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50. Internationales Wissenschaftliches Kolloquium
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V Chirikov / E. Kolev

COMPARISON OF CALCULATION AND TESTS FOR NATURAL FREQUENCIES OF BEAMS

INTRODUCTION

For the vast majority of elements of structures, buildings, machine links and robot arms, etc. from contemporary practical applications the classical Euler-Bernoulli theory, where the effects of rotary inertia and shear deformation are neglected, is a significant restriction for modeling.

Manipulators of nowadays robots can be given as an supporting example. Links of manipulators are heavy and bulky. They are much heavier than the load to carry they are disigned for, as soon as manipulator weight to load weight ratio is within the range from 30 to 50. The reason for such heavy links of manipulators is their stiffness properties. Designers stipulate links to be very stiff in order to secure the necessary precision of manipulator operations.

For the purposes of dynamic modeling links of industrial manipulators may be assumed to be short and thick beams, i.e. stubby beams. Disregard of rotary inertia and shear deformation effects in vibration of such objects leads to serious inadequacy of the applied models, especially important for the needs of effective manipulator control.

However, when accounting for these effects some considerable difficulties arise. In this respect we should mention clumsy control algorithms. They impede manipulator operation and obstruct effectiveness of robots application. The situation becomes more complex as a result of the fact that most precise calculations by contemporary numerical means give insufficient response, as far as the theory of Timoshenko [4] used in calculations is simply a first approximation. It is evident that further development of approximation methods for vibration analysis of stubby beams and bulky bodies is necessary, which would simplify numerical methods of control and bring closer to actual values in vibration processes.

1. STRANGE RESULTS FOR NUMERICAL EIGENFREQUENCIES OF STUBBY BEAMS

Natural frequencies of beams were thoroughly considered long time ago. Nowadays, nobody needs experimental verification of the Euler-Bernoulli beam theory, as it is finally proved that it yields a sufficient model for beams with relatively large slenderness ratios. However, the application of the modern theories for bulky elements of machines and structures with relatively low slenderness ratios is still ambiguous. Let us emphasize on a one ambiguity.

In vibration analysis of stubby free-free beam, manufactured out of ordinary steel, the eigenfrequencies of oscillation turn out to be higher than for the same fixed-fixed stubby beam. The divergence exacerbates significantly with diminishing of the slenderness ratio and with the increase of the frequency number. Such numerical results persist for various cross-sectional stubby beams modeled by different contemporary programs.

According to the physical sense the frequency increase means increase in stiffness of a flexible object. It turns out that a free stubby beam is stiffer than the same beam fixed at both ends. It can be asked then: why the structures are fixed for? Isn't it for making them stronger and hence more secure? Thus the results of calculation contravene the ordinary experience. The same situation can be observed in vibration analysis of pinned-free and fixed-pinned stubby beams.

2. CALCULATION AND TESTS FOR EIGENFREQUENCIES OF BEAMS

In the present survey an experimental verification of the real state of beams vibration and juxtaposition of them to numerical results is undertaken. For this purpose tests with series of beams under various boundary conditions were set up. The tests were conducted using up-to-date measuring system for acquisition and processing of vibration data. They were held in the Laboratory on Vibrations of Prof. Ozaki at Tokai University, Japan. During shock tests acquisition and processing equipment such as high sensitive (up to 36 kHz) and light-weight (up to 2 grams) accelerometers, laser technique and a frequency analyser (up to 100 kHz) were used. For forced vibration (shake) tests a shaker with a frequency range of excitation of up to 5000Hz was used. All such appliances are available at the Laboratory as usual equipment for conducting industrial research tests and for educational purposes as well.

2.1 EXPERIMENTAL SET OF BEAMS

A set of five steel (SS41) beams was designed and manufactured in order to conduct tests on vibration. The shape of beam cross-sections is rectangular with a depth of 5 mm and width of 10 cm. Their length varies as follows: 1400 mm, 1000 mm, 800 mm, 600 mm and 400 mm. A specialized Japanese producer of laboratory equipment guarantees high quality and precise dimensions of the samples. It is necessary to highlight here that dependent on the direction of vibration the same beam can be regarded as slender or as stubby. According to theory, for a stubby beam the length to depth ratio is less than 10, while for a slender beam it is much higher. That is why, if the direction of vibration is toward the depth of a beam with low length to depth ratio, the beam can be regarded as stubby and vice versa, if it is through the width of the same beam with big length to width ratio, it can be considered as slender.

