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Carsten Behn / Joachim Steigenberger

Adaptive Control and Worm-like Robotic Locomotion Systems revisited

1 Introduction

In this paper we introduce a certain type of mathematical models of worm-like locomotion systems and sketch the corresponding theory. Gaits from this theory can be tracked by means of adaptive controllers. Simulations are aimed at the justification of theoretical results.

2 Worm-like Robotic Locomotion System

The following is taken as the basis of our theory.

(i) A worm is a mainly terrestrial (or subterrestrical, possibly also aquatic) locomotion system characterized by one dominant linear dimension with no active (driving) legs or wheels.

(ii) Global displacement is achieved by (periodic) change of shape (such as local strain) and interaction with the environment (undulatory locomotion).

(iii) The model body of a worm is a 1-dimensional continuum that serves as the support of various physical fields.

The continuum in (iii) is just an interval of a body-fixed coordinate. Most important fields are: *mass,* continuously distributed (with a density function) or in discrete distribution (chain of point masses), *actuators*, i.e., devices which produce internal displacements or forces thus mimicking muscles, *surface structure* causing the interaction with the environment.

It is well known, that, if there is contact between two bodies (worm and ground), there is some kind of friction, which depends on the physical properties of the surfaces of the bodies. In particular, the friction may be anisotropic (depends on the orientation of the relative displacement). This interaction (mentioned in (ii)) could emerge from a surface texture as asymmetric Coulomb friction or from a surface endowed with scales or bristles (we shall speak of *spikes* for short) preventing backward displacements. It is responsible for the conversion of (mostly periodic) internal and internally driven motions into a change of external position (undulatory locomotion [11]), see [12].

In this paper only *discrete straight worms* shall be considered: chains of masspoints moving along a straight line, Fig. 1.

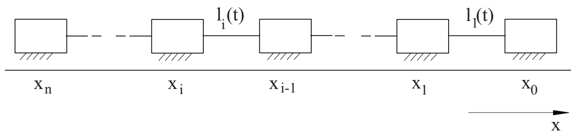

Figure 1: Chain of point masses with spikes.

First we focus on interaction via *spikes* since in this case a thorough kinematic theory is available. Later on we introduce Coulomb friction and try to analyze artificial worms as dynamical (control) systems.

3 Kinematics

The motions of the worm system, $t \longmapsto x_i(t)$, are investigated under the **general assumption** to be of differentiability class D^2, i.e.,

$$\begin{array}{l} x_i(\cdot) \in D^2(\mathbb{R}),\ i = 0, \dots, n, \\ x_i(\cdot) \text{ and } \dot{x}_i(\cdot) \text{ continuous, } \ddot{x}_i(\cdot) \text{ piecewise continuous } (\ddot{x}_i \in D^0). \end{array} \tag{1}$$

The spikes (attached to the points $\kappa \in \mathbf{K} \subset \{0, 1, \dots, n\}$) restrict the velocities of the contact points,

$$\dot{x}_\kappa \geq 0,\ \kappa \in \mathbf{K}. \tag{2}$$

This is a system of **constraints** in form of differential inequalities which the system's motions are subject to.

Let the distances of consecutive masspoints (= actual lengths of the links) be denoted $l_j,\ j = 1, ..n$, and the actual distance of the masspoint i from the head S_i,

$$l_j = x_{j-1} - x_j, \quad S_i = x_0 - x_i = \sum_{j=1}^{i} l_j \ . \tag{3}$$

Then there holds for the velocities

$$\dot{x}_i = \dot{x}_0 - \dot{S}_i,\ i = 0, \ldots, n\,,$$

and the constraint (2) yields $\dot{x}_0 - \dot{S}_\kappa \geq 0$, i.e., $\dot{x}_0 \geq \dot{S}_\kappa$ for all $\kappa \in \mathbf{K}$. This necessarily entails

$$\dot{x}_0 \geq V_0 := \max\{\dot{S}_\kappa \mid \kappa \in \mathbf{K}\}. \tag{4}$$

Consequently, the head velocity is

$$\dot{x}_0 = V_0 + w, \quad w \geq 0, \tag{5}$$

and for the others it follows

$$\dot{x}_i = V_0 - \dot{S}_i + w\,, \quad i = 0, \ldots, n. \tag{6}$$

Since w is a common additive term to *all* velocities $\dot{x}_i$, it describes a *rigid part of the motion* of the total system (motion at 'frozen' $\dot{l}_j$).

If $0 \in \mathbf{K}$ (head equipped with spike) then because of $S_0 = 0$ the head velocity is non-negative.

The coordinate and velocity of the **center of mass** are obtained by averaging the x_i and $\dot{x}_i$ (remind equal masses m for all i),

$$\begin{aligned} x^* &= x_0 - S, & S &:= \tfrac{1}{n+1} \sum_{i=0}^{n} S_i, \\ v^* &:= \dot{x}^* = W_0 + w, & W_0 &:= V_0 - \dot{S}. \end{aligned} \tag{7}$$

Now there are two representations of the velocities,

$$\dot{x}_i = \dot{x}_0 - \dot{S}_i = w + V_0 - \dot{S}_i. \tag{8}$$

They show that, alternatively, the head velocity $\dot{x}_0$ *together with* $\dot{S}_i$ (mind $\dot{S}_0 = 0$), or the rigid velocity part w *together with* $\dot{S}_i$ may serve as **generalized velocities** of the system (degree of freedom = $n + 1$).
When considering locomotion under external load it might be of interest to know which and how many of the masspoints κ with ground contact, $\kappa \in \mathbf{K}$, are at rest during the motion of the system, these are the **active spikes** which transmit the propulsive forces from the ground to the system. Now $\dot{x}_\kappa = V_0 - \dot{S}_\kappa + w$ together with $w \geq 0$ and $V_0 = \max\{\dot{S}_i \mid i \in \mathbf{K}\} \geq \dot{S}_\kappa$ imply

$$\dot{x}_\kappa = 0 \Longleftrightarrow w = 0 \wedge V_0 = \dot{S}_\kappa, \quad \kappa \in \mathbf{K}. \tag{9}$$

