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## Tense Distorted Condition of Bimorphous Piezoelectric Console Plate

### ABSTRACT

Problems of bending of transversely polarized bimorphic thin piezoelectric plate of the class 6mm are considered.

### THE EQUATIONS OF PROBLEM

The problem of bending of the thin bimorph piezoplate plates rigidly fastened among them through their faces is being considered. The plates are polarized on thickness. Constant potentials are set on the facing of bimorphs, i.e. the electric field is constant, and the reverse piezoelectric effect is not taken into account (Fig. 1).

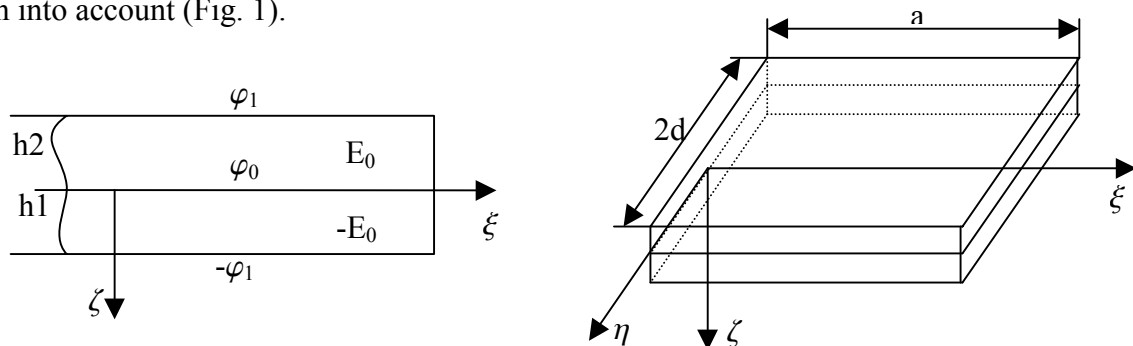


Fig. 1. The platte geometry

The equation of bending of a plate in the specified coordinate system looks like:

$$\Delta^2 \tilde{W} = 0, \quad \text{where } \Delta = \frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \xi^2}.$$

Expressions for generalized bending moments and shearing efforts for the chosen system of coordinates look like.

Generalized bending moments:

$$M_\xi = -D \left( \frac{\partial^2 \tilde{W}}{\partial \xi^2} + \nu \frac{\partial^2 \tilde{W}}{\partial \eta^2} \right) + h_1 h_2 S E_0, \quad M_\eta = -D \left( \frac{\partial^2 \tilde{W}}{\partial \eta^2} + \nu \frac{\partial^2 \tilde{W}}{\partial \xi^2} \right) + h_1 h_2 S E_0.$$

Generalized shearing efforts:

$$\tilde{N}_\xi = -D \frac{\partial}{\partial \xi} \left[ \frac{\partial^2 \tilde{W}}{\partial \xi^2} + (2-\nu) \frac{\partial^2 \tilde{W}}{\partial \eta^2} \right], \quad \tilde{N}_\eta = -D \frac{\partial}{\partial \eta} \left[ \frac{\partial^2 \tilde{W}}{\partial \eta^2} + (2-\nu) \frac{\partial^2 \tilde{W}}{\partial \xi^2} \right],$$

where

$$D = \frac{2h^3}{3} \left( C_{11} - \frac{C_{13}^2}{C_{33}} \right), \quad \nu = \left( C_{12} - \frac{C_{13}^2}{C_{33}} \right) \left( C_{11} - \frac{C_{13}^2}{C_{33}} \right)^{-1}, \quad S = e_{31} \left( 1 - \frac{C_{13}}{C_{33}} \right).$$

On the basis of the analysis model problems the decision of a problem of bending of bimorph piezoelectric plate rigidly jammed on the left edge is offered at the following boundary conditions:

$$\begin{aligned} \tilde{W}(0, \eta) = 0, \quad \frac{\partial \tilde{W}}{\partial \xi}(0, \eta) = 0, \quad \text{At } \xi = 0, \\ \frac{\partial^2 \tilde{W}}{\partial \xi^2} + \nu \frac{\partial^2 \tilde{W}}{\partial \eta^2} = \beta, \quad \frac{\partial^3 \tilde{W}}{\partial \xi^2} + (2-\nu) \frac{\partial^3 \tilde{W}}{\partial \xi \partial \eta^2} = 0, \quad \text{at } \xi = 1, \\ \frac{\partial^2 \tilde{W}}{\partial \eta^2} + \nu \frac{\partial^2 \tilde{W}}{\partial \xi^2} = \beta, \quad \frac{\partial^3 \tilde{W}}{\partial \eta^3} + (2-\nu) \frac{\partial^3 \tilde{W}}{\partial \xi^2 \partial \eta} = 0, \quad \text{at } \eta = \pm \alpha, 0 < \xi < 1. \end{aligned}$$

$\tilde{W}$  - function of a deflection,  $\beta$ - parameter determining bending moments caused by the influence of an electric field,  $\alpha$  - parameter determining a ratio of the sides of the plate.

## THE SOLUTION OF PROBLEM

The problem is solved on the basis of decomposition in the series of Furje. The decision looks like.

$$\begin{aligned} \tilde{W}(\xi, \eta) = \sum_{n=1}^{\infty} \frac{ch(\lambda_n \xi)}{ch \lambda_n} \{ A_n [th(\lambda \xi) - g_1 \xi] + B_n \xi [th(\lambda_n \xi) - g_2] \} \cos(\lambda_n \eta) + \\ + C_n \frac{ch(\chi_n \eta)}{ch(\chi_n \alpha)} \{ \eta \chi_n (\nu - 1) th(\chi_n \alpha) th(\chi_n \eta) + \chi_n \alpha (1 - \nu) - (1 + \nu) th(\chi_n \alpha) \} \cdot \\ \cdot \sin(\chi_n \xi) + F_3 \xi^3 + F_2 \xi^2 + F_1 \xi, \end{aligned}$$

where  $\lambda_n = \pi n / \alpha$ ,  $\chi_n = \pi n$  and factors  $g_n^{(1)}$  also  $g_n^{(2)}$  look like:

$$g_n^{(1)} = \frac{\lambda_n (\nu - 1) th(\lambda_n)}{\lambda_n (\nu - 1) - 2th(\lambda_n)}, \quad g_n^{(2)} = \frac{\lambda_n (\nu - 1) th(\lambda_n) - 2}{\lambda_n (\nu - 1) - 2th(\lambda_n)}.$$

Constant factors of the decision are determined from the reduced infinite system of the equations which turn out after substitution of the decision in boundary conditions.

The introduced form of the solution is tested on the basis of the solution of the several model problems conceding the solution in a self contained kind. Authors consider, that this approach to the solution of the indicated class of problems is more effective than finite elements method since allows calculating more precisely derivatives of the second and third order, i.e. the moments and shearing forces.

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