By weighing the beams on electronic balance was determined their mass density ρ, equal to $8000\frac{kg}{m^3}$.

2.2 SHOCK TESTS WITH A SET OF SLENDER BEAMS

First, shock tests with series of slender unrestricted free-free end condition beams were carried out. In these tests the beams suspended by a string were subjected to a shock by a hammer in different points along their center line, perpendicular to the plane formed by width b = 100 mm and length l of a beam, i.e. upon the wider side of a beam. High-sensitive and lightweight accelerometer registered vibrations in this direction, being firmly fixed to the beam surface during its motion. Through an amplifier and a controller the signal was fed into the analyser for processing, while the spectrum of the processed signal permanently appeared on the screen. In order to establish high accuracy of the received data, the place of shock and attachment of the accelerometer along the beam varied until achieving close results. The spectrum range for every single beam also varied during the tests for checking the obtained data. At the moment of best spectrum appearance the screen image was fixed and the pick numerical data were registered. The obtained results are in full compliance with the existing classical theory of slender beams, as presented in Fig. 1.

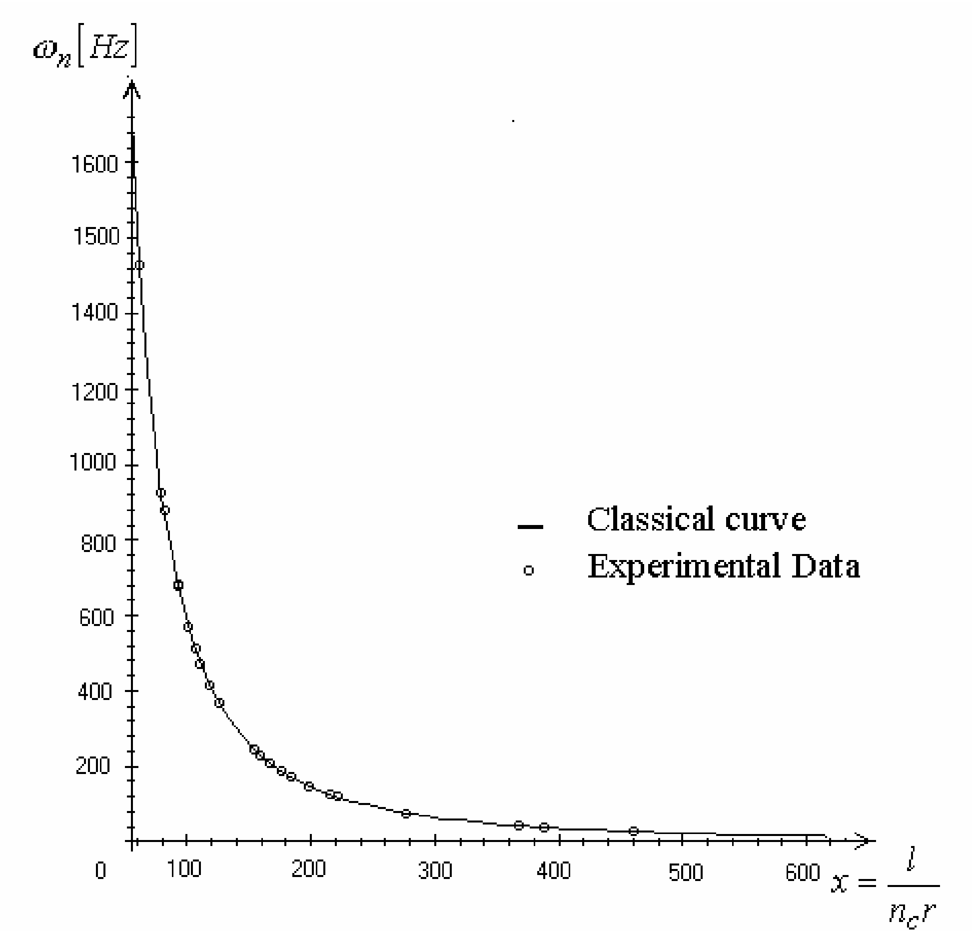


Fig. 1: Shock-experimental data vs. classical theory for slender free-free beams

The theoretical curve together with the specified experimental points demonstrates high precision of the experimental set up. The relative error in the theoretical and experimental results does not exceed 0.7 percent. This gives an opportunity to establish a correct value of modulus of elastisity E of the beam material. The received quantity appears to be within the range of known data for steel and is equal to 229GPa.