If the head is equipped with a spike, $0 \in \mathbf{K}$, then in view of $\dot{S}_0 = 0$ and the definition of V_0 in (4) it follows

$$\text{If } 0 \in \mathbf{K} \text{ then } \dot{x}_0 = 0 \Longleftrightarrow w = 0 \wedge \dot{S}_\kappa \leq 0 \text{ for all } \kappa \in \mathbf{K}. \tag{10}$$

The worm system is called to move under **kinematic drive** if by means of the actuators *all* distances $l_j, \ j = 1, .., n$, or, equivalently, the relative velocities $\dot{l}_j$ are prescribed as functions of t. Then the formulas make $\dot{S}_i = \sum_{j=1}^{i} \dot{l}_j$ and $V_0 = \max\{\dot{S}_\kappa \mid \kappa \in \mathbf{K}\}$ known functions of t. In the velocities (6)

$$\dot{x}_i = V_0(t) - \dot{S}_i(t) + w, \quad i = 0, \ldots, n,$$

the rigid part w is now the only free variable. This corresponds to the fact that for the system of $n+1$ masspoints on the $x-$axis ($DOF = n+1$) the distance relations (3),

$$x_{j-1} - x_j - l_j(t) = 0, \quad j = 1, \ldots, n, \tag{11}$$

now represent n independent *rheonomic holonomic constraints* which

shrink the degree of freedom to 1. w is the remaining generalized velocity of the worm system. The differential constraints (2) are, as before, satisfied by definition of V_0.
Once more, the rigid part w of the velocities keeps arbitrary in kinematics. So it seems promising to put it equal to zero, then all velocities of the masspoints are known functions of t. Putting $w = 0$ locks the single degree of freedom, the system has become a *compulsive mechanism with ground contact* (the latter causing locomotion).

There remains a nicely simple
Kinematical theory: (worm with kinematic drive and $w(t) = 0$)

Prescribe: $l_j(.) \in D^2(\mathbb{R}) : t \mapsto l_j(t) > 0, \quad j = 1, \ldots, n.$
Determine: $S_i := \sum\limits_{j=1}^{i} l_j, \quad V_0 := \max\{\dot{S}_\kappa \mid \kappa \in \mathbf{K}\} \in D^1(\mathbb{R}).$
Result: $x_0(t) = \int\limits_0^t V_0(s)ds, \quad x_j(t) = x_0(t) - S_j(t), \quad j = 1, \ldots, n.$

(12)

Clearly, the kinematical theory is valid iff at any time at least one spike is active. In applications it might be necessary to use a kinematic drive that ensures a prescribed number of spikes to be active at every time.

Example 3.1. *A worm system with $n = 2$ and $\mathbf{K} = \mathbf{N} = \{0, 1, 2\}$ is considered. We present (heuristic construction suppressed here) a kinematic drive such that at every time exactly one of the three spikes is active.*
Using the Heaviside function

$$h(a, b, \cdot) : \tau \mapsto h(a, b, \tau) := \begin{cases} 1, & \text{if } a < \tau \leq b \\ 0, & \text{else} \end{cases}$$

we define

$$
\begin{aligned}
l_1(t) &:= l_0 \Big[1-\varepsilon\big(1-\cos(\pi\, t)\big)\Big]\, h(0,2,t) + l_0\, h(2,3,t) \\
l_2(t) &:= l_0 \Big[1-\varepsilon\big(1-\cos(\pi\, t)\big)\Big]\, h(0,1,t) \\
&\quad + l_0\,[1-2\varepsilon]\, h(1,2,t) + l_0 \Big[1-\varepsilon\big(1+\cos(\pi\, t)\big)\Big]\, h(2,3,t)
\end{aligned}
\tag{13}
$$

on the primitive time interval $[0,T],\ T:=3$*, and then take their* T*-periodic continuation to* $\mathbb{R}^+$*. Here* $l_0 = 1$ *is the original length of the links, and* $l_0\varepsilon,\ \varepsilon = 0.2$*, is the amplitude of the length variation in time.*
Applying (12) *we obtain the results which are sketched in Fig. 2.*

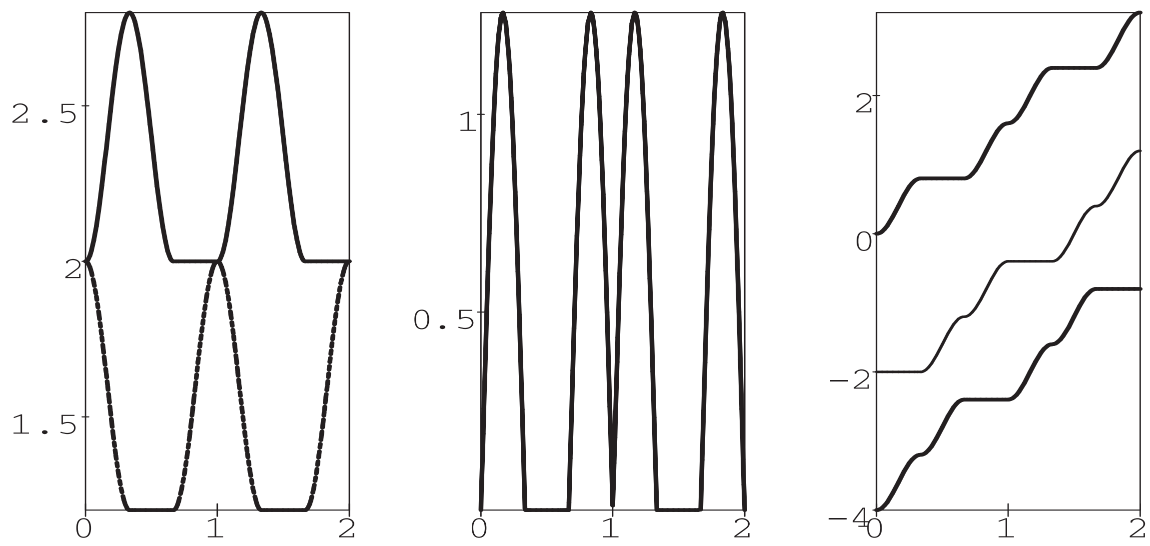

Figure 2: Left: Gait $l_{1,2}$ vs. t/T; middle: V_0 vs. t/T, right: $x_{0,1,2}$ vs. t/T.

