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Evolutionary Modelling in Economics: A Survey of Methods and Building Blocks

by

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Abstract

In this paper we present an overview of methods and components of formal economic models employing evolutionary approaches. This compromises two levels: (1) techniques of evolutionary modelling, including multi-agent modelling, evolutionary algorithms and evolutionary game theory; (2) building blocks or components of formal models classified into core processes and features of evolutionary systems - diversity, innovation and selection - and additional elements, such as bounded rationality, diffusion, path dependency and lock-in, co-evolutionary dynamics, multilevel and group selection, and evolutionary growth. We focus our attention on the characteristics of models and techniques and their underlying assumptions.

Key words: bounded rationality, evolutionary algorithms, evolutionary game theory, evolutionary growth, innovation, multilevel evolution, neo-Schumpeterian models.

JEL codes: B52, C60, C73

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1. Introduction

Bibliometric evidence suggests that evolutionary economics is dominated by appreciative theorizing, while formal and empirical analysis is less common (Silva and Teixeira, 2006). Nevertheless, there are fairly many formal contributions to evolutionary economics. They vary markedly with respect to the economic phenomena studied and techniques applied. Moreover, they lack the methodological consistency necessary for a systematic comparison or validity test. This may be confusing for researchers who try to grasp the principles of evolutionary modelling. A comparative study that evaluates a variety of techniques is missing. To fill this gap, the current paper presents an overview of methods and components of formal economic models employing evolutionary approaches.

The economy is generally recognized as a complex, hierarchical structure compromising various levels and subsystems, which are linked together through strong feedback mechanisms (Potts, 2000). Variation and selection processes occurring in any of these subsystems affect changes in the total environment. The global economy as an adaptive nonlinear network is a difficult subject for traditional formal modelling (Holland, 1988). The usual mathematical tools, as applied to economic analysis, exploit linearity, fixed points and convergence. These instruments are usually insufficient to deal with complexity of economic systems, path dependency, diversity and novelty. Evolutionary economics recognizes that the economy operates far from optimum (a global attractor) and that directions of economic changes depend on interactions of many elements that can act in parallel. It provides formal tools to capture these features.

It is possible to identify distinct developments within evolutionary modelling, namely evolutionary game theory (Friedman, 1991; Weibull, 1995; Samuelson, 1997; Fudenberg and Levine, 1999; Gintis, 2000), evolutionary computation techniques (Fogel, 2000; Eiben and Smith, 2003), and multi-agent based modelling (Wooldridge, 2002; Weiss, 1999, Tesfatsion and Judd, 2006). Various mathematical techniques are used, such as non-linear dynamic analysis (difference or differential equations), stochastic processes and evolutionary algorithms. Evolutionary game theory is an appropriate method for carrying out analysis at the most aggregate level. Dynamics are focused here on selection and are predominantly formalized with replicator dynamics. Other stochastic and deterministic selection equations are available, such as imitation, best response, mutator and adaptive dynamics, but they are rarely used in economic applications.

To encompass principles of disaggregation and micro-foundations, multi-agent simulation techniques can be adopted. These allow for modelling large numbers of boundedly rational agents, capable to engage in interactions with other agents and the environment. During the last two decades, multi-agent approaches have been increasingly used to model dynamic, decentralized economic systems. We review recent contributions to multi-agent modelling, ranging from models conforming to stylized facts ('macro regularities'), through coevolutionary models of supply and demand, to 'history friendly' models.

Evolutionary computation offers concrete techniques, such as genetic algorithms, learning classifier systems and genetic programming to study adaptive learning. Here, selection and variation operators govern the changes in the frequencies of individuals hosting distinct strategies in a series of succeeding generations (Riechmann, 2001). Notably, single individual does not change over time, but instead a population of individuals evolves due to a process of selective replication. Evolutionary algorithms may also be employed to deal with multi-agent systems. In this case, each agent is endowed with a set of decision rules, while the algorithm evolves the optimal rule for each individual in response to a (changing) environment.

Various formalisations of evolutionary-economic mechanisms have been proposed so far (Silverberg, 1988; Witt, 1993; Silverberg, 1997; Dosi and Winter, 2000; Kwasnicki, 2001, 2003; van den Bergh, 2004; Windrum, 2004; Dopfer, 2005; Hanusch and Pyka, 2007). Nevertheless, a general agreement on a categorization of building blocks has not yet been achieved. Here, we propose a classification of building blocks of evolutionary-economic models into the core elements diversity, innovation and selection, and additional features, namely bounded rationality, diffusion, path dependency and lock-in, co-evolutionary dynamics, multilevel and group selection, and evolutionary growth. Some of these building blocks or components have received much attention in formal models while others are less common. In the conclusions of the paper, we indicate which of the components show some convergence to a standard approach and which are characterised by heterogeneity.

In this paper we present a survey of formal modelling in evolutionary economics. We emphasize the assumptions of models and techniques without trying to be exhaustive in terms of applications and without systematically giving attention to specific insights of such applications. The organization of the reminder of the paper is as follows. In section 2, we discuss the main evolutionary modelling techniques. Next, in section 3 we examine the various components of evolutionary-economic models. Section 4 presents conclusions.

2. Evolutionary modeling techniques

2.1 Multi-agent simulations

Multi-agent simulations enable studying coordination processes, self-organization, distributed processing, micro diversity and innovation through recombination, all in a way that is far beyond the capabilities of any representative agent model (Potts, 2000). In early studies, the approach was employed to model social processes (Schelling, 1978; Axelrod, 1984; Arthur 1984). The most ambitious in this sense has been Epstein and Axtell's (1996) multi-agent 'Sugarscape' model, which integrates elements of demography, sociology, psychology and economics. The model runs show how from spatio-temporal interactions of agents a number of social phenomena emerge: the transmission of culture, the rise of conflicts, the spread of a disease, and migration. In economics the method of multi-agent simulations became more widely known through the work of Andersen et al. (1988) and Holland and Miller (1991). These authors proposed to view the economy as a complex, dynamic, and adaptive system with a large number of autonomous agents. In this respect, multi-agent simulations offer a unique tool for addressing interactions of heterogeneous, boundedly rational agents characterised by learning, increasing returns and path dependence.

Formally, agents can be defined as computational entities situated in some environment, capable of undertaking flexible autonomous actions with the objective of meeting their goals (Wooldridge, 1999). In particular, intelligent agents are characterised as capable of perceiving the environment and responding to it; of exhibiting goal-oriented behaviour, and of interacting with other agents. These interactions can take place indirectly through the environment in which agents are embedded, or in direct communication among agents (Weiss, 1999). Agents' interactions as well as feedback from aggregate (macro) to disaggregate (micro) phenomena are the sources of nonlinear dynamics.

There are no standard techniques for constructing and analysing agent-based models (see Epstein, 2007; Windrum et al., 2007). Weiss (1999) provides an overview of general attributes of a multi-agent system, listed in Table 1. The basic structure of such a system involves specifying: time, the number of agents, micro states (actions) that can be endogenously modified by agents, micro parameters containing information about agents' behavioural and technological characteristics, time independent variables governing the technological and institutional setup, the structure of interactions and information flows among agents, micro decision rules, and aggregate macro variables (Pyka and Fagiolo, 2005).

During the last two decades, the multi-agent approach has become a common way of modelling dynamic, decentralized economies. A number of models generating patterns consistent with empirical phenomena have been proposed (e.g., Gabriele, 2002; Fagiolo et al., 2004; Dosi et al., 2006). For instance, Fagiolo et al. (2004) develop an agent-based model whose simulations confirm stylised facts of product and labour markets, such as Beveridge, Wage and Okun's curve. The Beveridge curve predicts a negative relationship between rates of vacancies and unemployment, the Wage curve posits a negative correlation between levels of real wages and unemployment, and the Okun curve stipulates more than a proportional increase in a real GDP for every percentage-point of reduction in the unemployment rate. These macro regularities emerge in the model as an outcome of micro interactions among heterogeneous individuals (producers and workers). Agents' interactions underlie vacancy and wage setting mechanisms, matching and barging processes, demand and prices formation.

The competitiveness of firms, which depends on firm-specific labour productivities, forms a basis for selection to operate.

Table 1. An overview of attributes of multi-agent systems

	Attribute	*Range*
Agents	Number	Two or more
	Goals	Contradicting or complementary
	Architecture	Reactive (simple mapping of a signal into action) or deliberate response
	Abilities	Simple or advanced
Interactions	Frequency	Low or high
	Character	From pure observation over signal passing to sophisticated exchange of information (dialogue and negotiations)
	Persistence	Short term or long term
	Pattern (flow of data and control)	Decentralised or hierarchical
	Connections (structure of interactions)	Fixed or changeable
	Purpose	Competitive or cooperative
Environment	Changes in the environment	Foreseeable or unforeseeable
	Information	Uncertainty or full knowledge
	Stability	Static, dynamic, endogenous environment
	Availability of resources	Restricted or unrestricted

Source: adapted from Weiss (1999, p.4).

Recently, multi-agent modelling has become a formal tool in a new generation of evolutionary-economic models, know as 'history friendly' models (e.g., Malerba et al., 1999; Malerba and Orsenigo, 2001; Elliason and Taymez, 2000; Eliasson et al., 2004). History friendly models aim to capture qualitative theories about mechanisms and factors driving industry evolution, technological advances or institutional change (Malerba and Orsenigo, 2001). They rely on extensive analysis of empirical data and patterns of development in specific industries. For instance, Malerba et al. (1999, 2005) develop a multi-agent model of the evolution of the computer industry, and Malerba and Orsenigo (2001) of the pharmaceutical and biotechnology industry. In history friendly models, empirical data are used to calibrate parameters and behavioural rules while the evolvability of the resulting system is compared to the historical patterns of industry developments. Consequently, such models may be regarded as a method of validating results. Alternative approaches to empirical validation of multi-agent models include the indirect calibration approach and the Werker-Brenner approach (for a discussion on the strengths and weaknesses of each approach see Windrum et al., 2007).

Multi-agent models have been applied to modelling a wide range of topics: agent learning (Arthur, 1991; Ishibuchi et al., 2001; Klos and Nooteboom, 2001), the evolution of norms, and conventions (Axelrod, 1997; Thebaud and Locatelli, 2001; Hodgson and Knudsen, 2004), financial markets (Arthur et al., 1996; Caldarelli et al., 1998; LeBaron, 2001; Levy et al., 2000), diffusion of innovations and industry dynamics (Aversi et al., 1997; Gilbert et al., 2001; Windrum and Birchenhall, 1998, 2005; Saint-Jean, 2006; Schwoon, 2006), land use and environmental management (Paker et al., 2003), labour economics (Tassier and Menczer, 2001; Gabriele, 2002; Fagiolo et al., 2004), and environmental policies (Janssen and Jager, 2002; Carrillo-Hermosilla, 2006). Multi-agent models have been also applied to various markets, including the textile market (Brannon et al., 1997), fish market (Kirman and Vriend, 2001), wholesale electricity market (Bower and Bunn, 2001), and agricultural practices in a developing country (Lansing and Miller, 2004). For a more extensive discussion of multi-agent modelling, see Tesfatsion, (2001a), Axelrod (2003), Windrum (2004), Dawid (2006), Vriend (2006), and Epstein (2007). We will discuss aspects of some of the aforementioned models in greater detail later on in the paper.

