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An Adaptive Linearization Approach for Intelligent Sensor Devices

ABSTRACT

The study proposes a gradientless method for adaptive partitioning of the measurement range of industrial intelligent sensor devices. The method gives two benefits: first, low-cost microcontrollers could be used for hardware realization of the industrial sensor device; second, the optimal subdivision of the measurement range reduces the space in the memory of the microcontroller necessary for storage of the parameters of the linearized characteristic. The application of the method is illustrated for a sensor device with thermocouple.

1. INTRODUCTION

The proliferation of sensor and instrument busses has introduced new ways to interface and communicate with transducers. The widespread availability of microelectronics, computers, and networks provide a good opportunity to network large arrays of transducers to measure, characterize, model, and monitor many large structures, machinery, and mechanical systems. Nevertheless, these new ways have been useful only to segments of the transducer community. In addition, the increased use of large number of transducers has also created a need for keeping track of the transducers and their associated manufacturer data. The availability of economical off-the-shelf memory chips has helped to implement built-in electronic data sheets in small transducers. This has significant contribution in building smart transducers with self-identification capability through the use of electronic data sheets. The transducer community also recognized the need for a common way of connecting these smart transducers and thus began the work on the IEEE 1451 Smart Transducer Interface Standard.

The IEEE 1451 provides a set of common interfaces for connecting transducers (sensors and actuators) to existing instrumentation and control networks and lays a path for the sensor

community to design systems for future growth. It is intended to provide an easy upgrade path for connectivity of products from any manufacturer of transducers or networks.

In recent years the dominant claim in numerous European industrial projects was to reduce the complexity of intelligent sensor devices preserving their compatibility with the standard IEEE 1451. Main objective was to design a standard interface, that on the one hand is conformed with the constraints imposed by the restricted communication and data management resources, and on the other, allows for optimal implementation upon existing 8-bit microcontrollers.

The present work is devoted to a problem concerning the optimal design of low-cost microcontrollers and specialized sensor interface chips with incorporated microcontrollers for realization of intelligent temperature sensors [4, 6]. The constraints enforced from the architecture of the microcontrollers can be overcome using external software tools [7]. In this way, such complicated tasks as processing, correction and transmission of sensor signals can be reduced to more simple tasks easily solvable with cheaper microcontrollers.

The main idea in this contribution is to reduce the computational cost for correction of the sensor's functionality on the basis of an optimal piecewise linear approximation, constructed for the inverted sensor characteristic. The meaning behind the notion of optimality should be understood in the following sense: on the basis of an iterative procedure the measurement range is partitioned into a set of linear segments with irregular length so that the measurement error to be decreased under certain preliminary given threshold. Such conceptual rationalization tends to avoid the main drawback in many approaches for sensors' calibration shifting the necessary user-input from heuristically motivated number of segments to deterministically motivated threshold of the measurement error.

In a more rigorous context the computation of the final piecewise linear approximant is performed in two steps:

- (i) Point wise reconstruction of inverted sensor characteristic,
- (ii) Adaptive piecewise linearization of the inverse sensor characteristic.

In step (i), the sensor characteristics are computationally inverted using Newton-Raphson method for iterative evaluation of the reference points [2, 3]. On the basis of this reconstruction in step (ii), an iterative procedure for adaptive partitioning of the measurement range is employed and the optimal linear approximant is evaluated [1]. The procedure in step (ii) is gradientless and hence can be implemented for non-smooth sensor characteristics.

