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URN (Paper): urn:nbn:de:gbv:ilm1-2011iwk-005:9

56^{TH} INTERNATIONAL SCIENTIFIC COLLOQUIUM
Ilmenau University of Technology, 12 – 16 September 2011
URN: urn:nbn:gbv:ilm1-2011iwk:5

DESIGN AND SIMPLIFIED MANUFACTURING OF LARGE-DEFLECTIVE FLEXURE HINGES BASED ON POLYNOMIAL CONTOURS

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ABSTRACT

Material coherent flexure hinges with lumped compliance allow a specific geometrical design of monolithic compliant mechanisms and their deformation behavior. This contribution deals with the model-based design and simplified manufacturing of prismatic flexure hinges with symmetric polynomial notch contours to increase the motion range. Therefore the hinge geometry is described exemplarily by polynomial functions as well as simplified notch contours based on radii. FEM simulations are used to verify the determined polynomials and approximated contours regarding a minimal ratio of maximum stress to deflection depending on the polynomial order.

Index Terms - Compliant Mechanisms, flexure hinges, increased deflection, polynomial contours, simplified manufacturing

1. INTRODUCTION

Compliant mechanisms and flexure hinges have been the subject of various investigations in the past. They can be found in various areas of application. The microsystem technology (e.g. MEMS) and precision engineering (e.g. adjustment fixtures) can be seen as the key application areas. This is due to the fact that compliant mechanisms and hinges have only internal friction and therefore no wear, what is a desirable feature in these application areas (e.g. [1], [2]).

The term *compliant mechanism* is not consistently used in the literature. It is used to address specifically designed hinges as well as a variety of complex articulated flexible mechanical structures. In this paper the term *flexure hinges* will be used and understood as homogeneous, monolithic and bending articulated revolute joints with prismatic geometry and an elastic material behavior. The influence of the *notch contour* (cf. Figure 1) on the elastic properties is the focus of this contribution.

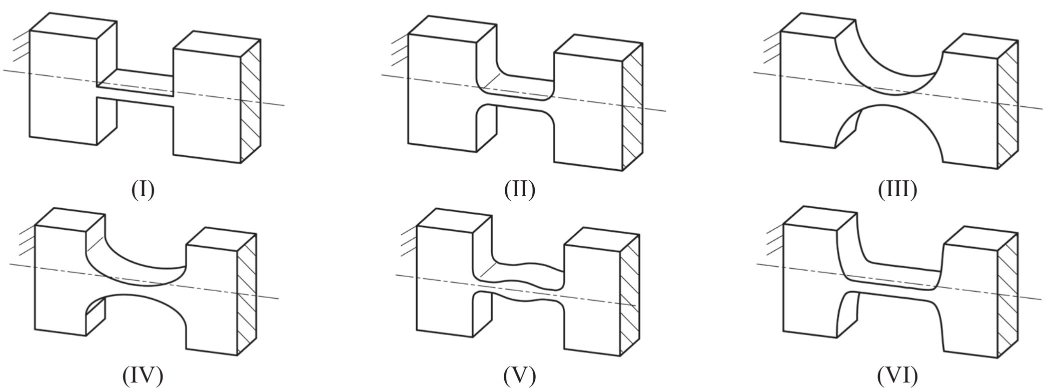

Figure 1: *Types of prismatic flexure hinges - (I) rectangular contour, (II) corner filleted contour, (III) circular contour, (IV) elliptical contour, (V) spline contour and (VI) polynomial contour*

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The influence of the hinge geometry on the deformation behavior of such flexure hinges has been investigated in several former contributions (e.g. [3], [4]). Starting with geometric primitives (circular and rounded corners) in early ones, notch contours with increasingly complex geometries (Bezier-curves, splines) have been considered in more recent contributions (e.g. [5], [6]). The aim of these investigations was usually to increase the intended elasticity, i.e. to reduce the resistance to the hinge deformation, according to a particular axis and to reduce the corresponding ratio of bending stress to deflection. Furthermore, the shift of the rotational axis (which is not consistently defined) was considered occasionally (e.g. [7], [5]).

Although the manufacturing of complicated optimized contours with irregular curve radii is possible with modern beam cutting and EDM-technologies, most applications that use such geometries are only prototypes. Causes for the low dissemination of optimized contours in comparison with common flexure hinges with e.g. circular contours, may be found in the simpler milling manufacturing (if applicable in one setting) and the difficulties of transferring the FEM-optimized geometry models into CAD models and CNC programs. Specifically the latter bears a risk of nullifying any complex optimizations by transcription errors between different models or programs which are added to the inevitable fabrication deviations.

This contribution presents an approach of modeling the flexure hinge contour on the basis of polynomial functions. This allows a good approximation of existing optimized hinge geometries and a comparably easy transfer to various software systems. With regard to milling an approach of further simplification by substituting the polynomial hinge contour by fillets of a few radii will be exemplified.

2. ASSUMPTIONS AND APPROACH

Subject of this paper is the geometrical design of a bending loaded, prismatic flexure hinge with a polynomial contour according to Figure 2. The main assumptions in respect to the further considerations are:

- a planar motion,
- a prismatic cross section,
- hinge dimensions: notch length l_G, notch height H_G, notch width $B = 0.5H_G$ and minimal notch height h_G,
- lumped distribution of compliance ($l_G = H_G$),
- a symmetrical, continuously differentiable, not undercut hinge geometry.