For graphical presentation in Fig.1. a special coordinate frame [1] with parameter x and a natural frequency ω_n along the axes was introduced. Along the horizontal axis the dimensionless value x characterizing geometrical attributes of beams, the applied support and number of frequency is designated. It means the next relation: $x = \frac{l}{n_c r}$, where r is a radius of gyration of a cross-sectional area.

For a simply supported beam for instance, the consecutive frequency-correlated numbers n_c are equal to the frequency number n, i.e. $n_c = n$. For fixed-fixed and free-free beams the consecutive frequency-correlated numbers are the following: 1.505619, 2.499753, 3.50001, 4.5, etc. For fixed-pinned or one-end pinned beams the consecutive numbers are 1.249876, 2.249999, 3.25, 4.25, etc. For cantilevers they are 0.5996864, 1.494176, 2.500247, 3.5, 4.5, etc. A general term for higher frequencies of all these series could be easily deduced. For fixed-fixed and free-free beams it is $n_c = n + 0.5$. For fixed-pinned or one-end pinned beams it is $n_c = n + 0.25$. For cantilever it is $n_c = n - 0.5$. Obviously, zero values of introduced frequency-correlated numbers related to the rigid body motions are not taken into account. For such values the parameter x is infinitely high. By definition the value of x could not be equal to zero. Moreover, it is advantageous to take the least value of x to be π at $n_c = 1$, that means the length of a beam is close to the beam height. For the least value of x at $n_c \neq 1$ we assume the related to the frequency-correlated number ratio π / n_c.

Along the ordinate axis the natural frequencies of beam vibrations ω_n are presented. Thanks to the introduced parameter x the classical natural frequency of every slender beam subjected to typical constraints can be depicted as follows

$$\omega_n = \frac{c}{r} \cdot \left(\frac{\pi}{x} \right)^2 \tag{1}$$

where we denote $c = \sqrt{\frac{E}{\rho}}$.

Apparently, we get the relation of quadratic hyperbola. For the convenience of results presentation let us use the next dimensionless parameter y:

$$y = \left(\frac{\pi}{x} \right)^2 \tag{2}$$

By its help the classical natural frequencies of vibration ω_n acquire the relation:

$$\omega_n = \frac{c}{r} \cdot y \tag{3}$$

2.3 SHOCK TESTS WITH A SET OF STUBBY FREE-FREE BEAMS

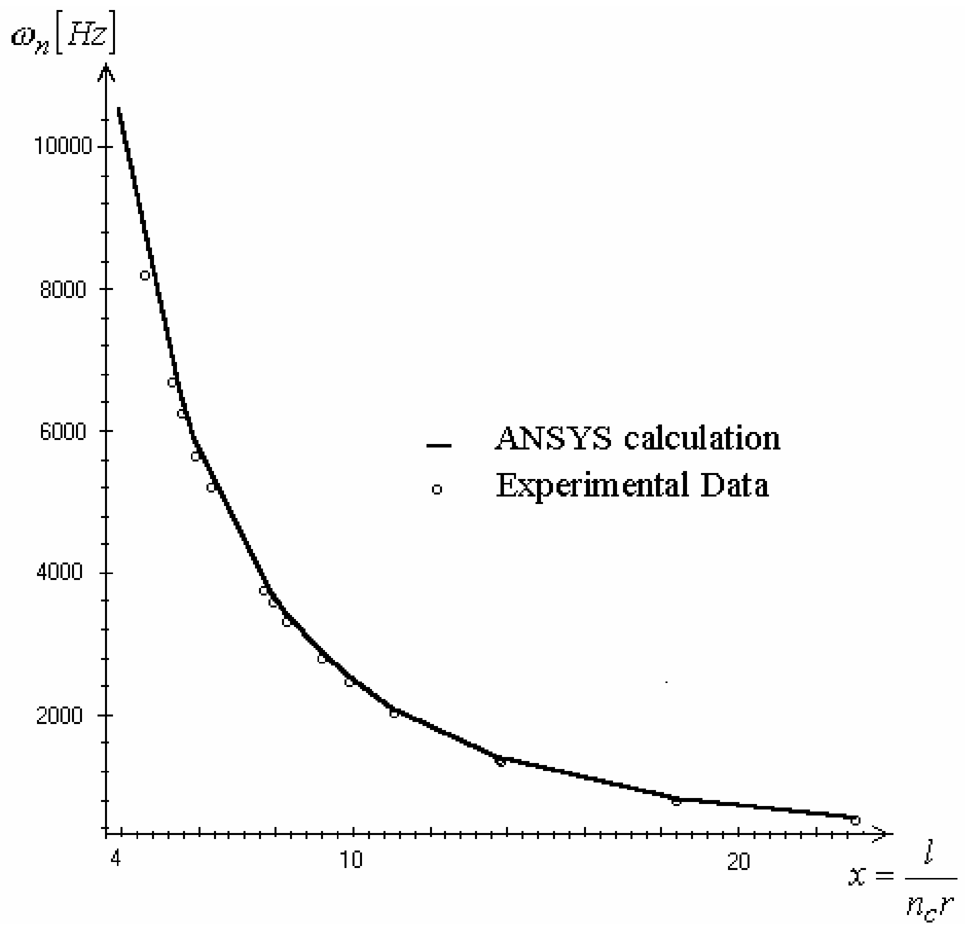
That after, the shock experiment was applied to series of stubby free-free beams [3]. For stubby beam tests the same set of beams was utilized. The difference was only in the change of shock and measuring direction. In these tests the sensor was put on the thin side of the beam profile of h = 5 mm and the measuring was carried out in the direction of width b of the cross-section, perpendicular to the length l of a beam. The shock was implemented in the same direction, that means the impact was exerted on the thinner side of a hung up beam. Here we rely on the fact that the width of the profile does not influence frequencies of transverse vibration of beams

The obtained data match well the data from the conventional theory for stubby free-free beams calculated by Master students Mr. Masuda and Mr. Hasegawa from the Department of Prime Mover Engineering of Tokai University, Japan and checked once more by Dr.-Ing. Emil Kolev from Technical University of Ilmenau, Deutschland with the ANSYS program package [6], as shown in Fig. 2. During calculations by the ANSYS program the Poisson's ratio is assumed 0.28. The experimental and the numerical data seem to be very close to each other, although the experimental data are a little bit lower. This is quite natural for FEM calculations [5]. Good correspondence of the compared data proves the validity of the theory of Timoshenko [4] for stubby beams with free ends.

Besides numerical results by the ANSYS program, the test frequencies of stubby beams were compared to the following approximate formula [3]:

$$\omega_n = \frac{c}{r} \cdot \frac{y}{\sqrt{1+4y}} . \tag{4}$$

It can be easily seen that the approximate formula (4) does not involve neither Poisson's ratio nor the shear coefficient. Nevertheless, the relative error between experimental data and approximate relation does not exceed 3 percent. Surely, Poisson's ratio has no substantial influence on the values of the experimental data obtained. Surely it is a particular feature of unrestricted free-free beams.