2. CALCULATION OF THE INVERSE CHARACTERISTICS

Consider the primary real variable x that possibly represents certain measurable quantity. The domain of x is called measurement range and will be denoted further with $M = [X_{\text{low}}, X_{\text{up}}] \in \mathbb{R}$. The values X_{low} and X_{up} denote the lower and upper boundary of the measurement range. The measurement usually is performed in terms of other variable y that depends on x through certain physical effect. Examples in this context are the Seebeck's effect, piezoresistive effect and so on. The dependence between y and x is given in form of a regression polynomial constructed on the basis of some representative data set. This polynomial is called characteristics of the measurement device and will be denoted in the following way: $y = f(x)$. Most frequently in the engineering practice it is necessary to know the inverse characteristic $f^{-1}(y)=x$ because it allows one to obtain closed form expressions for the error in the measurable quantity x . However, the problem to find $f^{-1}(y)$ given $f(x)$ is a non-trivial, since the regression map is a higher order polynomial (ergo 10 and above) and to invert it analytically is (almost) impossible. Much better approach is to invert numerically the polynomial using the Newton's method. The computational cost of such reconstruction is insignificant, since the characteristic of the measurement device represents dependence between scalar but not vector quantities.

From the available calibration standards it can be constructed the sequence $\{y_k\}$ at particular reference points in the interval $[f(T_{\text{min}}), f(T_{\text{max}})] \in \mathbb{R}$. Then the following problem could be formulated:

Problem A. Given the sequence $\{y_k\}_{k=1}^n$, and closed form analytic characteristics $y = f(x)$. Using certain iterative method find the sequence $\{x_k\}$ that constitutes the discrete representation of the inverse characteristics $f^{-1}(y)$.

To describe the algorithm for computation of $\{x_k\}$ we first transform the explicit function $y = f(x)$ into an implicit form $F(x, y) = f(x) - y = 0$ and further at every particular $y = y_k$ perform the following procedure:

1. Initialize the tolerance TOL, set the counter $j = 0$, and make an initial guess x_0 of the solution \hat{x}_k at y_k .
2. Start an iterative process and compute from $x^{(j)}$ the next approximation $x^{(j+1)}$ of \hat{x}_k according to the recurrence formula:

$$x_{j+1} = x_j - \frac{F(x_j, y_k)}{F'(x_j, y_k)} = x_j - \frac{F(x_j, y_k)}{f'(x_j)} \quad (1)$$

where $(.)' = d(.) / dx$.

3. Check for convergency according to

$$\left| \frac{F(x_j, y_k)}{F'(x_j, y_k)} \right| \leq \text{TOL} \quad (2)$$

If (2) is not satisfied, then go to step 2.

For the purpose of illustration consider the problem of calculating the characteristic inverse of the polynomial Callendar-Van-Dusen equation,

$$R_{\text{RTD}}(T) = R_0(1 - AT + BT^2 - 100CT^3 + CT^4) \quad (3)$$

where, R_{RTD} is the resistance in Ohms at temperature T , R_0 is the resistance at 0°C and T is the temperature in $^\circ\text{C}$. The primary variable is the temperature T measured in the interval $[T_{\text{low}}, T_{\text{up}}] \in \mathbb{R}$ with $T_{\text{low}} = 100^\circ\text{C}$, $T_{\text{up}} = 400^\circ\text{C}$. Provided the set of resistances $\{R_{\text{RTD}}^k\}_{k=1}^n$, the formula for iterative calculation of the inverse $T(R_{\text{RTD}})$ according to (1) is easily obtained as,

$$T_{j+1} = T_j - \frac{R_{\text{RTD}}^k - R_0(1 - AT_j + BT_j^2 - 100CT_j^3 + CT_j^4)}{R_0(-AT_j + 2BT_j - 300CT_j^2 + 4CT_j^3)} \quad (4)$$

Different results could be obtained for different values of the parameters in the Callendar-Van-Dusen equation. The table below provides some typical values for those parameters according to different standards.

3. ADAPTIVE APPROXIMATION OF THE INVERSE CHARACTERISTICS

The inverse characteristic of the sensor was computed as a solution of the Problem A. However, in most cases such kind of solution is meaningless. The reason roots in its point wise (local) character that requires immense amount of storage space thus reducing significantly the operational capacity of the sensor defined in terms of performance and accuracy. To resolve such kind of problematic another approach could be implemented. It is based on a piecewise linear approximation of the inverse characteristic. In essence, the whole measurement region is subdivided (partitioned) into segments where in every segment the inverse characteristics is approximated with a straight line (see Fig.1). The first order continuity requirements enforced on the boundary points lead to polygonal representation of the inverse characteristic, which is much more suitable for implementation in conventional microcontrollers.