As one can see, the cycle of active spikes is $\mathbf{1} \to \mathbf{0} \to \mathbf{2}$*.* ◇

4 Dynamics

The dynamics of the worm system are formulated by means of Newton's law for each of the masspoints. The following forces are applied to masspoint i, all acting in $x-$direction:

- g_i, the *external impressed* (physically given) *force* (e.g., resultant of weight and viscous friction: $g_i = -k_0\,\dot{x}_i - \Gamma_i$).

- μ_i, the *stress resultant* (inner force) of the links (let, formally, $\mu_0 = \mu_{n+1} := 0$).
- z_i, $i \in \mathbf{K}$, the *external reaction force* caused by the constraint (2), acting on the spiked masspoints.

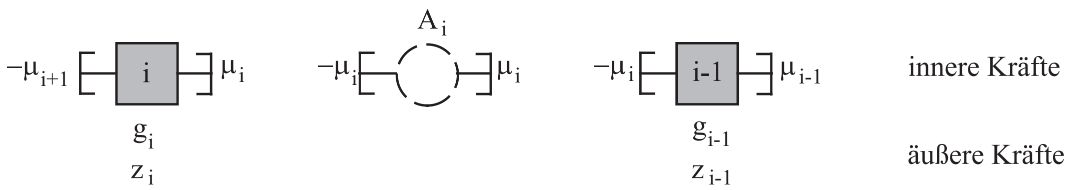

Figure 3: Masspoints with forces (A_i: actuator).

As the constraint (2) describes a *one-sided* restriction of $\dot{x}_\kappa$, velocity and reaction force are connected by a **complementary slackness-condition**:

$$\dot{x}_\kappa \geq 0, \quad z_\kappa \geq 0, \quad \dot{x}_\kappa \, z_\kappa = 0, \quad \kappa \in \mathbf{K}. \tag{14}$$

This means that $z_\kappa(t)$ must be zero if at time t the masspoint κ is moving forward, i.e., if the velocity inequality is *strict*, $\dot{x}_\kappa(t) > 0$ ("$\dot{x}_\kappa$ has *slack*"), whereas $z_\kappa(t)$ may have arbitrary non-negative values as long as $\dot{x}_\kappa(t) = 0$ (reaction force at resting spike). Positive $z_\kappa(t)$ implies $\dot{x}_\kappa(t) = 0$, simultaneous vanishing, $\dot{x}_\kappa(t) = \lambda_\kappa(t) = 0$ is possible (resting, non-active spike).

Newton's laws for the $n+1$ masspoints

$$\begin{aligned} m\ddot{x}_\kappa &= g_\kappa + \mu_\kappa - \mu_{\kappa+1} + z_\kappa, \quad && \kappa \in \mathbf{K}, \\ m\ddot{x}_i &= g_i + \mu_i - \mu_{i+1}, && i \in \mathbf{N} \setminus \mathbf{K} \end{aligned} \tag{15}$$

now appear as a system of equations that together with the slackness-conditions (14) and initial conditions for x_i and $\dot{x}_i, \; i = 0, \ldots, n$, governs the motions of the worm system.

An *actuator* is, first, a multipole with input activation signal and energy (immanent energy source - e.g., electrical battery, chemical agents - possible as well). Its output are forces, torques, displacements, twists, respectively, which depend on time and may be connected with the actual state or the state history of the system or with the system's rheological constitution. Often the internal dynamics of an actuator are not modeled, rather the output is connected with the input by means of working hypotheses: the multipole remains a black box. It becomes a (almost) white box if for its internal dynamics a (more or less crude) model is established which yields an output law.

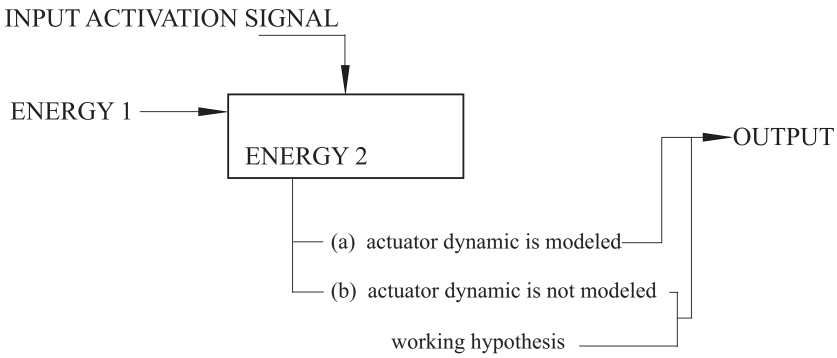

Figure 4: Actuator, schematically.

We hint at four physical models of actuators from literature:

a) in [9] with output force,

b) in [13] with output displacement,

c) in [12] a mixed case,

d) in [8] with output torque or rotation.

Here, our starting - point for introducing an actuator is a given output law. Let μ_i be qualified as impressed forces obeying the following law:

$$\mu_i(t, x, \dot{x}) := c_i\big(x_{i-1} - x_i - l_i^0\big) + k_i(\dot{x}_{i-1} - \dot{x}_i) + u_i(t). \qquad (16)$$

This mathematical relation describes the parallel arrangement of a linear-elastic spring with a constant stiffness c_i and original length l_i^0, a Stokes damping element with constant coefficient k_i, and a time-dependent force $u_i(t)$. The following figure (Fig. 5) shows the corresponding physical model of this actuator (cf. Fig. 3), where now the small circular box represents a non-modeled device generating the force $u_i(t)$.

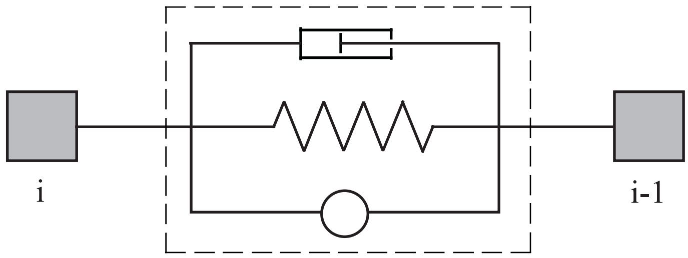

Figure 5: Actuator, general physical model.

