54. IWK

Internationales Wissenschaftliches Kolloquium
International Scientific Colloquium

Information Technology and Electrical Engineering - Devices and Systems, Materials and Technologies for the Future

Faculty of Electrical Engineering and Information Technology

Startseite / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=14089

TECHNISCHE UNIVERSITÄT ILMENAU

Impressum

Herausgeber:	Der Rektor der Technischen Universität Ilmenau Univ.-Prof. Dr. rer. nat. habil. Dr. h. c. Prof. h. c. Peter Scharff
Redaktion:	Referat Marketing Andrea Schneider Fakultät für Elektrotechnik und Informationstechnik Univ.-Prof. Dr.-Ing. Frank Berger
Redaktionsschluss:	17. August 2009

Technische Realisierung (USB-Flash-Ausgabe):
Institut für Medientechnik an der TU Ilmenau
Dipl.-Ing. Christian Weigel
Dipl.-Ing. Helge Drumm

Technische Realisierung (Online-Ausgabe):
Universitätsbibliothek Ilmenau
ilmedia
Postfach 10 05 65
98684 Ilmenau

Verlag:
isle
Verlag ISLE, Betriebsstätte des ISLE e.V.
Werner-von-Siemens-Str. 16
98693 Ilmenau

ISBN (USB-Flash-Ausgabe): 978-3-938843-45-1
ISBN (Druckausgabe der Kurzfassungen): 978-3-938843-44-4

Startseite / Index:
http://www.db-thueringen.de/servlets/DocumentServlet?id=14089

IN ADDITION TO SIMPLE MODELLING OF SOMMERFELD'S INTEGRALS IN THE CASE OF A VERTICAL HERTZ`S DIPOLE PLACED ABOVE A LOSSY HALF-SPACE

Milica Rančić, Predrag Rančić

Faculty of Electronic Engineering, University of Niš, Serbia

ABSTRACT

A review of simple expressions used for modelling of the Sommerfeld's integral kernel (SIK) in the case of a vertical Hertz`s dipole (VHD) placed above a homogenous and isotropic medium, is given in this paper. The results based on numerical calculations will be graphically presented and compared to the ones obtained by other authors that have applied a different evaluation method.

Index Terms – Vertical Hertz`s dipole, Sommerfeld`s integral kernel, Lossy ground

1. INTRODUCTION

Finite specific conductivity of the ground has an especially considerable influence on characteristics of antenna systems, primarily noticeable in their near field. Precise evaluation of the real ground`s influence on characteristics of wire structures located in the air or inside/on the ground, has been a subject of research by a pleiad of scientists for a whole century now. Starting point is usually assigned to the paper by Sommerfeld form 1909, [18], where he formulated the exact expression for the Hertz`s vector potential in the surroundings of the vertical Hertz`s dipole placed close to the boundary surface that divides two half-spaces (air and homogenous, linear and isotropic lossy medium). According to this formulation, the Hertz`s vector potential from the VHD can be expressed as a sum of two components: one that corresponds to a vertical Hertz`s dipole in the free space and the other one that describes the influence of the real ground. The later one can be split up in two parts: the first one that corresponds to the image in the flat mirror of the observed dipole and the second one that is not explicitly expressed by basic mathematical functions, but by a kind of improper integrals referred to as integrals of Sommerfeld`s type. Their integrand presents a product of the spectral reflection coefficient (SRC) and the standard potential kernel in the transformed domain of integration, which makes them unsolvable in a closed form. Consequently, researchers in this field have focused most of their efforts on finding models for as accurate and as simple possible approximate solving of this group of integrals, [1]-[16], [19].

Generally taken, the integrand of this type of integrals is a function of electrical parameters of the upper half-space (air for example) and the lower half-space (for example: homogenous, isotropic lossy half-space), position of the source (for example: VHD) and the position of the field point at which the potential is being calculated. Taking into account all of the above, as well as the range of integration (from zero to infinity), one could realize why solving this type of integrals presents such a complex problem of numerical integration. Consequently, the list of references dedicated to this issue is rather long, and the problem itself, even a hundred years after the pioneer work done by Sommerfeld, still very attractive ([4], [9]-[16], [19]).

Researchers have approached the problem in different ways, applying different methods and developing different approximate forms of the SIK with one single goal: to simply, but satisfyingly accurately solve mentioned integrals. Applied methods could be roughly classified into two groups. The first one involves some kind of, usually time consuming, numerical integration. The second one considers methods that involve some form of approximation of the SIK. These solutions, although approximate, give results that are for practical applications satisfyingly accurate. These methods could be classified as follows:

- Reflection coefficient method - RCM,
- Method of images,
- Method of the SRC approximation.

Schools of electromagnetic at the Faculty of Electronic Engineering in Niš, Serbia, and another one at the Electrotechnical Faculty in Belgrade made a significant contribution in this area of applied electromagnetics in the last thirty years. First steps were done for the purpose of designing SW and USW antennas of Radio Yugoslavia and MW vertical mast antenna with the elevated feeding of Radio Podgorica, Montenegro. All three groups of approximate methods were explored, but what is especially popular in the last ten years is the last mentioned approach – to approximate the spectral reflection coefficient.