In the context of this linear approximation two important questions can be addressed:

- (i) should the measurement interval be partitioned in a regular manner (i.e. in straight segments all with the same length);
- (ii) how much segments should be used in order to provide the piecewise linear approximation with required level of accuracy.

The answers of both questions - as the theory shows - are mutually dependent. That is to say, the partition of the measurement range into a preliminary fixed number of sub intervals with the same length does not necessarily provides "good" approximation. In addition for some cases, the approximation could be even worst (this depends on the regularity of the inverse characteristics). In order to provide a reasonable basis for non-regular subdivision of the measurement range it should be defined firstly a reliable criterion, which to rationalize the quality of the approximation and after that using this criterion to identify in an iterative fashion the number and dimensions of the elements in the partitioning. The estimation of the quality of approximation is given through the so called posteriori error. In our case this error is assumed to be the exact upper bound of the absolute approximation error.

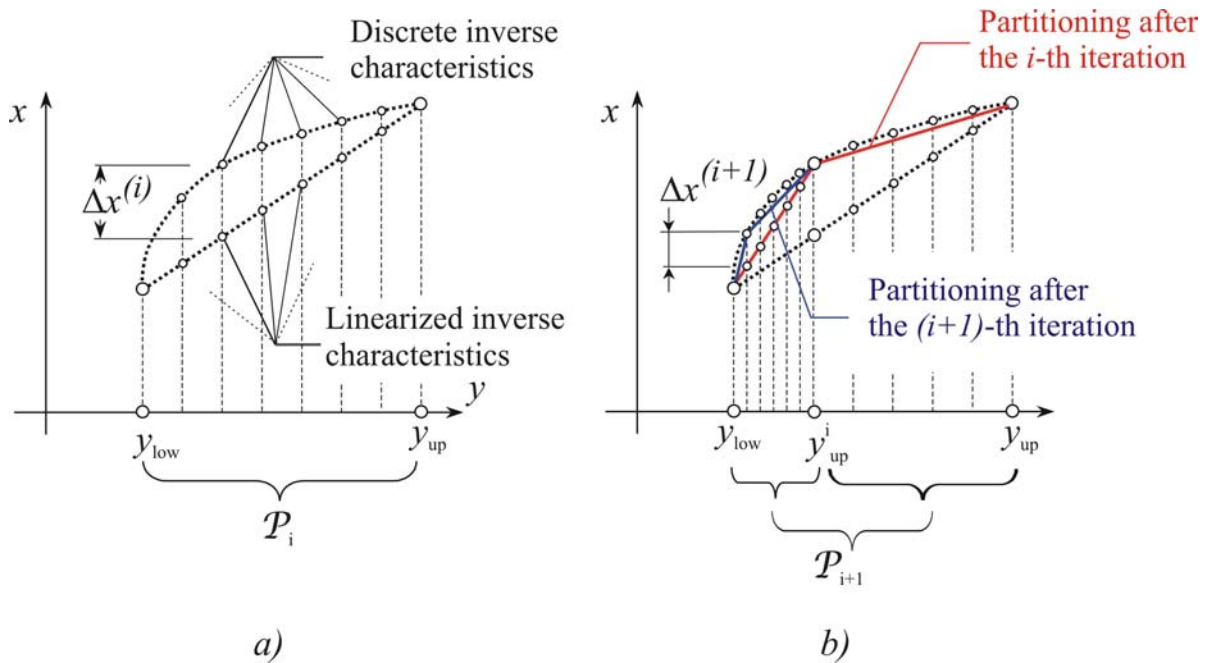


Figure 1. Graphical illustration of the adaptive algorithm. Starting with coarser partitioning \mathcal{P}_i (a) one calculates the upper boundary of the absolute approximation error $\Delta x^{(i)}$ and subdivides the measurement range $[y_{low}, y_{up}]$ into two parts where the linear inverse characteristics is constructed. (b) At the next iteration, $(i+1)$, again the exact upper bound of the absolute approximation error $\Delta x^{(i+1)}$ is calculated and the partitioning \mathcal{P}_0 is enriched to \mathcal{P}_{i+1} .