Under the assumption that all actuators have the same data (stiffness c, original spring-length l^0, damping coefficient k_{00}) and with $\mathbf{K} = \mathbf{N}$ the equations of motions follow from (15) in the actual form

$$\begin{aligned} m\ddot{x}_0 &= -c(x_0 - x_1 - l^0) - k_{00}(\dot{x}_0 - \dot{x}_1) - k_0\dot{x}_0 - u_1(t) - \Gamma_0 + z_0, \\ m\ddot{x}_j &= -c(2x_j - x_{j+1} - x_{j-1}) - k_{00}(2\dot{x}_j - \dot{x}_{j+1} - \dot{x}_{j-1}) + \\ &\qquad\qquad + u_j(t) - u_{j+1}(t) - k_0\dot{x}_j - \Gamma_j + z_j, \\ m\ddot{x}_n &= c(x_{n-1} - x_n - l^0) + k_{00}(\dot{x}_{n-1} - \dot{x}_n) - k_0\dot{x}_n + u_n(t) - \Gamma_n + z_n. \end{aligned} \tag{17}$$

The accompanying complementary slackness conditions can be satisfied through expressing the λ_i by means of the 'controller' (see [13])

$$z_i(f_i, \dot{x}_i) = -\frac{1}{2}\big(1 - sign(\dot{x}_i)\big)\big(1 - sign(f_i)\big)f_i, \quad i \in \mathbf{N}, \tag{18}$$

where f_i is the resultant of all further forces acting on the masspoint i. At this stage, the u_i are to be seen as prescribed functions of t - *offline-controls*, later on they will also be handled as depending on the state $(x, \dot{x})$ - *feedback, online-controls*.

If the actuator data are known (l^0, c and k_{00}) and n is small ($n = 2$), then an actuator input $u_i(t)$ can be calculated which controls the system in such a way as to track a preferred motion-pattern constructed in kinematical theory.

But, as a rule, the actuator data are not known exactly. Then an **adaptive control scheme** is required that, despite of this drawback, achieves tracking at least approximately. This will be presented in the next section.

5 Adaptive Control

A main problem is the lack of precise knowledge of the actuator data, moreover, the worm system parameter may be not exactly known as well. We have to deal with uncertain systems. Hence, we are not able to calculate force inputs u for the worm system to achieve a prescribed movement. We have to design a controller which generates the necessary output forces on its own to track a prescribed kinematical gait. This will lead us to an adaptive high-gain output feedback controller (learning controller). The aim is not to identify the actuator data or worm system parameter, but to simply control this system in order to track a given reference trajectory (kinematic gait) that is to achieve a desired movement of the system. We will not focus on exact tracking, we focus on the **$\boldsymbol{\lambda}$-tracking control objective** where we tolerate a pre-specified tracking error of size λ.

Precisely, given $\lambda > 0$, a control strategy $y \mapsto u$ is sought which, when applied to a MIMO control system $\dot{x} = f(x, u)$, $y = h(x)$, achieves λ-tracking for every reference signal $y_{\text{ref}}(\cdot)$, i.e., the following:

(i) every solution of the closed-loop system is defined and bounded on $\mathbb{R}_{\geq 0}$, and

(ii) the output $y(\cdot)$ tracks $y_{\text{ref}}(\cdot)$ with asymptotic accuracy quantified by $\lambda > 0$ in the sense that $\max\left\{0, \|y(t) - y_{\text{ref}}(t)\| - \lambda\right\} \to 0$ as $t \to \infty$.

The last condition means that the error $e(t) := y(t) - y_{\text{ref}}(t)$ is forced, via the adaptive feedback mechanism (controller (19)), towards a ball around zero of arbitrary small pre-specified radius $\lambda > 0$, see Fig. 6.

The following controller realizes our goal, for a mathematical proof see [3] and [4]:

$$\left.\begin{array}{rcl} e(t) & := & y(t) - y_{\text{ref}}(t)\,, \\ u(t) & = & -\gamma\left(k(t)e(t) + \frac{d}{dt}\big(k(t)e(t)\big)\right), \\ \dot{k}(t) & = & \max\left\{0, \|e(t)\| - \lambda\right\}^2, \end{array}\right\} \tag{19}$$

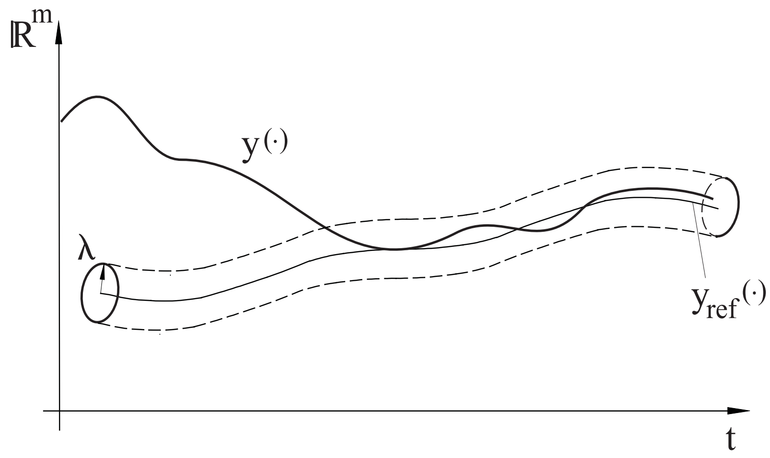

Figure 6: The λ-tube along the reference signal.

with any $k(0) \in \mathbb{R}$, $\lambda > 0$, $y_{\text{ref}}(\cdot) \in \mathcal{R}$ (a Sobolev-Space), $u(t)\,, e(t) \in \mathbb{R}^{\text{m}}$ and $k(t) \in \mathbb{R}$.

In application to our worm system we take as outputs the actual lengths of the links $y_1(t) := x_0(t) - x_1(t)$ and $y_2(t) := x_1(t) - x_2(t)$. The reference signal is $y_{\text{ref}\,1,2}(t) = l_{1,2}(t)$, control is $\big(u_1(t), u_2(t)\big)$.

The adaptor in (19) makes $k(t)$ monotonically increase as long as the tracking error is not permanently below the prescribed admissible tolerance (dead zone behaviour).

We choose the following data for all simulations in this paper. Additional data needed for simulations shall be given on the spot.