A lot of expressions, named models, were developed by the authors in order to substitute the original SRC. These models involve certain number of unknown constants that are determined in a very unique process of approximation. Since these models are not developed under any conditions regarding the type of ground below the analysed antenna structure, they can be characterized as general, which makes their applicability rather wide. Along generality, their very simple form, as well as proven accuracy, makes them very attractive for various applications. A part of these models will be presented further in the text, ([9]-[16]). Two groups of models will be proposed:

- The first one that is based on approximation of the spectral reflection coefficient using a rational function.
- The second one that consists of models that introduce a number of complex images whose position and weight coefficients are determined in the process of approximation. Since the models that will be proposed here introduce two images, they belong in a so-called two-image approximation (TIA) group.

Throughout the paper, these groups of models will be denoted as models of type R and E, respectively.

After describing the approximation procedure in detail, results of their validation will be graphically illustrated in the section "Validation of proposed models".

Based on presented results, corresponding conclusion will be made in the last section, followed by a list of relevant references.

2. THEORY

In the case of a VHD, fig.1, of moment $p_{z0} = I_k(z'_k)\mathrm{d}z'_k$, located in the air at height $z'_k \geq 0$ above a lossy ground, the Hertz`s vector potential only has a component along the z-axis, $\vec{\Pi}_{00} = \Pi_{z00}\,\hat{z}$, i.e.,

$$\Pi_{z00} = \frac{p_{z0}}{4\pi\underline{\sigma}_0}\left[K_0(r_{1k}) + S^v_{00}(r_{2k})\right], \qquad (1)$$

where: $K_0\left(r_{ik}\right) = \exp(-\underline{\gamma}_0 r_{ik})/r_{ik}$, $i = 1,2$, is the standard form of the potential kernel of the source and its image, r_{ik} - distance from the source, i=1, and its image, i=2, to the field point in air, i.e. $r_{1k} = \sqrt{\rho_k^2 + (z - z'_k)^2}$, $r_{2k} = \sqrt{\rho_k^2 + (z + z'_k)^2}$, $\rho_k = \sqrt{(x - x'_k)^2 + (y - y'_k)^2}$. $S^v_{00}(r_{2k})$ is the SIK of the following form,

$$S^v_{00}(r_{2k}) = \int_{\alpha=0}^{\infty} \widetilde{R}_{z10}(\alpha)\,\widetilde{K}_{00}(\alpha, r_{2k})\,\mathrm{d}\alpha\,, \qquad (2)$$

where $\widetilde{R}_{z10}(\alpha)$ and $\widetilde{K}_{00}(\alpha, r_{2k})$ are the SRC and the standard potential kernel in the transformed domain of the variable α, respectively, i.e.:

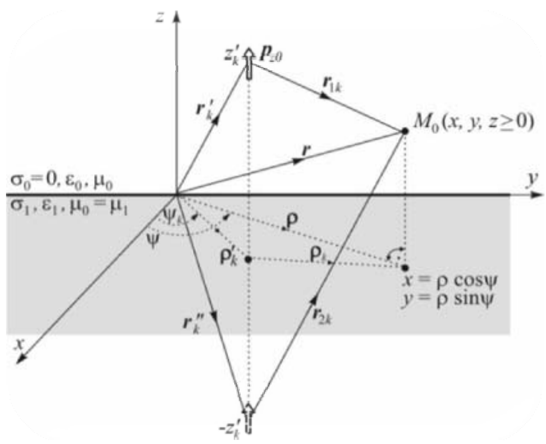

Figure 1 Illustration of the observed VHD above a lossy medium.

$$\widetilde{R}_{z10}(\alpha) = \widetilde{R}_{z10}(u_0) = \frac{\underline{n}^2\,u_0 - u_1}{\underline{n}^2\,u_0 + u_1}\,, \qquad (3)$$

$$\widetilde{K}_{00}(\alpha, r_{2k}) = \frac{e^{-u_0(z+z'_k)}}{u_0}\,\alpha\,J_0(\alpha\rho_k)\,, \qquad (4)$$

$$K_0(r_{ik}) = \int_{\alpha=0}^{\infty} \widetilde{K}_{00}(\alpha, r_{ik})\,\mathrm{d}\alpha\,, \quad i = 1,2\,, \quad u_i^2 = \alpha^2 + \underline{\gamma}_i^2\,,$$

$i = 0,1$ and $J_0(\alpha\rho_k)$ is zero-th order of the first kind Bessel's function.

