



# Jena Research Papers in Business and Economics

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02/2007

*Jenaer Schriften zur Wirtschaftswissenschaft*

Working and Discussion Paper Series  
School of Economics and Business Administration  
Friedrich-Schiller-University Jena

ISSN 1864-3108

**Publisher:**

Wirtschaftswissenschaftliche Fakultät  
Friedrich-Schiller-Universität Jena  
Carl-Zeiß-Str. 3, D-07743 Jena  
[www.wiwi.uni-jena.de](http://www.wiwi.uni-jena.de)

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# Sequencing Mixed-Model Assembly Lines: Survey, Classification and Model Critique

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## Abstract

Manufacturers in a wide range of industries nowadays face the challenge of providing a rich product variety at very low cost. This typically requires the implementation of cost efficient, flexible production systems. Often, so called mixed-model assembly lines are employed, where setup operations are reduced to such an extent that various models of a common base product can be manufactured in intermixed sequences. However, the observed diversity of mixed-model lines makes a thorough sequence planning essential for exploiting the benefits of assembly line production. This paper reviews and discusses the three major planning approaches presented in the literature, mixed-model sequencing, car sequencing and level scheduling, and provides a hierarchical classification scheme to systematically record the academic efforts in each field and to deduce future research issues.

*Keywords:* Mixed-model assembly lines; Sequencing; Mixed-Model Sequencing; Car Sequencing; Level Scheduling

## 1 Introduction

Since the times of Henry Ford and his famous Model-T, product requirements and thereby the requirements of production systems have changed dramatically. Originally, assembly lines were developed for a cost efficient mass production of a single standardized product. Nowadays, a multitude of options (e.g. manual or electric sunroof, air conditioning yes/no) is selectable by the customers, so that the manufacturers of these products need to handle a (theoretical) product variety which exceeds several billions of models (for detailed figures of European car manufacturers see, e.g. Pil and Holweg, 2004; Meyr, 2004; Röder and Tibken, 2006). To enable such a highly diversified product portfolio without jeopardizing the benefits of an efficient flow-production, so called mixed-model assembly lines are utilized. Practical applications stem not only from the automobile industry, but also from many segments of consumer goods industries, e.g. consumer electronics, white goods, furniture and clothing (see Sarker and Pan, 2001; Boysen et al., 2006b).

In a mixed-model assembly line, the application of flexible workers and machinery leads to a substantial reduction in setup times and cost, so that different products can be jointly manufactured in intermixed product sequences (lot size of one) on the same line. In addition to flexible

resources being available, the production processes of manufactured goods require a minimum level of homogeneity. Thus, there usually exists a common base product, which is customizable by the (de-)selection of optional features out of a pre-specified set of options.

In addition to the long- to mid-term assembly line balancing problem (see Baybars, 1986; Scholl and Becker, 2006; Becker and Scholl, 2006; Boysen et al., 2006a+b), mixed-model assembly lines give rise to a short-term sequencing problem, which has to decide on the production sequence of a given number of model copies within the planning horizon, e.g., one day or shift.

Although (almost) any intermixed sequence of models is technically feasible, its significant economic impacts necessitate a thorough planning. In particular the labor utilization at workstations and the spreading of material demand are determined by the model sequence and are, hence, in the center of two different general objectives (c.f. Bard et al., 1994).

- *Work overload:* The installation of varying options typically leads to variations in processing times at work stations. In automobile production, for instance, the installation of an electrical sunroof requires a different amount of time than that of a manual one. If several work intensive models follow each other at the same station, work overloads might occur, which need to be compensated, e.g., by additional utility workers. Work overloads can be avoided if a sequence of models is found, where those models which cause high station times alternate with less work-intensive ones.
- *Just-in-Time objectives:* JIT-centric sequencing approaches focus on the deviating material requirements. Different models are composed of different product options and thus require different materials and parts, so that the model sequence influences the progression of material demands over time. As an assembly line is commonly coupled with preceding production levels by means of a JIT-supply of required materials (e.g. Monden, 1998), the model sequence needs to facilitate this. An important prerequisite for JIT-supply as stated in literature (e.g. Joo and Wilhelm, 1993) is a steady demand rate of material over time, as otherwise the advantages of JIT are sapped by enlarged safety stocks that become necessary to avoid stock-outs during demand peaks. Accordingly, JIT-centric sequencing approaches aim at distributing the material requirements evenly over the planning horizon.

These two basic objectives (minimizing work overload and leveling part usage) were taken up in three alternative sequencing approaches discussed in literature:

- *Mixed-model sequencing:* This approach aims at avoiding/minimizing sequence-dependent work overload based on a detailed scheduling which explicitly takes operation times, worker movements, station borders and other operational characteristics of the line into account.
- *Car sequencing:* To avoid the significant effort of data collection that accompanies mixed-model sequencing, car sequencing attempts to minimize sequence-dependent work overload in an implicit manner. This is achieved by formulating a set of sequencing rules of type  $H_o : N_o$ , which postulate that among  $N_o$  subsequent sequence positions at most  $H_o$  occurrences of a certain option  $o$  are allowed. If a sequence is found which does not violate such rules, work overload can be avoided. Even if avoidance is not fully possible, the work overload is supposed to be the lower the fewer rules are violated.
- *Level scheduling:* While the first two approaches aim at minimizing violations of capacity constraints, level scheduling seeks to find sequences which are in line with the JIT-philosophy. For this purpose “ideal” production rates are defined and models are sequenced in such a manner that deviations between actual and ideal rates are minimized. Although the majority of papers focus on the demand rates of material, the same principles can also be employed to smoothen capacity utilization.

All three sequencing approaches are discussed in this paper. Section 2 establishes the exact scope of this review and identifies the common structural characteristics of all reviewed models. The subsequent sections 3 to 5 address the alternative model sequencing approaches individually and provide a tuple-notation (e.g. see Graham et al., 1979) to classify relevant problem extensions. While existing survey papers (mixed-model sequencing: Yano and Bolat, 1989; car-sequencing: Solnon et al., 2006; level scheduling: Kubiak, 1993; Dahmahl and Kubiak, 2005; Boysen et al., 2006c) exclusively focus on one sequencing approach separately, this work aims at providing an integrated review and classification, which is of particular value for two reasons:

- As both objectives (minimizing work overload and leveling part usage) proved to be highly important for practical applications, hybrid sequencing approaches were developed to optimize both goals simultaneously. These approaches can only be captured in an integrated review and are, thus, considered in Section 6.
- By investigating the relationship between the three sequencing approaches important fields of future research can be identified, as is discussed in Section 7.

The primary aim of this paper is to provide a comprehensive and integrated analysis of sequencing approaches for mixed-model assembly to line out the status quo as well as current and future challenges in this vast field of research.

## 2 Scope of the review

The assembly lines considered in this paper consist of multiple stations arranged along some kind of transportation system, e.g., a conveyor belt, which is steadily moving workpieces from station to station (paced assembly lines). Workers escort the workpieces while processing a set of pre-specified tasks in each production cycle. As soon as all tasks are completed, the worker returns upstream until he reaches the next workpiece or the border of his station. W.l.o.g. it is assumed that the transportation system moves from left to right. Additionally, the following assumptions are presupposed:

- There are no buffers between stations. Thus, the production sequence is determined prior to the launch such that a reordering or preemption of jobs is impossible.
- The workpieces have a fixed location on the transportation system, only their orientation may change.
- The model-mix, i.e. the demand for models throughout the planning horizon, is known with certainty and not subject to changes (static problem), so that there are no rush orders.
- Multiple models contain different materials and require different tasks with individual processing times, such that the demands for material and the utilization of the stations' capacities may change from model to model.
- It is supposed that there are no disturbances, like machine breakdowns or material stock-outs, so that a resequencing (see Coffman et al., 1985; Inman, 2003; Ding and Sun, 2004) is not considered.

As a direct consequence of the restrictions stated above, the massive body of literature investigating throughput estimation or buffer allocation in unpaced buffered assembly lines (see the review papers of Dallery and Gershwin, 1992; Papadopoulos and Heavey, 1996; Gershwin, 2000) is excluded from the survey.

Furthermore the review is restricted to those solution concepts – combining a (theoretical) problem type considered, the mathematical model derived and the (exact or heuristic) solution procedure

$M$	set of models with $m \in M$
$T$	number of production cycles with $t = 1, \dots, T$
$K$	number of stations with $k = 1, \dots, K$
$O$	set of options with $o \in O$
$P$	set of parts (materials) with $p \in P$
$d_m$	demand for copies of model $m$
$a_{mp}$	demand of model $m$ for part $p$
$a_{mo}$	binary coefficient: 1, if model $m$ contains option $o$ ; 0, otherwise
$p_{mk}$	processing time of model $m$ at station $k$
$r_p$	target consumption rate of part $p$
$r_m$	target production rate of model $m$
$H_o : N_o$	sequencing rule: at most $H_o$ out of $N_o$ successively sequenced copies may require option $o$
$BI$	big integer
$x_{mt}$	binary variable: 1, if model $m$ is produced in cycle $t$ ; 0, otherwise
$w_{kt}$	continuous variable: work overload occurring at station $k$ when the $t$ -th unit is processed
$s_{kt}$	continuous variable: position of the operator in station $k$ when cycle $t$ begins
$y_{mt}$	integer variable: total cumulative production quantity of model $m$ up to cycle $t$
$z_{ot}$	binary variable: 1, if a rule with respect to option $o$ is violated in cycle $t$ ; 0, otherwise

Table 1: Notation

applied – which treat the core problem of model sequencing, i.e. the assignment of model copies to the production cycles available.