Accordingly, let the measurement range is subdivided into N subintervals. Let us denote the i -th subinterval with $T_i = [y_{low}^i, y_{up}^i] \subset M$, where $i = 1, 2, 3, \dots, N$. Solving the Problem A (see Sect. 2) in every subinterval we could obtain the inverse characteristics in point wise form $\{y_k\}_{k=1}^m \rightarrow \{x_k\}_{k=1}^m$ where m determines the desired number of reference points $\{y_k\}_{k=1}^m$ per subinterval. Let also for the same sequence of reference points the linear approximation of the inverse function is calculated and given as $\{y_k\}_{k=1}^m \rightarrow \{\bar{x}_k\}_{k=1}^m$. For example in the subinterval T_i this linear approximant is given through

$$\begin{vmatrix} y^{(i)} & \bar{x}^{(i)} & 1 \\ y_{low}^{(i)} & x_{low}^{(i)} & 1 \\ y_{up}^{(i)} & x_{up}^{(i)} & 1 \end{vmatrix} = 0 \quad (5)$$

The posteriori approximation error for the subinterval T_i is defined as

$$\varepsilon = \sup_{T_i} \{ |x_k - \bar{x}_k| \} \quad (6)$$

Disposing on this posteriori error estimation, the partitioning refinement process has the following structure:

1. Construct an initial coarse partitioning P_0 representing sufficiently well the measurement range. Put an iteration indicator $k = 0$.
2. Build for the partitioning P_k the piecewise linear approximant and compute for every subinterval the posteriori error estimate $\Delta x^{(i)}$.
3. If in every subinterval the posteriori error estimate is smaller than the preliminary specified threshold ε_{TOL} , then STOP. Otherwise, in all segments where $\varepsilon_{TOL} < \Delta x^{(i)}$, refine the partitioning and construct the next subdivision P_{k+1} . Replace k by $k+1$ and return to step 2.

The algorithm is conceptually clear. The only remark that can be done, concerns the last sentence in 3. The refinement process in this case is gradient less and is based on the following intermediate step: the interval where posteriori error requirement is not satisfied is divided in two, at the point y_k where the absolute error $|x_k - \bar{x}_k|$ is maximal. With such strategy the process of partitioning refinement goes subinterval by subinterval, providing smoothness of the computation and avoiding the instabilities due to round off errors.

4. ILLUSTRATIVE EXAMPLE

Consider a thermocouple with characteristics given by the polynomial-exponential regression,

$$E(t) = \sum_{i=1}^n c_i (t_{90})^i + \alpha_0 \exp\{\alpha_1 (t_{90} + a_2)^2\} \quad (7)$$

where, t_{90} is the temperature in °C, c_i are the polynomial coefficients, α_i are the coefficients in the exponential term, E is the output voltage in μV and n is the order of interpolation polynomial. The values $\{c_i\}_{i=1}^n$ and $\{\alpha_j\}_{j=1}^3$ are tabulated depending on the measurement range $[t_{\min}, t_{\max}]$ and the type of thermocouple. We apply the algorithm for an interpolation polynomial of 8-th order in the measurement range $t \in [0, 1372]^\circ\text{C}$ for thermocouple of type-K (Chromel/Alumel) with the following values of regression coefficients:

- Polynomial counterpart:

c1	c2	c3	c4
-1,7600413686e01	3,8921204975e01	-1,8558770032e-02	9,9457592874e-05
c5	c6	c7	c8
3,1840945719e-07	-5,6072844889e-10	5,6075059059e-13	-3,2020720003e-16
c9	c10		
9,7151147152e-20	-1,2104721275e-23		

- Exponential counterpart:

α_0	α_1	α_2
1,185976e02	-1,183432e-04	-1,269686e02

For the first partition, the inverse characteristic $t_{90} = t(E)$ is calculated at 11 (including both boundaries) points from the measurement range $t \in [0, 1372]^\circ\text{C}$. Accordingly, the obtained values (shown only 9 of them) for an admissible error of $1 \cdot 10^{-6}$ in the iterative Newton method are,

$t_{90}^i \cdot 10^3, ^\circ\text{C}$	0	0,1339	0,2702	0,4016	0,5307	0,6599	0,7916	0,9275	1,0682	...
$E^i \cdot 10^4, \text{mV}$	0	0,5489	1,0977	1,6466	2,1955	2,7443	3,2932	3,8420	4,3909	...