3. SIK MODELS

Let`s consider a VHD placed in the air at arbitrary height $z'_k \geq 0$ along the z-axis of the Descartes' coordinate system above the LHS treated as a homogenous and isotropic medium of known electrical parameters (σ_1 -specific conductivity, $\varepsilon_1 = \varepsilon_{r1}\,\varepsilon_0$ - permittivity, $\mu_1 = \mu_0$ - permeability, $\underline{\sigma}_1 = \sigma_1 + \mathrm{j}\omega\varepsilon_1 = \mathrm{j}\omega\underline{\varepsilon}_{r1}$, complex specific conductivity, $\underline{\gamma}_i = \alpha_i + \mathrm{j}\beta_i = \mathrm{j}\omega(\underline{\varepsilon}_{ri}\,\mu_i)^{1/2} = (\mathrm{j}\omega\underline{\sigma}_i\,\mu_i)^{1/2}$, $i = 0,1$ - complex propagation constant, $\underline{\varepsilon}_{r1} \approx \varepsilon_{r1} - \mathrm{j}60\sigma_1\lambda_0$ - complex relative permittivity, $\underline{n} = \underline{\gamma}_1/\underline{\gamma}_0 = \sqrt{\underline{\varepsilon}_{r1}}$ - refractive index, and $\omega = 2\pi f$ - angular frequency). The influence of the finite ground conductivity, above which the VHD is considered, that is expressed by the SIK, will be further analyzed in the paper. Based on expressions (2-4) one can understand the complexity of this problem. Since it is the case of integrals that are not solvable in a closed form, one of most common approaches to their solution is approximation of the integrand that presents a product of the SRC, Eq.(3), and the standard potential kernel, Eq.(4). Group of models denoted as models of type R approximate the

SRC using a rational function formed in different ways that will be presented in the following text:

3.1. Models of type R

Model R1: A model R1 is a function of variable u_0 with two unknown complex constants that are obtained matching the SRC at certain characteristic points, [11], [10], [14]. Mathematical form of the model can be generally written as follows:

$$\tilde{R}_{z10}(u_0) \cong B + A_1 \frac{\underline{\gamma}_0}{u_0}. \qquad (5)$$

Application of this model replaces the lossy ground`s influence by an image in the flat mirror of the VHD with a weight coefficient B, and the second continual image with weight coefficient $A_1 \underline{\gamma}_0$ that spreads from the first image position to infinity (fig.2).

Approximation procedure is performed without any limitations as to the values of the electrical parameters of the ground nor as to the position of the observed VHD. Previously mentioned complex constants B and A_1 are obtained matching expressions (3) and (5) at points $u_0 \to \infty$ and $u_0 = \underline{\gamma}_0$, respectively, which yields:

$$B = R_\infty, \qquad (6a)$$

$$A_1 = R_0 - R_\infty, \qquad (6b)$$

where

$$R_\infty = \frac{\underline{n}^2 - 1}{\underline{n}^2 + 1}, \text{ and} \qquad (6c)$$

$$R_0 = \frac{\underline{n} - 1}{\underline{n} + 1}. \qquad (6d)$$

Substituting (5) into (2), and adopting obtained expressions (6a-d), the following model of the SIK is obtained:

$$S_{00}^v(r_{2k}) \cong B\, K_0(r_{2k}) + A_1\, \underline{\gamma}_0 L(r_{2k}) = \\ = R_\infty K_0(r_{2k}) + (R_0 - R_\infty)\, \underline{\gamma}_0 L(r_{2k}), \qquad (7)$$

where the integral kernel $L(r_{2k})$ is

$$L(r_{2k}) = \int_{v=z+z'_k}^{\infty} \frac{e^{-\underline{\gamma}_0 \sqrt{\rho_k'^2 + v^2}}}{\sqrt{\rho_k'^2 + v^2}}\, \mathrm{d}v \cong \\ \cong -\int_{v=0}^{z+z'_k} K_0(r_{2kv})\, \mathrm{d}v - \\ - \frac{\pi}{2}[N_0(\beta_0 \rho'_k) + \mathrm{j}\, J_0(\beta_0 \rho'_k)]$$

where $N_0(\beta_0 \rho'_k)$ - Neumann`s function of the first kind and zero-th order.

Model R2: The complex constant A_1 in Eq.(5) could be also determined matching the SRC at some other characteristic points. A number of models is develop-

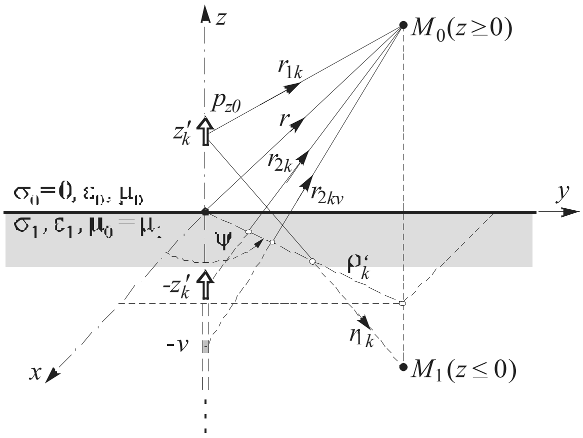

Figure 2 Illustration of the geometry of the VHD placed above a lossy half-space and position of images for models of type R.

ed this way more or less improving the accuracy of the approximation but still keeping it simple and with an unlimited application. For example, it is possible to choose matching points so that they correspond to zeros of the SRC, $\tilde{R}_{z10}(u_0 = u_B) = 0$, i.e. a so-called Brewster`s point, $u_B = \underline{\gamma}_0 / \sqrt{\underline{n}^2 + 1}$, as it has been done in [9]. This way, the following is obtained for the constant A_1:

$$A_1 = -R_\infty / \sqrt{\underline{n}^2 + 1}. \qquad (8)$$

Adopting (8) and substituting it into (5), and then the obtained SRC model back into (2), the following SIK model is obtained:

$$S_{00}^v(r_{2k}) \cong B\, K_0(r_{2k}) + A_1\, \underline{\gamma}_0 L(r_{2k}) \\ = R_\infty K_0(r_{2k}) - \frac{R_\infty}{\sqrt{\underline{n}^2 + 1}}\, \underline{\gamma}_0 L(r_{2k}). \qquad (9)$$

