $b/a = 0.6$,
$N = 400$, $f = 0{,}99$ AND DIFFERENT RATIO d/a .

d/a	C'/ε (CSM)	C'/ε (FEM)
0.1	9. 003 460	9. 036 503
0.2	5. 594 840	5. 611 849
0.3	4. 362 240	4. 377 404
0.4	3. 702 160	3. 714 565
0.5	3. 281 660	3. 291 804
0.6	2. 985 970	2. 994 868
0.7	2. 764 370	2. 772 461
0.8	2. 590 740	2. 597 928
0.9	2. 450 140	2. 456 694
1	2. 333 400	2. 339 855
1.5	1. 951 380	1. 956 814
2	1.733 010	1. 737 931
3	1. 481 880	1. 486 579
4	1. 335 780	1. 340 620
5	1. 237 390	1. 242 589
6	1. 165 350	1. 171 039
7	1. 109 650	1. 115 901
8	1. 064 880	1. 071 779
9	1. 027 870	1. 035 459
10	0. 996 578	1. 004 912

3.2. Two wire line with rectangular cross section above conducting plane

Electric field distribution in vicinity of transmission line with rectangular cross section above conducting plane is presented in Fig. 5.
The comparison of the results for capacitance per unit length obtained by CSM and FEM is presented in Table 3.

The capacitance per unit length and the characteristic impedance of the line are presented in Figs. 6 and 7.

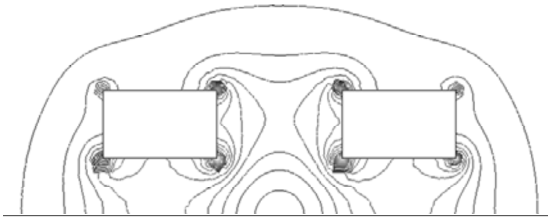

Fig. 5. Equienergetic curves.

TABLE 3

THE CAPACITANCE PER UNIT LENGTH, FOR $b/a = 0.6$, $h/a = 0.5$, $N = 400$, $f = 0,99$ AND DIFFERENT RATIO d/a.

d/a	C'/ε (CSM)	C'/ε (FEM)
0.1	9. 659 090	9. 689 761
0.2	6. 287 500	6. 303 868
0.3	5. 090 390	5. 103 830
0.4	4. 464 270	4. 478 042
0.5	4. 076 210	4. 085 872
0.6	3. 811 490	3. 819 550
0.7	3. 619 450	3. 627 574
0.8	3. 474 000	3. 481 567
0.9	3. 360 300	3. 367 516
1	3. 269 210	3. 276 130
1.5	2. 998 940	3. 005 132
2	2. 869 990	2. 875 789
3	2. 752 130	2. 757 759
4	2. 700 730	2. 705 996
5	2. 673 590	2. 678 923
6	2. 657 490	2. 663 702
7	2. 647 140	2. 653 467
8	2. 640 090	2. 646 221
9	2. 635 070	2.641 196
10	2. 631 360	2.637 636

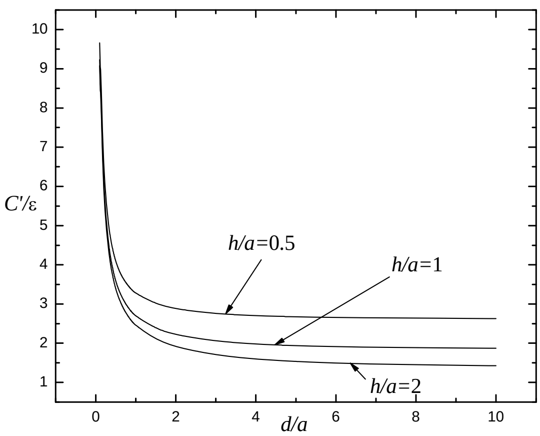

Fig. 6. Capacitance per unit length of the line, when $b/a = 0.6$ and different ratio h/a.