Fiq.2: Shock-experimental data vs. ANSYS data for stubby free-free beams

2.4 TESTS WITH A SET OF STUBBY FIXED-FIXED BEAMS

On the basis of the very good correspondence between the theory and tests for both slender and stubby free-free beams we switch over to shock testing of a set of stubby fixed-fixed beams. For this purpose two massive vices were placed facing one another. They assured firm fixture of the beam ends and with their own eigenfrequencies do not contribute to the range of measured vibration data. In this case the beam of $l = 1.4$ m length, by clamping it ends, serves as fixed-fixed beams of $l = 1$ m and $l = 0.8$ m. For the tests with shorter fixed-fixed beams a follow-up in length beam from the set was chosen. Thus, the already tested beams were used for assessment of the test quality.

At the very beginning the laser technique was applied for testing, but after some trials it was rejected. The reason for this is the high sensitivity of the laser technique that lead to bad records. Even in tests by shock upon the wide side of the beam it was impossible to register the proper value of the fundamental natural frequency for slender fixed-fixed beam, while for stubby fixed-fixed beam the first frequency could not be indicated at all. After a substitution of the laser by a lightweight accelerometer the records for slender beam became clearer.

After this change the fundamental eigenfrequency start to be measured with high accuracy. The data obtained were compared with the previously received data for slender free-free beams, as their eigenfrequencies are equal to eigenfrequencies of slender fixed-fixed beams. After experimental achieving of such a coincidence a good assurance of clamped boundary conditions was established. It was the landmark for passing on to substantial tests for a stubby fixed-fixed beam. Unfortunately, at the very first shock upon the thin side of the beam the difficulty to achieve eigenfrequencies for stubby fixed-fixed beams became evident. Even fundamental eigenfrequencies were very hard to register. Moreover, the registered frequency values appeared to be much lower than those calculated by ANSYS.