2.2 Evolutionary computation

Evolutionary computation offers algorithms based on the mechanisms of natural selection and genetics, such as genetic algorithms (Back, 1996; Mitchell, 1996; Goldberg, 1989), genetic programming (Banzhalf et al., 1989), evolutionary programming (Back, 1996), learning classifier systems (Lazi et al., 1998; Bull, 2004) and evolutionary strategies (Beyer and Schwefel, 2002).[1] These techniques are increasingly applied to evolutionary-economic modeling (see Arifovic, 2000; Dawid, 1999). In evolutionary computation models[2], individuals do not change over time but a population evolves due to selective replication and variation processes. Riechmann (1999) has proposed to interpret selective replication and variation operators in terms of socio-economic interactions, namely as learning by imitation (selective replication), learning by communication (crossover) and learning by experimentation (mutation).

Central to all techniques in evolutionary computation is the search process for better solutions. The process involves generating new options with mutation and recombination operators. A mutation operator is always stochastic. It acts by changing a value of a random characteristic of an individual with some positive probability. Recombination (crossover) merges information (characteristics) from two parent codes into an offspring code. The important difference between mutation and recombination is that mutation is a unary operator; it requires one object as an input, while crossover is typically (i.e. in biology) a binary operator applied to two objects (parents). In addition, the possibility of recombination with

[1] Since evolutionary programming and evolutionary strategies are rare in economic applications, we do not discuss them further here.
[2] Excluding multi-agent models, which employ evolutionary algorithms.

more than two parents is also possible in a socioeconomic or technological context (see Eiben, 2000). This creates a very wide spectrum of innovation outcomes.

The process of selective replication transfers a set of individuals hosting distinct strategies from one generation to the next. In evolutionary algorithms, selection consists of two processes: parent and survival selection (Eiben and Smith, 2003). The role of parent selection is to stimulate better individuals to become parents of the next generation. Parent selection is typically probabilistic: better quality individuals have a higher chance to reproduce. For instance, parents may be selected in proportion to their relative fitness (a quality measure assigned to each solution). The approach is also known as roulette wheel selection; the chance of selecting a particular parent may be envisaged as spinning a roulette wheel, where the size of each pocket is proportional to the parent's fitness. Other types of selection mechanisms are linear sorting and tournament selection. According to the first, an algorithm sorts all individuals based on their fitness and then assigns a selection probability to each individual according to its rank. Alternatively, in tournament selection an algorithm chooses randomly two parents and creates an offspring of the fitter parent. Subsequently, parents are returned to the initial population. The process is repeated n times to create a succeeding population of n offsprings.

The second type of selection is survival selection (often deterministic). Here, offspring compete for a place in the next generation based on their fitness. Two options may be distinguished: a new population can be constructed from a set of parents and offspring, referred to as fitness bias selection, or solely from the offspring population known as age bias selection.

It is worth mentioning that evolutionary algorithms may be employed to model individual learning in multi-agent systems. In such models, each agent observes a representation of the current state and undertakes an action according to a selected decision rule (from a finite set of rules). After all agents undertake their decisions, payoffs are revealed, and the effectiveness of rules is evaluated. The most effective rules have a higher chance to be selected in the future. Over time an evolutionary algorithm evolves the optimal rule or set of rules in response to a changing environment.

Genetic algorithms

Holland (1975, 1980, 1992), inspired by genetic processes, developed the Genetic Algorithm (GA) method. Initially, it was regarded as a means of studying adaptive behaviour. A simple genetic algorithm is characterised by a population of binary strings (of equal length), i.e. sequences of 0s and 1s, like $\{0,1,0,1,1\}$. Alternatively, a string can be presented as a sequence of real values. A GA operates as follows: from an initial parent population some strings are chosen with a probability proportionate to their fitness. Offspring are created by applying variation operators to the selected parents: mutation 'flips' the value of any bit-string with some positive probability, while recombination (crossover) switches sequences of consecutive bits between two parents' strings. A new generation is then created from parent and offspring populations (or only from the offspring population). The process is repeated a finite number of times until convergence occurs or some (other) stopping rule is satisfied.

Arifovic (1994) proposes an augmented GA with an additional election operator. This operator tests newly generated offspring before they are permitted to become members of the succeeding population. It compares the fitness of a potential offspring with the fitness values of its parents: if the offspring has the higher fitness than one or both of its parents, it replaces the parent with the lowest fitness; otherwise both parents go to the next generation.

Holland (1975) suggested that the analysis of the GA could be simplified with the use of the Schemata Theorem. The theorem provides a formula for assessing how a number of 'instances' of certain schemas (strings) in a population varies over time as a result of fitness proportional selection, one-point crossover and point mutation (Eiben and Smith, 2003). A schema is a string built of elements {0,1,#}. The equivalence class is a set of strings that match a schema wherever it has 0s or 1s, and which can take any symbol 0 or 1 where the schema has "#" (Sargent, 1993; Birchenhall et al., 1997). An example of a schema could be {0,1,0,#,0,1} with the corresponding equivalence class [{0,1,0,1,0,1},{0,1,0,0,0,1}]. The theorem uses specific terminology: order - the number of defined positions i.e. 0s or 1s, for instance {0,1,0,#,0,1} has order 5; length - a distance between the first and the last defined position: in our example it is 6-1=5, and schema's fitness – the average fitness across all strings in a schema's equivalence class. Schemata Theorem then states that short, low order schemata of above average fitness increase their number of instances within a population from one generation to the next.

Genetic algorithms are widely employed in evolutionary modelling. The string representation offers a convenient way to code: consumer preferences (Aversi et al. 1997), production designs (Windrum and Birchenhall, 1998, 2005), firm routines (Kwasnicka and Kwasnicki, 1992), production rules in cobweb model (Arifovic, 1994, 1995; Dawid and Kopel, 1998; Franke, 1998), production functions (Birchenhall, 1995; Birchenhall et al., 1997); pricing strategies (Curzon Price, 1997), and strategies in a Prisoners Dilemma (Axelrod 1987, Miller, 1996). For instance, in the Cobweb model developed by Arifovic (1994) each binary string represents a single decision rule concerning the production quantity. The role of selection and variation operators is to update firms' decision rules and ultimately to evolve the optimal rule. In particular, crossover and mutation generate new ideas by recombining and varying already existing rules.

An example of the application of a genetic algorithm to the iterated Prisoner's Dilemma tournaments is presented in Axelrod (1997, chapter 1). The author simulates repeated tournaments played during a finite time by a population of evolving automata. Each individual player is endowed with a single string of the length of 70 bits (initially generated randomly), where bits correspond to strategies: cooperate (C) or defect (D) depending on the outcome of three previous moves.[3] Individuals play the iterated Prisoner's Dilemma in pairwise encounters and score the average payoff over all the games they participate in. The single iteration payoff is given by:

Table 2. Payoffs (row, column) in the Prisoner Dilemma

		Column Player	
		Cooperate	Defect
Row Player	Cooperate	3,3	0,5
	Defect	5,0	1,1

Source: Axelrod (1997, p.16).

Subsequently, the relatively successful strategies are randomly paired to produce offspring for the next generation with the use of crossover and mutation. Relatively fitter strategies produce more offspring per mating. Results reveal that the most successful strategies that evolved over generations resemble the Tit-for-Tat (TFT) strategy. The TFT cooperates on the first move and later it imitates the strategy played by the opponent in the previous move. Axelrod (1997, chapter 2) replicates the tournaments in the present of noise, where with 1 percent chance an opposite to intended strategy is implemented. In this context, the Generous TFT and the Contrite TFT turn out to be more effective than the simple TFT in restoring mutual cooperation after unintended defection by one of the players. The Generous TFT cooperates whenever the opponent cooperated in the previous move; if the opponent defected the GTFT cooperates with a certain percent probability, while the Contrite TFT does not respond to the other player's defection after its own unintended defection.

Learning classifier systems

A classifier system was designed by Holland (1975) as an adaptive system where rules are activated depending on the state of the environment. Each rule consists of a condition-action part (for example 'if X appears-then do Y'). Classifier conditions are strings of symbols {0,1,#}, while actions are expressed as binary strings. Classifier systems work as follows. First, the state of the environment is coded on a binary string and transmitted to the system. If a condition part of a rule matches the message from the environment, the rule enters a

[3] Each time step one of four possible outcomes is realised: (C,C), (D,D), (C,D), and (D,C).

competition with other rules that have satisfied this condition. The outcome of the process depends on strengths of rules, which indicates a rule's past performance. The strengths are updated over time with a learning algorithm (e.g., the traditional bucket-bridge or *Q*-learning algorithm). In a second stage, a genetic algorithm is run on the population of rules to generate new and delete poorly performing rules with the use of one-point crossover and bitwise mutation. The purpose of employing classifier systems is to create a cooperative set of rules that together solve the problem (Bull, 2004).

Classifier systems are typically employed to model agent's adaptive behaviour (Marimon et al., 1990, Arthur, 1991; Arthur et al., 1996; Vriend, 1995; Kirman and Vriend, 2001). For instance, Arthur et al. (1996) develop a model in which a classifier system is used to simulate agents' behaviour in an artificial stock market. The model describes *N* agents choosing between investing in a stock and leaving money in the bank at a fixed interest rate. Agents make their investment decisions by attempting to forecast the future returns on the stock. Each agent is endowed with a set of *M* predictors, which are condition-forecast rules. An agent chooses *H* of the most accurate from active predictors, i.e. from predictors whose condition part matches a current state of the market. He computes the next period price and dividend by combining statistically the forecast parts of *H* selected predictors. Subsequently, he computes expected returns. Depending on his current holdings, an agent announces how many assets he wants to sell or buy. Under the condition that total asset demand meets supply (the number of shares issued) bids and offers are matched. After the market clears, the next period's price and dividend are revealed, and the precisions of the forecast rules (in predicting prices) are updated.

Genetic programming

Genetic programming (GP) represents the youngest technique in the artificial intelligence and computational literature. It was developed by Koza (1992, 1994) and builds on the concept of functions applied to arguments; these functions are organized into trees, whose nodes are described with a set of basic functions (e.g., the arithmetic, Boolean, relation, if-then operators) plus some variables and constants $\{+,-,*,/,\ldots,OR, AND, NOT,>,<,=, \ldots v_1,v_2,v_3\ldots c_1,c_2,c_3\ldots\}$ (see Dosi et al., 1999). Operators have connections to other operators or variables. Variables, which have no further connection, constitute 'leaves' of the tree.

GP algorithm proceeds by evaluating each solution according to the fitness function and selecting the best solutions for 'reproduction'. In order to generate new solutions, the fittest among the existing ones are modified and recombined. For example, crossover operates by selecting randomly two nodes in the parents' trees and swapping the sub-trees, which have such nodes as roots. The idea of generating new, possibly better functions or trees in GP is similar to GA.

