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CONTROL OF A LOADING BRIDGE USING NONLINEAR MODEL PREDICTIVE CONTROL

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ABSTRACT

In this paper, we apply a nonlinear model predictive control (NMPC) approach to control a highly nonlinear loading bridge system. A multiple shooting combined with a collocation on finite elements is used to realize the NMPC. That means, the multiple shooting algorithm is used to convert the optimal control problem to a nonlinear program (NLP). Thus, the degree of freedom of this NLP consists of a parameterized controls and initial conditions of the state trajectories in each subinterval. The collocation on finite elements is used to compute the state variables and their gradient at the end of each subinterval. Applying this approach to control the loading bridge shows a high accuracy and computation efficiency for the integration of the model equation. The controlled loading bridge is considered to be disturbed in each feedback measurement. The numerical solution is realized in the framework of the numerical algorithm group (NAG) and IPOPT to solve the NLP problem and in C/C++ for the rest of computation.

Index Terms**—** Loading bridge, nonlinear model predictive control, multiple shooting, collocation on finite elements.

1. INTRODUCTION

Model predictive control (MPC) or receding horizon control (RHC) is considered as one of the most important control algorithms. It has been applied in almost all of industrial fields such as petro-chemical, biotechnical, electrical and mechanical processes. Many different algorithms have been presented in the literature to discuss the MPC. These algorithms differ only amongst themselves in the model used to represent the process in the noise and the cost function to be minimized. MPC are used in many complex application such as control of diversity of process ranging robot manipulators [1], clinical anaesthesia [2], cerement industry [3], chemical engineering application [4, 5] and steam generator or servos [6].

The corner stone of the NMPC is solving the nonlinear optimal control repeatedly. Simple optimal control problems are always treated with indirect method which applies the first order optimality conditions of variation [7]. Within the indirect method, the inequality constraint will be converted into an equality constraint and solution of optimal control problem will be transformed to the solution of a two point boundary value problem and then be solved numerically. On the other hand, more complicated optimal problem is normally solved by the direct methods which reformulate at the beginning the optimal control problem into nonlinear programming (NLP) problem. This NLP can be solved by sequential quadratic programming (SQP). Where inequality and equality constraints and highly nonlinear complex optimal control problems can be treated and we can also deal with highly nonlinear complex control problems [8]. In all direct methods, a parametrization and discretization methods to states and control are used. Basically three different classifications of direct methods are found in the literature to solve the optimal control problem [9]; sequential approach (e.g. single direct method), simultaneous approach (e.g. collocation on finite elements) and hybrid method (e.g. direct multiple shooting) which combines the advantages of the simultaneous method with major advantages of the sequential method, for more details the reader may refer to [10, 11].

In this paper, we use a new direct approach to the solution of a NMPC problem which is a challenging and highly nonlinear loading bridge system. This approach is a combination of multiple shooting with collocation on finite elements. The multiple shooting is used to convert a NOCP into a NLP problem, and then the collocation on finite element is used for the integration of the ODE system and the computation of the gradients required. This control strategy possesses a higher efficiency, since it requires a smaller amount of computation expense compared with the existing NMPC algorithms.

This paper is organized as follows, Section 2 reviews the issues of the NMPC. In Section 3 the existing method of multiple shooting will be addressed. In section 4 the combined approach of multiple shooting and collocation on finite elements is presented. Section 5 presents the simulation results optimal and model predictive control of the loading bridge and in Section 6 the work is concluded.

2. MODEL PREDICTIVE CONTROL

The idea of the model (based) predictive control (MPC or MBPC) is to create a formulation that solves on-line the finite horizon optimal control problem subject to system dynamics and constrains involving states and controls [4, 12, 13, 14, 15, 8, 16].

The explicit system model is used to predict the process output at a future time horizon by finding a control sequence that minimize a certain objective function using the theory of the optimal control [17]. Let us assume that the system that we want to control is explicitly described by a set of differentiable algebraic equation (DAE) with initial condition $x(t_0) = x_0$. Then the optimal control of the system in the *prediction* horizon ($T_p = t_f - t_0$) according the optimal control theory is $u(t)$. Based on measurements obtained at t, the optimal controller $u(t)$ predicts the future dynamic behavior of the system state over the prediction horizon T_p such that the open-loop performance objective functional is optimized. Using the MPC, first the optimal control $u(t)$ is applied to the system over the control horizon ($T_c < T_p$). If there are no disturbances and no model-plant mismatch, and if we can do the optimization for infinite horizons, then we can apply the input $u(t)$ for the system for all times $t \geq t_0$. But when we have disturbances and model-plant mismatch to the system, then applying the input function to system is not possible in general. Therefore, a feedback mechanism will be implemented, that means, the open loop manipulated input function will be implemented only until the next feedback measurement is available. Fig (1) shows the basic principle of the model predictive control. Fig.