- worm system: $m_0 = m_1 = m_2 = 1$, $c = 10$, $k_{00} = 5$;
- environment: $k_0 = 0$, $\Gamma_{1,2,3} = 2.7$ (ensures kinematical theory to be dynamically feasible);
- reference gait: (13) with $l_0 = 2$, $\varepsilon = 0.2$;
- controller: $k(0) = 0$, $\lambda = 0.2$, $\gamma = 100$;

For numerical reasons we use the smooth approximation sign$(x) \approx \tanh(100\,x)$.

Simulation 1: We use the ideal spikes, i.e., (18) with controller (19).

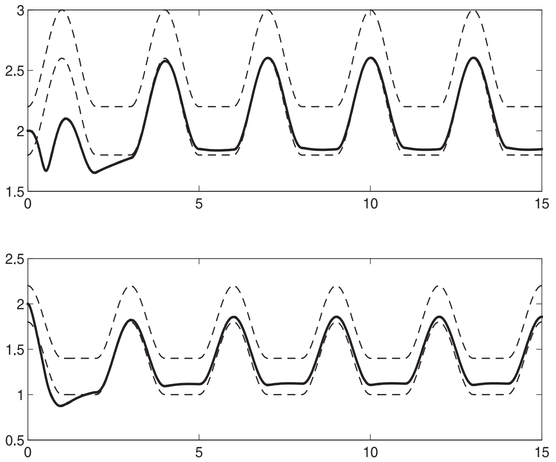

Figure 7: Output and tubes.

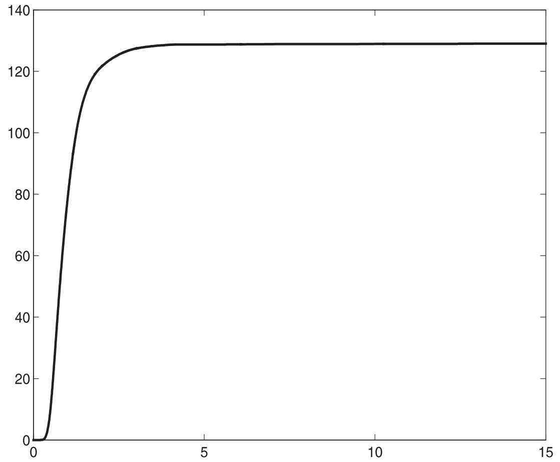

Figure 8: Gain parameter.

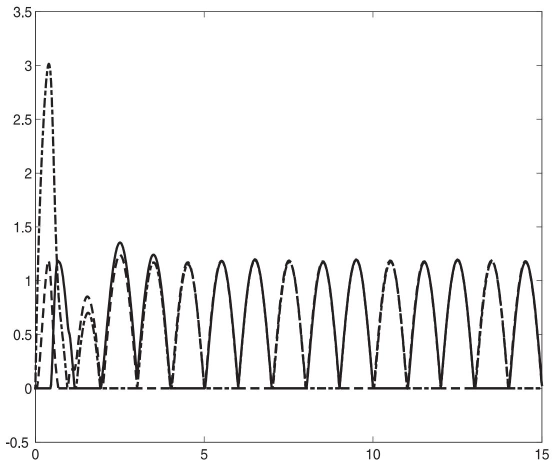

Figure 9: Velocities.

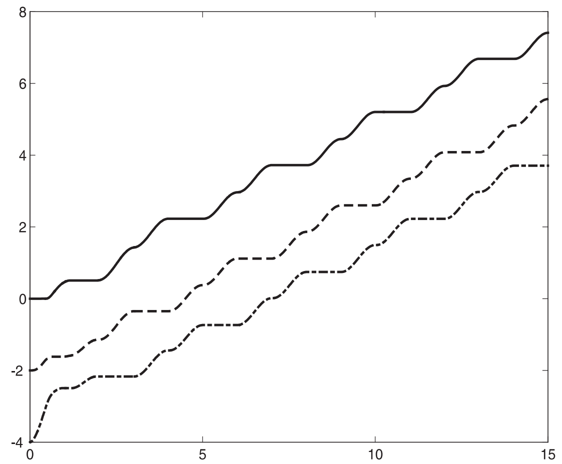

Figure 10: Worm motion.

Fig. 8 shows the monotonic increase of $k(t)$ towards a limit k_∞. But if some perturbation would repeatedly cause the output to leave the λ-strip then $k(t)$ took larger values again and again. $\diamond$

For practical reasons $k(t)$ must not exceed a feasible upper bound. That is why we introduce an improved adaptation law, see [6], which let $k(t)$ decrease as long as further growth is not necessary. We distinguish three cases: 1. increasing $k(\cdot)$ while e is outside the tube, 2. constant $k(\cdot)$ after e entered the tube - no longer than a pre-specified duration t_d of stay, and 3. decreasing $k(\cdot)$ after this duration has been exceeded.

For instance:

$$\dot{k}(t) = \begin{cases} \gamma \left(\|e(t)\| - \lambda \right)^2, & \|e(t)\| \geq \lambda \,, \\ 0, & \left(\|e(t)\| < \lambda \right) \wedge (t - t_E < t_d) \,, \\ -\sigma \, k(t), & \left(\|e(t)\| < \lambda \right) \wedge (t - t_E \geq t_d), \end{cases} \tag{20}$$

with given $\sigma > 0$, $\gamma \gg 1$, and $t_d > 0$, whereas the entry time t_E is an internal time variable.

Simulation 2: Again, we use the ideal spikes but now controller (19) with adaptor (20) ($\sigma = -0.2$, $t_d = 1$).

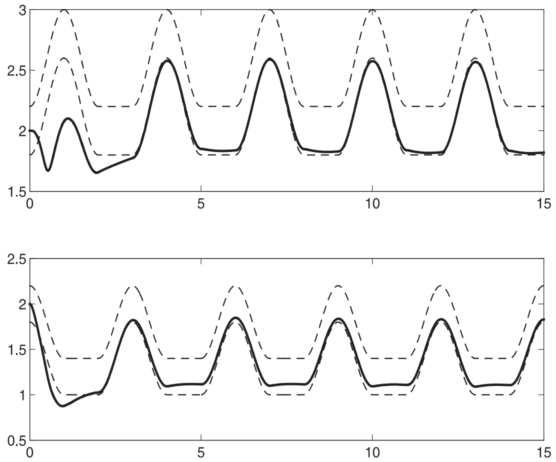

Figure 11: Output and tubes.