3.2. Models of type E

Besides previously proposed models that are in a form of a rational function, another approach, widely analysed in the literature ([1], [4]-[7]), considered the approximation of the SRC using an exponential function or an exponential series. First attempts were done towards realizing such approximations that introduce one or more images at complex distances. This resulted in many approximate solutions whose accuracy was dependable on the number of images, the refractive index, the position of the dipole and the position of the point where the field was calculated. The same idea laid at the basis of the so-called *image theory approximation* that considers introduction of a system of images with complex weight coefficients positioned at complex distances that are determined in the process of approximation. This means that the second continual image in the case of models of type R is replaced by one or more images at complex distances (fig.3). Application of these models and development of new ones of similar form, is especially popular in the recent years in the area of modelling the EM field of atmospheric discharge, ([17], [19]).

A few variants of models that consider an approximation of the SRC using a function consisting of one unknown constant term and an exponential part with another two unknown constants will be proposed in this section. The approximation procedure is rather simple, as the one from the previous chapter, the models are also very simple and their application gives highly accurate results in both the near and the far-field of the dipole. These conclusions were made based on numerous analyses performed by the authors, some of which were published in [10], [13], [14].

If the SRC is assumed in an approximate form of an exponential function with a constant term, a so-called – TIA (*two-image approximation*), than the general model is mathematically expressed as:

$$\widetilde{R}_{z10}(u_0) \cong B + A_1\, e^{-(u_0-u_{0c})\underline{d}}\,, \tag{10}$$

where B, A_1 and $\underline{d}$ - unknown complex constants and u_{0c} - characteristic value of variable u_0 in the range of integration. When (10) is substituted into (2), the following general TIA model for calculating the SIK is obtained:

$$S_{00}^{v}(r_{2k}) \cong B\,K_0(r_{2k}) + A\,K_0(r_{3k})\,, \tag{11}$$

where $r_{3k} = \sqrt{\rho_k'^2 + (z + z_k' + \underline{d})^2}$, presents the distance between the second image and the observed point M_0 in the field and $A = A_1 \exp(u_{0c}\,\underline{d})$.

Different possibilities of SIK modelling formed choosing different values for the u_{0c} will be analysed further in the text.

<u>Model E1</u>: Adopting $u_{0c} = \underline{\gamma}_0$ and matching the expression (10) at $u_0 = \underline{\gamma}_0$ and $u_0 \to \infty$, and the first derivative of the same expression at $u_0 = \underline{\gamma}_0$, the following values for the unknown complex constants in (10) are obtained:

$$B = R_\infty\,, \tag{12a}$$

$$A_1 = R_0 - R_\infty\,, \tag{12b}$$

$$\underline{d} = (1 + \underline{n}^{-2})/\underline{\gamma}_0\,. \tag{12c}$$

Substituting (12a-c) into (10), and then into (11), the following TIA form for the SIK is obtained:

$$S_{00}^{v}(r_{2k}) \cong R_\infty\,K_0(r_{2k}) + (R_0 - R_\infty)\,e^{\underline{\gamma}_0 d}\,K_0(r_{3k})\,. \tag{13}$$

When calculating the SIK using the expression (13), the absolute value of the complex distance $d = |\underline{d}|$ can be used instead of its complex value $\underline{d}$.

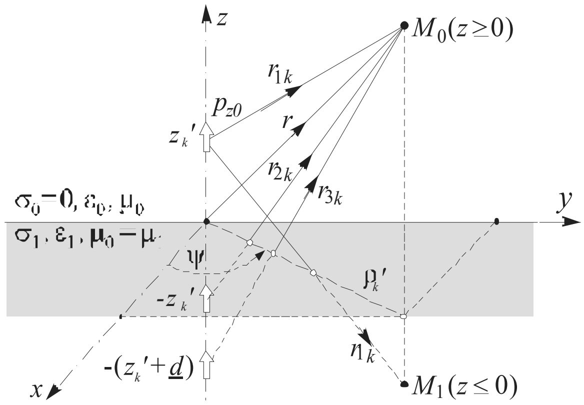

Figure 3 Illustration of the geometry of the VHD placed above a lossy half-space and position of images for models of type E.

This solution was obtained based on numerical experiments that have shown that this modified model gives results of highest accuracy in both the near and the far-field of the VHD, see [10]. In further analysis in this paper, only the modification of the E1 model will be considered.