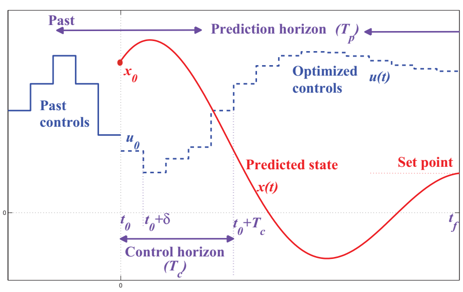

Fig. 1. Principle of model predictive control (MPC)

2 shows the basic structure of the MPC implementation to various dynamic processes. We use the system model to predicts the future plant outputs, based on past and current values and optimal future control creation as well as the system constraints.

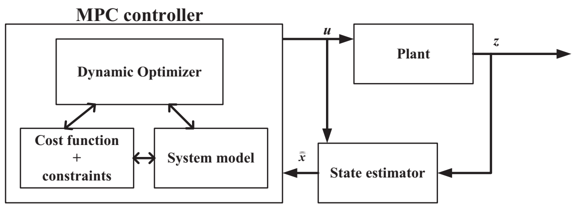

Fig. 2. Basic MPC control loop.

If there are no disturbances on the system state, the feedback at every sampling time is not necessary since the prediction is exact. When disturbances are considered, however, the states should be measured and feeded back each control horizon. In this paper we will assume that the disturbance $d(t)$ is added to states loading bridge system at the moment of measurement feedback. Fig. 3 shows an example of NMPC in which an additive disturbance d_k is added to the state at every control horizon.

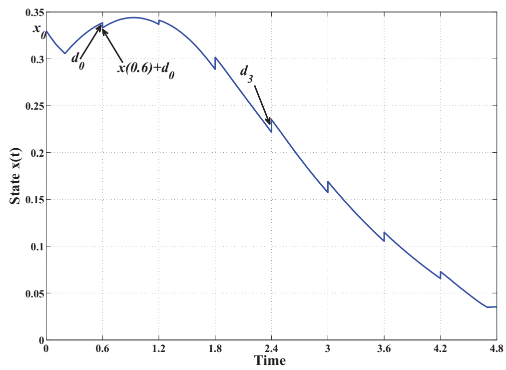

Fig. 3. Disturbance representation of system model

In this paper, we apply a new nonlinear model predictive control (NMPC) approach to control a loading bridge. A multiple shooting combined with a collocation on finite elements is used to realize the NMPC. The optimal control problem will be transformed into NLP using the method of direct multiple shooting which discertizes the prediction horizon into equal subintervals and parameterizes both the control trajectories and initial conditions of the state trajectories in each subinterval.

3. DIRECT MULTIPLE SHOOTING

General optimal control problem to be solved using NMPC can be defined as:

$$\begin{aligned}
&\min_{x,u} \quad \left(\int_{t_0}^{t_f} L(x(t), u(t), t) dt \quad + E(x(t_f)) \right). \\
&\text{s.t.} \\
&\dot{x}(t) = f(x(t), u(t), t), \qquad t \in [t_0, t_f]. \qquad (1) \\
&x(t_0) = x_0. \\
&g(x(t), u(t), t) \geq 0, \qquad t \in [t_0, t_f]. \\
&r(x(t_f)) = 0,
\end{aligned}$$

where L and E are scalar functions and t_0 and t_f are initial and final time of the receding horizon, respectively, $x(t) \in R^n$ and $u(t) \in R^m$ are the state and control vectors, respectively, with $f : R^n \times R^m \to R^n$, $x(t_0) \in R^n$ is an inial state vector, $g : R^n \times R^m \to R^o$ is a path constraint.