All presented models share the following characteristics: The planning horizon is divided into  $T$  production cycles (with  $t = 1, \dots, T$ ) and for each model  $m \in M$  the demand  $d_m$  at the end of the planning horizon is given and has to be met. It follows that the sum over model demands is equal to the number of production cycles available,  $\sum_{m \in M} d_m = T$ . The assignment decision is represented by binary variables  $x_{mt}$ , which indicate whether a copy of model  $m$  is produced in cycle  $t$  ( $x_{mt} = 1$ ) or not ( $x_{mt} = 0$ ):

$$x_{mt} \in \{0, 1\} \quad \forall m \in M; t = 1, \dots, T \quad (1)$$

Furthermore, in each production cycle  $t$  exactly one model  $m$  is produced:

$$\sum_{m \in M} x_{mt} = 1 \quad \forall t = 1, \dots, T \quad (2)$$

Finally, over all production cycles the demand for model copies  $d_m$  (with  $\sum_{m \in M} d_m = T$ ) is to be satisfied:

$$\sum_{t=1}^T x_{mt} = d_m \quad \forall m \in M \quad (3)$$

The notation used throughout this paper is summarized in Table 1.

### 3 Mixed-model sequencing

#### 3.1 Problem statement

The line balance of a mixed-model assembly system is typically determined on the basis of a so called joint precedence graph, where the diverging processing times of the respective models are averaged (see van Zante-de Fokkert and de Kok, 1997; Boysen et al., 2006e). In order to avoid excessive capacities, the cycle time is then determined such that it is observed on average over all models. As a consequence, the processing times of some models are higher than the cycle time, whereas that of others are lower. If several models with higher processing times follow each other at the same station, the worker will not be able to return to the left-hand border before the

next workpiece has arrived and thus be consecutively moved towards the right-hand border of the station. This finally results to a work overload whenever the operations of a workpiece can not be finished within the station's boundaries. Depending on the exact type of boundaries considered (see section 3.2.1), this might necessitate one of the following reactions (cf. Wild, 1972, p. 164; Scholl, 1999, p. 69): (i) The whole assembly line is stopped until all stations have finished work on their current workpiece, (ii) utility workers support the operator(s) of the station to finish work just before the station's border is reached, (iii) the unfinished tasks and all successors are left out and executed off-line in special finishing stations after the workpiece has left the last station of the line, or (iv) the production speed is accelerated at the risk of quality defects.

To avoid such costly compensations, mixed-model sequencing searches for sequences where those models with high processing times alternate with less work-intensive ones at each station (Wester and Kilbridge, 1964). For this purpose, models are scheduled at each station and cycle, by explicitly taking into account processing times, worker movements, station borders and further operational characteristics of the line. As a reference point for our classification, we present a basic model which rests on the following additional assumptions (see Domschke et al., 1997; Scholl, 1999, p.96):

- Working across the stations' boundaries is not possible (closed stations).
- The constant velocity of the conveyor belt is normalized to one.
- The operators return with infinite velocity to the next workpiece. This is an adequate simplification whenever the conveyor speed is much slower than the walking speed of workers.
- Work overload is measured by the time or space necessary to complete all work in excess of the respective station's right border (without compensation).
- A work overload has no impact on the succeeding station. Thus, the model assumes that the work overload is either compensated by an utility worker or by accelerating processing velocity.

On the basis of these assumptions a model formulation is given by objective function (4) and constraints (1)-(3) and (5)-(8). Similar formulations can be found in Yano and Rachamadugu (1991), Bard et al. (1992) as well as Scholl et al. (1998).

$$\text{Minimize } MM(X, S, W) = \sum_{k=1}^K \sum_{t=1}^T w_{kt} \quad (4)$$

subject to (1)-(3) and

$$s_{k,t+1} \geq s_{kt} + \sum_{m \in M} p_{mk} \cdot x_{mt} - w_{kt} - c \quad \forall \quad k = 1, \dots, K; t = 1, \dots, T \quad (5)$$

$$s_{kt} + \sum_{m \in M} p_{mk} \cdot x_{mt} - w_{kt} \leq l_k \quad \forall \quad k = 1, \dots, K; t = 1, \dots, T \quad (6)$$

$$s_{kt} \geq 0, w_{kt} \geq 0 \quad \forall \quad k = 1, \dots, K; t = 1, \dots, T \quad (7)$$

$$s_{k1} = 0, s_{k,T+1} = 0 \quad \forall \quad k = 1, \dots, K \quad (8)$$

Objective function (4) minimizes the total work overload. The constraints (5) guarantee that processing of a model copy in cycle  $t+1$  by station  $k$  cannot start before this station has completed the preceding unit in cycle  $t$ . Work is restricted to the stations' borders by constraints (6) and the nonnegativity constraints for  $s_{kt}$  in (7). Equations (8) ensure that the line is in a initial state prior to and at the end of the planning horizon.

In extension to this basic model, a wide range of further operational characteristics and objectives are regarded in the literature. In the following, a classification of mixed-model sequencing is introduced which systematically covers all known extensions.

## 3.2 A classification scheme for mixed-model sequencing

In a previous work, Bard et al. (1992) introduce a preliminary framework for characterizing mixed-model sequencing problems, which is, however, limited to a small subset of operational characteristics discussed in the literature. Thus, a new classification scheme is required, which is constructed as follows:

- The basic model presented in Section 3.1 is chosen as a reference when classifying problem characteristics, so that, if not further stated, it is supposed that the assumptions of the basic model apply and only deviations from this reference model are explicitly provided.
- The classification scheme has been adopted from the widely accepted and successful scheme for machine scheduling of Graham et al. (1979). Other tuple notations which successfully helped structuring complex research fields are, e.g., provided by Brucker et al. (1999) for project scheduling, Dyckhoff (1990) for cutting and packing, and Boysen et al. (2006a) for assembly line balancing. In a tuple notation respective objectives and operational characteristics are referenced by a symbolic notation, so that in spite of the multitude of possible properties of sequencing problems, a particular model can be briefly described by a tuple.
- Any mixed-model sequencing problem will at least consist of three basic elements: Operational characteristics of the stations, characteristics of the assembly line as a whole and an objective to be optimized. Accordingly, the presented classification will be based on those three elements which are noted as tuple  $[\alpha|\beta|\gamma]$ , where:

$\alpha$	characteristics of stations
$\beta$	characteristics of the assembly line
$\gamma$	objectives

*Remark:* One major advantage of the tuple-notation is that any default value, represented by the symbol  $\circ$ , can be skipped when a tuple is actually specified. As explained above, the default values of all attributes constitute the basic model. In the following notation, the symbol  $*$  always indicates that for the respective attribute the alternative values (except for  $\circ$ ) do not exclude each other and can be combined arbitrarily. As all attribute values are chosen such that they are unique, it is not necessary to specify the attribute designators within the tuples.

### 3.2.1 Characteristics of stations

The operational characteristics  $\alpha$  associated with the stations are represented by the six attributes  $\alpha_1$  to  $\alpha_6$ :

**Station boundaries**  $\alpha_1 \in \{\circ, open^\lambda\}$ :

$\alpha_1 = \circ$  If stations are *closed*, the operators must work within the boundary limits of the station. A crossing of station borders for the assembly is not permitted. Closed stations are necessary whenever the working area requires specific environmental conditions, for example, in the case of paint shops or heating chambers (cf. Scholl, 1999, p. 17). Where helpful for immediately interpreting a tuple, we also use  $\alpha_1 = closed$ .

$\alpha_1 = open^\lambda$  The boundaries of *open* stations can be crossed to a certain extent, which is usually subject to some technical constraint, e.g., the limited range of power tools or of the material handling system. A further differentiation can be made by  $\lambda \in \{\circ, left, right\}$  (see Thomopoulos, 1967):

$\lambda = \circ$ : Both station boundaries are open.

$\lambda = left$ : Open-to-the-left stations.

$\lambda = right$ : Open-to-the-right stations.

**Reaction on imminent work overload**  $\alpha_2 \in \{\circ, off, stop, var^\lambda\}$ : The type of reaction on imminent work overload can have a significant impact on the scheduling decision.

$\alpha_2 = \circ$  In the default case, it is assumed that work overloads do not affect starting times of successive stations, so that in spite of an overload at station  $k$  the successive station  $k + 1$  can start processing the workpiece as soon as it reaches its left station border (provided that it finished work on the preceding workpiece). This is always the case if (i) work overload is compensated by the timely assignment of a utility worker, who helps to finish the work within the station's boundaries (see Tsai, 1995), (ii) the processing velocity is increased so that the work is finished in time or (iii) the unfinished tasks are left out and completed later at end-of-line repair shops or special in-line repair stations (see Buzacott, 1999). Irrespective of the measure taken, the line can continue processing work pieces. Therefore, we also use  $\alpha_2 = cont$  where this seems to be a helpful additional information.

$\alpha_2 = off$  The workpiece is taken off the transportation system, e.g., for disposal or off-line completion, so that successive stations have empty cycles.

$\alpha_2 = stop$  The line is stopped as soon as an unfinished workpiece reaches a station's definite border, which induces idle time at all other stations, and is, for instance, the typical compensation in the Japanese automobile industry (cf. Tsai, 1995).