<u>Model E2</u>: This variant of models of type E is formed adopting $u_{0c} = 0$ and matching the (10) at $u_0 = 0$ and $u_0 \to \infty$, and its first derivative at $u_0 = 0$. This way, new values for the constants B, A_1 and $\underline{d}$ are obtained and SIK model becomes:

$$S_{00}^{v}(r_{2k}) \cong B\,K_0(r_{2k}) + A_1\,K_0(r_{3k})\,, \tag{14}$$

where:

$$B = R_\infty\,, \tag{15a}$$

$$A_1 = -(1 + R_\infty)\,, \tag{15b}$$

$$\underline{d} = \frac{1}{\underline{\gamma}_0\,R_\infty}\sqrt{\underline{n}^2 - 1}\,. \tag{15c}$$

The same solution has been already proposed in [19] but those authors have developed it using a different approach. Also, a decade later, other authors from [20] obtained the model of the same form in that way that they firstly developed the SRC into the Taylor`s series around $u_{0c} = 0$, then adopted only the first two terms of the series, then reorganized the obtained solution and finally expressing it in an exponential form. As it will be further shown, the proposed E2 model gives results of low accuracy except in the case of a highly dielectric medium, although it is widely applied in the literature.

4. VALIDATION OF PROPOSED MODELS

The validity of models proposed previously will be numerically verified in this section. A great number of numerical experiments gave results that will be, in order to certify the accuracy of SIK calculation, compared to the highly accurate ones (relative error around 10^{-6}) obtained by a software package from

[8]. In the following text, comparison of obtained results will be graphically illustrated, presenting the normalized modulus $S_{00}^{v}(r_{2k})/\beta_0$, for $z=0$, taking the position of the VHD, z'_k/λ_0, as a parameter in all calculations. Calculations were done for different sets of electrical parameters of the lossy ground (relative permittivity of the lossy ground $\varepsilon_{r1}\in[1,81]$ and its normalized specific conductivity $\sigma_1\lambda_0\in[0,\infty)$) in order to evaluate the validity range of proposed models.

Firstly, the results obtained applying models denoted as R1 and R2 will be presented in comparison to highly accurate ones from [8]. In fig.4, the normalized modulus of the SIK as a function of the radial distance $\beta_0\rho'_k$ of the point M_0 placed on the boundary surface between two mediums ($z=0$). The position of the VHD is parameter whose values are varied in the range of: $z'_k/\lambda_0 = 0, 0.05, 0.10, 0.15, 0.20$ i 0.25. Relative permittivity of the lossy ground is $\varepsilon_{r1}=2$, and figures correspond to the following values of the imaginary part of the complex relative permittivity: $\varepsilon_{i1}=10^{-3}$, 2 and 600.

Results of the same analysis are presented in fig.5, but setting the relative permittivity to a higher value, $\varepsilon_{r1}=10$. Each figure corresponds to a different value of the imaginary part of the complex relative permittivity: $\varepsilon_{i1}=10^{-3}$, 10 and 600, keeping the same values of all the above mentioned parameters from the previous example.

Based on presented results of analysis of models of type R, one could conclude that both models are, aside from simple, satisfyingly accurate in a wide range of values of the electrical parameters of the LHS and regardless of the VHD position. Also, they have general character in a sense that their application is not constrained by anything since they were all developed without introducing any limitations, which is not the case with some other models proposed in the literature.

Fig.6, present results of similar analysis but when models of type E are employed, i.e. models E1 and E2. For the sake of establishing the accuracy of these models, the results from [8] were also given in figures. Again, the position of the VHD is a variable parameter in all examples taking values from the range of: $z'_k/\lambda_0 = 0, 0.05, 0.25, 0.50$ and 1.00. Relative permittivity of the lossy ground is $\varepsilon_{r1}=2$, and figures correspond to the following values of the imaginary part of the complex relative permittivity: $\varepsilon_{i1}=10^{-3}$, 2 and 600. Corresponding results obtained when the relative permittivity is set to $\varepsilon_{r1}=10$, and values of the imaginary part of the complex relative permittivity to $\varepsilon_{i1}=10^{-3}$, 10 and 600, are shown in fig.7, respectively.

As it can be seen from presented diagrams, accordance of the results obtained applying the model E1 with those from [8] is much better than in the case of results that gives employment of the model E2. Drastic error of SIK evaluation is especially noticeable when the value of the LHS`s specific conductivity is increased. Apart from the proven accuracy of the model E1 regardless of the type of the LHS or VHD