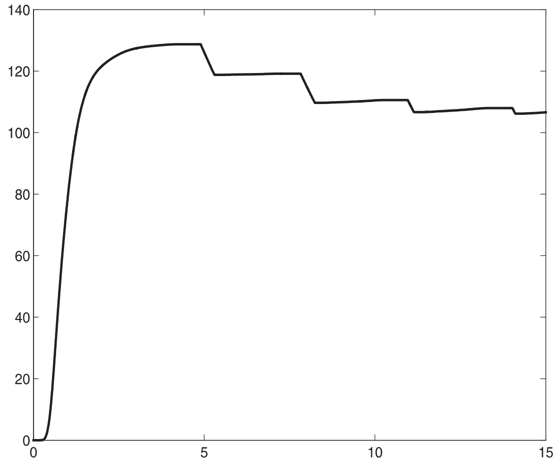

Figure 12: Gain parameter.

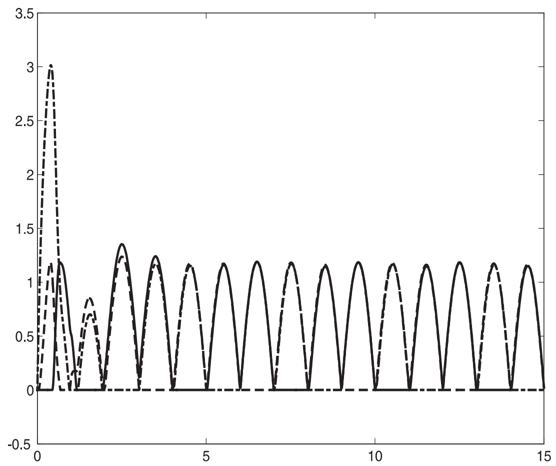

Figure 13: Velocities.

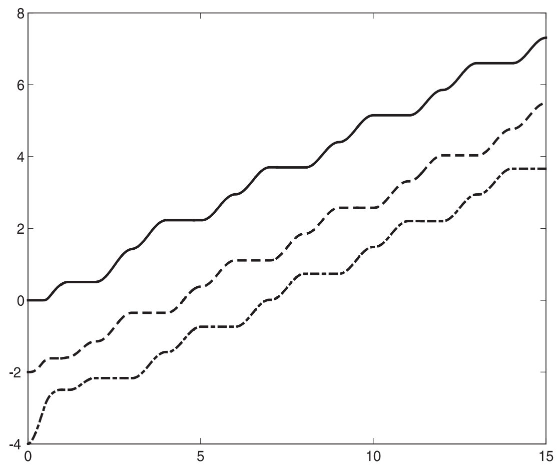

Figure 14: Worm motion.

Fig. 12 indicates a limit $k(\infty)$ smaller than that in Fig. 8. This means that the maximal $k \approx 130$ is needed only for to force the output to enter the λ-strip $\diamond$

6 Friction

In order to envisage practical failing of the ideal spikes (finite strength) they could be replaced by assuming stiction combined with Coulomb sliding dry friction. First, we present a mathematical model for the Coulomb laws that is both theoretically transparent and handy in computing. It is by far simpler than various sophisticated laws in literature (e.g. [1], [2], [7], [10]) but it well captures stick-slip effects in application to worm dynamics, see [5] and [14]. We note that this modelling makes friction a function of two arguments, and not only depending on the single argument velocity as preferred by most authors. Let

$$F(f,v) := \begin{cases} F^-, & v < -\varepsilon \vee \big(|v| \le \varepsilon \wedge f < -F_0^-\big), \\ -f, & |v| \le \varepsilon \wedge f \in \big[-F_0^-, F_0^+\big], \\ -F^+, & v > \varepsilon \vee \big(|v| \le \varepsilon \wedge f > F_0^+\big). \end{cases} \tag{21}$$

Think of F acting on a masspoint whose dynamics are $\dot{x} = v$, $m\dot{v} = f + F$. Then (21) essentially indicates the mutual cancellation of forces if the point is at rest and $|f|$ is bounded by $F_0^\pm$, and a constant 'braking' of magnitude $F^\pm$ during motion. Different $F^\pm$, $F_0^\pm$ values characterize a friction anisotropy. A suitable $\varepsilon > 0$ replaces the computer accuracy and mimics the vague processes at small velocities.

Using the Heaviside function h from Example 3.1 F can be given in the closed form (disregarding its values at $v = \pm\varepsilon$)

$$\begin{aligned} F(f,v) &= -f\, h(-\varepsilon,\varepsilon,v)\, h(-F_0^-,F_0^+,f) \\ &\quad +F^-\left\{h(-\infty,-\varepsilon,v) + h(-\varepsilon,\varepsilon,v)\, h(-\infty,-F_0^-,f)\right\} \\ &\quad -F^+\left\{h(\varepsilon,\infty,v) + h(-\varepsilon,\varepsilon,v)\, h(F_0^+,\infty,f)\right\}. \end{aligned} \tag{22}$$

In order to avoid difficulties in computing caused by jumps of the h-function we turn to a smooth mathematical model (in the sense of an approximation). Basically, we use a $\tanh$-approximation of the sign-function sign$(x) \approx \tanh(A\,x)$ with some sufficiently large $A \gg 1$.

The smooth mathematical model then is

$$\begin{aligned} F(f,v) &= -f\, H(-\varepsilon,\varepsilon,v)\, H(-F_0^-,F_0^+,f) \\ &\quad +F^-\left\{H(-\infty,-\varepsilon,v) + H(-\varepsilon,\varepsilon,v)\, H(-\infty,-F_0^-,f)\right\} \\ &\quad -F^+\left\{H(\varepsilon,\infty,v) + H(-\varepsilon,\varepsilon,v)\, H(F_0^+,\infty,f)\right\}, \end{aligned} \tag{23}$$

where F is now presented as a $\mathcal{C}^\infty$-function in closed analytical form,

$$H(a,b,x) := \frac{1}{2}\left\{ \tanh\big(A\,(x-a)\big) + \tanh\big(A\,(b-x)\big)\right\} \tag{24}$$

is the smooth approximation of $h(a,b,x)$. We use $A = 100$ in the sequel.