GP as a member of the evolutionary algorithm family shares some properties with GA. Formally, GP is a variant of GA characterised by a different data structure. In addition, the two approaches differ with respect to the application area: GP is used to seek models with maximum fit to the environment, while GA aims to find an optimal solution (Eiben and Smith, 2003). Working with GP allows for more flexibility: trees take a form of complex structures with nested components, while the size of trees may vary within a population. As opposed, a GA population consists of fixed-length binary strings. Nevertheless, the complex structures of GP may hinder their usefulness, in particular making interpretation of results difficult (Arifovic, 2000).

Genetic programming has been employed in a number of economic applications, for instance, to evolve an optimal price-setting rule (Dosi et al., 1999) an optimal trading rule (Neely et. al., 1997; Allen and Karjalainen, 1999), and to model speculators' adaptive behaviour (Chen and Yeh, 2000). Dosi et al. (1999) employ GP to model firms' behaviour in a complex monopolistic environment where the parameters of the demand function and cost vary constantly. Every time step, each firm selects one pricing rule to maximise its profits. Firms set prices simultaneously and independently of each other. After every firm announces its price, the average price and the corresponding demand are computed. Initially, pricing rules are generated randomly. During consecutive iterations, the probability of each rule being chosen is proportional to the payoffs it cumulated in the past iterations. Learning takes a form not only of adaptive selection but also of a search process for more successful functions: the generation of new pricing rules emerges from recombination (crossover) of the most successful strategies. In simulations, a mark-up type of pricing turns out to dominate among emerging rules.

2.3 Evolutionary game theory

Evolutionary game theory studies the strategic behaviour of boundedly rational players. Individuals are drawn randomly from large populations and have little or no information about the game (Weibull, 1998). The finite set of strategies is given at the outset, while the equilibrium is defined in terms of these strategies (pure strategies) or their combinations (mixed strategies). Friedman (1991) identifies formal ingredients of evolutionary game models:

- Spaces of states and strategies. First, interacting populations indexed $k=1,..,K$ should be defined, where each member chooses a strategy from a finite number of possible actions. Any point r^k of the N-simplex $S^k:=\{x=(x_1,x_2,\ldots x_N): x_i>0,\Sigma_i x_i=1)\}$ represents the fractions of population k employing strategy i.
- Fitness functions. This assigns payoffs to the strategy r^i depending on the current state s. Formally, it can be denoted: $f: S*S \rightarrow R^K$ with $f(r,s):=(f^1(r^1,s)\ldots f^K(r^K,s))$.

- Systems of ordinary differential equations. The final element concerns the evolution of state s over time. The dynamic structure is defined in terms of time derivatives: $\dot{s}=(\dot{s}^1,..\dot{s}^K)$ with $\dot{s}^K:=(\dot{s}^K_1,..,\dot{s}^K_N):=(ds^k_1/dt,...ds^K_N/dt)$. It can be simplified as $F:S\rightarrow R^{NK}$: $\dot{s}=F(s)$.

Additional restrictions are required for $F:S\rightarrow R^{NK}$ to be admissible:

- $\Sigma^N F^k_i(s)=0$ for all $s \in S$ and $k=1,..N$
- $S^k_i=0$ implies $F^k_i(s)=0$
- F is continuous and differentiable on S

Differential equations specifying dynamics can take the form of either deterministic or stochastic differential equations. Both types are discussed below.

Replicator dynamics

Evolutionary game dynamics describes how the frequencies of various strategies within a population change over time according to their payoffs (fitness). The payoffs depend on the strategies of other players, and thus on the frequencies of these strategies within a population. Since these frequencies change according to the payoffs, this creates a feedback loop mechanism (Samuelson, 1997). Replicator dynamics is often applied to capture this. It goes back to Fisher (1930), who claimed: 'the rate of increase of fitness of any species is equal to the genetic variance in fitness'; and it was first formalized by Taylor and Jonker (1978). Replicator dynamics governs the selection process ensuring that units with above-average fitness increase their frequency in a population. It applies to any population divided into types E_1 to E_n, with corresponding frequencies x_1 to x_n, space ($\Sigma_i x_i=1$). According to the replicator model, individuals meet each other in random encounters. Whenever an individual of i-type meets individual of j-type, the payoff to i is a_{ij}. The motion for the frequency of type i is governed by (Hofbauer and Sigmund, 1998):

$$\dot{x}_i = x_i((Ax)_i - x^T Ax).$$

where Ax_i is the expected payoff for an individual of type i given by an n x n payoff matrix $A=(a_{ij})$, and $x^T Ax$ is the average payoff. The frequency of type i increases in the population if its payoff exceeds the average payoff in the population.

A fitness function, as applied to model pairwise interactions, often takes a linear form. In the context of games with interactions occurring in groups with more than two members, fitness may be expressed as a nonlinear function of the frequencies (Nowak and Sigmund, 2004). Replicator dynamics is then re-written as:

$$\dot{x}_i = x_i(f_i(x) - \bar{f}(x))$$

where $f_i(x)$ is a fitness function and $\bar{f}(x)=\Sigma_i x_i f_i(x)$ is the average fitness.

Foster and Young (1990) were the first to introduce a stochastic term into replicator dynamics. They claim that the biological model on which replicator dynamics is based is inherently stochastic in nature so that not every encounter between *i*-type and *j*-type individuals must result in exactly the same change in fitness. Under the assumption of a large population size and frequent interactions, Foster and Young approximate any source of variability in the payoffs by a continuous-time Wiener process:

$$\dot{x}_i(t)=x_i(t)[Ax(t)\Delta t - x(t)^T Ax(t)\Delta t + \sigma(\Gamma(x)\Delta W(t))_i]$$

where $x(t)=[x_1(t),..x_n(t)]^{\mathrm{T}}$ is the proportion of different strategies. $W(t)$ is a continuous, white-noise process with a zero mean and an unit rate covariance matrix; $\Gamma(x)$ is continuous in x and has the property $x^T\Gamma(x)=[0,0,..,0]^T$. The stochastic version of replicator dynamics is suitable for models where random perturbations constantly affect the selection process and thus system dynamics.

Other selection dynamics

Replicator dynamics describes one of many possible transmission mechanisms. Hofbauer and Sigmund (1998, 2003) suggest other selection dynamics, such as best response, Brown-von Neumann-Nash, imitation, mutator, and adaptive dynamics (see also Nowak and Sigmund, 2004). For instance, best response dynamics requires certain cognitive capabilities: agents need to recognize a best reply to the mean population strategy. From this perspective, imitation of a rival's strategy in pairwise comparisons offers more realistic accounts for modelling social interactions. On the other hand, in complex, uncertain and rapidly changing environments individuals often find it difficult to copy the desired behaviour. Mutator dynamics may be employed to depict selection occurring with errors. Finally, adaptive dynamics is useful for modelling adaptive learning in a homogenous population, where almost all individual use the same strategy and only a small number of agents ('mutants') use alternative strategies. The equation captures the process of myopic search, where mutants explore the immediate surrounding of the incumbent strategy (Hofbauer and Sigmund, 1998). These various selection dynamics are discussed in more details below.

(1) Best response dynamic

Best response dynamics may be applied to model myopic behaviour of rational agents. It is derived under the assumption that in large populations a small fraction of individuals revise their strategies and choose the best reply to the population mean strategy x:

$$\dot{x}=\beta(x)-x$$

where $\beta(x)$ denotes the set of best replies b to strategy x such that $z^TAx \leq b^TAx$ for any $z, x, b \in S^n$. The best reply does not have to be unique.

(2) Smoothed best replies

Best reply dynamics can be approximated by smooth dynamics such as the logit dynamics (in order to ensure a unique solution) for $\varepsilon>0$:

$$\dot{x}_i = \frac{e^{a_i(x)/\varepsilon}}{\sum_j e^{a_j(x)/\varepsilon}} - x_i$$

for $\varepsilon \to 0$, this converges to best response dynamics.

(3) The Brown-von Neumann-Nash dynamics

The Brown-von Neumann-Nash dynamics is defined as:

$$\dot{x}_i = k_i(x) - x_i \Sigma_j k_j(x)$$

where $k_i(x)=max(0, a_i(x)-x^T a(x))$ denotes the positive part of excess payoff for strategy i. This equation ensures that if there exists a strategy j with the excess payoff higher than i*'s*, the frequency of strategy i will decrease in a population. The equation defines innovative better reply dynamics.

(4) Imitation dynamics

The frequency of certain strategies can increase in a population through imitation. Imitation dynamics is derived under the assumption that an individual selects randomly another player in the population and decides whether to adopt his strategy. It takes a form:

$$\dot{x}_i = x_i \Sigma_j [f_{ij}(x) - f_{ji}(x)] x_j$$

where f_{ij} is the rate at which a player of type j adopts type's i strategy.

The simplest rule, proposed by Hofbauer and Sigmund (2003), is 'imitate the better'. In this case the rate depends only on the payoffs achieved by the two players:

$$f_{ij}(x)=f(a_i(x),a_j(x)) \begin{cases} =0 \text{ for } a_i(x) < a_j(x) \\ =1 \text{ for } a_i(x) > a_j(x) \end{cases}$$

The frequency of strategy i increases if i's payoff exceeds j's (the term $[f_{ij}(x)-f_{ji}(x)]$ is in this case equal to 1). Alternatively, the switching rate may depend on the payoff difference i.e. $f_{ij}(x)=f(a_i(x),a_j(x))=\varphi[a_i(x)-a_j(x)]$ with a monotonically increasing function φ. The dynamics then follow:

$$\dot{x}_i = x_i \Sigma_i \psi[a_i(x)-a_j(x)] x_j$$

where $\psi(.)$ is an increasing and odd function. The equation may be interpreted as players imitating strategies of other agents with a probability proportional to the expected gain from switching.

(5) Selection-mutation dynamics

Replicator dynamics describes selection without any drift or mutation. To allow for errors to occur during the process models of selection-mutation can be employed, such as mutator and replicator-mutator dynamics. According to mutator dynamics the processes of replication and mutation take place one after another (sequentially), while a replicator-dynamic equation assumes that mutation occurs during the replication process. Fischer (2005) has proposed mutator dynamics of the form:

$$\dot{x}_i = x_i((Ax)_i - x^T Ax) + \mu(1/n - x_i))$$

where μ is a mutation probability, and n the number of strategies. The component μ/n depicts the rate at which individuals change their strategies 'away' from x_i and μx_i is the rate at which individuals change strategies to x_i.

Mutator dynamics can be also expressed as (Helbing, 1995; Brenner, 1998):

$$\dot{x}_i = x_i(Ax)_i - x^T Ax) + \Sigma_j[x_j\, q_{ji} - x_i\, q_{ij}]$$

where q_{ij} is a mutation probability from strategy i to j, and q_{ji} from j to i. The first term on the right-hand side depicts replicator dynamics, and the second term describes the process of mutation as a sum of the probabilities of flow towards and away from the strategy x_i.