$\alpha_2 = var^\lambda$  Overloads are (seen to be) compensated by variable station borders. Typically, the decision on the stations' length is part of assembly line balancing (see Boysen et al., 2006b) and, thus, already fixed for short-term model sequencing. Nevertheless, such sequencing models can be utilized to either decide on the station extents on the basis of a representative medium-term model mix as an addition to the balancing information or as a surrogate model, e.g., for assembly lines with open stations (Dar-El and Nadivi, 1981). A further differentiation is provided by  $\lambda \in \{\circ, late\}$  (e.g. Goldschmidt et al., 1997):

$\lambda = \circ$ : An *early start model* presupposes fixed left borders of stations, so that each stations may only expand in downstream direction. Each worker starts processing in the first cycle at the left border of his station (reference point 0). When the operator has finished his work he walks back and starts work, when another workpiece has already been launched into the station area. Otherwise, he goes back to the beginning of the station (reference point 0) and waits for the next workpiece.

$\lambda = late$ : In a *late start model* the stations expand in both directions. Workers who completed their tasks move back until they reach the subsequent workpiece, even if it puts them beyond their respective reference point 0, instead of incurring idle time by waiting at left station borders.

**Processing time**  $\alpha_3 \in \{\circ, t^{sto}\}$ : In reality, task times can vary over time, for instance due to complicated manual operations or defaults of machinery (see Tempelmeier, 2003). Sequencing models can consider this phenomenon in different ways.

$\alpha_3 = \circ$  The stochastic nature of task times can be neglected, so that processing times are considered to be static and deterministic. This assumption is especially justified if variations are small, e.g., due to reliable machinery or simple manual tasks (cf. Boysen et al., 2006b).

$\alpha_3 = t^{sto}$  Stochastic processing times: It is assumed that variations of processing times follow a known (or even unknown/partially known) distribution function.



**Concurrent work**  $\alpha_4 \in \{\circ, cc\}$ : Concurrent work enables the worker(s) of a stations to start processing although the previous station has not finished its work on the respective workpiece. This necessitates open stations ( $\alpha_1 = open$ ) as well as workpieces of an appropriate size, so that workers do not impede each other (e.g. Macaskill, 1973), as in the final assembly of cars (Falkenauer, 2005; Boysen et al., 2006a).

$\alpha_4 = \circ$  Concurrent work is not accounted for.

$\alpha_4 = cc$  The sequencing approach considers *concurrent* work.

**Setups**  $\alpha_5 \in \{\circ, setup^\lambda\}$ : A mixed-model assembly line necessitates a considerable reduction of setup times and costs. Otherwise, a production of different models in an intermixed sequence is utterly impossible. Nevertheless, short setup operations, which consume just a fraction of the cycle time, may be relevant, e.g., due to tool swaps (Burns and Daganzo, 1987).

$\alpha_5 = \circ$  Neither setups times nor costs are considered.

$\alpha_5 = setup^\lambda$  Setup operations\* are considered; with  $\lambda \in \{time, cost\}^*$ :

$\lambda = time$ : Setup *times* are considered.

$\lambda = cost$ : Setup *costs* are considered.

**Parallel stations**  $\alpha_6 \in \{\circ, par^\lambda\}$ : Tasks with comparatively large processing times may lead to inefficient line balances, as they force the cycle time to be inefficiently large inducing idle times at other stations. Thus, it might be preferable to install parallel stations, which alternately process identical work contents (e.g. Pinto et al., 1981; Bard, 1989). Parallel stations can be implemented in two different ways (see Inman and Leon, 1994): In *spacial* parallelization, the respective stations are located side by side and are alternately fed with workpieces over a switch. In *chronological* parallelization, a number, say  $p$ , of operators or teams of operators work at the same segment of the serial line covering  $p$  sequential stations. The teams process the workpieces for  $p$  cycles such that they circulate within the line segment each team being responsible for one out of  $p$  successive workpiece.

$\alpha_6 = \circ$  No parallelization is considered.

$\alpha_6 = par^\lambda$  *Parallel* stations are considered; with  $\lambda \in \{\circ, chr\}$

$\lambda = \circ$ : Parallel stations are located side by side.

$\lambda = chr$ : Parallel stations are realized in a chronological manner.

### 3.2.2 Characteristics of the line

The properties of the assembly line can be classified by five attributes  $\beta_1$  to  $\beta_5$  of the second tuple entry  $\beta$ .

**Number of stations**  $\beta_1 \in \{\circ, n\}$ : A real-world assembly line is composed of more stations than one. Nevertheless, a restriction to a given number  $n$  of stations or even a single station might be of value, e.g. for bound computation (Bolat and Yano, 1992a+b; Yano and Rachamadugu, 1991).

$\beta_1 = \circ$  An arbitrary number of stations can be regarded.

$\beta_1 = n$  The number of stations is restricted to a given *positive integer* value  $n$ .

**Homogeneity of stations**  $\beta_2 \in \{\circ, div\}$ : In practical cases, an assembly line may consist of stations with diverging characteristics. For instance, open and closed stations can be mixed throughout the line, which Dar-El (1978) refers to as hybrid lines. The majority of existing sequencing approaches presuppose stations with homogeneous characteristics, which is very limiting for practical applications (Kim et al., 1996).

$\beta_2 = \circ$  All stations have the same characteristics.

$\beta_2 = div$  The assembly line consists of stations with *diverging* characteristics.

*Remark:* If  $\beta_2 = div$  the different station characteristics can be separated by a semicolon within a tuple. For instance, the case  $[closed; open | div | ]$  describes an assembly line, which consists of a mixture of closed and open stations.

**Launching discipline**  $\beta_3 \in \{\circ, vl\}$ : The intervals at which workpieces are launched down the line influence the efficiency of the system. Two main launching strategies can be distinguished (cf. Wester and Kilbridge, 1964):

$\beta_3 = \circ$  *Fixed* rate launching: Consecutive units are placed on the line at the same intervals equal to cycle time  $c$ . An advantage of this launching strategy is that output quantities  $q$  within a given time frame of length  $T$  can be easily calculated by  $q = \lfloor \frac{T}{c} \rfloor$ .

$\beta_3 = vl$  A *variable* rate launching augments the flexibility of operating the line. The launching interval can be dynamically adapted to avert idle times and work overloads.

**Return velocity**  $\beta_4 \in \{\circ, fin\}$ : Whenever a worker has finished all operations, he needs to return upstream to start processing the next workpiece. In the real world, to cover a distance needs some fraction of time, in any case. Nevertheless, in a mathematical model two kinds of premises with regard to the return speed are possible:

$\beta_4 = \circ$  If workers are considerably faster than the movement of the line, return times of workers can either be neglected (e.g., Bard et al., 1992) or treated as fixed and directly added to station times (e.g. Bolat et al., 1994). Because the assumption of infinite return speed of workers simplifies analysis and, in many cases, is a slight relaxation of reality only, most approaches act on this assumption.

$\beta_4 = fin$  If return times vary considerably from cycle to cycle and worker to worker, e.g., due to different processing times and, hence, different walking distances, an approach for mixed-model sequencing should take *finite* return speeds into account.

**Line layout**  $\beta_5 \in \{\circ, u, feeder\}$ : This attribute accounts for the layout of the line:

$\beta_5 = \circ$  The stations are arranged in a serial manner along the flow of the line.

$\beta_5 = u$  A *U-shaped* line layout with crossover stations is used (cf. Miltenburg and Wijngaard, 1994; Monden, 1998). In principle, the physical layout of the line is not relevant for the sequencing decision. However, a U-line allows operators to work on more than one workpiece per cycle at different positions on the line, because crossover stations have access to two *legs* of the U-shaped line simultaneously. This influences the sequencing problem considerably.

$\beta_5 = feeder$  One or more feeder lines flow into a main line (see Groeffin et al., 1989). Peculiarities for model sequencing stem, for instance, from the fact, that operators may perform tasks on both the feeder and the main line at their contact point (Lapierre and Ruiz, 2004).

*Remark:* If other special layouts with a considerable influence on the sequencing problem are introduced, the classification can easily be extended.

### 3.2.3 Objectives

Finally, the optimization will be guided by some objective which evaluates solutions. In the case of multi-objective optimization more than a single objective can be selected out of the set  $\gamma \in \{\circ, len, thru, idle, dis, stop, \star, Co(\lambda)\}^*$ .