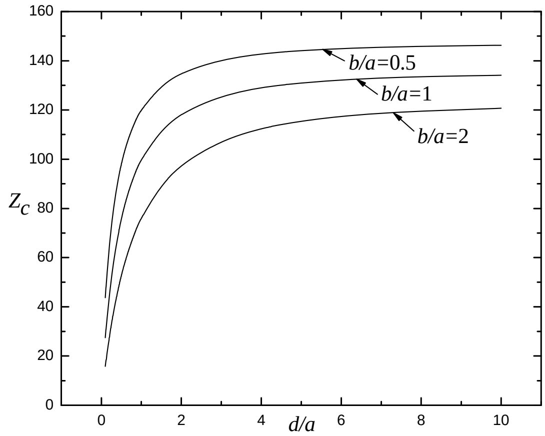

Fig. 7. Characteristic impedance of the line, when $h/a = 0.5$ and different ratio b/a.

3.3. Two wire line with rectangular cross section above dielectric surface

Transmission line with rectangular cross section above dielectric surface is presented in Fig. 8.

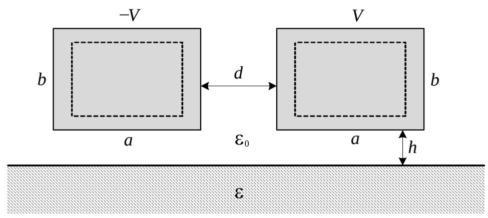

Fig. 8. Transmission line with rectangular cross section above dielectric surface.

Green's function of potential of the FS including symmetric component is

$$G_i = \frac{1}{4\pi\varepsilon}\left(\begin{array}{l} \ln\dfrac{(x+x_i)^2+(y-y_i)^2}{(x-x_i)^2+(y-y_i)^2}+ \\ +\alpha\ln\dfrac{(x+x_i)^2+(y+y_i)^2}{(x-x_i)^2+(y+y_i)^2} \end{array} \right), \qquad (3)$$

where $\alpha = (1-\varepsilon_r)/(1+\varepsilon_r)$ and ε_r is relative dielectric constant of dielectric surface.

The comparison of the results for capacitance per unit length obtained by CSM and FEM is presented in Table 4.

The characteristic impedance Z_c and the effective dielectric constant ε_{eff} can be calculated as:

$$Z_c = \frac{\sqrt{\varepsilon_{eff}}}{c\,C'} \qquad (4)$$

and

$$\varepsilon_{eff} = \frac{C'}{C'_0}, \qquad (5)$$

where c stands for the velocity of electromagnetic waves in free space and C'_0 is capacitance per unit length of the line without dielectric surface.

TABLE 4

THE CAPACITANCE PER UNIT LENGTH, FOR $b/a = 0.6$, $h/a = 0.5$, $\varepsilon_r = 5$, $N = 400$, $f = 0{,}99$ AND DIFFERENT RATIO d/a.

d/a	C'/ε (CSM)	C'/ε (FEM)
0.1	9. 392 980	9. 423 349
0.2	6. 003 570	6. 020 979
0.3	4. 788 990	4. 801 904
0.4	4. 145 760	4. 156 529
0.5	3. 740 990	3. 750 178
0.6	3. 459 960	3. 475 057
0.7	3. 252 010	3. 264 704
0.8	3. 091 080	3. 098 081
0.9	2. 962 310	2. 969 162
1	2. 856 570	2. 863 114
1.5	2. 519 140	2. 524 779
2	2. 332 280	2. 337 622
3	2. 120 530	2. 125 762
4	1. 996 380	2. 001 866
5	1. 910 920	1. 916 744
6	1. 846 720	1. 853 119
7	1. 795 780	1. 802 769
8	1. 753 850	1. 761 556
9	1. 718 400	1. 726 905
10	1. 687 820	1. 697 128