We discertize the moving horizon into N finite subintervals $[t_i, t_{i+1}]$, where $t_0 < t_1 < ... < t_N = t_f$ and $i = 0, 1, ...N$. We use the approach in [16] to solve the finite optimal control defined problem 1. This approach converts the optimal control problem into a nonlinear programming (NLP) problem:

$$\begin{aligned}
&\min_s F(s). \\
&\text{s.t.} \quad \begin{aligned} G(s) &= 0 \\ H(s) &\leq 0 \end{aligned}
\end{aligned} \qquad (2)$$

where the vector $s = [s_0^x, s_0^u, s_1^x, s_1^u, ..., s_N^x, s_{N-1}^u]$, $G(s) = \begin{pmatrix} s_0^x - x_0 \\ s_1^x - x(t_1; s_0^x, s_0^u) \\ \vdots \\ s_{N-1}^x - x(t_1; s_{N-2}^x, s_{N-2}^u) \\ r(x_N) \end{pmatrix}$ and $H(s) = \begin{pmatrix} g(s_0^x, s_0^u) \\ \vdots \\ g(s_N^x, s_N^u) \end{pmatrix}$, here s_i^x denotes the parameterized initial values of the discretized system states and s_i^u denotes the parameterized controls in the subinterval $[t_i, t_{i+1}]$. For simplicity, we assume here that controls are parameterized as piecewise constant. We solve the set of ordinary differential equations (ODEs) in the equality constraint $G(s) = 0$ and compute the state trajectories and the and state values at the nodes $i = 0, ..., N$ using the collocation on finite elements.

4. DIRECT MULTIPLE SHOOTING COMBINED WITH COLLOCATION ON FINITE ELEMENTS

To solve this NLP problem the method of sequential quadratic programming (SQP) approach such as interior point optimizer (IPOPT) [18] we need at the first step to compute all of the information needed for each SQP. We use the method of collocation on finite elements to compute state trajectories and the values of the state trajectories ant the discertized nodes. In each element we use M collocation (interpolating) points and the orthogonal Lagrange polynomials (T) to compute state trajectory [19]. Therefor, the state trajectories in each subinterval (element) will be:

$$\begin{aligned}
x_i(t) &= \sum_{j=0}^{M} T_{ij}(t).x_i(t_{ij}). \\
\text{where} \quad T_{ij}(t) &= \prod_{\substack{k=0 \\ k \neq j}}^{M} \frac{t - t_{ik}}{t_{ij} - t_{ik}}
\end{aligned} \qquad (3)$$

Fig. 4 shows the principle of collocation on 3 elements, where the number of collocation points in one element is $M = 3$. Here the indices j and k denote the finite element and the collocation point in each finite element, respectively. We consider each shooting subinterval to be equal each finite element and thus the number of finite elements is equal to the number of shoots for discretizing the dynamic system over the time horizon. An element means a time interval in which both the collocation and shooting will be carried out for the discretiztion.

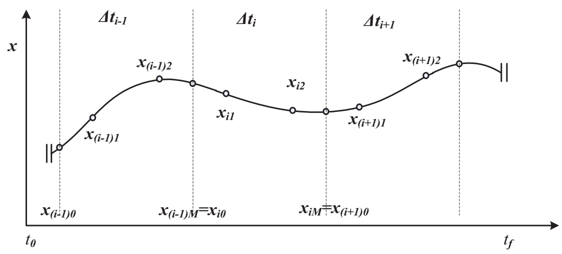

Fig. 4. Principle of collocation on finite elements

To compute the approximated $x(t)$ we have to determine the orthogonal Lagrange polynomial $T_{ij}(t)$ and to solve the collocation points $x_i(t_{ij})$. We use the transformed locations of Legendre polynomials' roots to represent the time values t_{ik}, $i = 0, ..., N$, $k = 0, ..., M$ [20, 19, 21]. In addition, to determine the values $x_i(t_{ij})$ from the ODE, we need to rewrite Eq. (3) and combine the ODE equation $\dot{x}_i(t) = f(x_i(t), u_i(t), t)$, $x_i(t_i) = s_i^x$, $t \in [t_i, t_{i+1}]$ with Eq. (3), then we yield

$$\frac{d}{dt}\left(\sum_{j=0}^{M} \prod_{\substack{k=0 \\ k \neq j}}^{M} \frac{t - t_{ik}}{t_{ij} - t_{ik}} .x_i(t_{ij}) \right) - f(x_i(t), s_i^u, t) = 0 \qquad (4)$$