In population genetics, biochemistry, and models of language learning the replicator-mutator equation is used: $\dot{x}_i = \sum_j x_j f_j(x) q_{ij} - \bar{f}(x) x_i$ (Bürger, 1998; Komarowa, 2004; Nowak and Sigmund, 2004). The mutation matrix $Q=[q_{ij}]$ is a stochastic matrix, where each entry is a probability that replication of i will result in j, with $\Sigma_j q_{ij}=1$. The replicator-mutator contains both replicator dynamics and quasi-species equations as special cases. If the matrix Q is an identity matrix, the equation reduces to replicator dynamics (perfect learning). Second special case of replicator-mutator dynamics is the quasi-species equation, which describes deterministic mutation-selection dynamics on a constant fitness landscape. The fitness values are independent here of the frequencies of other strategies in a population. Formally, the quasi-species equation takes a form: $\dot{x}_i = \sum_j x_j f_j q_{ij} - \bar{f} x_i$, where f_i is a reproductive rate (fitness) of strategy i and $\bar{f} = \sum_i x_i f_i$ is the average fitness.

(6) Adaptive dynamics

Adaptive dynamics requires a population, in which almost all individuals use a strategy p. The population can be invaded by a strategy q if a payoff for an individual playing the strategy q while all other play p exceeds the payoff he would receive from playing the strategy p. Adaptive dynamics takes a form:

$$\dot{p} = \frac{\partial f(q,p)}{\partial q}\Big|_{q=p}$$

The function $f(q,p)$ denotes the payoff for an individual playing the strategy q in a homogenous population with the strategy p. The derivative of this function determines the direction of the mutant's advantage.

Stochastic dynamics

Stochastic dynamics provide an alternative approach to model economic phenomena. While evolutionary dynamics concerns the evolution of strategies (frequencies), stochastic equations (e.g., Markov processes, a master equation, the Polya urn- see below) deal with the evolution of probabilities of states. For instance, according to a Markov process, a probability of transition from state x to y at time t is conditional on all past states, but it can be reduced to a probability that is conditional only on the state visited in the previous time $t-1$:

$$Pr(X_t=y \,|\, X_{t-1}=x,..,X_0=x_0)=Pr(X_t=y \,|\, X_{t-1}=x)$$

Economic variables modeled as Markov processes are 'memory-less': their values depend solely on the values in the previous period. For instance, in Nelson and Winter (1982, chapter 6) describe changes in the industry as being generated by probabilistic transition rules: search and investment rules applied to each individual firm. The transition rules are mostly implicit; a firm's current state (defined in terms of production techniques and capacity utilization) and values of environmental variables are mapped into the new industry state. Wheeler et al. (2006) offer another application of a discrete-time Markov chain to model adaptive learning in the context of the Cobweb model.

A master equation is a special case of a Markov chain (in a finite time space). It may be employed to model agents' discrete choices. The equation describes transition probability based on probabilities of flows into and out of the set of states. Formally, it can be written down as (Aoki, 1996; 117):

$$\partial P(x',t)/\partial t = \sum_{x \neq x'} P(x,t)\omega(x'|x,t) - \sum_{x \neq x'} P(x',t)\omega(x|x',t)$$

where $P(x, t)$ denotes a probability of being in state x at time t, while $\omega(x'|x,t)$ a transition rate from state x to x'. The first term is the sum of probability of flows into state x', while the second is the probability of flow out of state x'. Weidlich and Braun (1992) employed the master equation to model competition among firms. They assume a number of firms producing a single commodity differentiated with respect to quality. Transition rates govern the unit changes of variables, such as supplied quantities, price, and quality. On the demand side, a population of consumers consists of homogenous individuals. Each consumer may own either none or one of the commodities. Here, the transition rate determines changes between states: 'owner' and 'nonowner'.

Alternatively, the Polya urn may be employed for modelling system dynamics. This approach refers to an urn that is filled with balls of two colours. Each time one ball is drawn randomly: the selected ball is returned to the urn, while an additional ball of the same colour is added. The probability of adding a ball of a particular colour equals exactly the proportion of balls of this colour in the urn. Alternatively, Arthur et al. (1987) propose a framework, where a probability of adding a ball of type *j* is an arbitrary function of the colour frequencies. It augments the standard Polya urn with a perturbation component. Formally, the urn consists of *w* balls of *n* colour, where a vector $X_n=\{X_n^1, X_n^2,.., X_n^N\}$ describes the proportions of balls of colours 1 to *N* respectively. At each time, one ball is added; the probability that it is a ball of a colour *i* is equal to $q_n^i(X_n)$. The frequency of the *i*-colour ball is:

$$X_{n+1}^i = X_n^i + 1/(w+n)[q_n^i(X_n) - X_n^i] + 1/(w+n)\,\mu_n^i(X_n)$$

Here, $\mu_n^i(X_n) = \beta_n^i(X_n) - q_n^i(X_n)$, while $\beta_n^i(X_n)$ equals 1 with a probability $q_n^i(X_n)$ and 0 otherwise. The Polya urn mechanism as described refers to a non-linear Polya process (Arthur et al., 1987). Dosi et al. (1994a) apply the general urn scheme to modelling technology choice, and Fagiolo (2005) to coordination games.

2.4 The Price equation

A model often used in evolutionary analysis is the Price equation (Price, 1970). It provides a complete description of evolutionary change under any condition (Frank, 1995). The model requires a population of heterogonous individuals index by *i*. It takes the form of:

$$\overline{w}\,\Delta\bar{z} = Cov(w_i,z_i) + E(w_i,\Delta z_i)$$

Here, $\Delta\bar{z}$ depicts a change in the average characteristic (trait) over generations according to $\Delta\bar{z} = \Sigma q_i' z_i' - \Sigma q_i z_i$, where q_i is the frequency of the type *i* with the characteristic z_i in the parent population, and q_i' the frequency of the type *i* with the characteristic z_i' in a descendant (offspring) population; Δz_i measures the change in the trait value for the type *i* as $\Delta z_i = z_i' - z_i$. In addition, the frequency of type i in the offspring population is proportional to the relative fitness of the type i in the parent population: $q_i' = q_i w_i/\overline{w}$, where w_i stands for the fitness of *i* type and $\overline{w}$ denotes the average fitness of the population. In the Price Equation the covariance term depicts a change in the character due to successful reproduction, while the expectation term measures the fitness weighted by a change in the character over generations.

The Price equation is often mistaken for being a generally applicable analytical tool, while its role is solely to decompose evolutionary change. Ultimately, the equation is an identity or mathematical tautology (Grafen, 2000). Van Veelen (2005) suggests to clearly distinguish between statistical and probability (stochastic) analysis. He claims that the Price equation can be employed to address two types of questions. First, it can be used to assess a possibility (likelihood) of certain modelling assumptions being correct. Alternatively, one may employ the equation to make interferences given a set of assumptions and mechanisms underlying a theoretical (evolutionary) model.

The components of the Price equation are open to a wide variety of interpretations (Frank, 1995). For instance, the equation may decompose the evolutionary process into selection and transmission. Alternatively, the covariance and expectation terms can be construed as effects of between- and within- group selection on average trait frequency in a population. Metcalfe (2002) notes that many authors in evolutionary economics have carried out analyses of economic change consistent with the Price equation without even realizing. Using the Price equation, Andersen (2004) decomposes a change in the mean productivity in the Nelson and Winter's (1982) model into selection $Cov(w_i,z_i)$ and innovation $E(w_i,\Delta z_i)$. Here, z_i is interpreted in terms of productivity of a firm i's capital stock, Δz_i as the change in productivity between two periods, and w_i as the reproduction coefficient defined in terms of firm i's growth rate.

The Price equation describes the selection process assuming that it acts on a single trait (characteristic). Thus, the analysis requires isolating the effect of this trait from multiple of other effects on the fitness. Alternatively, the conceptual analysis is a method for analyzing selection acting on multiple characters. It integrates the covariance approach (Price, 1972) and the selection-gradient method (Lande and Arnold, 1983). The conceptual analysis involves multiple regressions in which both individual and group characters, including aggregate characters denoting group means and quantifiable group properties that cannot be obtained solely from measurements of group members, are treated as individual traits and are included as independent variables (Heisler and Damuth, 1987).

3. Building blocks of evolutionary-economic models

In this section we present an overview of components of formal models in evolutionary economics. The following categorization is employed: (1) diversity, (2) bounded rationality, (3) innovation, (4) selection, (5) diffusion, (6) path dependence and lock-in, (7) co-evolutionary dynamics, (8) multi-level and group selection, and (9) evolutionary growth.

3.1 Diversity

Central to any evolutionary model is a heterogeneous population i.e. a population characterised by internal diversity. Diversity relates to progress through Fisher's principle: 'The greater the genetic variability upon which selection for fitness may act, the greater the expected improvement in fitness' (Fisher 1930). In evolutionary-economic frameworks, diversity is formalised in a number of different ways. In evolutionary computation models, populations consist of individuals hosting distinct strategies. Here, each individual can produce a single type of behaviour only, but yet different individuals may produce different behaviours, referred to as 'developmental coin flipping' (Bergstrom and Godfrey-Smith, 1998). Alternatively, individuals may do different things on different occasions. An individual exhibiting a variety of behaviour within his lifetime has been referred to as the **'**individual behaviour mixing' (Bergstrom and Godfrey-Smith, 1998)**.** The latter approach is used extensively in evolutionary game settings, where it is formalised with the notion of mixed strategies. Finally, in multi-agent systems, agents may differ with respect to behavioral rules, knowledge, goals, physiological features (e.g., vision and energetic efficiency in the Sugarscape model, Epstein and Axtell, 1996) or signals. This creates a wide spectrum of opportunities to realize heterogeneity.

The concept of diversity can be elaborated as having three properties: variety, balance, and disparity (Stirling, 2004, 2007). Variety is defined as the number of categories into which a population can be partitioned; the greater the number of options in a portfolio, the greater its diversity. Balance relates to the distribution of shares of each category in a portfolio; for a particular portfolio of a given variety, the more equal are the fractional contributions of each option, the more even is the distribution and the greater is diversity. Finally, disparity refers to the degree to which options differ; it captures the distance between categories. Disparity is a qualitative property, which represents a rather subjective and context-dependent aspect of diversity.

Stirling suggests a simple diversity measure that combines these components. It takes the form of multiplicative function, representing an integrated diversity heuristic measure D (Stirling, 2007):

$$D = \sum_{i,j(i \neq j)} d_{ij}^{\alpha} (p_i p_j)^{\beta}$$

Here d_{ij} is the distance in a Euclidean disparity-space between options i and j, and p_k is the frequency of element k in the population. The α and β may take any of possible permutations of 0 and 1. In the reference case, if α and β are both equal to 1 the measure captures balance- and disparity-weighted variety. If $\alpha=0$ and $\beta=1$ the index reduces to balance-weighted variety, while if $\beta=0$ and $\alpha=1$ to disparity-weighted variety. For $\alpha=0$ and $\beta=0$ the measure depicts scaled variety.