- $\gamma = \circ$  Minimize work overload: This objective minimizes the time (or space) by which station borders would be exceeded if no type of compensation was carried out. As work overload is avoided by the assignment of utility workers, this objective is also referred to as “minimization of total utility work” (e.g. Scholl et al., 1998). Where helpful to avoid misunderstandings or to emphasize this objective, we also use  $\gamma = wo$ .
- $\gamma = len$  Minimize line length: This objective can be followed whenever station borders are not yet fixed ( $\alpha_2 = var$ ). It is a surrogate for minimizing investment cost of the transportation system, which are considered to raise in proportion to an increase in length (Bard et al., 1994).
- $\gamma = thru$  The objective “minimize throughput time”, which is defined as time interval between the launching of the first workpiece and the finishing of the last, is highly correlated (see Dar-El and Nadivi, 1991; Bard et al., 1992) to the objective “minimize line length”. This objective also depends on non-fixed station lengths ( $\alpha_2 = var$ ).
- $\gamma = dis$  Minimize maximum displacement of workers from their respective reference point. This objective depends on non-fixed station lengths ( $\alpha_2 = var$ ). A model with this objective is especially used as a surrogate model. For instance, a model for the case [*var* | | *dis*] can be used to represent a real world case of [*open, stop* | | *stop*] as an increasing displacement of workers enlarges the risk of workers hindering each other and, thus, the risk of a line stoppage (cf. Okamura and Yamashina, 1979).
- $\gamma = idle$  Minimize total idle time: Idle time represents unused (non-productive) capacities of machines and workers. They occur whenever a worker has to wait for the workpiece to reach his station ( $\alpha_1 = \circ$ ) or if the worker has to wait for the preceding station to finish work if concurrent work is not possible ( $\alpha_1 = open$  and  $\alpha_4 = \circ$ ). Idle time in itself (if not used as a surrogate objective, see Domschke et al., 1997, p. 264; Scholl, 1999, p.101) has its justification if unused capacities can be profitably utilized for performing other work (Scholl, 1999, p. 21).
- $\gamma = stop$  Minimize the duration of line stoppages: During the time a line is stopped no workpieces can be completed. Thus, this objective minimizes opportunity costs for lost sales.
- $\gamma = \star$  Some other kind of (surrogate) objective trying to increase the utility of the line and/or sequence is considered.
- $\gamma = Co(\lambda)$  Moreover, all of the named time- or space-oriented objectives can be weighted with costs. If a sequencing model aims at minimizing costs, this is abbreviated with  $\gamma = Co$ . Then, the phenomenon, which caused these costs is specified in brackets. For example, a tuple [ | | *Co(idle)*] represents a model, which minimizes total costs of idle time.

*Remark:* In a multi-criteria optimization approach, objectives are separated by a semicolon. For instance, a tuple [ | | *Co(idle); \star*] represents a model, which minimizes cost of idle time and another type of direct or opportunity cost.

In Table 2, the research on mixed-model sequencing is classified according to the presented scheme by assigning the tuple-notation to each relevant publication and its contribution to the research field. If more than one problem version is treated in a paper, the most complex tuple or, if this is

Publication	Notation	Contribution
Bard et al. (1992)	$[open, var^{late}   vl   len], [open, var^{late}   vl   thru]$	M
Bolat (1997)	$[   fin   ]$	M, B, HM
Bolat and Yano (1992a)	$[   1   ]$	M, P, HS
Bolat and Yano (1992b)	$[   1   ]$	P
Bolat et al. (1994)	$[setup     wo; Co(\star)]$	M, B, E, HS
Boysen (2005)	$[par     ]$	M, P, HM
Celano et al. (2004)	$[stop   fin, u   stop]$	M, HM
Chutima et al. (2003)	$[   t^{sto}   thru]$	HM
Dar-El (1978)	$[closed; open, var^{late}   div, vl, fin   len; thru]$	HS
Dar-El and Cother (1975)	$[var     len], [open, var     len]$	B, HS
Dar-El and Cucuy (1977)	$[var   1   len]$	HS
Dar-El and Nadivi (1981)	$[open, var     len]$	HS
Decker (1992, 1993a)	$[open     ]$	HI
Domschke et al. (1997)	$[     Co(wo; \star)]$	M
Felbecker (1980)	$[open, cc     idle]$	M, HS
Goldschmidt et al. (1997)	$[open, var^{late}   1   len]$	P
Kim et al. (1996)	$[closed; open, var, setup^{time}   div   len]$	HM
Koether (1986)	$[open     Co(wo; idle)]$	C
Macaskill (1973)	$[open, cc     wo; idle]$	P, S
Okamura and Yamashina (1979)	$[open, var   fin   dis]$	HI, HS
Sarker and Pan (1998, 2001)	$[open, var   vl, fin   Co(wo; idle)]$	M
Schneeweiß and Söhner (1991)	$[over     ]$	HS, HI
Scholl (1999)	$[     ]$	M, B, E, HM
Scholl et al. (1998)	$[     ]$	M, HM
Sumichrast et al. (2000)	$[     ]$	C, HM
Thomopoulos (1967)	$[open     Co(wo; idle)]$	HS
Tsai (1995)	$[   1, fin   ], [open   1, fin   dis]$	P, E
Wester and Kilbridge (1964)	$[   1   wo; idle]$	HS
Xiaobo and Ohno (1994)	$[stop     stop]$	B, E
Xiaobo and Ohno (1997)	$[stop     stop]$	B, HM, E
Xiaobo and Ohno (2000)	$[stop   fin   stop]$	P, B, HS
Yano and Bolat (1989)	$[     ]$	M
Yano and Rachamadugu (1991)	$[   1   ], [     ]$	M, E, HS
Yoo et al. (2005)	$[cont; stop   fin   idle; stop]$	M, HM

Table 2: Overview on mixed-model sequencing research

not unique, several tuples are reported. Further, we distinguish between the following contributions made by the respective publications:

M	mathematical model	B	bound computation
HI	heuristic improvement procedure	HS	start heuristic for initial solution
HM	meta-heuristic	E	exact solution procedure
P	properties of model/solution space	C	comparison of procedures

## 4 Car sequencing

### 4.1 Problem statement

Instead of a detailed scheduling of work content, car sequencing considers and controls the succession of work intensive product options (e.g., sunroof, air conditioning) in order to avoid work overload. A set of product options can be subject to sequencing rules, which restrict the maximum number of occurrences within a subsequence of a certain length. The car sequencing problem then seeks to

find a sequence of models which meets the required demands for each model without violating the given sequencing rules.

Car sequencing originally stems from practical applications of the automobile industry (e.g. Parrello, 1988; Nguyen, 2005) and was first formulated by Parrello et al. (1986), however, the approach can also be applied to mixed-model assembly problems in other industries.

The sequencing rules are typically of type  $H_o : N_o$ , which means that out of  $N_o$  successive models only  $H_o$  may contain the option  $o$  in order to avoid work overload. Such rules are derived from considering the capacity situation at the stations as expressed by Drexel and Kimms (2001a) as follows:

*“Assume that 60% of the cars manufactured on the line need the option ‘sun roof’. Moreover, assume that five cars (copies) pass the station where the sun roofs are installed during the time for the installation of a single copy. Then, three operators (installation teams) are necessary for the installation of sun roofs. Hence, the capacity constraint of the final assembly line for the option ‘sun roof’ is three out of five in a sequence, or 3:5 for short.”*

With the help of these sequencing rules, the car sequencing problem can be formulated as a constraint satisfaction problem  $CS$ , consisting of constraints (1)-(3) and (9), which enforce the observance of sequencing rules (see Drexel and Kimms 2001a):

$$\sum_{t'=t}^{t+N_o-1} \sum_{m \in M} a_{mo} \cdot x_{mt'} \leq H_o \quad \forall o \in O; t = 1, \dots, T - N_o + 1 \quad (9)$$

Recent publications investigate the applicability of combinatorial optimization techniques for solving instances of car sequencing (Gottlieb et al., 2003; Gagné et al., 2006; Gravel et al., 2005; Flidner and Boysen, 2006). For this purpose, the  $CS$  problem has to be transformed into an optimization problem  $CO$ . Such an optimization model has the advantage of finding a model sequence which minimizes violated rules whenever a solution without violations is not existent. A widespread suggestion for an appropriate  $CO$  is known as the “sliding windows” technique, in which a penalty cost of one is assigned to any violation of a restriction (see Gottlieb et al., 2003; Gravel et al., 2005). This approach tends to double-count violations and further weights violations differently depending on their occurrence in the sequence. Thus, Boysen and Flidner (2006a) as well as Flidner and Boysen (2006) propose an alternative objective function in order to mend the defects. Instead of counting the number of constraints violated by option occurrences, here only the occurrences are counted which actually lead to a violation. The respective optimization model consists of the constraints (1)-(3) and (11) based on additional binary variables  $z_{ot}$  as well as objective function (10):

$$\text{Minimize } CO(X, Y) = \sum_{o \in O} \sum_{t=1}^T z_{ot} \quad (10)$$

subject to (1)-(3) and

$$\sum_{m \in M} \sum_{t'=t}^{\min\{t+H_o-1; T\}} a_{mo} \cdot x_{mt'} - \left(1 - \sum_{m \in M} a_{mo} \cdot x_{mt}\right) \cdot BI \leq H_o + BI \cdot z_{ot} \quad \forall o \in O; t = 1, \dots, T \quad (11)$$

The feasibility problem  $CS$  acts as the base model for the classification of car sequencing in the next section.

## 4.2 A classification for car sequencing

The different car sequencing approaches especially distinguish themselves with respect to the *objective* function and the *operational characteristics* considered. Thus, these two elements are to be specified by a tuple  $[\alpha|\beta]$ :

$\alpha$  objectives  $\beta$  operational characteristics

*Remark:* As in the previous classification, the following special symbols are used in the tuple-notation:

- skipped default value \* combination of attributes possible
- ★ some further (not classified) characteristic or objective

### 4.2.1 Objectives

This tuple entry merely distinguishes whether the feasibility or the optimization problem is considered:  $\alpha \in \{\circ, Opt^\lambda\}$

$\alpha = \circ$  The feasibility problem of car sequencing is considered and, thus, no objective function is optimized.

$\alpha = Opt^\lambda$  The optimization version of the problem, which aims at minimizing violations of sequencing rules, is considered with  $\lambda \in \{\circ, act, \#, ind, \star\}$ :

$\lambda = \circ$ : The number of violated  $N_o$ -subsequences is to be minimized, which is known as the “sliding window” approach (e.g. Gottlieb et al., 2003; Gravel et al., 2005).

$\lambda = act$ : The number of option occurrences, which actually lead to a violation of an  $N_o$ -subsequence is to be minimized (e.g. Boysen and Fliedner, 2006).