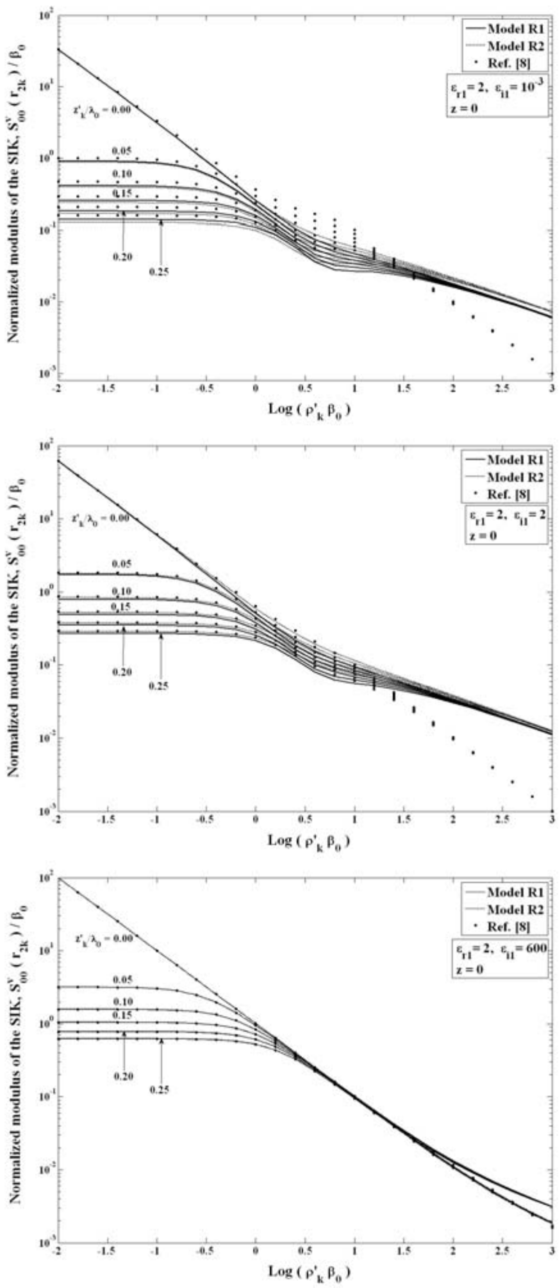

Figure 4 Modulus of the normalized SIK, at plane $z=0$, versus normalized radial distance. VHD position, z'_k, and complex electrical permittivity, $\underline{\varepsilon}_{r1}$, are parameters. Comparison of three models.

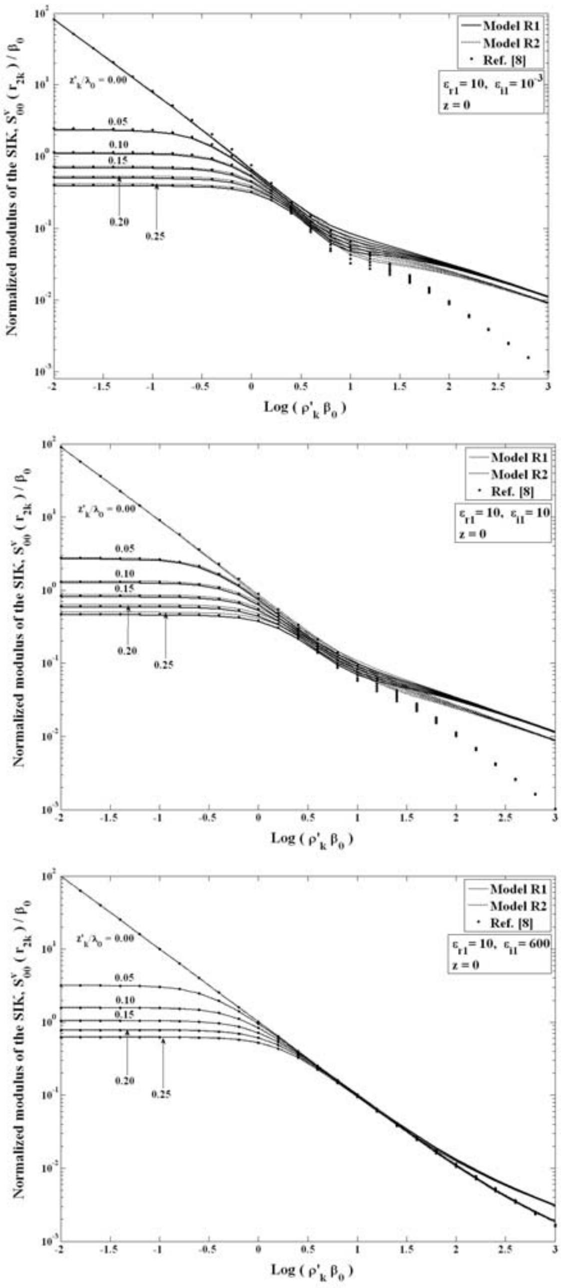

Figure 5 Modulus of the normalized SIK, at plane $z = 0$, versus normalized radial distance. VHD position, z'_k, and complex electrical permittivity, $\underline{\varepsilon}_{r1}$, are parameters. Comparison of three models.