For the purpose of statistical analysis, a number of other diversity measures have been proposed (Theil, 1967**;** Weitzman, 1992, 1998a; Önal, 1997; Frenken et al., 1999; Saviotti, 2001). However, Stirling (2007) claims that most of these measures are not very balanced. For instance, an entropy-based index is a dual measure combining diversity and balance, while the Weitzman index is limited to disparity. Entropy-based measures, such as the Shannon and the Simpson indexes, compute the statistical variety on the basis of the frequency distribution of discrete variables. The Shannon index is defined as $H= -\sum_{i=1}^{n} p_i \ln(p_i)$, where n is the number of species, and p_i is the share of the ith species. $H=0$

indicates the lowest diversity (Önal, 1997). The Simpson index takes the form of the sum of the squared shares of each option in the portfolio: $H = \Sigma_i p_i^2$. A related entropy measure was proposed by Önal (1997) for the purpose of creating a more operationally and computationally convenient index. It defines the structural diversity index as: $V(x) = 1 - \frac{1}{2(n-1)} \Sigma_{i,j} |s_i - s_j|$ (n is the number of species, and s_i, s_j are shares of i and j species respectively). For a given pair of groups i and j $|s_i - s_j|$ measures the relative diversity between the two groups. Maximum diversity occurs when all groups in an assembly have equal numbers of elements, while a minimum value is realized if one group contains all of the elements.

Alternatively, Weitzman's index (1992, 1998a) emphasises distance between entities. The measure can be applied to both discrete and continuous variables. It classifies entities in groups based on their dissimilarity through a distance measure d. Formally, diversity $V(S)$ is the solution of the recursion: $V(S) = max_{y \in S} (V(S \backslash y) + d(S \backslash y, y))$, where $S \backslash y$ stands for a set S without a member y and $d(S \backslash y, y)$ captures the distance between this set and y. The Weitzman's index addresses disparity alone; it does not account for the relative abundance of different options within a population.

Several studies have employed these diversity measures: Saviotti and Trickett (1992) in a study of helicopters, Bourgeois et al. (2005) for refinery processing, Frenken and Nuvolari (2004) for the steam engine, and Frenken and Windrum (2005) for microcomputers and laptops. Frenken et al. (1999) use both the entropy and Weitzman's diversity measure to analyze the evolution of technology in four industries: aircrafts, helicopters, motorcycles and microcomputers. They define a population of products in terms of the distribution of product characteristics. Changes of variety in each particular industry are investigated as changes in the composition of the population structure over time (measured with diversity indexes). The results reveal a tendency for decreasing variety towards product standardization for helicopters and microcomputers and increasing variety for aircrafts and motorcycles.

3.2 Innovation

Innovation is an inherent feature of any evolutionary system. It is essential for diversity creation. Although it is intrinsically uncertain, and for this reason in most evolutionary economic models treated as stochastic, it would be incorrect to consider the process as totally random. Innovations may be expected to occur in a systematic manner, namely preceded by the cumulativeness of relevant technical advances. In addition, some view the innovative processes as following relatively ordered technological pathways, for instance: Nelson and

Winter's (1977) natural trajectories; Sahal's (1985) technological guidepoints, and Dosi's (1982) technological paradigms (see Silverberg and Verspagen, 2003).

Technological evolution may be brought about by a series of incremental improvements in already existing designs or by the introduction of a design radically different from the latest technological achievement. Mokyr (1990) distinguishes in this respect between micro and macro inventions. The former refers to small and incremental steps to improve a design in line with artifacts developed under the current paradigm. Macro-inventions concern the introduction of radically new ideas without a clear precedent, which disturb the existing economic structures and dependencies. Formally, incremental innovations may be seen as continuous changes in product characteristics (incremental improvements in technical or service attributes), while radical innovations are discontinuous changes (Savotti and Meltcafe, 1984).

A number of studies examine the notion of recombinant innovation (Weitzman 1998b, Olson and Frey, 2002; Tsur and Zemel, 2006, Van den Bergh 2008). Weitzman presents a formal model in which the number of new combinations is a function of the number of existing ideas. He shows that if this number is the only limiting factor in knowledge production, super-exponential growth may result. Olson and Frey (2002) connect Weitzman's recombinant growth with Schumpeter's view of the entrepreneur, who innovates by combining existing ideas or technologies in a convex way. They demonstrate that the resulting combinatory process is constrained by following factors: convexity implies exhaustion of technological opportunities; the cost of combining ideas increases with distance (disparity) between them and thus profit maximization requires combining ideas that are technologically close; social acceptance constrains or prohibits certain combinations; and a ruling technological paradigm limits the scope for recombinant growth. In line with this, van den Bergh (2008) develops a model to derive optimal diversity in the presence of the trade-off between increasing returns to scale and benefits of recombinant innovation.

Evolutionary models emphasise the importance of innovative activities in driving industry dynamics. In evolutionary game theoretical settings, innovation typically transforms a firm as a whole. For instance, each innovation may be associated with a new vintage of capital (e.g., Iwai, 1984 a,b; Silverberg and Lehnert 1993; Silverberg and Verspagen, 1994a,b, 1995a). In this context, Iwai (1984a) develops a capital vintage model to examine how dynamic interactions between the equilibrating force of imitation and the disequilibrating force of innovation shape the evolutionary pattern of an industry. The market consists of M firms (active and potential producers) and n production methods with corresponding unit costs c_i (c_n>..>c_1). Firms face two alternatives, namely innovate or imitate the technology exhibiting a lower than their current cost of production. It is assumed that each firm has a small but equal chance of successful innovation at every point at time. If innovation occurs, it creates a new cumulative frequency $F_t(C_N)=1/M$, where C_N denotes the unit cost of the best production method that is technologically possible at time t. The relative frequency of firms with the unit cost equal to c or lower than c changes according to:

$$\Delta F_t(c)=\{\mu F_t(c)(1- F_t(c)) +vM(1-F_t(c))(1/M)\} \Delta t$$

where μ and v are indices of the effectiveness of firm's imitation and innovation activities, respectively; $v\Delta tM$ denotes the probability that an innovation is carried out successfully by one of the firms over a small time period Δt.

In micro-simulation models of industry dynamics, each firm is engaged in the search process for better solutions. In Nelson and Winter's (1982) pioneering model search is modeled as a two-stage random process: in the first stage, imitation and innovation draws determine the firm's probability of undertaking R&D activities (0 or 1). If a firm i gets an imitation draw, then in the second stage it copies the industry's best practice. If it gets an innovation draw, it samples productivity A from a distribution of technological opportunities $F(A;\ t,\ A_{it})$, where A_{it} is firm i's current productivity level. Finally, if a firm obtains a combination of imitation and innovation draws, its new productivity level is determined by: $A_{i(t+1)}=Max(A_{it},\ \overline{A}_t,\ \overline{A}_{it})$, where A_{it} is firm i's current productivity level, $\overline{A}_t$ is the best practice productivity level at time t, and $\overline{A}_{it}$ is a random variable resulting from the innovation draw.

In Nelson and Winter's model firms are treated as a single unit of selection. Alternatively, a firm can be treated as a multi-operation unit (e.g., Kwasnicki and Kwasnicka 1992; Chiaromonte and Dosi, 1993; Dosi et al., 1994b; Dosi et al., 2006). For instance, in Kwasnicki and Kwasnicka (1992) model of industry dynamics, each firm is characterised by two types of routines: active ones employed in everyday practice, and latent ones stored but not actually applied. Each set of routines is divided into separate segments, consisting of similar routines employed by firms in different domains of their activities. New routines evolve due to recombination, mutation, transition or transposition. With a certain probability the lth routine in the kth sector changes (mutation) or the segment k of a firm-unit i is recombined with the segment k of a firm-unit j (recombination). Alternatively, a single routine may be transmitted from another firm (transition) or within a single firm a latent routine can be transposed from a latent into an active state (transposition).

To model a myopic search for better solutions in a technology context an NK-model may be employed (Altenberg, 1997; Auerswald et al., 2000; Frenken and Nuvolari, 2004). Here, N stands for the number of elements, while K denotes complexity of the system (interdependence of dimensions). Each element has its own sub-function(s) within the system. It is assigned a fitness value w_n drawn randomly from the uniform distribution [0,1]. Elements in NK system are interdependent; these dependencies are often referred to as 'epistatic relations'. If a value of a particular element changes, the change affects both the fitness (and functioning) of this element and the fitness (and functioning) of elements that are interlinked with it. The total fitness of the system changes according to the average fitness of its elements:

$$W(s)=\frac{1}{N}\sum_{n=1}^{N} w_n(s)$$

where w_n denotes fitness of n element. In this context, search is modelled as a trail and error process. Each time step a value of one of elements is mutated and the fitness of the system before and after mutation is compared. If the average fitness has increased, mutation continues, otherwise the state of the system is brought back to the previous configuration. The process is repeated until an optimum (local or global) is reached.

Modelling innovations on the supply side is well established in the evolutionary economics' literature. On the contrary, conceptualising innovations on the demand side has not led to a common approach. An interesting attempt to formalise evolving preferences in an abstract model has been undertaken by Potts (2000).[4] The author sketches eight ways in which the schematic preferences, coded on a string, may evolve with the use of a genetic algorithm. In the context of an agent choosing a set of goods from the available set {a,b,c,d,…}, the change in his preferences may be captured with (# has the meaning 'I do not care'):

1. Point mutation: <aaab> → <aaaa>
2. Cross over: <aabc><bbcc> → <aacc>
3. Inversion <abca> → <acba>
4. Slide <##aabbcc##> → <aaaabb####>
5. Reclustering <abcabcaabc> → <aaabbbccc>
6. Emergence/Closure <aaaaa###> → <aaaaa>
7. Higher or lower specification: <aabb##>→ <aabbc#>; <aabb##> → <aab###>
8. Birth or death: <...> → <aabbc#>; <aabb##> → <…>

The list can be augmented with other mechanisms corresponding to genetic processes. In addition to the point mutation and recombination, insertion and deletion are distinguished (in genetics). Insertion implies adding a string to the existing sequence of code. Deletions characterize the reverse process, the loss of a string of code (Nowak, 2006). New solutions may also result from hybridization of existing ideas, a process know as multi-parent recombination in evolutionary computation, or modular evolution in biology. In particular, modular evolution is the source of radical innovations in both natural and social-technological

[4] For a model of endogenous preference change see, for instance, Aversi et al. (1997).

history, and Watson (2006) theoretically supports this by formally showing that modular evolution can realize more complex systems than gradual evolution.

3.3 Bounded rationality

The notion of bounded rationality originated in the 1950's from Herbert Simon's critique of 'economic man'. Simon (1955, 1956) proposed the concept of bounded rationality which applies to the conditions of extensiveness, complexity and uncertainty (Hodgson, 1997). Under extensiveness, information may be readily accessible and comprehensible, even though time and other resources are required to obtain it. Complexity stipulates the existence of a gap between the computational capacity of an agent and the complexity of his environment. Under uncertainty, agents have difficulties in acquiring crucial information and assessing the probabilities over the future events. In these cases, individuals are likely to exhibit habits and rule-driven behaviour. The assumption of bounded rationality prevails in evolutionary game theory. Agents are assumed here to have little or no knowledge about the game. They are incapable to anticipate actions of other agent or consequences of their own decisions. They may engage in myopic search for better solutions, imitate the most frequent behaviour. Various forms of replicator dynamic equations have been proposed to model boundedly rational behaviour in section 2.3. Conlisk (1996) offers an extensive overview of different types of bounded rationality in economic models.