Substituting the M collocation points in Eq. (4) and writing it in a matrix from leads to:

$$F(x_{i0}, X_{ik}, s_i^u) = \dot{W}_{ik}X_{ik} + \dot{W}_{i0}I_M x_{i0} - f_i(x_i(t_{ik}), s_i^u) = 0 \quad (5)$$

where $W_{ik} = \begin{bmatrix} T_{i1}(t_{i1}) & \cdots & T_{iM}(t_{i1}) \\ \vdots & \ddots & \vdots \\ T_{i1}(t_{iM}) & \cdots & T_{iM}(t_{iM}) \end{bmatrix}$, $W_{i0} = \begin{bmatrix} T_{i1}(t_{i0}) & \cdots T_{iM}(t_{i0}) \end{bmatrix}^T$, I_M is a unit matrix and $X_{ik} = \begin{bmatrix} x_i(t_{i1}), \cdots, x_i(t_{iM}) \end{bmatrix}^T$. We solve now the nonlinear Eq. (5) on the collocation points $x_i(t_{i1}), \cdots, x_i(t_{iM})$, by using the Newton-Raphson method or one of roots finding techniques and we define, also, the beginning of each finite element to be equal to the last collocation point of the previous element, that means

$$\begin{aligned} &t_{i0} = t_{(i-1)M}, \quad i = 1, ...N, \\ &t_{00} = t_0, \\ &t_{(N-1)M} = t_N = t_f. \end{aligned} \quad (6)$$

As the next step, the gradients $\frac{\partial F}{\partial s}$, $\frac{\partial G}{\partial s}$ and $\frac{\partial H}{\partial s}$ need to be computed. By finding first order Taylor-expansion of Eq. (5) leads to

$$\frac{\partial F}{\partial xi0}\Delta x_{i0} + \frac{\partial F}{\partial X_{ik}}\Delta X_{ik} + \frac{\partial F}{\partial s_i^u}\Delta s_i^u = 0 \quad (7)$$

According Eq. (7), the sensitivities can be computed by

$$\frac{\Delta X_{ik}}{\Delta x_{i0}} \cong \frac{\partial X_{ik}}{\partial x_{i0}} = -\left(\frac{\partial F}{\partial X_{ik}}\right)^{-1} \frac{\partial F}{\partial x_{i0}} \quad (8a)$$

$$\frac{\Delta X_{ik}}{\Delta q} \cong \frac{\partial X_{ik}}{\partial s_i^u} = -\left(\frac{\partial F}{\partial X_{ik}}\right)^{-1} \frac{\partial F}{\partial s_i^u} \quad (8b)$$

where $\{F : R^n \times R^{Mn} \times R^m \to R^{Mn}\}$. Equation (8) is a linear equation system and thus can be solved by a LU factorization using the forward and backward substitution. From the solutions of Eq. (8), the last column of the Jacobian matrices $\frac{\partial X_{ik}}{\partial x_{i0}} = \begin{bmatrix} \frac{\partial x_{i1}}{\partial x_{i0}} & \cdots & \frac{\partial x_{iM}}{\partial x_{i0}} \end{bmatrix}^T$ and $\frac{\partial X_{ik}}{\partial s_i^u} = \begin{bmatrix} \frac{\partial x_{i1}}{\partial s_i^u} & \cdots & \frac{\partial x_{iM}}{\partial s_i^u} \end{bmatrix}^T$ lead to the sensitivity $\frac{\partial G}{\partial s}$. The sensitivity $\frac{\partial H}{\partial s}$ can be obtained from the direct differentiation of the states with respect to s.

The solution using the multiple shooting method depends mainly on the SQP iteration. Inside each SQP iterate gradient values of the objective function and Jacobian of the constraints as well as the approximated Hessian need to be computed. We use three collocation points to solve the model equations and compute the sensitivities. If a higher accuracy is needed, more collocation points should be used, but this will lead to more computation time. With the definition of the last point of an element as the initial point of the next element i.e. $t_{iM} = t_{(i+1)0}$, the locations of the collocation points mapped with the shifted roots of Legendre polynomials are $t_{i0} = t_i$, $t_{i1} = 0.127(t_{i+1} - t_i) + t_i$, $t_{i2} = 0.5635(t_{i+1} - t_i) + t_i$ and $t_{i3} = t_{i+1}$.