$\lambda = \#$ : The number of all excessive options in violated  $N_o$ -subsequences is to be minimized (e.g. Bolat and Yano, 1992b).

$\lambda = w$ : An individual weight for each additional excessive occurrence per option is given as an input data (e.g. Parrello et al., 1986, Smith et al., 1996).

$\lambda = \star$ : Some other kind of related objective not specified within this classification is considered (e.g. Perron and Shaw, 2004; Hindi and Ploszajski, 1994, who insert “empty” models, whenever a violation impedes, and minimize the number of empty cars scheduled).

### 4.2.2 Operational characteristics

The operational characteristics of the assembly line relevant for car sequencing are classified by the four attributes  $\beta_1$  to  $\beta_4$  of the second tuple entry  $\beta$ .

**Number of options**  $\beta_1 \in \{\circ, n\}$ : Usually, more than one selectable option is offered to the customers. Nevertheless, a restriction to a given number of options or even a single one might be of value, e.g. for bound computations (see Boysen and Fliedner, 2006a).

$\beta_1 = \circ$  An arbitrary number of options can be considered.

$\beta_1 = n$  The number of considered options is restricted to a given *positive integer* value  $n$ .

**Hard and soft sequencing rules**  $\beta_2 \in \{\circ, mix\}$ : In practical applications, e.g. at the French car maker Renault, it is often desirable to differentiate between hard and soft sequencing rules (cf. Nguyen, 2005; Gagne et al., 2006; Solnon et al., 2006). Hard rules are related to critical options and have to be met in any case, whereas soft rules may be violated if necessary. In principle, this can be seen as a combination of the feasibility (*CS*) and the optimization (*CO*) version. At Renault the two types of rules are employed to differentiate between crucial operations, whose cumulative succession may cause a full-stop of the line and less critical operations (Solnon et al., 2006):

$\beta_2 = \circ$  Only one type of rule, either hard or soft, are considered. The feasibility problem *CS* presupposes only hard constraints whereas the optimization problem *CO* acts on the assumptions of soft constraints.

$\beta_2 = mix$  Both hard and soft constraints are considered.

**Kind of sequencing rules**  $\beta_3 \in \{\circ, min^\lambda, max^\lambda, cross\}^*$ : In addition to conventional  $H_o : N_o$  rules, other kinds of sequencing rules might be considered by a car sequencing approach (e.g. Hindi and Ploszajski, 1994; Puchta and Gottlieb, 2002).

$\beta_3 = \circ$  Only conventional  $H_o : N_o$  rules are considered. Where helpful for immediately interpreting a tuple, we also use  $\beta_3 = conv$ .

$\beta_3 = min^\lambda$  The minimum number of direct successions is restricted with  $\lambda \in \{\circ, opt\}$ :

$\lambda = \circ$ : At least  $H_o$  consecutive empty cycles have to occur between two occurrences of the same option  $o$ .

$\lambda = opt$ : At least  $H_o$  consecutive cycles containing option  $o$  have to occur between two empty cycles.

*Remark*: Both minimum rules are special cases of conventional  $H_o : N_o$  rules. A minimum of  $H_o$  consecutive empty cycles is equivalent to a conventional sequencing rule of kind  $1 : H_o + 1$ . The same applies to a minimum of  $H_o$  consecutive option occurrences but, beforehand, the respective option occurrences have to be inverted.

$\beta_3 = max^\lambda$  The maximum number of direct successions is restricted with  $\lambda \in \{\circ, opt\}$ :

$\lambda = \circ$ : At most  $H_o$  consecutive empty cycles may occur between two occurrences of the same option  $o$  (e.g. Parrello, 1988).

$\lambda = opt$ : At most  $H_o$  consecutive cycles containing option  $o$  may occur between two empty cycles.

*Remark*: Both maximum rules are special cases of conventional  $H_o : N_o$  rules. A maximum of  $H_o$  consecutive option occurrences is equivalent to a conventional sequencing rule of kind  $H_o : H_o + 1$ . The same applies to a maximum of  $H_o$  consecutive empty cycles but, beforehand, the respective option occurrences have to be inverted.

$\beta_3 = cross$  *Cross-ratio constraints* control the occurrence of interrelated options. For instance, a constraint “ $(A, B) 1 : N$ ” means that no workpiece with option  $A$  is allowed in any partial sequence with  $N$  positions that contains one or more copies of option  $B$ , whereas the occurrences of option  $B$  are independent of  $A$  (cf. Solnon et al., 2006).

Publication	Notation	Contribution
Bolat and Yano (1992a)	$[Opt \mid 1]$	P
Bolat and Yano (1992b)	$[Opt^\# \mid 1]$	P
Boysen and Fliedner (2006a)	$[ \mid 1]$	P, B
Boysen and Fliedner (2006b)	$[Opt^{act} \mid ]$	M
Butaru and Habbas (2005)	$[ \mid ]$	HS, CP
Davenport and Tsang (1999)	$[ \mid ]$	HI, CP
Davenport et al. (1994)	$[ \mid ]$	HI, CP
Dincbas et al. (1988)	$[ \mid ]$	CP
Drexler and Jordan (1995)	$[ \mid ]$	P, CP
Fliedner and Boysen (2006)	$[Opt^{act} \mid ]$	M, B, P, E
Gagné et al. (2006)	$[Opt \mid mix, fix]$	M, HM
Gent (1998)	$[ \mid ]$	P
Gottlieb et al. (2003)	$[Opt \mid ]$	M, HI, HM
Gravel et al. (2005)	$[Opt \mid ]$	M, HM
Hindi and Ploszajski (1994)	$[Opt^* \mid fix]$	HS
Kis (2004)	$[ \mid ]$	P
Parrello et al. (1986)	$[Opt^w \mid ]$	CP
Perron and Shaw (2004)	$[Opt^* \mid ]$	HI
Regin and Puget (1997)	$[ \mid ]$	CP
Smith (1996)	$[ \mid ]$	CP
Solnon (2000)	$[Opt \mid ]$	HM
Smith et al. (1996)	$[Opt^w \mid 1]$	M, HS, HM
Warwick and Tsang (1996)	$[Opt^{act} \mid ]$	B, HM
Yano and Bolat (1989)	$[Opt^w \mid ]$	B, HS

Table 3: Overview on car sequencing research

**Assignment restrictions  $\beta_4 \in \{\circ, fix, range\}^*$ :** Assignment restrictions constrict the production cycles available for an assignment of model copies. These restrictions can also occur in the other sequencing approaches, but, up to now, have merely been considered in car-sequencing (e.g. Bergen et al., 2001; Nguyen et al., 2005).

$\beta_4 = \circ$  Assignment restrictions are not considered.

$\beta_4 = fix$  For a subset of production cycles the models to be produced are already determined, so that only the remaining cycles are available for a model assignment. This may be the case, if workers from consecutive shifts take over work on the fly and production runs continuously over time. In this case, the continuous sequencing problem has to be broken down in virtually separated problems. In order to consider the interdependencies between adjacent shifts, the ultimate models of the preceding shift are taken over as the fixed start-up production cycles of the subsequent shift (Hindi and Ploszajski, 1994; Nguyen, 2005; Solnon et al., 2006).

$\beta_4 = range$  Each model receives a pre-specified range of cycles it can be assigned to. For instance, such constraints might be necessary to meet requirements of quality control (e.g. Bergen et al., 2001; Solnon et al., 2006).

Table 3 summarizes the literature on car sequencing with the help of the developed tuple-notation. The abbreviations for the research contributions specified are the same as in the classification of mixed-model sequencing (see section 3.3). Additionally, the contribution: CP (constraint programming) is considered (for a review on constraint programming and the car sequencing problem see Brailsford et al., 1999).



## 5 Level scheduling

### 5.1 Problem statement

As part of the famous “Toyota Production System” level scheduling received wide attention in research (see the surveys by Kubiak, 1993; as well as Dahmala and Kubiak, 2005) and practical applications (e.g. Monden, 1998; Duplaga et al., 1996, Mane et al., 2002). This approach aims at evenly smoothing the material requirements induced by the production sequence over time, so that a just-in-time supply of material is facilitated and safety stocks are minimized. For that purpose, each material receives a (theoretical) target consumption rate, which is determined by distributing its overall demand evenly over the planning horizon. Hence, a sequence is sought where actual consumption rates of materials are as close as possible to target rates.

Consider a set  $M$  of models each of which consist of different parts  $p$  (with  $p \in P$ ). The production coefficients  $a_{mp}$  specify the number of units of material  $p$  needed in the assembly of one unit of model  $m$ . The target consumption rate  $r_m$  per production cycle is then calculated as follows:

$$r_p = \frac{\sum_{m \in M} a_{mp} \cdot d_m}{T} \quad \forall p \in P \quad (12)$$

Together with the integer variables  $y_{mt}$ , which represent the total cumulative production quantity of model  $m$  up to cycle  $t$ , the part-oriented level scheduling can be modeled as follows (e.g. Joo and Wilhelm, 1993; Monden, 1998; Bautista et al., 1996):

$$\text{Minimize } LS^P(X, Y) = \sum_{t=1}^T \sum_{p \in P} \left( \sum_{m \in M} a_{mp} \cdot y_{mt} - t \cdot r_p \right)^2 \quad (13)$$

subject to (1)-(3) and

$$y_{mt} = \sum_{t'=1}^t x_{mt'} \quad \forall t = 1, \dots, T \quad (14)$$

The objective function (13) aims at minimizing the sum over all deviations of actual from ideal cumulative demands per production cycle  $t$  and part  $p$ . The additional constraints (14) determine the cumulative demands of binary variables  $x_{mt}$  and are introduced for matters of convenience.