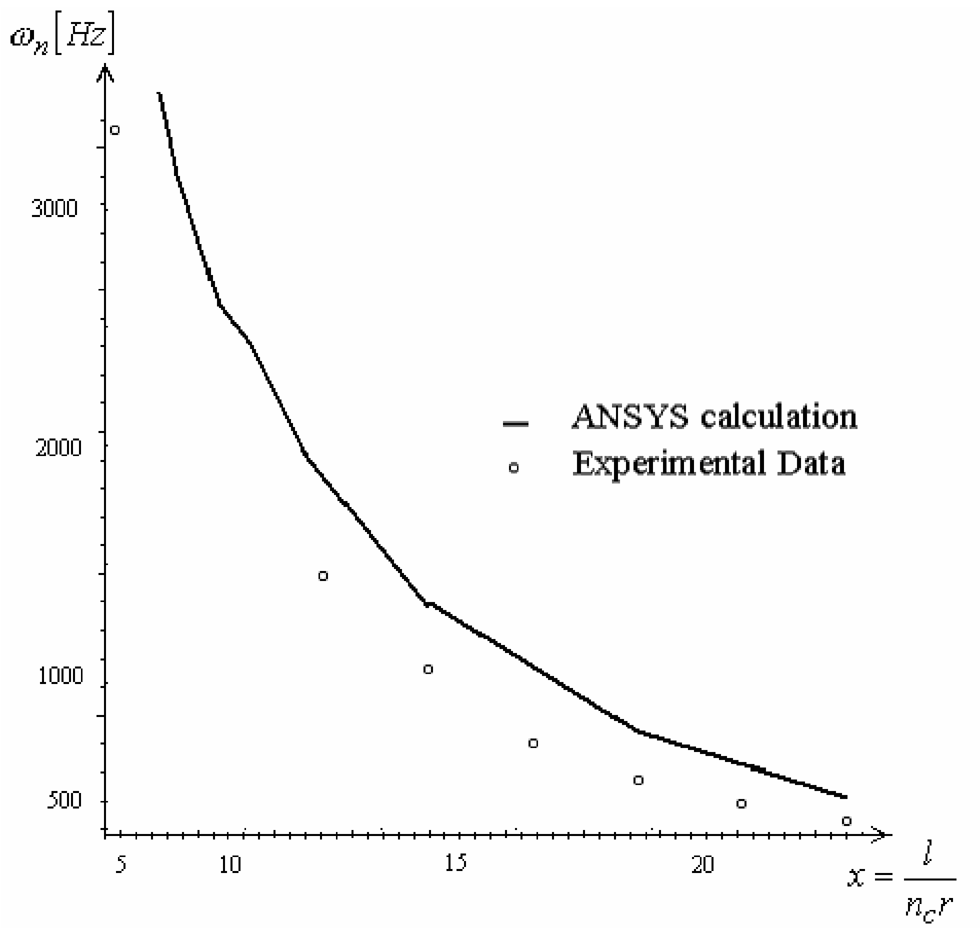


Fig. 3 Shock-experimental data vs. ANSYS data for stubby fixed-fixed beams

The obtained divergence over 30 percent between experimental and numerical data received by the ANSYS program can be qualified as enormous. We should emphasize heavily the fact, that the ends fixture was not a reason for that divergence. The actual reason is the small amplitudes of natural vibrations of stubby fixed-fixed beams in respect to slender fixed-fixed beams, which vibrations come through the base as an echo. A more thorough analysis reveals that the data regarded as first frequencies of stubby fixed-fixed beams actually are exactly the values of the fifth natural frequency of the respective slender beam.

To avoid the influence from low frequencies of slender both ends clamped beams it was necessary to set up a shake experiment. The alternative solution is to use series of quadratic or round cross-section beams. The latter option means much bigger weight of each beam and higher costs of research.

The advantage of shake tests is that the vibration generator consecutevely excitates vibration of an object with different frequencies in a specified direction of motion. When the frequency of excitation become equal to the natural frequency of the tested object, then the amplitude of the response increases. The enhanced amplitude at resonance frequency indicates the exact value of the eigenfrequency of an object. .

Difficulties to set up a shake experiment for both ends clamped stubby beam are the dimensions of a vibration generator platform and its frequency range of excitation. The dimensions of the platform must be roughly 1m × 1m to let in a one-meter long stubby beam with fixtures. The range of excitation must be not less than 5000Hz in order to achieve the real picture of natural frequencies for the available beams.

An available vibration generator (shaker) at the Laboratory has a sufficient platform, but operates at a low range of excitation, namely up to 250Hz. Another available shaker sustaining a high level of excitation of up to 5000Hz has a small platform. Under these existing circumstances the only way to continue the research of the boundary conditions impact upon eigenfrequencies of beams is to study a one-end clamped beam, i.e. a stubby cantilever.

3 TESTS WITH A SET OF STUBBY CANTILEVER BEAMS

To confirm the shock test results with stubby cantilever, obtained at the Technical University of Varna, Bulgaria [2], the new set of tests with series of cantilevers was carried out at the Laboratory on Vibratiion at Tokai University. The tests were easily implemented by using the same equipment, as in the case of tests with fixed-fixed beams.

3.1 SHOCK TESTS WITH STUBBY CANTILEVERS

Shock tests with cantilevers were conducted by fixing one end of the beam by a massive vice and executing a knock by a hammer upon the free end of the cantilever. Unfortunately, like in the case of both ends fixed beam the registered data were significantly influenced by the response of the respective slender beam that comes through the base as an echo. Although the first frequencies from the shock tests with cantilevers were inapplicable because of the above mentioned influence, the second frequencies appeared to be well exposed. To clarify the registered data shake tests were additionally carried out as well.

3.2 SHAKE TESTS WITH STUBBY CANTILEVERS

The vibration generator platform of the available shaker that generates a high level of excitation allows installing of only one fixture. Thus, instead of fixed-fixed beams the shake tests can be conducted for cantilevers.

During the preparation of such tests special attention was payed to the beam fixture to the platform. Its design was improved in order to avoid lateral movements and to prevent influence of slender beam eigenfrequencies on the responce. Besides, in each test special measures were undertaken for balancing a beam with fixture on the vibration platform. Such measures are necessary for protecting the beam responce from noise encountered from bending of the mechanical system.