In standard economics, the analysis of choice under uncertainty relies on expected utility theory, which goes back to Von Neumann and Morgenstern (1944). The theory is based on three axioms on preferences: ordering, continuity and independence. It has been shown that in certain applications, individual decisions are inconsistent with these axioms. Behavioural economics seek to provide a more realistic account of decision-making by incorporating psychological insights into the theory of choice. The contributions to behavioural economics are numerous. Crucial ones include: prospect theory (Kahnemann and Tversky, 1979), quasi-hyperbolic discounting as an alternative to traditional exponential discounting (Thaler, 1981, Prelec and Lowenstein, 1992, Frederick et al., 2002) social preferences (Guth et al., 1982), regret theory (Bell, 1985; Loomes and Sugden, 1986), and case-based theory (Gilboa and Schmeidler, 1995). In particular, prospect theory of Kahnemann and Tversky (1972) has received much attention. It builds upon the premise that individuals evaluate differently losses and gains relatively to a situation-specific reference point. The theory of social preferences is inspired by the evidence that players tend to sacrifice to reduce inequality of payoffs and are likely to reciprocate behaviours that have benefited them. Regret theory assumes that whenever the outcome of the prospect is worse than expected a sense of disappointment is generated, while in case the outcome of the prospect is good, a person experiences elation. Finally, case based theory suggests that people choose acts based on their performance in similar problems in the past. It provides insights into habit formation. In short, theories in behavioural economics offer interesting alternatives to formalise bounded rationality of individuals. However, there is little or no guidance when to use each of these models (Fudenberg, 2006). Most theories are derived for a specific context, relying on unobservable data (e.g., mental states, reference points in prospect theory). This makes their application not straightforward (Pesendorfer, 2006).

In the context of studies of firm and organizational behaviour, bounded rationality has taken the form of rules and routines. Nelson and Winter (1982) claim that firms operate, to a large extent, according to decisions rules that are not consistent with profit maximization but instead take the form of complex patterns of routinised behaviour. Heuristics, cognitive and learning processes are crucial for decision-making. In particular, imitation is an important mechanism underlying firms' behaviour in models of technology diffusion. It allows saving on costs of individual learning, experimentation or searching by exploiting information already acquired by others (horizontal and vertical transmission). In the context of social interactions, imitation can take a form of either copying the' the most successful' or 'the majority' strategy. Copying 'the most successful' is also known as prestigious-bias transmission; it occurs, when individuals seek to copy the most influential, knowledgeable or skillful behaviour (Henrich et al., 1999). Copying the majority strategy has been termed by Boyd and Richardson (1985) as conformist transmission. It refers to a propensity of an individual to adopt cultural traits that are most frequent in the population.

Imitating the most successful or majority strategy in a population requires the assumption of common knowledge. One way to deal with this rather unrealistic setting is to limit the environment in which agents operate (Kirman, 1997). This can be achieved by assuming that individual interact with a limited number of agents, for instance through networks. Networks play an important role in facilitating communication, specialisation of competences, standardization of complementary technologies, and flow of knowledge. In this context, a number of studies have been devoted to the analysis of firms and industries as networks and to organizational and strategic arrangements within specific networks (see, Malerba, 2006).

Networks have been increasing applied to model a broad array of socio-economic phenomena, such as social interactions (Axelrod, 1997; Jansen and Jager, 2002; Morone and Taylor, 2004), technological innovation and diffusion (Silverberg and Verspagen, 2003, Cowan and Jonard, 2004, Cowan et al., 2006). Network structures range from percolation models (Antonelli, 1996; Solomon, 2000; Conlisk et. al., 2001; Silverberg and Verspagen, 2003), network neutral nets (Plourabove et. al., 1998), to graphs (Watts and Strogatz, 1998; Cowan, 2004, see Frenken, 2006). These models are referred to as static: the analysis is carried out for a given network structure, which does not change over time. In particular, graphs are popular in evolutionary-economic modelling. They compromise Ising models, small world models and random graphs. In Ising models agents are located at fixed points in a regular integer space, and they are connected to their *n*-nearest neighbours only. In small world models some agents can interact with farther than neighbouring sites. The network structure in small world models is characterised by high cliquishness i.e. high density of agents' interactions, and short average path lengths between agents (Cowan and Jonard, 2000). Alternatively, in random graph models agents are connected with some positive probability regardless of their location; the networks have no explicit psychical space. Watts and Strogatz's (1998) proposed a one-parameter random graph model compromising these three approaches. A parameter p, reflecting a probability of connecting a random agent to each link within the network, is used to scale between the regular and random graph (e.g., p=0 the Ising model, p=1 for the random graph).

Finally, dynamic network models may be employed to study the process of networks formation (pioneering contributions by Jackson and Wolinsky, 1996 and Bala and Goyal, 2000a). They are generally classified into directed and non-directed graphs. In directed graphs one player may be connected to a second without the second being connected to the first, while in non-directed graphs links are necessarily reciprocal (Jackson, 2005). Consequently, in non-directed graphs creation of a new link requires mutual agreement between two agents. For instance, Bala and Goyal (2000b) develop a noncooperative model of network formation, where communication between agents is costly and not fully reliable. A pair of individuals decides whether to create a mutual link, in which case both agents can share information. In this context, authors analyse which configurations ensure stable and efficient networks (i.e. Nash networks).

3.4 Selection

Selection in the most simple form can be understood in terms of picking a subset from a certain set of elements according to a criterion of preference, referred to as subset selection (Price, 1995). Alternatively, selection can be seen by analogy with natural selection as the outcome of two independent processes, namely replication of an encoded instruction set, and interaction of entities with their environment[5] causing differential replication (Knudsen, 2002). If the second process applies, a population of offspring is not a subset of parents but consists of new entities. Similar to Price (1995), we can describe a general selection process that unifies subset and natural selection as follows. Formally, a set P includes w_i units of entities with value x_i (for all i) for some characteristics x. A set P' is composed of new entities

[5] Or with other entities in case of selection in social and economic systems.

corresponding to entities of P. Selection on the set P in relation to the property x can then be defined as a process of producing the corresponding set P' such that w_i' is a function of x_i. According to subset selection $w_i' \leq w_i$, while $x_i = x_i'$. These assumptions are not required in the case of natural selection.

An early discussion in evolutionary economics focused on firms being selected by the market, in the sense of surviving competition, with possible effects on profit seeking or even maximizing behavior (Alchian, 1950, Friedman, 1953, Winter, 1964). In later models of industry dynamics, selection was formalized with replicator dynamics by analogy with natural selection. Here, technology diffusion is treated as an outcome of selective competition between rival technologies, where selection covers both traditional types of competitiveness e.g., price competition and product differentiation (e.g., Nelson and Winter, 1982; Iwai, 1984a,b; Soete and Turner, 1984; Silverberg et al., 1988; Metcalfe, 1988). The second type of competition may be referred to as Schumpeterian; firms compete by offering new, improved product characteristics or services, which enable them to capture some temporary monopoly rents (Savotti and Pyka, 2004). Replicator types of dynamics, however, ignore the possibility of mistakes, imperfect learning, and costly experimentations to occur during the selection and replication processes. Alternative models of selection dynamic exist (discussed in section 2.3), although these have seen little application to economic phenomena. Important exceptions are Foster and Young (1990), Canning (1992), Young (1993), and Kandori et al. (1993), who propose models of adaptive learning in the context of repeated 2x2 games. Here, mistakes by players constantly disturb the process of learning and thus selection dynamics.

The fundamental and secondary theorems of natural selection offer complementary perspectives to replicator dynamics approaches to analyze the aggregate patterns of change in the industry structure (Meltcafe, 1994, 1998). Both theorems are special cases of the more general Fisher's principle (e.g., Edwards, 1990; Findley 1990, 1992; see also section 3.1). The fundamental theorem claims that the rate of improvement in the mean characteristic in a population is proportional to the variance of this characteristic. It may be applied to capture the structural change in an industry, whose average growth rate evolves according to (Meltcafe, 1998):

$$\frac{dg}{dt} = \Sigma_i s_i (g_i - g) g_i = V_s(g)$$

The average growth of the industry equals to the weighted average of firm growth rates: $g = \sum_i s_i g_i$, where s_i and g_i are firm's i market share and growth rate.

The secondary theorem is an extension of the fundamental theorem, where the variance is replaced by a covariance term. The rate of change in the mean characteristic equals here the covariance between this characteristic and the population mean fitness

(Robertson, 1968). With the use of this theorem the evolution of the average unit cost can be expressed as (Meltcafe, 1998):

$$\frac{d\overline{h}_s}{dt}=\Sigma_i\, s_i(g_i\text{-}g)h_i=C_s(g_i,h_i)$$

Here h_i is firm's i unit cost of production, and $\overline{h}_s$ is the average cost. This equation assures that the rate of change of the mean characteristic is equal to the covariance between growth rates and unit costs at the firm level. The secondary theorem can be reduced to the fundamental theorem, by assuming that the trait is the fitness itself (Meltcafe, 1998).

Note that although selection environments are often modeled as being constant, this does not need to be the case. For example, the dynamics of consumer preferences may alter the selection environment for firms, leading to demand-supply coevolution (see section 3.7). Alternatively, selection may be modeled as a two-stage or a multi-level process: internal and external to the firm. Internal selection concerns selection of routines at the level of a firm, while external selection is typically understood in terms of market selection (Kwasnicki and Kwasnicka, 1992; Lazaric and Raybaut, 2005). For instance, in Kwasnicki and Kwasnicka (1992) each firm searches for new routines (or new combinations) to increase its overall competitiveness. After a firm has made decisions concerning the production process, its performance is subject to external (market) selection. As a result, a firm's market share depends on relative prices, relative values of products, and the market saturation level. For more general discussion on multi-level evolution see section 3.8.

3.5. Diffusion

Diffusion of a technology, product or behaviour over time typically follows a sigmoid (*S*) curve: the diffusion rate first rises, at initially low but increasing adoption rates, leading to a period of relatively rapid adoptions. Later, the diffusion rate starts to decline, slowly approaching satiation. In general, models of technology diffusion aim at explaining the logistic patterns of the diffusion process. For overviews see Metcalfe (1988), Silverberg et al. (1988), Geroski (2000), and Manfredi et al. (2004).

The diffusion process in the context of demand dynamics is driven by the progressive dissemination of information about technical and economic characteristics of products within a population of potential adopters (Silverberg et al., 1988). The minimal structure of such a model requires distinguishing between mutually exclusive sub-groups of users and non-users, while the analysis of model dynamics focuses on the spread of information from adopters to non-users. Within this category, several types of models can be distinguished.

According to the epidemic model (the seminal work is by Mansfied, 1961), technology spreads like a disease. An individual adopts a particular technology after having had contact with the 'infected population' i.e. individuals who already have adopted the innovation. The framework explains patterns of innovation diffusion from the date of its first implementation (not invention) by some percentage of users. The evolution of the number of adopters follows the pattern given by: $y(t)=N(1-exp[-\alpha t])$, where: N is the number of potential adopters, while α denotes the percentage of the population that has learned about a new technology. The model applies to a situation in which information spreads from a central source.