In practical applications, where products may consist of thousands of different parts, the resulting problem instances of  $LS^P$  are barely solvable to optimality. Accordingly, literature proposes a class of simplified approximate models, which, under specific prerequisites, are claimed to be sufficient to level part usages without explicitly considering the materials contained in products. This is said to be the case, whenever either all models require approximately the same number and mix of parts (Miltenburg, 1989, p. 193) or part usages are distinct (Kubiak, 1993, p. 261). The objective of these model-oriented level scheduling problems is to achieve a constant production rate  $r_m$  for each model  $m$ :  $r_m = d_m/T \quad \forall m \in M$ . Thus, the objective (13) is replaced by the new objective function (15):

$$\text{Minimize } LS^M(X, Y) = \sum_{t=1}^T \sum_{m \in M} (y_{mt} - t \cdot r_m)^2 \quad (15)$$

subject to (1)-(3) and (14)

As the model  $LS^M$  is the most simple level scheduling model, it is used as a basis for the following classification.

## 5.2 A classification for level scheduling

The different approaches to level scheduling can be distinguished with respect to the *objective* function and the *operational characteristics* of the production system. Thus, these two elements are specified by tuple  $[\alpha|\beta]$ :

$\alpha$  objectives  $\beta$  operational characteristics

*Remark:* As in the previous classifications, the symbol  $\circ$  represents the default and is skipped when a tuple is actually specified.

### 5.2.1 Objectives

Level scheduling always aims at adjusting an actual production schedule to a level target schedule, so that several underlying indicators (e.g., material usage, production rate, workload distribution). Deviations can be penalized with different *weighting* functions, which in turn can be consolidated to a global objective value by different *aggregation* functions. Thus, the actual objective is characterized by three attributes:  $\alpha \in \{\alpha_1, \alpha_2, \alpha_3\}$

**Indicator  $\alpha_1 \in \{P, \circ, W\}$ :** In general, production is defined as an input-output process. Incoming materials are transformed with the help of diverse resources into finished products. Thus, all three elements (input, transformation, output) can possibly be chosen as the indicator which is to be leveled according to the JIT-principal.

- $\alpha_1 = P$       Input: Traditionally, JIT aims at leveling the *parts* supply, so that (work-in-progress) inventory quantities are minimized. Thus, the material demand induced by the model sequence on the assembly line can be considered explicitly.
- $\alpha_1 = \circ$       Output: Although level scheduling primarily seeks to ensure a leveled material usage, the literature suggests that, under special conditions, it is sufficient to level the production rate of *models* without considering material demands explicitly. If useful to clarify presentation, we also use  $\alpha_1 = O$ .
- $\alpha_1 = W$       Transformation: The basic principles of level scheduling are also adopted by researchers to level the *workload* at the stations of the line or at preceding production levels. This type of level scheduling can be seen as an approximate alternative to the other workload oriented approaches of mixed-model sequencing and car sequencing. A level scheduling model for workload smoothing can be easily obtained by re-interpreting the parts  $p$  of model  $LS^P$  as stations and the material coefficients  $a_{mp}$  as the processing time of model  $m$  at station  $p$ , respectively (see Engel et al., 1998).

**Weighting function  $\alpha_2 \in \{euc, abs, \circ\}$ :** The minimization of deviations between actual and target production schedules is just an approximate objective for the underlying cost of late and early material supplies without its own economical justification. Thus, in principle, every possible metric for penalizing deviations is imaginable. The following weighting functions are suggested in literature:

- $\alpha_2 = euc$       *Euclidean* weighting function.
- $\alpha_2 = abs$       A rectilinear weighting function measures *absolute* deviations.
- $\alpha_2 = \circ$       In the default case *squared* deviations are considered.

**Aggregation function**  $\alpha_3 \in \{\text{minmax}, \circ\}$ : Finally, the deviations (of all productions cycles and materials or stations) have to be aggregated to a global objective value.

$\alpha_3 = \text{minmax}$  The *maximum* deviation between actual and target production schedule is to be *minimized*.

$\alpha_3 = \circ$  The *sum* of all separate deviations is to be *minimized*.

*Remark:* There exist some multi-objective approaches in the literature, which consider more than one level scheduling objective simultaneously. To cover these approaches in our classification, multiple objectives are separated by a semicolon. For example, the following tuple  $[P; O, \text{abs}, \text{minmax}]$  specifies a multi-objective approach, which on the one hand minimizes the sum of squared deviations of the material usage and on the other hand minimizes the maximum absolute deviation of the models' production rates.

### 5.2.2 Operational characteristics

Production systems can be considered in different levels of detail. To denote the actual characteristics covered by a level scheduling approach, the attributes  $\beta = \{\beta_1, \beta_2\}$  are employed.

**Number of production levels**  $\beta_1 \in \{\circ, n^{\lambda, \vartheta}\}$ : Most production systems consist of multiple levels. Nevertheless, under the assumption of a final assembly which instantaneously *pulls* its material through the whole supply chain, it may be sufficient to solely consider the final production stage. A level production schedule at the final assembly automatically induces a level production schedule at all preceding stages, if a small lot production is directly triggered by the respective material usages (Bautista et al., 1996a). However, multiple production levels can also be considered explicitly. In this case, the deviations of all production levels are included within the objective function (e.g. Miltenburg and Sinnamon, 1989, 1992, 1995).

$\beta_1 = \circ$  Only the final level of the production system is considered.

$\beta_1 = n$  Multiple production levels are covered explicitly; with  $\lambda \in \{\circ, 2, 3, \dots\}$  and  $\vartheta \in \{\circ, w\}$ :

$\lambda = \circ$ : An arbitrary number of different production levels can be considered.

$\lambda \in \{2, 3, \dots\}$ : The number of production levels covered is exactly  $\lambda$ .

$\vartheta = \circ$ : The deviations at different production levels are weighted equally.

$\vartheta = w$ : Each production level receives an individual weight for its deviations.

**Number of workstations**  $\beta_2 \in \{\circ, k^\lambda\}$ : Although a real-world assembly line consists of more than one station, it can be treated as a single station  $[P]$ , if each material is only consumed at one of the original stations and workpieces may not change positions on the line. Under these two prerequisites, the actual consumption of parts is only delayed by a constant time span. This constant delay has no effect on the leveled part usage. In contrast, stations have to be considered explicitly, if a material is consumed at more than one stations (Xiaobo and Zhou, 1999; Xiaobo et al., 1999). In this case, the points in time materials are actually consumed at the stations have to be considered in detail  $[P|k]$ .

$\beta_2 = \circ$  Stations are not considered explicitly.

$\beta_2 = k$  Multiple stations are covered explicitly with  $\lambda \in \{\circ, 2, 3, \dots\}$ :

$\lambda = \circ$ : An arbitrary number of stations can be covered.

$\lambda \in \{2, 3, \dots\}$ : The number of stations covered is exactly  $\lambda$ .

In Table 4, the existing literature on level scheduling is classified according to our tuple-notation. The abbreviations for the specified contributions of each paper are the same as in the previous classifications (see section 3.3). Additionally, the contribution MAS (multi-agent system) is considered.

Publication	Notation	Contribution
Aigbedo (2000)	[   ]	M, P
Aigbedo (2004)	[ $P, euc$   ]	P
Aigbedo and Monden (1996)	[ $O, euc; P, euc$   ]	C
Aigbedo and Monden (1997)	[ $O, euc; P, euc; W, euc$   $k, n$ ]	M, HS
Balinski and Shahidi (1998)	[   ], [ $abs, minmax$   ], [ $euc$   ]	M, P
Bautista et al. (1996a)	[ $P$   ]	M, HS, HI, E
Bautista et al. (1996b)	[   ], [ $abs, minmax$   ], [ $P, euc$   ]	M, P
Cakir and Inman (1993)	[ $P, abs$   ]	M, HS
Caridi and Sianesi (2000)	[ $P$   $n^{4,w}$ ]	M, MAS
Cheng and Ding (1996)	[   ]	M, HS, C
Ding and Cheng (1993a+b)	[   ]	M, HS
Ding et al. (2006)	[ $O; P; W$   $k$ ]	P, V, HS
Duplaga et al. (1996)	[ $P$   ]	HS
Duplaga and Bragg (1998)	[ $P$   ], [ $P, abs$   $k$ ]	C
Engel et al. (1998)	[ $W$   $k$ ]	M, HS
Inman and Bulfin (1991)	[   ], [ $abs$   ]	M, HS
Inman and Bulfin (1992)	[ $P$   $n^{4,w}$ ]	M, HS
Korkmazel and Meral (2001)	[ $O; W$   $k$ ]	M, HS, E, C
Kovalyov et al. (2001)	[   ], [ $rekt$   ], [ $minmax$   ], [ $abs, minmax$   ]	M, P
Kubiak (2003a)	[   ]	P
Kubiak (2003b)	[ $abs, minmax$   ]	M, P
Kubiak and Sethi (1991)	[   ]	M, E
Kubiak and Sethi (1994)	[   ]	M, E
Kubiak et al. (1997)	[ $P, abs, minmax$   $n^w$ ], [ $P$   $n^w$ ]	M, P, E
Kurashige et al. (1999, 2002)	[ $W, euc$   $k$ ]	M, HS, HM
Leu et al. (1996)	[ $P$   ]	HM
Leu et al. (1997)	[ $P$   ], [ $P; W$   ]	M, HS
Mane et al. (2002)	[ $P, euc$   ]	V
Merengo et al. (1999)	[ $W, abs$   $k$ ]	HS
Miltenburg (1989)	[   ]	P, HS
Miltenburg (2001)	[ $P; W$   $n^w, k$ ]	M
Miltenburg and Goldstein (1991)	[ $P; W$   $n^w, k$ ]	M, HS, P
Miltenburg and Sinnamon (1989)	[ $P$   $n^{4,w}$ ]	M, HS
Miltenburg and Sinnamon (1992)	[ $P$   $n^w$ ]	M, HS
Miltenburg and Sinnamon (1995)	[ $P$   $n^{4,w}$ ]	P
Miltenburg et al. (1990)	[   ]	M, E
Monden (1998)	[ $P$   ]	HS
Morabito and Kraus (1995)	[ $P$   $n^{4,w}$ ]	M, P, HS
Moreno and Corominas (2006)	[ $abs, minmax$   ]	M, E
Ng and Mak (1994)	[   ]	M, B, E
Steiner and Yeomans (1993)	[ $abs, minmax$   ]	M, P, E
Steiner and Yeomans (1996)	[ $P, abs, minmax$   $n$ ]	M, P, E
Sumichrast and Clayton (1996)	[ $P$   ], [ $W$   $k$ ]	C
Sumichrast and Russel (1990)	[ $P, abs$   ]	C
Sumichrast et al. (1992)	[ $P$   ], [ $W$   $k$ ]	HS, C
Ventura and Radhakrishnan (2002)	[ $W$   ]	M, HS, B
Xiaobo et al. (1999)	[ $P, abs$   $k$ ]	M, HS
Xiaobo and Zhou (1999)	[ $P, abs$   $k$ ]	HS, HM
Zaramdini (2003)	[ $P; W$   $k$ ]	M, HM
Zeramdini et al. (2000)	[ $P; W$   $k$ ]	HS