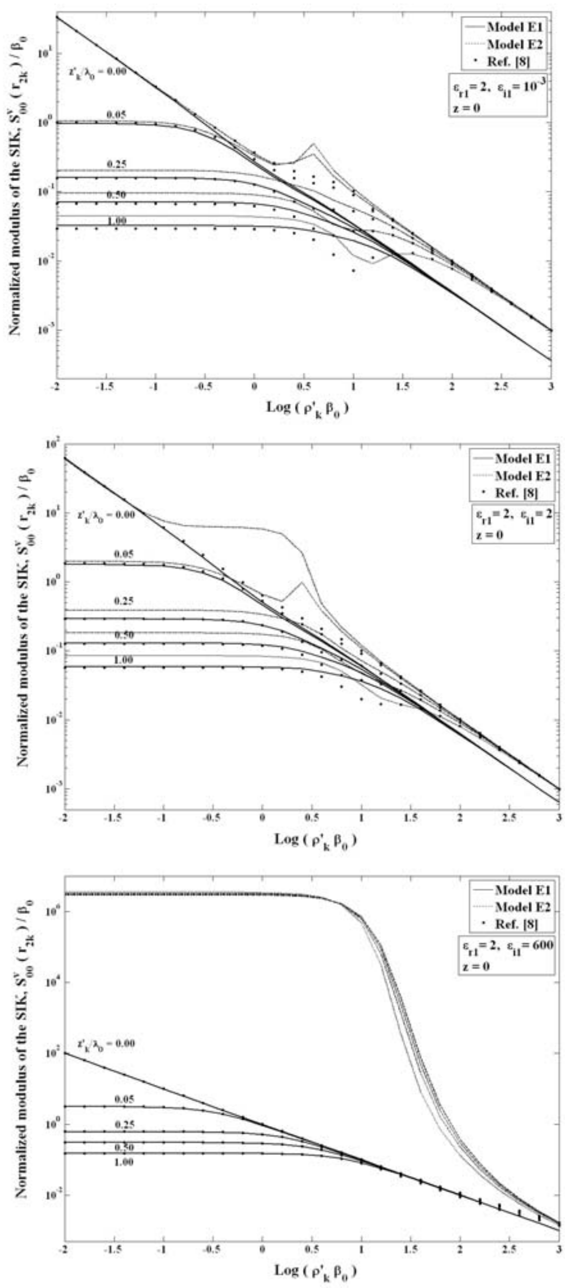

Figure 6 Modulus of the normalized SIK, at plane $z = 0$, versus normalized radial distance. VHD position, z'_k, and complex electrical permittivity, $\underline{\varepsilon}_{r1}$, are parameters. Comparison of three models

position, it is also noticeable that the accuracy of SIK calculations is now, using the same model significant not only in the surroundings of the VHD, but also in its far-field.

5. CONCLUSION

A procedure of deriving a certain number of models developed for the purpose of approximate calculation of one type of Sommerfeld`s integrals was presented in detail in this paper. Since these models are not developed under any conditions regarding the type of ground below the analysed VHD, they can be characterized as general, which makes their applicability rather wide. Along generality, their very simple form, as well as proven accuracy, makes them very attractive for various applications.

Proposed models are classified in two groups:

- Models of type R: based on approximation of the SRC using a rational function.
- Models of type E: based on approximation of the SRC using a so-called two-image approximation.

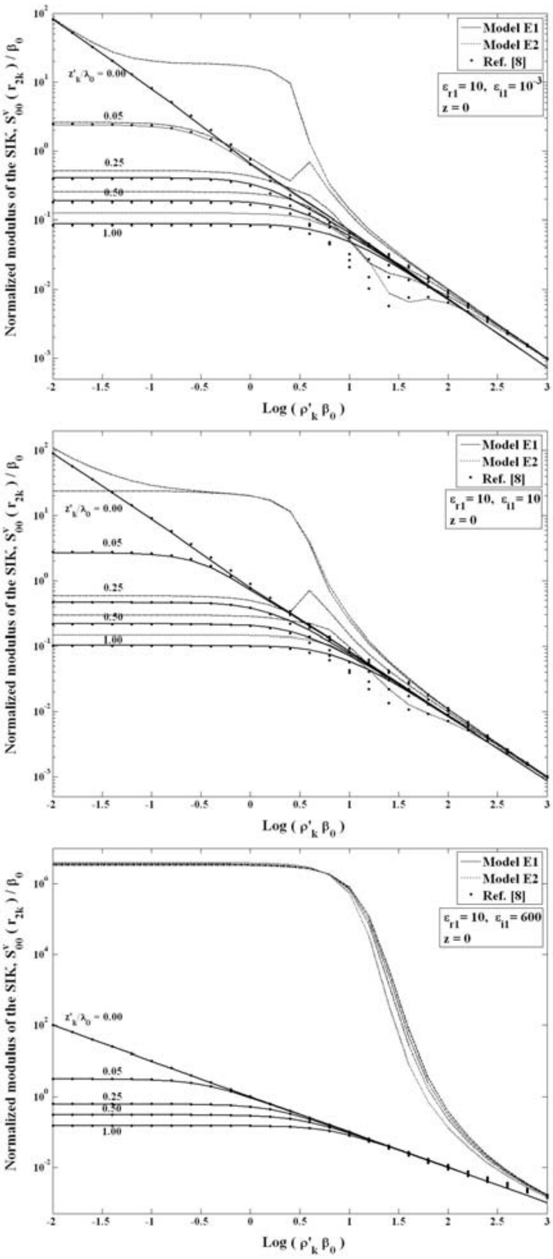

Figure 7 Modulus of the normalized SIK, at plane $z = 0$, versus normalized radial distance. VHD position, z'_k, and complex electrical permittivity, $\underline{\varepsilon}_{r1}$, are parameters. Comparison of three models.

After thorough analysis, the model denoted as E1 stands out in every sense, along with the R1 and R2. Poor accuracy of model E3 was confirmed against highly accurate results from [8]. Based on all presented numerical results one can conclude that both models of type R and especially the model E1 could be successfully used for the purpose of determination of integral characteristics of different wire structures placed above the LHS. Model E1 could be also applied in the field of antenna modelling of atmospheric discharge.