Alternatively, 'word of mouth models' account for direct communication between users: they independently contact non-users with a positive probability β. The process of diffusion follows an *S*-curve over time: the rate of infection increases as a population of users gradually rises (increasing the aggregate source of information) until it reaches the maximum. Then it starts declining as non-users become more hard to find and therefore to infect.

Mixed information source models combine the epidemic and the word of mouth approaches. The information spreads with a probability equal to a constant rate at which an individual learns about new technology from the central source plus a flexible rate at which an individual learns about novelty from other users: $\alpha+\beta y(t)$ (see Bass, 1969).

Finally, the probit model was developed for the analysis of individual adoptions. A simplified version of this approach assumes that individuals differ in some characteristic x, which are randomly distributed in a population according to a function $f(x)$. Only individuals whose characteristic value exceeds a threshold level x^* adopt the innovation. Over time technology gets cheaper and the threshold value falls. As a consequence, more people have a chance to adopt it. If the distribution underlying $f(x)$ is normal the gradual movement of the threshold level across the distribution generates the *S*-shaped diffusion curve.

The aforementioned models have been criticized for lacking a description of individual decision-making. They do not provide insight into how the possible saturation level is reached or determined. In addition, adopters are exposed homogenously to the source of information. Recent models put the emphasis on the behavioural aspects of consumers' decision-making processes, in particular on the role of imitation. Agents may imitate behaviour of other individuals (e.g., the information cascades), of the neighbouring sites if a game has a spatial dimension (agents are located on a grid), or of individuals that belong to their social network (e.g., Jansen and Jager, 2002; Alkemand and Castaldi, 2005; Delre et al., 2006). For instance, Delre et al. (2006) develop a multi-agent model, where adoption decisions depend on agents' personal networks and external marketing efforts. The results suggested that the speed of diffusion is highly sensitive to the network structure and the degree of consumer heterogeneity. See also section 3.3. for discussion on networks in economics.

Evolutionary graph theory may provide interesting insights for studying the effect of the population structure on diffusion. Individuals are placed here on the vertices of the graph and are connected by edges. Edges denote reproductive rates at which individuals place offspring into adjacent vertices. The analysis of the fixation probability indicates how likely is that a single mutant (placed randomly within the network) may take over a whole population (Nowak, 2006, chapter 8). In this context, some graphs act as suppressors or amplifiers of selection. In particular, amplifiers structures increase the probability of fixation of advantageous mutants (with high relative fitness) and reduce the probability of fixation of disadvantageous mutants. The superstar, funnel and metafunnel are examples of such amplifier structures (Lieberman et. al., 2005). Evolutionary dynamics on graphs have been applied to study social games (e.g., Prisoner Dilemma, Dove and Hawk) in spatially structured populations.

3.6 Path-dependency and lock-in

Economic systems are characterised by various reinforcement and feedback mechanisms that explain why after a system moves on a particular path of development, it may be difficult to change the direction of a process. Feedback mechanisms associated with increasing returns may arise from economies of scale, learning by doing, technological interrelatedness, the accumulation of knowledge and experience, and agglomeration or spillover effects (see Arrow, 1962; Arthur, 1988; Meltcafe, 1994). These are typically mechanisms associated with supply-side dynamics. In addition, increasing returns on the demand side play a role, in particular network externalities, informational increasing returns, imitation and bandwagon effects, learning-by-interacting, and external influences like advertising, education (Katz and Shapiro, 1985; Lundvall, 1988).

Increasing returns are the sources of lock-in and path dependence. A simple model illustrating dynamics in the presence of increasing returns was developed by Arthur (1989). This model considers two technologies, *A* and *B,* competing for adoption by two types of economic agents: an agent *R,* who has a natural or intrinsic preference for technology *A*, and an agent *S* having a natural inclination to chose technology *B*. Choices are made sequentially; each time a randomly drawn type of agent (either *R* or *S*) decides which technology to adopt by comparing payoffs from two technology variants. The returns from adoption of a particular technology depend on the number of its previous adopters. This causes increasing returns to scale: the more adopted, the more attractive is a technology. It is a self-reinforcing mechanism, which may be the source of lock-in: once a certain technology becomes dominant; subsequent adoptions will most likely be of the same type enhancing its leading position.

Witt (1997) notes that the resulting lock-in is critically dependent on the assumption of an infinitely growing population of adopters. This, together with the presence of only two types of agents and specific interactions between adopters (imitation), prevents model dynamics from exhibiting cyclic or more complex behaviour. If a finite or constant population is assumed, an unstable fixed point rather than an inescapable state of lock-in results. Arthur and Lane (1993), Kirman (1993) and Dosi et al. (1994a) show that lock-in is not a necessary outcome if interactions between agents take a different form than in the basic Arthur model. For instance, Dosi et al. (1994a) reformulate Arthur's model with the generalised Polya urn schemes approach. Here, new adopters choose the technology used by the majority of a sample m of other adopters with probability α, while with probability $1-\alpha$ they adopt the technology used by the minority. Due to the presence of a stochastic factor, technology shares never converge to either 0 or 1, ensuring co-existence of variety. In addition, Leydesdorff and Besselaar (1998) use Arthur's model to demonstrate that under the assumption of limited cognitive capabilities of individuals, i.e. agents being unable to perceive small differences in the adoption rate below a certain threshold, lock-in disappears.

Path dependence and lock-in are important features of technological change in the context of environmental regulation. Problems of lock-in and unlocking policy are closely related to the difficulty of making a transition to sustainable systems in energy, transport and agriculture (Unruh, 2000; van den Bergh et al., 2006, van den Bergh, 2007). Lock-in does not need to be permanent. Assuming that everyone switches, the change from an inferior state is possible (Arthur, 1994). For instance, actors might coordinate their decision to adopt a new technology when they recognize that coordinated action yields special benefits (Foray, 1997). In line with the above remarks, Witt (1997) argues that the capacity to pass a "critical mass threshold" in terms of the number of potential adopters of a market alternative is the key to the success of unlocking the market. He notes that in fact governments and innovating firms take account of the critical mass phenomenon. For instance, with promotion campaigns firms undertake efforts to convince potential adopters that others are already about to adopt the new variant in order to stimulate coordinated adoption decisions.

Since the seminal work by David (1985) and Arthur (1988, 1989), lock-in and path dependence have received increasing attention in the context of policy studies in multi-agent models (Janssen and Jager, 2002; Carrillo-Hermosilla, 2006, Schwoon, 2006). For instance, Carrillo-Hermosilla (2006) develops a framework in which a public authority representing the collective interest of society tries to guide the market (individual decisions) by supporting the socially preferable technology with a subsidy. The conditions are investigated under which escaping a lock-in of environmentally unstable practices is possible. It is further examined whether a system can move between equilibria (i.e. be un-locked) without a need for public intervention, and if the timing and the direction of these spontaneous transitions would be socially optimal.

3.7 Coevolution

The term coevolution refers to a situation when two or more evolutionary systems are linked together in such a way that each influences the evolutionary trajectory of the others. It is achieved through reciprocal selective pressures among evolving populations. Notably, linking an evolutionary to a non-evolutionary system does not produce strict co-evolutionary dynamics but co-dynamics of sub-systems (Winder et al., 2005).

Coevolutionary dynamics underlie the process of change in the economic system. Different sub-systems (market, technology, institutions, scientific knowledge, etc.) and within them different groups of entities (producers, consumers, policymakers, universities, etc.) co-evolve leading to irreversible changes in socio-technological trajectories (see van den Bergh and Stagl, 2004; Geels, 2005; Loorbach and Rotmans, 2006). Nevertheless, there are relatively few contributions to coevolutionary modelling available. Most formal applications focus on demand-supply coevolution (Janssen and Jager, 2002; Windrum and Birchenhall, 1998, 2005; Saint-Jean, 2006; Schwoon, 2006, Safarzynska and van den Bergh, 2007). Models of other types of coevolutionary dynamics exist, but are rare. For instance, Noailly (2003, chapter 5) develop a formal coevolutionary framework to analyze the effect of human activity (total pesticide use) on the size and the composition of pest, while Malerba et al. (2005) propose a history friendly model that captures coevolution of computer and semiconductor industries.

A simple evolutionary model of industry dynamics reduces the consumer side to the selection environment, while it assumes processes of innovation creation and selection to be independent (Schot, 1994). As opposed, a coevolutionary model accounts for the process of reciprocal developments and adaptations between heterogeneous groups of consumers and producers. For instance, in a coevolutionary model developed by Saint-Jean (2006), the probability that a consumer adopts a particular good depends on the distinct product characteristics and the relative weights a consumer assigns to each of them. Characteristics to which consumers assign relatively high weights are considered as their priorities. Every period firms invest in quality improvements. Each firm reallocates R&D budget towards characteristics that are priorities for consumers and in which a firm has reached a (sufficiently) high performance level. On the other hand, consumers' preferences evolve over time in response to technological advances and changes in the industry structure. These mechanisms create strong feedbacks between supply and demand. In this context, Saint-Jean discusses policy lessons for innovation diffusions.

In a coevolutionary model by Windrum and Birchenhall (1998, 2005), each firm aims to offer a product design maximizing the average utility of a randomly selected consumer class. The notion of consumer classes is crucial for the emergence of distinct niches. Consumers can move between classes, depending on how well they are served by the incumbent firms. In order to improve its competitiveness, each firm engages in product innovations. It implements a new design only if it yields a higher utility of its target class than the current design. Evolving consumer preferences influence the direction of such product innovations. Formally, firms compete by offering distinct designs or different points in a multi-dimensional (service characteristic, price) space. Their success depends on realizing a utility of the target consumer class *i* above the average level:

$$\varphi_{i,\,t+1} = \varphi_{it} * (w_{it} \colon W_t)$$

where $\varphi_{it}=G_{it}/G$; G is the total number of consumers; G_{it} is the number of consumers in class *i* at time *t;* w_{it} denotes the average utility in the *i* class in time *t*; and W_t is the average level of utility across classes. Consequently, technological change (product succession) is modelled here as an outcome of coevolutionary process involving interactions between consumers and producers.

Building upon Windrum and Birchenhall (1998, 2005), Safarzynska and van den Bergh (2007) propose an agent-based model of demand-supply coevolution to assess the probability of market lock-in depending on various increasing returns to scale. A technological trajectory arises from the interplay of incremental and radical innovations. Evolving consumer preferences affect the direction of innovative activities of firms. For instance, the introduction of a new design is preceded by marketing research to evaluate the consumers' capacity to adopt a novelty. The impact of alternative demand side specifications on the direction of innovative activities of firms is examined and turns out to be important for overall system dynamics.

3.8 Multi-level evolution and group selection

The economy can be seen as a complex, hierarchical structure compromising various levels and subsystems linked together through strong feedback mechanisms Norgaard, (1984), for instance identifies the subsystems: knowledge, values, organization, technology and environment. The micro-interactions among heterogeneous elements lead to the emergence of a higher structure, while variation and selection processes occurring in any of the subsystems affect changes in the total environment. In this context, Potts (2000) has called for a new evolutionary microeconomics based on the technique of discrete, combinatorial mathematics of graph theory. A standard graph theory model is described by the elements $S=(V,E)$ *S*-system, *V*-elements, *E*-connections. According to Potts, connections are crucial for the analysis of dynamics, complexity and system change. Due to the introduction of connections, the notions of emergence and hierarchy can be combined into a single construct, namely a hyperstructure. Formally, this requires recognizing that a system itself can be an element of a higher-level system while an element may itself be a system at a lower level ($S^n=V^{n+1}$).