5. LOADING BRIDGE

We consider a loading bridge with the mechanical setup as shown in Fig. 5. It consists of a cart which can be moved along a metal guiding bar by means of a transmission belt. A winch drive is mounted on top of the cart to change the length of a rope. The control task

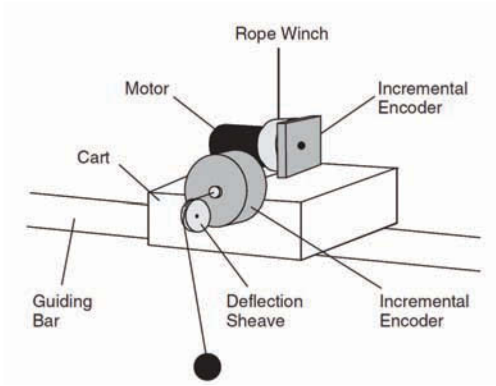

Fig. 5. Elements of the loading bridge

is to move the cart by means of the transmission belt to defined point at the metal guiding bar. In addition the movement should be carried-out with a minimum energy. The optimal control problem is formulated as

$$\min_{x,u} \ \frac{1}{2}\int_0^5 (x^T(t)Qx(t) + u^T(t)Ru(t))dt. \quad (9a)$$

s.t.

$$\dot{x}_1 = x_2. \quad (9b)$$

$$\dot{x}_3 = x_4. \quad (9c)$$

$$\dot{x}_5 = x_6. \quad (9d)$$

$$\dot{x}_2 = \frac{(u_1 - F_r x_2)(m_2 + \frac{\theta}{R_T^2}) + g m_2 \frac{\theta}{R_T^2}\sin(x_3)\cos(x_3)}{(m_1 + m_2\sin^2(x_3))(m_2 + \frac{\theta}{R_T^2}) - m_2^2\sin^2(x_3)} - \frac{(u_2 - F_{Tr}x_6)m_2\sin(x_3) + x_4^2 x_5 m_2 \frac{\theta}{R_T^2}\sin(x_3)}{(m_1 + m_2\sin^2(x_3))(m_2 + \frac{\theta}{R_T^2}) - m_2^2\sin^2(x_3)}. \quad (9e)$$

$$\begin{aligned} \dot{x}_4 = &\frac{(u_2 - F_{Tr}x_6)m_2\sin(x_3)\cos(x_3)}{x_5[(m_1 + m_2\sin^2(x_3))(m_2 + \frac{\theta}{R_T^2}) - m_2^2\sin^2(x_3)]} \\ &- \frac{g\sin(x_3)[m_1(m_2 + \frac{\theta}{R_T^2}) + m_2\frac{\theta}{R_T^2}]}{x_5[(m_1 + m_2\sin^2(x_3))(m_2 + \frac{\theta}{R_T^2}) - m_2^2\sin^2(x_3)]} \\ &- \frac{x_4^2 x_5 m_2 \frac{\theta}{R_T^2}\sin(x_3)\cos(x_3)}{x_5[(m_1 + m_2\sin^2(x_3))(m_2 + \frac{\theta}{R_T^2}) - m_2^2\sin^2(x_3)]} \\ &- \frac{2x_4x_6[(m_1 + m_2\sin^2(x_3))(m_2 + \frac{\theta}{R_T^2}) - m_2^2\sin^2(x_3)]}{x_5[(m_1 + m_2\sin^2(x_3))(m2 + \frac{\theta}{R_T^2}) - m_2^2\sin^2(x_3)]} \\ &- \frac{(u_1 - F_r x_1)(m_2 + \frac{\theta}{R_T^2})\cos(x_3)}{x_5[(m_1 + m_2\sin^2(x_3))(m_2 + \frac{\theta}{R_T^2}) - m_2^2\sin^2(x_3)]} \end{aligned} \quad (9f)$$

$$\dot{x}_6 = \frac{(u_2 - F_{Tr}x_6)(m_1 + m_2\sin^2(x_3))}{(m_1 + m_2\sin^2(x_3))(m2 + \frac{\theta}{R_T^2}) - m_2\sin^2(x_3)} + \frac{g m_1 m_2 \cos(x_3) + x_4^2 x_5 m_1 m_2}{(m_1 + m_2\sin^2(x_3))(m2 + \frac{\theta}{R_T^2}) - m_2\sin^2(x_3)} - \frac{(u_1 - F_r x_2) m_2 \sin(x_3)}{(m_2 + \frac{\theta}{R_T^2}) - m_2^2\sin^2(x_3)}. \quad (9g)$$

$$x_1(5) = 0.4 \text{ and } x_5(5) = 1, \quad (9h)$$

$$x(0) = [0.5\ 0\ 0\ 0.6\ 0]^T. \quad (9i)$$

$$[0\ -1\ -1\ -1\ 0\ -1]^T \le x \le [1\ 1\ 1\ 1\ 1\ 1], \quad (9j)$$

$$[-22.5\quad -3.75]^T \le u \le [22.5\quad 3.75]^T. \quad (9k)$$

where the states x_1, x_2, x_3, x_4, x_5, and x_6 are the position of the cart , the velocity of the cart, the angle of the rope, the angular velocity of the rope, the length of the rope in m and the differentiation of the length of the rope, respectively. The controls u_1 and u_2 are the driving force of the cart and the winch, respectively. The parameters of the loading bridge model Eqs. (9) are shown in Table 1. For more details on this example see [22].