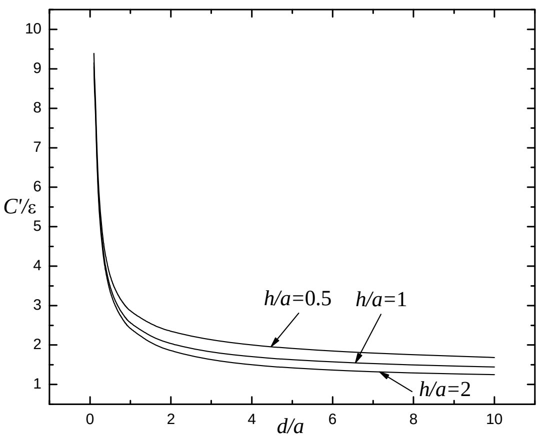

Fig. 8. Capacitance per unit length, when $b/a = 0.6$.

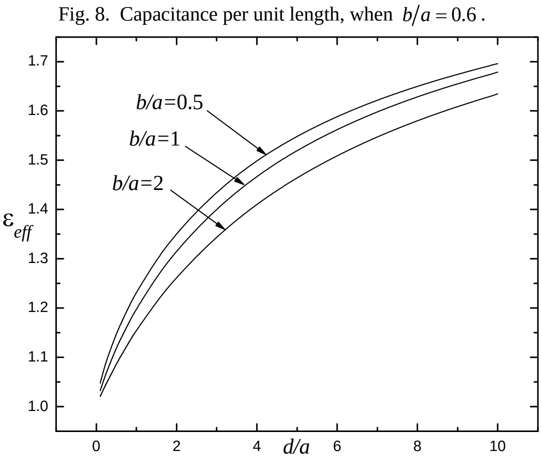

Fig. 9. Effective dielectric constant of the line, when $h/a = 0.5$.

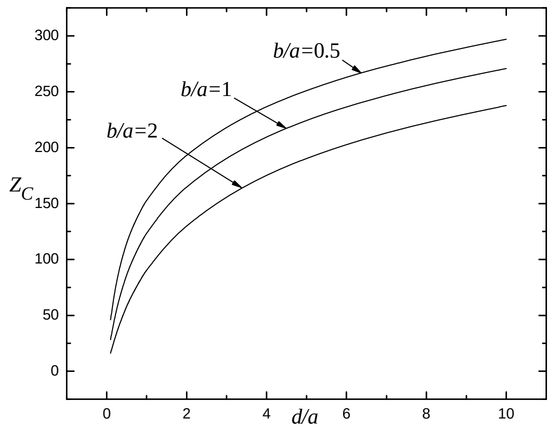

Fig. 10. Characteristic impedance of the line, when $h/a = 0.5$.

3.4. Microstrip line

Microstrip line is presented in Fig. 11.

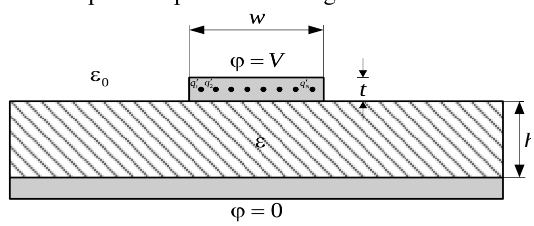

Fig. 11. Microstrip line.

Green's function of potential of the FS is

$$G_i = \frac{1}{4\pi\varepsilon}\left\{ \ln\frac{(x-x_i)^2+(y+y_i)^2}{(x-x_i)^2+(y-y_i)^2} + \right.$$
$$\left. +\sum_{m=1}^{m=\infty}(\alpha)^m \ln\frac{(x-x_i)^2+(y+h+t/2+2hm)^2}{(x-x_i)^2+(y-h-t/2+2h(m-1))^2} \right\} \quad (6)$$

where

$$\alpha = (1-\varepsilon_r)/(1+\varepsilon_r)$$

and ε_r is relative dielectric constant of dielectric layer.

The characteristic impedance Z_c and the effective dielectric constant ε_{eff} can be calculated such as in the previous example, expressions (4) and (5).