Gunderson and Holling (2001) develop an alternative complexity model build upon the notion of resilience: panarchy. The idea of panarchy combines the concept of space/time hierarchies with the notion of the adaptive structures. Here, elements of a complex adaptive system, which emerge through local interactions among various components, are nested in one another in a hierarchy. The framework may be applied to evolving systems: economic, ecological or social. For instance, nature (forests, lakes) and humans (cultures, governance structures) can be interlinked through the panarchy in never-ending adaptive cycles of growth, accumulation, restructuring, and renewal. The approach has seen formalisation through multi-agent evolutionary models (e.g., Jansen and Carpenter, 1999).

A multilevel theory of evolution that is receiving much attention presently is built on combination of individual and group selection (Wilson and Sober, 1994; Wilson, 2002; Wilson, 2006; van den Bergh and Gowdy, 2008). Group selection theory tries to elucidate emerging phenomena by taking into account individual and group level processes framed in a multi-level model. There are many relevant models available now (see Bergstrom, 2002; Garcia and van den Bergh, 2007). The minimal structure of a group selection model requires defining a reproducing population composed of groups characterised by more intense or regular interactions among members than with outsiders. Two main approaches can be identified to attain a group formation for the next generation. In a haystack or migration pool type of models, after reproducing, groups are pooled together and then randomly sampled. Alternatively, in propagule pool types of models, groups are formed solely on the basis of a single parent group; in this case offspring are continuously added to the parent group that splits into two after reaching a certain size (Bowles et al., 2004; Trauslen and Nowak, 2006). The second approach makes selection more effective. To further increase the effectiveness of group selection, non-random assortment typical of cultural and economic systems may be included (Bergstrom, 2003).

A wide range of techniques can be used to build a group selection model, such as difference and differential equations, deterministic and stochastic models, spatial models and multi-agent frameworks. For instance, Henrich (2004) decomposes mechanisms underlying a spread of altruistic genes into between-group and within-group components with the use of the Price equation (see section 2.4). Alternatively, Trauslen and Nowak (2006) employ a multi-agent simulation technique. They assume a large population of individuals divided into groups. Within each group, individuals meet in pairwise encounters and play a Prisoner Dilemma game. The realized payoffs determine their fitness and the speed at which individuals reproduce. If a group reaches a certain size, it can split into two. Conditions are then derived for cooperation (altruism) to prevail in such a setting.

Group selection has not been employed in many economic applications. Nevertheless, it may provide the basis for explaining the emergence and evolution of all sorts of institutions.

For instance, selection on the group level may contribute to better understanding of the processes of replication of successful and extinction of ineffective institutions.

3.9 Evolutionary growth

Endogenous growth theories (e.g., Romer 1986, 1990, Grossman and Helpman 1991a,b) try to explain the rate of technological progress referring to human capital or a ratio of skilled labour devoted to R&D research. New growth theories devote more attention to the importance of creativity and innovations in the process. For instance, as a consequence of research activities, new types of capital goods may emerge. From this perspective, long term growth relies on the increasing variety of intermediate products (e.g., Romer, 1990). Under the assumption of diminishing returns of capital, an increase in the variety raises the economy's production potential as the initial capital stock is spread over a larger number of products (see, Aghion and Hovitt, 2006). Alternatively, the driving force behind long term growth may be innovations in productive efficiency or incremental quality improvements. For instance, Aghion and Howitt (1992) develop a model embedding Schumpeter's idea of creative destruction, where the expected growth rate of the economy depends upon the economy-wide amount of research. Each innovation is regarded here as an act of creation aimed at capturing monopoly rents, but which simultaneously destroys rents that motivated the previous discovery. The model relies on a temporal equilibrium, a representative agent and rational expectations, so that it cannot be categorised as an evolutionary-economic approach.

Evolutionary economics instead calls for micro-foundations of growth theories. Models developed in an evolutionary spirit describe diversity of production techniques at the level of individual firms. Opportunities of innovation can be brought about any time, as entities (agents, firms) are constantly involved in search activities. The analysis focuses on structural change and differential growth of a population of firms. In the classic evolutionary model of growth by Nelson and Winter (1982, chapter 12), heterogeneous firms produce the same homogenous product but with different techniques. Dynamics are driven by investment rules and search processes applied to each individual firm. Firm *i*'s desired expansion or contraction (of the capital stock *K*) at time *t* is determined by gross investment $I()$, the output per unit capital A_{it}, price P_t, profit on capital Π_{it}, the depreciation rate of the capital δ, the production cost c, and the market share Q_{it}/Q_t:

$$K_i(t+1)=I(P_tA_{i(t+1)}/c\ ,\ Q_{it}/Q_t,\ \Pi_{it},\ \delta)\ K_{it}+(1-\delta)K_{it}$$

Industry output results from aggregations over individual firms' production levels: $Q_t=\Sigma_iQ_{it}$.

Nelson and Winter built their evolutionary growth model from the bottom-up. They carried out simulations of micro data, which generated patterns consistent with observed macro aggregates. The model initiated a new phase in evolutionary growth theorizing. Later contributions to evolutionary growth theory can be categorised into models following the Nelson and Winter's perspective of micro foundations and evolutionary growth theories formulated at the macro level (Silverberg and Verspagen, 2005). Within modeling in the spirit of micro foundations, two distinct approaches can be identified: (1) Silversberg and Verspagen capital-vintage type of models (e.g., Silverberg and Verspagen, 1994a,b; 1995; Iwai, 2000); and (2) Dosi-type of models (Chiaromonte and Dosi, 1993; Dosi et al., 1994b, Fagiolo and Dosi, 2003), where the single economy is divided into two sectors: the industry fabricating inputs for production and the industry manufacturing final goods. In these models, dynamics at the firm level underlie the growth rate of aggregate output. The common modelling technique is computer simulations. Models differ in the degree of complexity, technology representation, firm behaviour rules. In addition, an extension to a multi-country framework is possible. For instance, Silverberg and Verspagen (1995) develop an evolutionary framework of endogenous growth to explain the convergence between countries' productivity levels. In each country there are q firms producing a homogenous good from a variable number of different types of capital goods. Technological progress is due to stochastic innovation processes: a probability of a successful innovation depends on firm i's research specific R&D level h_i. The firm's specific R&D level is augmented by its distance from the world best practice frontier to allow for cross-county knowledge spillovers:

$$h'_i=h_i(1+\kappa \ln(a^*/a_i^*))$$

where a^* denotes the best world-wide practice technology in terms of labour productivity, a_i^* is the best technology of firm i, and κ is a parameter. Consequently, firms' R&D potential depend on the catch-up term to the world best practice. During model simulations, the convergence among countries productivity levels has been observed.

Contributions to the macro approach to evolutionary growth do not include micro foundations explicitly. Here, dynamics are analysed at the sector or industry level. Different techniques are employed: analytical methods and computer simulations (Silverberg and Verspagen, 2005). The aggregate growth rate of output may be driven by an increase in labour productivity (Conlinsk, 1989; Silverberg and Lehnert, 1993, Meltcafe et al. 2006) or by a growing variety of the economic system (Saviotti and Pyka, 2004, 2008). For instance, Saviotti and Pyka (2004) develop a model in which the emergence of new products and services allows for a continuation of economic development. Here, an industry is defined as a collection of firms producing variants of goods with different characteristics along the same dimensions of the characteristics space. The growth rate of the number of firms in each industry depends on firms' entry and exit, and thus on the size of the potential market, financial availability, the intensity of competition, and a number of mergers and acquisitions. For each industry there exists a saturation level: once it is reached, firms search for new niches, i.e. they innovate radically by offering a new product in the characteristic space. As a result, new sectors emerge and old ones disappear.

To conclude, the list of feasible assumptions and modelling techniques in evolutionary growth theory have not been yet exhausted. In particular, recent models of growth through variety suggest interesting directions for further research.

4. Conclusions

This paper has reviewed techniques and components of evolutionary modelling in economics. The main techniques, namely multi-agent simulations, evolutionary computation, and evolutionary game theory have been described in some detail. In addition, an overview of components or theoretical building blocks of evolutionary economic models has been provided.

The number of evolutionary contributions to multi-agent modeling has increased significantly in recent years. However, establishing a common rule of model specification, conducting simulations and validating results have not yet been achieved. In addition, evolutionary algorithms to study population learning have become increasingly popular. They are predominantly being used to generate innovations, namely through the use of variation operators (mutation and crossover). Finally, within evolutionary game theory, replicator dynamics is the most popular variant of dynamic equations. Others, such as best response, imitation dynamics are less frequently used. Here, selection may dominate system dynamics rendering convergence to a single strategy, since no mechanism generating diversity (the emergence of new strategies) is required. Among evolutionary dynamic equations, only mutator, mutation-replicator and adaptive dynamics allow for errors to occur during the process of replication.

In this paper we have discussed components of formal models like diversity, bounded rationality, innovation and selection, and additional elements, such as diffusion, path dependency and lock-in, co-evolutionary dynamics, multilevel and group selection, and evolutionary growth. There is no agreement on how to conceptualise and formalise (dimensions of) diversity and bounded rationality; concepts are often tailored to the application context or they are determined by pragmatic reasons such as computational convenience. Most of the identified diversity measures fail to account for variety, balance and disparity. Stirling (2007) measure seems to provide the most general framework thus far.

Building blocks like innovations, diffusion and evolutionary growth are well established in evolutionary economics. Innovation is essential for attaining diversity within a population. The specific approaches for modelling innovative activities of firms depend on the formal technique applied, and whether a firm is treated as a single unit or as multiple units of selection. Diffusion is crucial for the information flow and dissemination of a new good or technology into a population. It relates closely to path dependence and lock-in. In this context, various models have been developed to study un-locking policies. Finally, evolutionary growth theory tries to illuminate the economic progress based on stochastic technical change, recombinant selection and capital accumulation. This approach contributes markedly to opening the 'black-box' of standard growth theories, providing more realistic micro foundations and offering a tool for complex system simulations.

In short, evolutionary models of industry (supply side) dynamics have converged to a certain standard. Such a standard is missing for the demand side. However, a full understanding of economy as a complex evolving system requires accounting for interdependencies among various groups and entities, including consumers. This can be only achieved if consumer and producers attain equal balance, especially in the context of coevolutionary interaction models. These, are however still very uncommon. In addition, a group selection approach has been rarely applied to modelling economic phenomena, although it potentially provides a concrete formal theory of selection at multiple levels (individual and group). This could enhance an understanding of the emergence and evolution of human organizations and institutions.

All in all, we can conclude that many concepts still remain in an immature stage of conceptualisation. To foster their development, an attempt to adopt well established theories from other (sub)disciplines (physics, complexity science, psychology, behavioural economics) has been undertaken. However, this process is far from complete. Formalizing the abstract concept of multi-level transition, incorporating results from behavioural economics and translating self-organization and network frameworks to an economic context are the important challenges ahead.

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