Table 4: Overview on level scheduling research

## 6 Classifying hybrid model sequencing approaches

Both “minimization of work overload” and “leveling part usage” turned out to be valuable but often conflicting objectives for model sequencing. Consequently, several hybrid approaches have been developed. In order to classify those approaches, the three classifications are unified to a single hierarchical classification scheme. At the first level the distinction between the three model sequencing approaches is made. This is clarified by the abbreviations: MM (mixed-model sequencing), CS (car sequencing), and LS (level scheduling). At the second level, the respective approaches are denoted with their tuple-notation. Thus, a hybrid approach combining a model-oriented level scheduling objective with car-sequencing constraints (e.g. Drexel and Kimms, 2001a+b) can be specified by  $CS[[]] + LS[[]]$ .

In addition, the following further operational characteristics and objectives other than those already mentioned are especially relevant for practical applications:

- *Setup operations:* The manufacturing process of a car includes three stages: body shop, paint shop and final assembly (cf. Inman and Schmeling, 2003; Meyr, 2004). In some cases those shops are closely coupled by the same conveyor belt, so that a once scheduled model sequence moves unaltered through the whole plant (if at all, intermediate buffers, so called selectivity banks, see Spieckermann et al. (2004), between the shops allow for a limited relocation of models). That is why model sequencing in the automobile industry has to take care for the concerns of the paint shop and the final assembly, concurrently, whereas the body shop is said to be uncritical (cf. Gagné et al., 2006; Solnon et al., 2006). In the paint-shop a switch to a different color necessitates a flush out of the old paint before starting to apply the new color, so that sequence-dependent setup costs are evoked. In the literature, sequencing approaches can be found which exclusively deal with the paint-shop problem (e.g. Lustig and Puget, 2001; Spieckermann et al., 2004), and others, which couple car sequencing and paint shop batching by a multi-objective approach (e.g. Bolat, 1994; Inman and Schmeling, 2003; Gagné et al., 2006; Solnon et al., 2006). A related question is considered by McMullen et al. (2003), who minimize the number of tool swaps stemming from setup operations.
- *Due dates:* Especially in an assembly-to-order environment (see Mather, 1989), model copies are often subject to due dates. Thus, model sequencing might have to regard these due dates of models, for instance, by minimizing the number of late models (see Lovgren and Racer, 2000; Zhang et al., 2000). This presupposes that the model sequence is determined for a comparatively long planning horizon (for an example see Bergen et al., 2001). In the automobile industry model sequencing is conducted once per day or shift (see Boysen et al., 2006d). Within such short time frames due dates seem to be of minor relevance, and are mainly influenced by preceding planning tasks like master scheduling which specifies the shift a model is produced in (see Ding and Tolani, 2003; Bolat, 2003; Boysen et al., 2006d).
- *Assembly line balancing:* The short-term decision problem of model sequencing is heavily interdependent with the long- to mid-term assembly line balancing. The line balance decides on the assignment of tasks to stations and thus determines the work content and material usage per station and model. This decision constitutes the input data of model sequencing. Thus, the amount of overload resulting from a planned model sequence by itself is a measure of efficiency for the achieved line balance. That is why some authors have proposed a simultaneous consideration of both planning problems (McMullen and Frazier, 1998a; Kim et al., 2000b+c, 2006; Miltenburg, 2002; Sawik, 2002; Bock et al., 2006). A simultaneous approach is, however, only viable under very special conditions as both planning problems have completely different time frames. The balancing decision has a typical planning horizon of several month, so that daily model mix is typically not known at this point in time. Detailed forecasts of future model sales are often bound to heavy inaccuracies. It can thus be more meaningful to generally anticipate the sequencing decision at the higher planning level

as part of a hierarchical planning approach (Domschke et al., 1996; Scholl, 1999, ch. 3.4; Boysen et al., 2006d).

Table 5 summarizes the hybrid approaches presented in literature by stating the (hierarchical) tuple-notation, additional aspects (if present) and the respective methodological contribution (see section 3.3.).

## 7 Discussion of models

Open research can be divided into two categories: (i) research needs within the respective model family, which can be identified with the help of our classification scheme-based literature analysis, and (ii) research needs regarding the relation of models.

### 7.1 Research needs within a model family

**Mixed-model sequencing** is discussed in the literature since its first formulation by Wester and Kilbridge (1964). Due to its claim of deriving detailed model schedules a wide range of problem extensions have been introduced and studied by research thus far. Nevertheless, the following operational characteristics might deserve a closer investigation:

- Stochastic task times ( $\alpha_3 = t^{sto}$ ), which frequently occur under manual labor, have, except for the fuzzy-set approach of Chutima et al. (2003), hardly ever been considered.
- Special layouts such as U-shaped assembly lines ( $\beta_5 = u$ ) are widely applied in many industries (see Miltenburg, 2000), but have so far been covered only by Celano et al. (2003). This also holds for feeder lines ( $\beta_5 = feeder$ ), which, in spite of their practical importance (see Boysen et al., 2006b) have not yet been considered at all.
- In real world applications assembly lines often consist of a mix of open and closed stations (see Kim et al., 1996). Moreover, the reactions on imminent work overload ( $\alpha_2$ ) and the ability of concurrent work ( $\alpha_4 = cc$ ) can diverge between different segments of an assembly line. Up to now, only very few approaches consider diverging station characteristics ( $\beta_2 = div$ ).

Moreover, all operational characteristics can arise (nearly) in any combination, so that mixed-model sequencing requires flexible solution procedures, if to solve real-world assembly problems.

The **car sequencing problem** has only recently gained wider attention due to its practical relevance. As a consequence, future research efforts are required from both a methodological and a model oriented point of view. Very recently traditional combinatorial optimization detached the dominance of constraint programming as recommendable solution techniques. This development could be furthered by the construction of even more efficient heuristic and exact solution procedures. Nevertheless, a combination of optimization techniques and constraint programming seems especially promising. For instance, a coupling of the “specification method” of Drexel et al. (2006) and the branch and bound approach of Fliedner and Boysen (2006) could result to even more powerful exact solution procedures. As car sequencing focuses on the combinatorial aspect of the sequencing decision, a direct consideration of line characteristics seems impractical. Nevertheless it needs to be investigated, how different line characteristics translate into sequencing rules and to what extent they might be modeled, e.g. via cross-ratio constraints ( $\beta_3 = cross$ ) or assignment restrictions ( $\beta_4 = range$ ).

The **level scheduling problem** has been extensively studied since the 1990s and plenty powerful heuristic and exact solution procedures have been developed for any meaningful tuple. Thus, open problems for future research should mainly deal with the question of finding the best solution concept for different practical applications.