Shake tests with series of stubby cantilevers reveal for sure fundamental eigenfrequencies of vibration. Some second frequencies of the cantilevers were discovered by those tests as well. After the acquisition and processing of vibration data for fundamental and second frequencies, the results obtained are presented in Fig.4 and Fig.5. Figure 4 depicts the distribution of experimental results for the fundamental eigenfrequency of tested cantilevers in respect to the approximate formula (4) obtained for stubby free-free beams, while Fig. 5 presents the same correspondence for second frequencies of stubby cantilevers. Both graphs explicitly demonstrate the increase of eigenfrequencies of stubby cantilevers over stubby free-free beams that match the previous results obtained in TU-Varna, Bulgaria. As before, the experimentally determined enhancement of eigenfrequencies contradicts the preliminary calculation results obtained by the ANSYS package for stubby cantilevers, which yield a substantial downturn in eigenfrequencies with respect to stubby free-free beams. The same conclusion can be made in the case of stubby fixed-fixed beams.

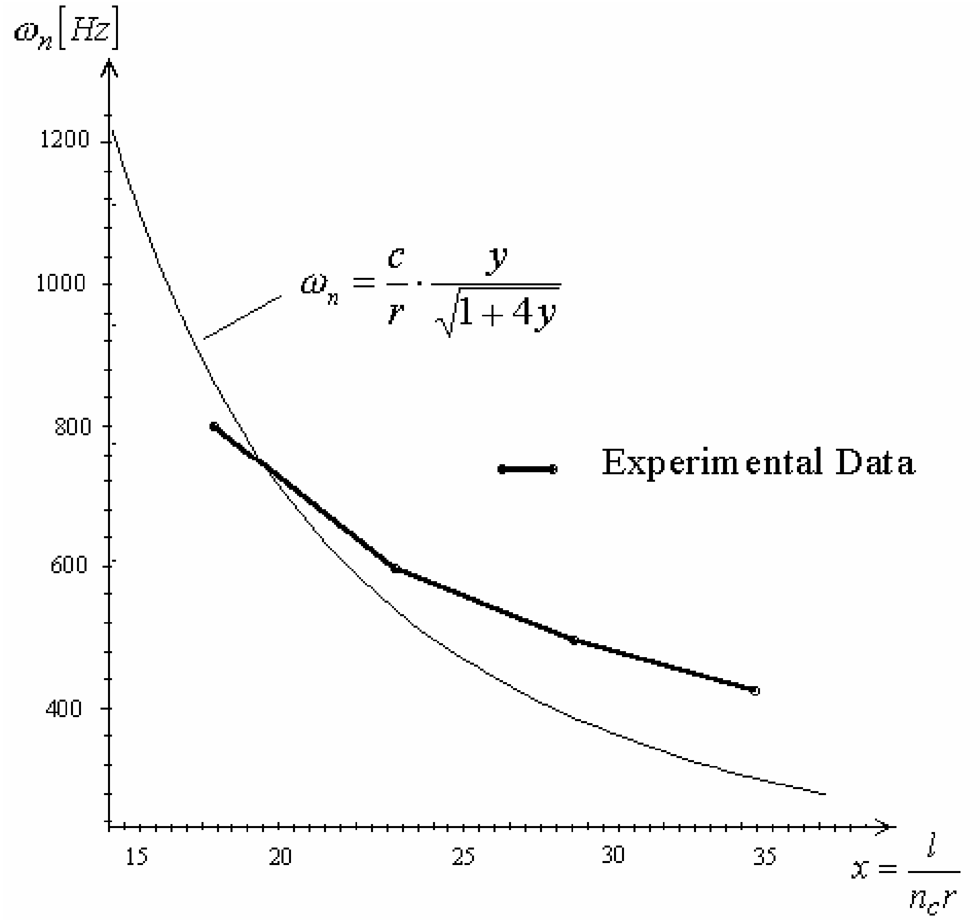


Fig. 4. First eigenfrequencies for stubby cantilevers vs. stubby free-free beams

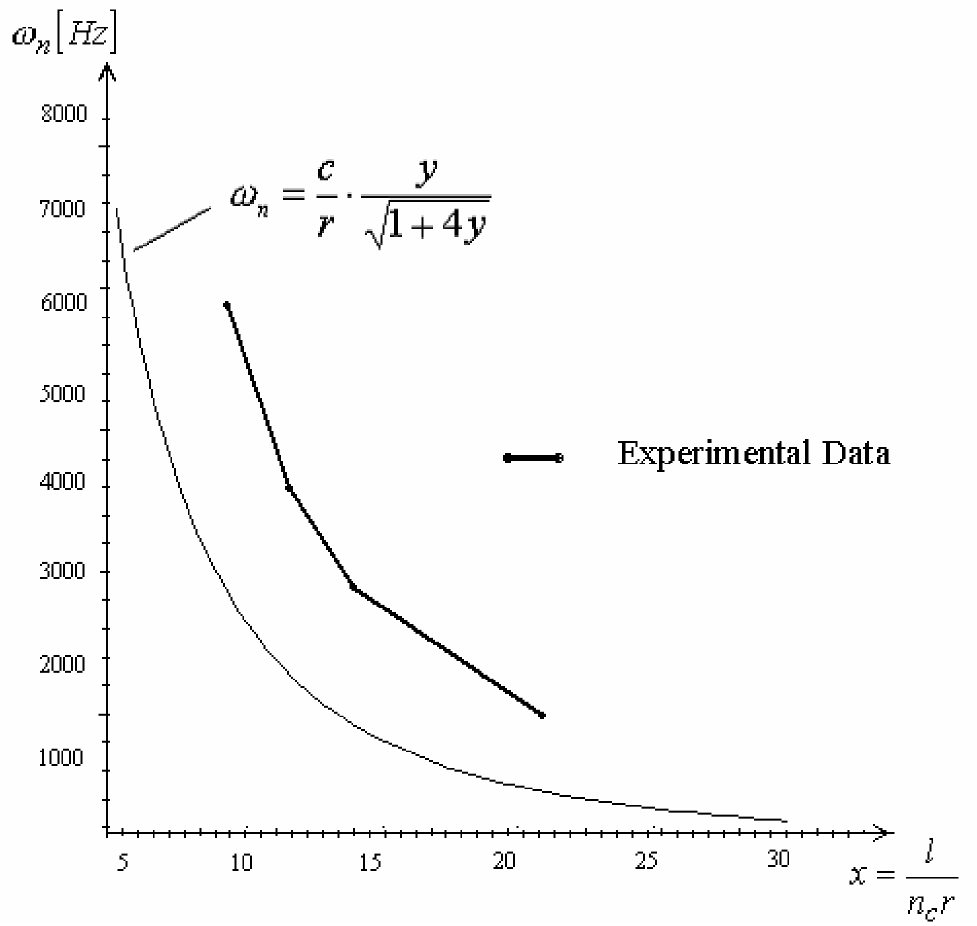


Fig. 5. Second eigenfrequencies for stubby cantilevers vs. stubby free-free beams

4. CONCLUTIONS

The test data obtained for stubby free-free beams prove the validity of the theory of Timoshenko for unrestricted bodies.

Unfortunately, during the survey of stubby fixed-fixed beams the registered data were significantly influenced by the response of the respective slender beams that comes through the base as an echo. This reveals the reason why until now it was difficult to confirm or to reject the theory of vibration for restricted stubby beams, as it was difficult to reveal the actual response of restricted stubby beams with fixed end conditions. Surely, the final conclusion for the validity of the vibration theory for stubby beams was made leaning only on eigenfrequencies of unrestricted beams with free end conditions.

The comparison between experimentally received data for stubby free-free and stubby cantilever beams discloses superiority in natural frequencies for cantilevers. It seems to be quite natural, as cantilevers are more restricted in motion in respect to relevant free-free end condition beams. Besides, we can expect now that eigenfrequencies of a fixed-fixed beam to be higher than both free-free and cantilever beams.

To confirm this statement on the next stage of research we may focus on carrying out additional shock and shake tests for stubby cantilevers and fixed-fixed beams with round cross section. By these means we can avoid the influence of higher eigenfrequencies of a slender beam on eigenfrequencies of a stubby beam. The real picture of stubby beams vibration would be of great importance for disclosing the real oscillation behavior of robot arms and bulky structures.

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