The iterative process of subdivision continues further till the absolute measurement error in every subinterval is not below a user prescribed value. For instance for absolute measurement error of $0,05^\circ\text{C}$ the number of subintervals necessary to build the linear approximant of the sensor characteristics is 48, for error of $0,1^\circ\text{C}$ this number is 45 and so on. The number of sub-intervals necessary to accommodate a user-defined absolute error in the temperature can increase exponentially (as shown in Fig. 2) hence the importance of possible optimal subdivision as the

presented here is significant.

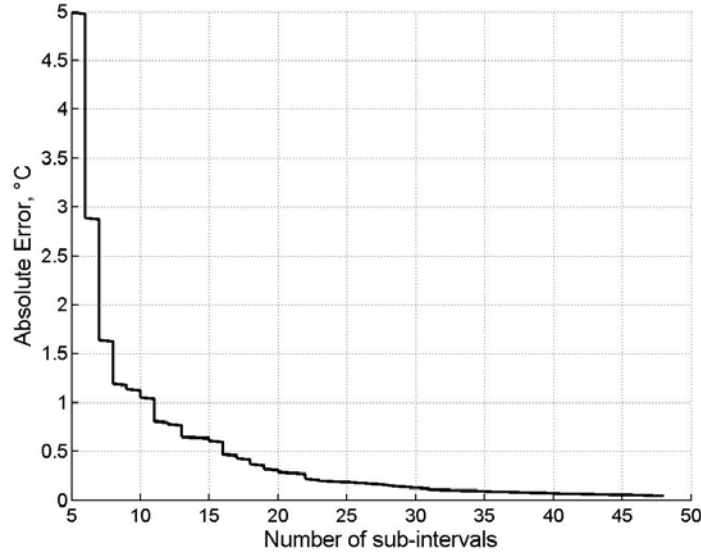


Figure 2. Dependence between the absolute measurement error and number of sub-partitions. In order to accommodate the preliminary prescribed overall measurement error one iteratively increases the number of sub-partitions. We underline that the length of sub-partitions is not the same.

5. CONCLUSIONS

The paper deals with an efficient algorithm for adaptive piecewise affine approximation of the inverse sensor characteristics. Although the discussion and the computational examples address the construction of adaptive approximants for temperature inverse characteristics, the algorithm could be easily extended to other process parameters, even when the approximation is based on the best fit-type linearization in the partitioning

References:

- [1] Ainsworth, M. & Oden, J. (2000), *A Posterior Error Estimation in Finite Element Analysis*, John Wiley & Sons: New York.
- [2] Bertsekas, D. (1996), *Constrained optimization and Lagrange multiplier methods*, Athena Scientific.
- [3] Bertsekas, D. (1999), *Nonlinear Programming*, Athena Scientific.
- [4] Brignell, J. (1991), Software techniques for sensor compensation, *Sensors and Actuators A*, 25-27, 29-35.
- [5] Hill, J., Szewczyk, R., Woo, A., Hollar, S., Culler, D. & Pister, K. (2000), System architecture directions for networked sensors, in 'Proceedings of the 9th International Conference on Architectural Support for Programming Languages and Operating Systems ASPLOS IX, Cambridge MA', pp. 93-104.
- [6] Huddleston, C. (2003), 'Digital signal controllers turn thermocouples into superstars', *Sensors Magazine* 20(1).
- [7] Pallas-Areny, R. & Webster, J. (2001), *Sensors and Signal Conditioning*, John Wiley & Sons: New York.

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