Publication	Notation	Additional aspects	Contribution
Bard et al. (1994)	$MM[var   fin   len] + LS[M, abs   ]$	–	M, B, E, HM
Bergen et al. (2001)	$CS[Opt^*   max, range]$	setup operations, parallel lines, diversity of work at stations	HI, E, B, CP
Bock et al. (2006)	$MM[open, cont; of f   vl   Co(wo; \star)]$	assembly line balancing; worker assignment	M, HM
Bolat (1994)	$MM[     Co(\star)]$	minimize setup cost	M, B, E
Celano et al. (1999)	$MM[stop     stop] + LS[P, abs   ]$	accommodation of permanent stuff among each other	HM
Chew et al. (1992)	$CS[Opt   mix, range] + LS[P   ]$	minimize setup cost	HM
Cordeau et al. (2006)	$CS[Opt   mix, fix]$	minimize number of setups	HM
David and Chew (1995)	$CS[Opt   mix, range] + LS[P   ]$	minimize setup cost	HM
Drexel and Kimms (2001a+b)	$CS[   ] + LS[   ]$	–	M, B
Drexel et al. (2006)	$CS[   ] + LS[abs   ]$	–	M, B, E
Gagné et al. (2006)	$CS[Opt   mix, fix]$	minimize number of setups	M, HM
Hyun et al. (1998)	$MM[   ] + LS[abs   ]$	minimize setup cost	M, HM
Kara et al. (2006)	$MM[var     \star]$	u-line balancing	HM
Kim et al. (2000b, 2006)	$MM[var     \star]$	u-line balancing	HM
Kim et al. (2000c)	$MM[     ]$	assembly line balancing	HM
Kotani et al. (2004)	$MM[cont; stop   fin   stop] + LS[P   ]$	–	M, HS, HM
Lochmann (1999)	$MM[open     wo; idle] + LS[abs   ]$	–	M, HM, C
Lovgren and Racer (2000)	$LS[P   ]$	minimize number of late models	M, HI
Mansouri (2005)	$LS[   ]$	minimize number of setups	HM
McMullen (1998, 2001a+b), McMullen et al. (2000), McMullen and Frazier (2000), McMullen and Tarasewich (2005)	$LS[   ]$	minimize number of setups	HM
Miltenburg (2002)	$LS[P   n^w]$	U-line balancing	HM
Mohammadi and Ozbayrak (2006)	$LS[W   k]$	minimize number of setups	HM
Ponambalam et al. (2003)	$MM[   ] + LS[P   n^{4,w}]$	minimize number of setups	HM
Prandstetter and Raidl (2005a+b)	$CS[Opt^\#   fix]$	minimize number of setups	M, HM
Puchta and Gottlieb (2002)	$LS[P   ] + CS[Opt^w   conv; min^{opt}; max^{opt}]$	–	HI, HM
Ribeiro et al. (2006)	$CS[Opt   mix, fix]$	minimize number of setups	HM
Risler et al. (2004)	$CS[Opt   mix, fix]$	minimize number of setups	HM
Tamura et al. (1999)	$LS[P; W   k]$	synchronization of parallel line segments	M, HS, HM, E
Tavakkoli-Moghaddam and Rahimi-Vahed (2006)	$MM[   ] + LS[abs   ]$	minimize setup cost	M, HM
Yu et al. (2006)	$MM[var     thru] + LS[ euc   ]$	–	HM
Zhuqi and Shusaku (1994)	$LS[P, abs   ]$	intervals of material supply	HS

Table 5: Overview on hybrid model sequencing

	workload-oriented approaches	material supply-oriented approaches
high	mixed-model sequencing (MM)	—
	car sequencing (CS)	part-oriented level scheduling (LS <sup>P</sup> )
low	workload-oriented level scheduling (LS <sup>W</sup> )	model-oriented level scheduling (LS <sup>M</sup> )

Figure 1: Classification of model sequencing approaches

## 7.2 Research needs regarding the relations of models

Although hybrid models have been developed which consider both capacity and material aspects, the exact relationship between the sequencing approaches has hardly been studied. This is surprising as they are proposed for the same or at least very similar production environments. Open research questions in this field are outlined in the following. For this purpose, the alternative sequencing approaches are arranged in a portfolio matrix with regard to the general objective and the level of planning detail considered (Figure 1).

Among the **workload-oriented approaches**, mixed-model sequencing covers the problem in greatest detail as it provides an explicit schedule of all models at stations. If these time- or space-related measures are weighted with their respective costs (cf. Thomopoulos, 1967; Bolat et al., 1994; Sarker and Pan, 2001), there is a direct mapping between the objective of the sequencing approach and the companies’ general objective of profit maximization, provided that the model sequence has no impact on sales. In contrast to that, car sequencing observes capacity on a more aggregated level. Here, time- or space-related measures like “minimizing the work overload” are only considered indirectly via sequencing rules.

A different, yet also indirect approach is pursued by the workload-oriented level scheduling which smoothes cumulative stations times. Unlike the former two, this approach ignores capacity constraints completely and is thus on an even more aggregate level.

The level of planning detail does not yield general conclusions regarding the superiority or inferiority of models. On the one hand, a more detailed model might capture the “true” economic objective and all influencing factors better, on the other hand, the effort for data collection increases. If this data cannot be anticipated appropriately, the quality of plans resulting from more detailed models is not necessarily better. This general trade-off in modeling can only be decided with respect to specific real-world case.

However, scientific research can assist practitioners by quantifying this trade-off in simulation settings adopted from representative real-world assembly lines. By doing so, important questions regarding the suitability of models in real-world situations can be answered:

- What is the most appropriate objective function for car sequencing? Up to now, there are some alternative objective functions (see section 4.2.1) but very few advice on which objective function to choose. Some first attempts on answering this question can be found in Bolat and Yano (1992b) as well as Flidner and Boysen (2006), but a comprehensive computational comparison should reveal further insights.
- Not only the objective function is an important part of car sequencing being an appropriate aggregate model for the “minimization of work overload”, but also the question of how to derive sequencing rules, so that maintaining them actually minimizes work overloads, seems most important. Present research completely ignores this question and presupposes given



sequencing rules. Only Bolat and Yano (1992a) face this question, but their research acts on the very limiting assumption of merely one zero-one option per station (only two recurrent station times). How sequencing rules have to be derived, when more than two station times (due to more than one option per station) accrue, is completely up to future research.

- How good is the workload-oriented level scheduling ( $[W|]$ ) compared to mixed-model and car sequencing in minimizing work overloads in practical settings? Preliminary results are provided by Sumichrast et al. (1992), who schedule resulting model sequences obtained by workload-oriented level scheduling in a simulated real-world production environment. Unfortunately, they do not compare these results to mixed-model and car-sequencing, so that further research seems recommendable.

Among the **material-oriented sequencing** approaches, model-oriented level scheduling is an approximate model for the part-oriented level scheduling, which is supposed to be appropriate whenever:

- “Products require approximately the same number and mix of parts.” (Miltenburg 1989, p. 193).
- “Outputs [of preceding production levels] required for each different product are distinct.” (Kubiak 1993, p. 261).

Both prerequisites seem quite limiting with regard to today’s markets, in particular in an assembly-to-order environment (see Mather, 1989). In such a setting, there is no compelling reason why customers’ choices should either result in products requiring (almost) the same number and mix of parts or completely different parts. Moreover, a reasonable application of model-oriented level scheduling presupposes that actually more than one copy at least of some products is to be produced. Otherwise all products compete for the same middle position within the sequence, so that a meaningful regulation of part usage is ruled out. In many fields of application, however, the product variety is so extraordinary large (see Section 1) that not a single model is produced more than once within a shift (Meyr, 2004).

The computational studies of Sumichrast and Russel (1990) and especially Zhu and Ding (2000) reveal considerable deviations from the solutions of part-oriented level scheduling and thus further underline the limited applicability of model-oriented level scheduling.

Part-oriented level scheduling is in itself merely a surrogate model for the underlying economic factors, as a leveled distribution of the material demands does not yield a direct economic value. It is nevertheless said to facilitate a JIT-supply, as the need for costly safety-stocks and flexible capacities is reduced. This raises the question for the real-world prerequisites under which part-oriented level scheduling is a suited surrogate model to minimize the actual model sequence-dependent costs of material supply. Preliminary computational studies on the material flows resulting from optimal sequences of part-oriented level scheduling in simulated production environments are provided by Sumichrast and Clayton (1996) as well as Sumichrast et al. (2000).

Part-oriented level scheduling seems especially adequate whenever material demands are directly pulled throughout the whole production system. This assumption is generally fulfilled if preceding production levels are located in immediate vicinity of the final assembly and are directly coupled via a Kanban system or feeder lines. Today’s trend of reducing vertical integration, however, leads to a decrease in the number of parts produced in-house. In the automobile industry, the majority of parts are delivered Just-in-Time or even Just-in-Sequence (see Meyr, 2004) by trucks in discrete time intervals. Whenever production stages are more loosely coupled, the adjustment towards an ideal production rate seems much less relevant. In the extreme case, if parts are delivered only once prior to each production shift, part-oriented sequencing becomes dispensable. Instead master scheduling should aim at smoothing part usage evenly over all shifts (see Ding and Tolani, 2003).

If parts are delivered repeatedly during a shift, a leveling of materials needs to consider delivery quantities and intervals (e.g. Pleschberger and Hutomi, 1993; Aigbedo, 2004). Such a detailed

approach which on the one hand directly addresses the costs associated with material supplies in a multi-stage production system and on the other hand regards diverging requirements of different materials is completely missing up to now and would be a valuable contribution of future research.

## 8 Conclusion

This paper gives a comprehensive review of the three major approaches for sequencing mixed-model assembly lines as well as related multi-criteria and hybrid problems. A hierarchical classification scheme is developed, which covers all proposed problem extensions in a systematic manner. The classification provides insights in the status quo of research in each field, but also allows a comparison of the different approaches with regard to the level of planning detail and the actual problem characteristics considered. As was established, there seems to be a dire need for theoretical and empirical results concerning the relationship between the three approaches and the resulting consequences for business practise. In addition to the tremendous academic effort spent on describing the mathematical properties of alternative models and deriving suitable solutions procedures, there is an apparent lack of empirical research evaluating the goodness of fit of alternative sequencing approaches for real-world applications. Therefore, contributions which provide insights into this complex matter are to be seen as especially valuable.

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