6. REFERENCES

[1] P.R.Bannister, "Summary of Image Theory Expressions for the Quasi-Static Fields of Antennas at or above Earth's Surface", Proc. of the IEEE, Vol.67, No.7, pp.1001-1008, July 1979.

[2] A.Banos, Dipole Radiation in the Presence of a Conducting Half-space, Pergamon Press, NY, 1966.

[3] D.C.Chang, and J.R.Wait, "Theory of a Vertical Tubular Antennas Located above a Conducting Half-space", IEEE Trans. on AP, Vol. AP-18, pp.182-188, 1970.

[4] A.R.Djordjevic, M.B.Bazdar, V.V.Petrovic, D.I.Olcan, T.K.Sarkar, and R.F.Harrington, AWAS for Windows, Version 2.0, Analysis of Wire Antennas and Scatterers – Software and User's Manual, A.H., 2002.

[5] I.V.Lindell, and E.Alanen, "Exact Image Theory for the Sommerfeld Half-Space Problem, Part II: Vertical Electric Dipole", IEEE Trans. on AP, Vol.32, No.8, pp.841-847, August 1984.

[6] S.F.Mahmoud, "Image Theory for Electric Dipoles above a Conducting Anisotropic Earth", IEEE Trans. on AP, Vol. AP-32, No.7, pp. 679-683, 1984.

[7] V.V. Petrović, "Analysis of Wire Antennas in the Presence of Real Ground Using the Method of Images", PhD Thesis, Electrotechnical Faculty of Belgrade, 1996.

[8] V.V.Petrović, Program SOMMINT, Private communications, Electrotechnical Faculty of Belgrade, 2005.

[9] M.P.Rančić, and P.D.Rančić, "An Approximate Model for Analyzing Vertical Antennas Above a Ground Plane", 12th Telecom. Forum TELFOR'04, Nov. 23-25, Beograd, Serbia, 2004. http://www.telfor.org.yu/telfor2004/radovi/PEL-9-28.pdf

[10] M.P.Rančić, and P.D.Rančić, "Vertical Hertz`s Dipole above a Lossy Half-Space: Review of Different Models for Sommerfeld`s Integral", Conf. Proc. XLIX YU Conf. ETRAN'05, Vol. 2, Budva, Serbia, pp. 272-275, June 5-10, 2005. (in Serbian)

[11] M.P.Rančić, and P.D.Rančić, "Vertical Linear Antennas in the Presence of a Lossy Half-Space: An Improved Approximate Model", Int. J. of Electr. and Com - AEÜ, Vol. 60, No. 5, pp. 376-386, May, 2006.

[12] M.P.Rančić, and P.D.Rančić, "EM field of Vertical Hertz's Dipole (VHD) Placed above a Lossy Half-Space: Approximate Expression for the Sommerfeld's Integral", CD Proc. of 51st IWK, Poster session 2 - Wireless Communication System and Sensor Technologies, N°4, Ilmenau, Germany, September 11-15, 2006.

[13] M.P.Rančić, and P.D.Rančić, "Vertical Dipole Antenna above a Lossy Half-Space: Efficient and Accurate Two-Image Approximation for the Sommerfeld's Integral", CD Proc. of EuCAP 2006, Session 2PP1A - Antenna Theory and Modelling, (Ref. N^{o}362128), Nice, France, November 6-10, 2006.
[14] M.P.Rančić, "In the Addition to Modelling of the Real Ground Influence on Characteristics on Vertical Wire Structures in the Air", Master Thesis, Faculty of Electronic Engineering, University of Niš, Niš, Serbia, December 2008.
[15] P.D.Rančić, and M.I.Kitanović, "A New Model for Analysis of Vertical Asymmetrical Linear Antenna above a Lossy Half-Space", Int. J. Electron. Commun. AEÜ, Vol. 51, No. 3, pp.155-162, 1997.
[16] P.D.Rančić, "H.F. Yagi Antennas Above a Conducting Half-Space", FACTA Universitatis, Series: Electronics and Energetics, Vol. 14, No. 2, pp.145-166, 2001. http://factaee.elfak.ni.ac.yu/fu2k12/fu01.html.
[17] P.D.Rančić, and V.Javor, "Vertical Monopole Antenna Model of the Lightning Discharge Current Two Image Approximation of Sommerfeld`s Integral Kernel", 16th COMPUMAG'07 Conf., CD Proc. pp. 1197-1198, Aachen, Germany, June 24-28, 2007.
[18] R.M.Shubair, and Y.L.Chow, "A Simple and Accurate Complex Image Interpolation of Vertical Antennas Present in Contiguous Dielectric Half-Spaces", IEEE Trans. on AP, Vol.41, No.6, pp.806-812, June 1993.
[19] A.N.Sommerfeld, "Über die ausbreitung der wellen in der drahtloslen telegraphie", Ann. der Physik, Vol. 28, pp.665-736, 1909.
[20] C.Yang, and B.Zhou, "Calculation Methods of Electromagnetic Fields Very Close to Lightning", IEEE Trans. on EMC, Vol.46, No.1, pp.134-141, February 2004.