Remark 6.1. *We would like to point out the following*

- $F_0^- \gg 1$ *leads us to the theory of ideal spikes,*
- *a very small* F_0^- *corresponds with a broken spike.* $\diamond$

Again, adaptive control has to be used when considering uncertain or randomly changing friction data (rough terrain). Successful application is shown by the following simulation results: 1. stiction only , 2. sliding friction only, and 3. both.

Simulation 3: Here, we consider only stiction, i.e., $F^+ = F^- = 0$ for (23). We choose $F_0^- = 15$ and $F_0^+ = 3$. Applying controller (19) with adaptor (20) ($\sigma = -0.2$, $t_d = 1$) yields:

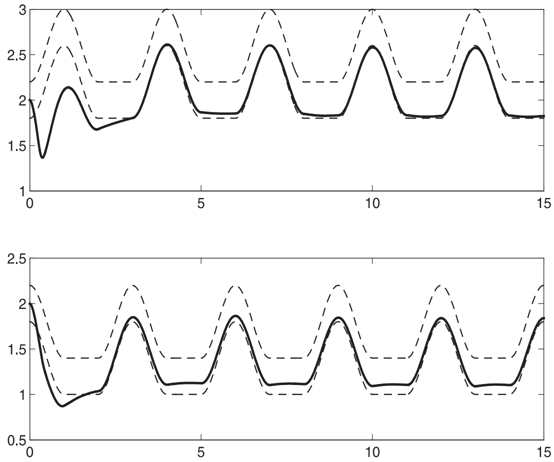

Figure 15: Output and tubes.

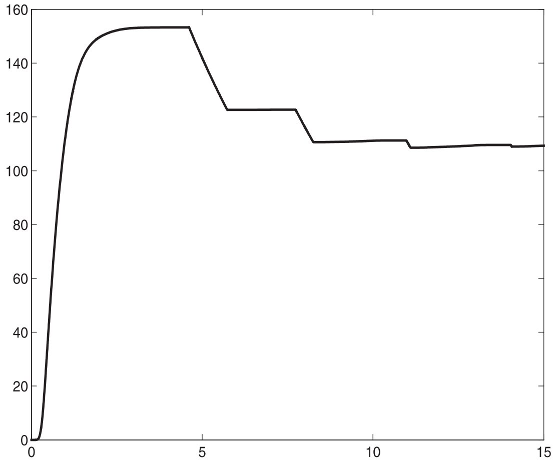

Figure 16: Gain parameter.

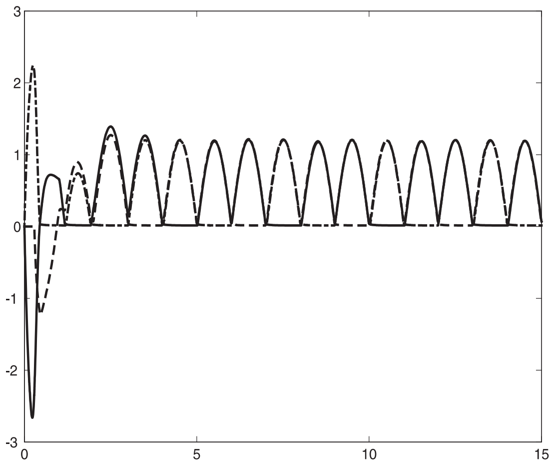

Figure 17: Velocities.

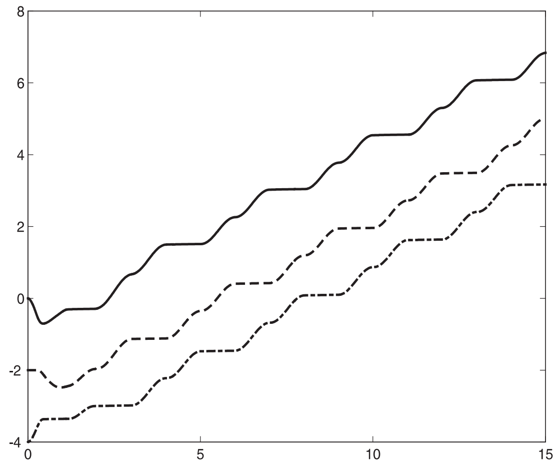

Figure 18: Worm motion.

There are some short backward motions at the beginning, afterwards the motion coincides with that of Simulations 1 and 2. $\diamond$

Simulation 4: Now consider only sliding friction, i.e., $F_0^+ = F_0^- = 0$. We choose $F^- = 15$ and $F^+ = 3$. Again applying controller (19) with adaptor (20) ($\sigma = -0.2$, $t_d = 1$) yields:

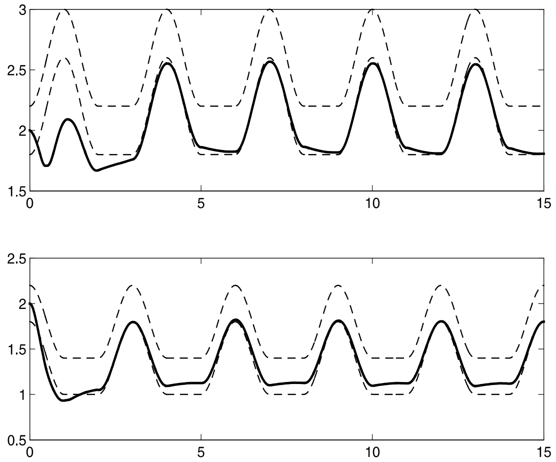

Figure 19: Output and tubes.

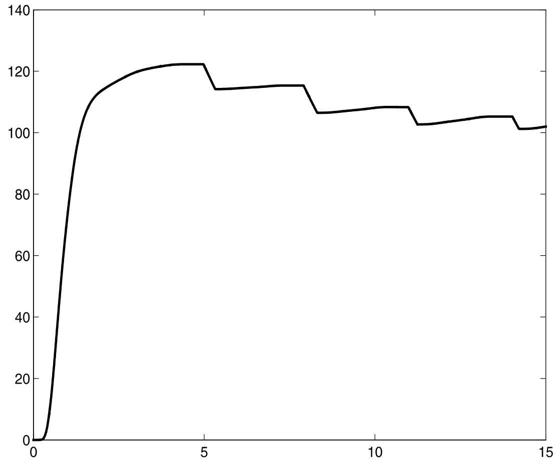

Figure 20: Gain parameter.

