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## PROCEDURE OF THE CALCULATION OF THE COMBINED AND EXPANDED UNCERTAINTY OF THE TYPE B IN MEASUREMENT RESULT AT THE ABSENCE OF THE INFORMATION ABOUT COMPONENTS DISTRIBUTION

*Abstract* The presenting work describes the difficulties of the procedure of the calculation combine and expanded uncertainty of type B components proposed in the Guide [1]. The procedure of the evaluation of the combined uncertainty of type B of the measurement result under the assumption of the absence of information about the components distribution is proposed. This procedure is based on the concept of the worst probability distribution.

The evaluation of the measurement uncertainty consists of the three basic stages: 1) the establishment of the uncertainty components, their distribution density and also other parameters, including of their cross-correlation; 2) the computational procedures of the determination of the parameters of the uncertainty of the result of measurement; 3) interpretation and presentation of the obtained measurement results and their uncertainties in the required form.

The basic problem of the known procedure [1] of the uncertainty evaluation lay in the next: 1) this procedure does not consider some number of summarized components, and 2) the value of uncertainty is calculated on the basis of the classical procedure of randomization. It is well known that a small measurement results number has a strong influence at the value of confidence coefficient, which determines the estimation of the expanded uncertainty of type A. From other side in practice during the real measurement process always only the unique copy of each measuring meters are used and measurement is made only in the unique measuring circuit. Therefore in such measurements the conditions of randomization are not satisfied. In this case each component of the type B measurement uncertainty in the measurement process is considered constant, but with unknown value. For the purpose of analysis simplification the time, temperature etc. parameter drifts are not considered.

It is known that the density distributions of different parameters of measuring devices characterize the instrument type (so-called general population of this type instruments), but not the concrete version of it. The mentioned above density distribution is very important for the measurement instrumentation producer. However, the producer usually does not indicate these distributions in the documentation, but he obviously gives only the bound ( $\pm d_i$ ) values of the instrument parameter  $D_i$ . During the measurement process each parameter takes only one unique value, by the other words the value of the parameter is the sample by one element size of the general population. This relates to the all parameters of all used instruments.

For the purpose of the determination of the combined expanded uncertainty of the type B it is necessary to have the distribution density of summary effect caused by the different influences. If the summary result of different uncertainty sources is determined by enumerating the convolution of the

density distributions of each source then the obtained result should be interpreted as the expanded uncertainty of the type B, which would take place under conditions that the measurement is realized using all general population of assigned type of the each instrument.

Assumption of uniform distribution as the worst distribution of influence value is the main problem of the proposed in the Guide [1] procedure of the type B combine and expanded uncertainty calculation. This assumption is correct in the information theory of the measurements, where is applied to the measurement data set, but is not true for the unique unknown value. I.e., the uniform distribution can be used for the statistical summing up of type A components at the absence knowledge about their component distributions. In [1] implicitly supposed that all components of the type B (from different sources) with the known bound values are the certain general population representatives, and therefore it can be used a uniform distribution. In practice the components of the type B even in the theoretical sense does not form the general population. Therefore this approach is not justified and available and the enumerating of the summary effect and have to be determined by another way.

We proposed such approach, which is been based on the adoption of the worst distribution of the type B with the known bound values of uncertainty influence source. As the worst distribution would be accepted discontinuous distribution in the form of two delta-functions which are placed at the bound value points. By the way, precisely, the similar approach was used in the well known work of Taylor [2]; however, for the summing up of random components, i.e. the type A.

With this assumption, the procedure of the expanded uncertainty of the type B evaluation is reduced to the convolution calculation of the corresponding number of discontinuous distributions mentioned above. This convolution does not present computational problems here, since the values of summary uncertainty are equal to all possible algebraic sums and differences of the component bound values. If a component number is equal to  $m$ , their boundary values are equals to  $d_1, d_2, \dots, d_m$ , then, taking into account the symmetry of distribution, all possible positive values  $d_{s_j}$  of discrete convolution can be determined using the following recursion formula

$$i =: 2 \dots m; \quad j =: 2 \dots 2^{m-2};$$

$$d_{s_1} = d_1; \quad d_{s_{i,j}} = \left| d_{s_{i-1,j}} + d_i \right|; \quad d_{s_{i,j+2^{m-2}}} = \left| d_{s_{i-1,j}} - d_i \right|.$$

The negative values of discrete convolution have the opposite sign. After the sorting of the obtained values the accumulated distribution  $P(d_{s,k})$  (in the range  $P \geq 0,5$ ) can be determined after the expression

$$P(d_k) = 0,5 + \frac{k}{2^m}; \quad k =: 1 \dots 2^{m-1}.$$

Using these formulas it is possible to calculate the confidence interval. Can be shown that for the confidence level of  $P=0,9$  expanded uncertainty may be calculated only in the case of 5 or more components, for the confidence level of  $P=0,95$  it can be calculated in the case of 6 and more components, for the confidence level of  $P=0,99$  it can be calculated for 8 components. In all other cases it is necessary to use either a smaller level of confidence or to use a bound value as the sum of the bound values of components.

## References

- [1] Guide of the Expression of Uncertainty In Measurement. International Organisation for Standardisation. Switzerland, 1993, 1995.
- [2] Taylor J.R. An Introduction to Error Analysis. The study of Uncertainties in Physical Measurements. University Science Books Mill Valley, California 1982.

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