Fig. 6 shows the system states and controls as the result from the solution of the finite optimal control problem Eqs. (9) by defining $Q = \text{dig}([200\ 0.5\ 0.5\ 0.5\ 1\ 1])$, $R = \text{dig}([0.05\ \ 0.05])$. To solve problem (9) we apply the optimization technique presented in Sections 3 and 4 by dividing the time horizon into 20 subintervals. The resulted NLP includes 189 variables with 149 constraints. We used the IPOPT 3.4.0 to solve the NLP and NAG mark 8 to solve the discertized model equations and compute the sensitivities. The computation was done using a PC with an intel processor "Pentium 4, 3 GHz and 1G Byte RAM". The solution took 570 ms and gave the of the objective function value with $J = 0.9993$. Fig. 7 shows the system states and controls if we apply the model predictive methodology presented in Section 2 with prediction (optimization) horizon $T_p = 5s$ and control horizon $T_c = 2s$. We assume that a disturbance d is added to system states at the beginning of each repeated optimization process.

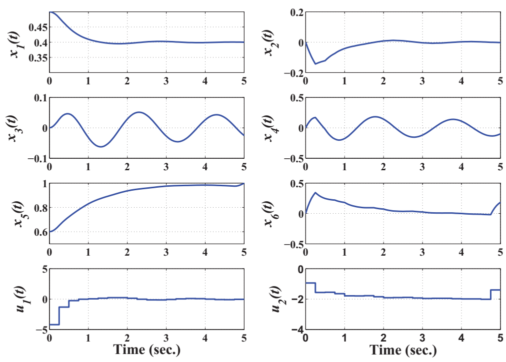

Fig. 6. Optimal control problem solution (states and controls) of loading bridge, $x_i(t)$, $i = 1, ..., 6$ and $u_1(t)$ and $u_2(t)$

Table 1. Parameters of the loading bridge model

Parameter	Description	Value
m_1	Cart mass	5.5 kg
m_2	Winch mass	0.2 Kg
F_r	Friction constant on the cart	13 N
F_{tr}	Friction constant on the winch	2 N
θ	Moment of inertia of the winch	2.25×10^{-4} kg.m^2
g	Gravitational acceleration	9.81 m/s^2
R_T	Winch radius	3 cm

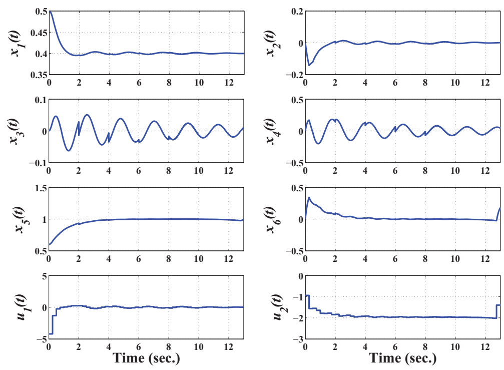

Fig. 7. MPC solution (states and controls) of the loading bridge, states: $x_i(t)$, $i = 1, ..., 6$, and controls: $u_1(t)$ and $u_2(t)$

6. CONCLUSION

In this paper we implemented a novel algorithm for NMPC to highly nonlinear loading bridge system. This approach is a combination of the multiple shooting method and the collocation method. That means the optimal control problem will be converted into NLP using multiple shooting method and then the function values and gradients required in the NLP will be computed using collocation method. We used piecewise constant for controls and the three-point-collocation for states to parameterize the vector of optimization variables. The implementation was done with the framework of the numerical algorithm group (NAG) and IPOPT in the C/C++ environment. In addition, the simulation results of the loading bridge was presented in both cases offline optimization and receding horizon control. Since this NMPC algorithm is more efficient for a large-scale NMPC problem, we conclude, also from the results, that this algorithm is well suited for the control of the loading bridge.

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