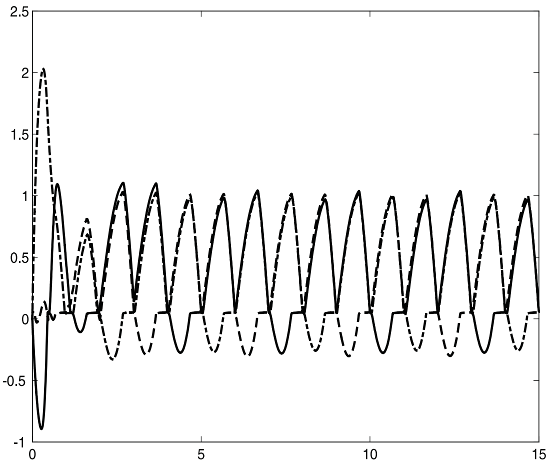

Figure 21: Velocities.

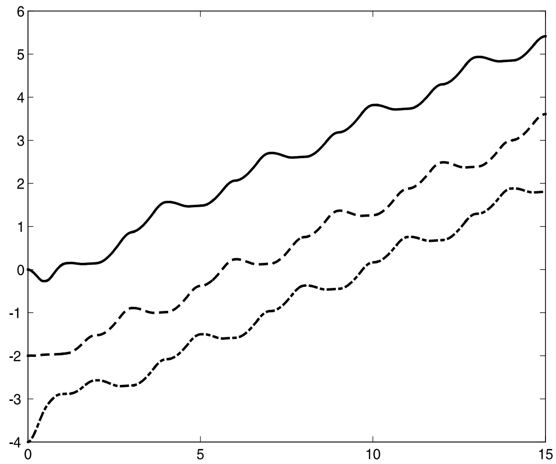

Figure 22: Worm motion.

There is again a good tracking of the desired gait from kinematical theory. But we observe an unsatisfactory external behaviour of the worm, obviously owing to the cancellation of stiction. $\diamond$

Simulation 5: Now, applying (23) with $F_0^- = 15$, $F_0^+ = 3$, $F^- = 10$ and $F^+ = 1$ yields with the same control data:

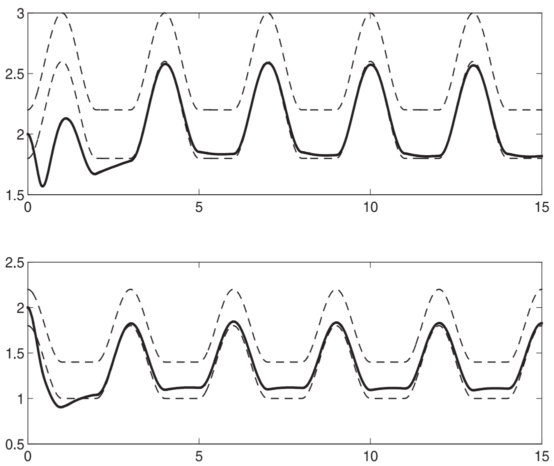

Figure 23: Output and tubes.

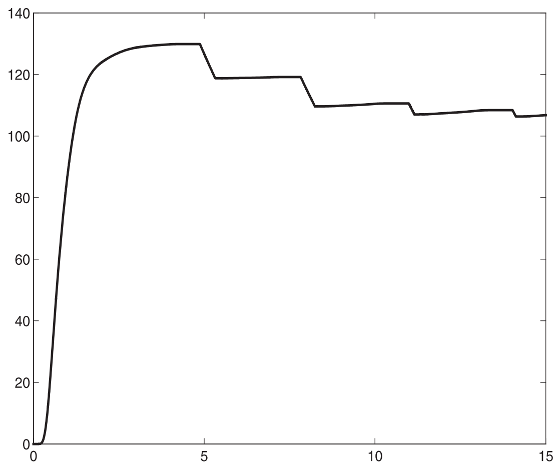

Figure 24: Gain parameter.

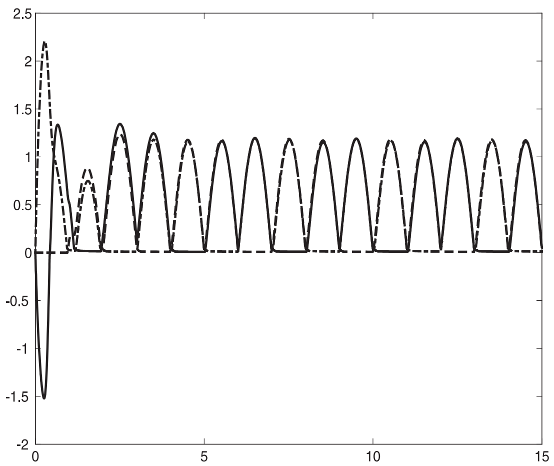

Figure 25: Velocities.

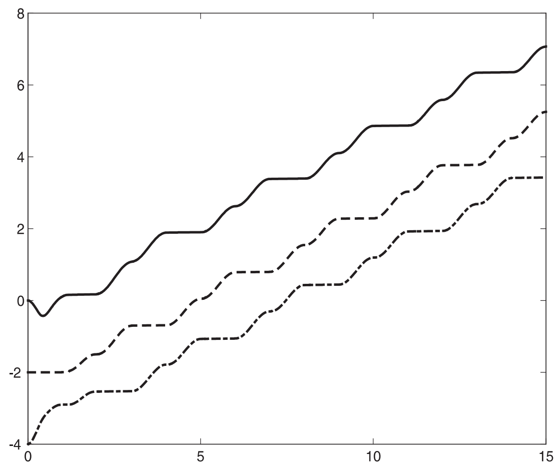

Figure 26: Worm motion.

Good behaviour, comparable with that in Simulation 3, but now a bit smaller velocities. ◇

7 Outlook & Conclusion

The foregoing considerations have shown that adaptive control is promising in application to worm-like robotic locomotion systems. In particular, it allows, based on the dynamical motion equations, to track desired movements from kinematical theory. Improved adaptive controllers are useful and should be developed further. Future tasks are

- tracking under friction which randomly changes online, possibly coupled with appropriate change of gaits ('gear shift'),

- worm systems with $n > 2$,

- experiments to validate theory.

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