Approximate analytical expressions for calculating of the effective dielectric constant and the characteristic impedance of microstrip line are [11]:

$$\varepsilon_{eff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left[\left(1+12\frac{h}{w}\right)^{-1/2} + 0.04\left(1-\frac{w}{h}\right)^2\right], \quad (7)$$

$$Z_C = \frac{60}{\sqrt{\varepsilon_{eff}}}\ln\left(\frac{8h}{w}+\frac{w}{4h}\right), \quad (8)$$

$$\text{for } \frac{w}{h} < 1$$

and

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2\sqrt{\left(1 + 12\frac{h}{w}\right)}}, \qquad (9)$$

$$Z_C = \frac{377}{\sqrt{\varepsilon_{eff}}}\left[\frac{w}{h} + 1.393 + 0.667\ln\left(\frac{w}{h} + 1.444\right)\right]^{-1}, \qquad (10)$$

$$\text{for } \frac{w}{h} > 1.$$

The comparison of the results for the characteristic impedance and the effective dielectric constant of microstrip line obtained by CSM, FEM and these analytical expressions are presented in Tables 5 and 6.

TABLE 5
THE CHARACTERISTIC IMEDANCE, FOR $\varepsilon_r = 5$, $N = 500$, $m = 20$, $t/w = 5 \cdot 10^{-7}$ AND DIFFERENT RATIO w/h.

w/h	Z_C (CSM)	Z_C (FEM)	Z_C (Exp.)
0.1	146. 507	144. 702	145. 929
0.2	122. 623	121. 170	121. 747
0.3	108. 632	107. 347	107. 702
0.4	98. 770	97. 553	97. 815
0.5	91. 163	89. 986	90. 207
0.6	84. 989	83. 815	84. 044
0.7	79. 812	78. 656	78. 880
0.8	75. 367	74. 235	74. 448
0.9	71. 486	70. 368	70. 579
1	68. 053	66. 947	66. 897
1.5	55. 299	54. 244	54. 489
2	46. 872	45. 859	46.120
3	36. 212	35. 256	35. 457
4	29. 668	28. 752	28.897
5	25. 212	24. 328	24. 430

TABLE 6
THE EFFECTIVE DIELECTRIC CONSTANT, FOR $\varepsilon_r = 5$, $N = 500$, $m = 20$, $t/w = 5 \cdot 10^{-7}$ AND DIFFERENT RATIO w/h.

w/h	ε_{eff} (CSM)	ε_{eff} (FEM)	ε_{eff} (Exp.)
0.1	3. 249 750	3. 233 540	3. 246 620
0.2	3. 302 380	3. 290 970	3. 307 270
0.3	3. 340 410	3. 334 020	3. 351 550
0.4	3. 375 310	3. 370 940	3. 388 010
0.5	3. 407 100	3. 404 310	3. 420 000
0.6	3. 436 730	3. 435 740	3. 449 240
0.7	3. 464 680	3. 464 630	3. 476 740
0.8	3. 491 270	3. 492 120	3. 503 200
0.9	3. 516 680	3. 518 440	3. 529 070
1	3. 541 030	3. 543 790	3. 554 700
1.5	3. 649 900	3. 656 490	3. 666 670
2	3. 741 140	3. 751 560	3. 755 930
3	3. 884 540	3. 901 730	3. 894 430
4	3. 991 160	4. 015 440	4. 000 000
5	4. 073 050	4. 104 880	4. 084 650

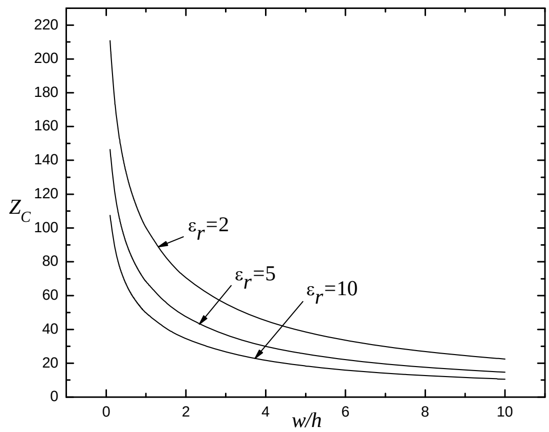

Fig. 12. Characteristic impedance of the line from Fig. 11.

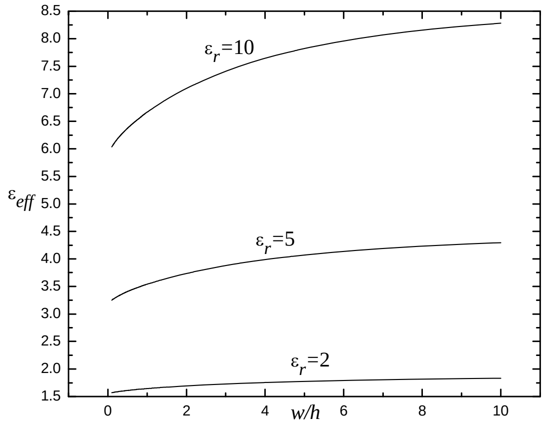

Fig. 13. Effective dielectric constant of microstrip lines.

4. CONCLUSION

In this paper, the application of Charge Simulation Method for the calculation of the per unit length parameters of the transmission lines with rectangular cross section is presented. The results for the capacitance per unit length, characteristic impedance and effective dielectric constant of some characteristic examples, two wire lines, two wire lines above conducting plane, two wire line above dielectric surface and microstrip line are shown. The very good agreement of the results obtained with CSM and the results obtained with Femlab and proper analytical expressions is noticed. CSM can be successfully implemented for solving of these and similar, and more complex problems such as coupled microstrip lines, striplines or lines with asymmetrical geometry. This will be considered in one of the following papers.

5. REFERENCES

[1] Software package Femlab, Version 2.3.0.145., http://www.femlab.com.

[2] N.H. Malik, A Review of the Charge Simulation Method and its Application, IEEE, Transactions on

Electrical Insulation, February 1989, Vol. 24, No 1, pp. 3-20.
[3] J.V. Surutka, D.M. Velickovic, Some Improvements of the Charge Simulation Method for Computing Electrostatic Fields, Bulletin LXXIV de l'Academie Serbe des Sciences and Arts, Classe des Sciences Techniques, 1981, No. 15, pp. 27-44.
[4] D.M. Velickovic, A.M. Milovanovic, Approximate Calculation of Capacitance, Proc. 1998 VII International IGTE Symposium on Numerical Field Calculation in Electrical Engineering, Graz, pp. 339-344.
[5] K.L. Kaiser, Electrostatic Discharge, Taylor & Francis Group, Boca Raton, FL., 2006.
[6] S.M. Musa, M.N.O. Sadiku, Calculating the Capacitance of Shielded Microstrip Lines, Proceedings of the COMSOL Conference, Boston, 2007.
[7] B.M. Koprivica, A.M. Milovanovic, Modeling of Coplanar Waveguide in COMSOL Multiphysics, Proceedings of the COMSOL Conference, Budapest, 2008.
[8] K.C. Gupta, R. Garg, A. Bahl, P Bhartia, Microstrip Lines and Slotlines 2nd Edition, Artech House, Norwood, 1996.
[9] A.M. Milovanovic, B.M. Koprivica, M.D. Veskovic, The Capacitance of Two Wire Line with Rectangular Cross Section, 2nd International Conference on Electrical Systems Design and Technologies, Hammamet Tunisia, November 8-10, 2008, CD Proceedings.
[10] K.L. Kaiser, Transmission Lines, Matching, and Crosstalk, Taylor & Francis Group, Boca Raton, FL., 2006.
[11] A.R. Djordjevic, Elektromagnetika, Akademska misao, Beograd, 2008.