The attached segments are assumed to be ideal rigid and the considerations are limited to the notch contour. One end of the hinge is assumed to be fixed to the frame and free end is loaded with a bending moment M_{bz} or a single force F (applied by a displacement v). The material is assumed for all calculations as steel with linear elastic material behavior and the properties: E = 200 GPa, μ = 0.3 and ρ = 7.85 gcm^{-3}.

The aim of the proposed approach is to minimize the stress-deflection ratio. The obtained results are compared to a circular (*contour 1*) and an elliptical hinge geometry (*contour 2*, with semi-axis $r_a = 2r_b = H$). These simple contours are used as reference contours and are well known and described in the literature. The shift of the rotational axis during deflection is not regarded.

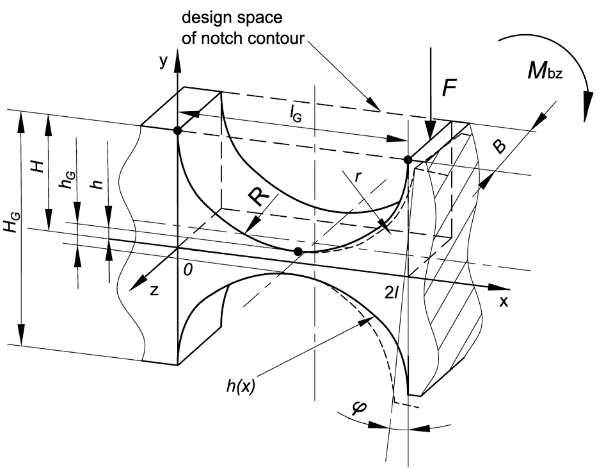


Figure 2: *Convention of the parameters of the flexure hinge with the notch contour h (x) and the – as rigid assumed – attached segments (shown is the circular reference contour with the radius R = H).*

The approach utilizes polynomial functions to describe the notch contour. Polynomials are mathematically simple to formulate and allow the adjustment and optimization of the hinge contour depending on the order and the coefficients of the polynomial function.

On the basis of biquadratic polynomials that are analytically determinable by the boundary conditions, polynomials of higher orders are calculated by FEM simulations.

2.1. Approach of polynomial notch contours

The symmetric biquadratic polynomial $h(x)$ (eq. 1) describes the notch height of the considered flexure hinge. The boundary conditions for the determination of the five coefficients of this function are indicated in Figure 2 by the default curve points $(0, H)$, (l, h) and $(2l, H)$.

$$h(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 \tag{1}$$

For a maximization of the deflection angle φ of the hinge loaded with a constant bending moment with dimensionless length variables normalized to the height $H = 1$ of the hinge (eq. 2),

$$\varphi(2l) = v'(2l) \xrightarrow{a_i} max. \quad (i = 0,..,4) \tag{2}$$

and assuming the same geometry and material properties, the equation for the deformation results in a function only dependent on the height $h(x)$, [8].

$$v''(x) = \frac{12\,M_{bz}}{E\,B}\frac{1}{h(x)^3} \tag{3}$$

The resulting coefficients from a_0 to a_3 (cf. Table 1) of the polynomial function are dependent on the half minimal notch height h and the polynomial coefficient a_4.

Table 1: *Coefficients of the polynomial of 4th order h(x) and values for predetermined h and a_4 (cf. eq. 1)*

Coefficient	Conditional equation	Value (h = 0.1H and a_4 = 0.9)
a_0	1	1
a_1	$-2(1+a_4-h)$	-3.6
a_2	$1+5a_4-h$	5.4
a_3	$-4a_4$	-3.6
a_4	$\geq -1+h$	0.9

According to the coefficient a_4 different notch contours can be calculated (cf. Figure 3). It can be shown that the maximum deflection can be achieved for $a_4 = 0.9$, [8].

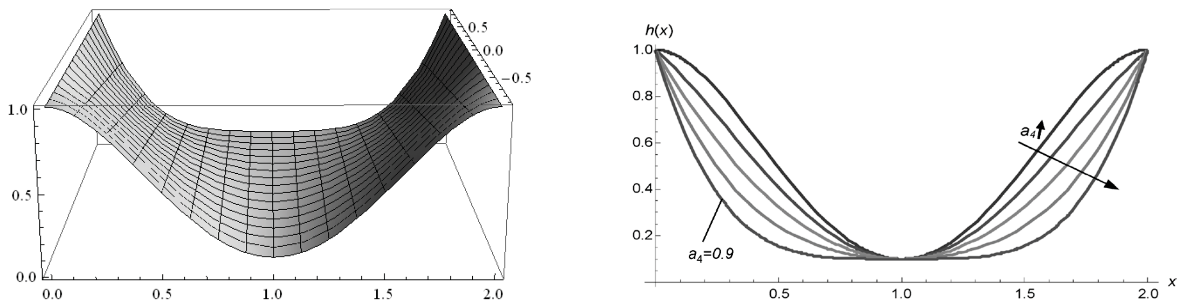


Figure 3: *Notch contour h(x) of half flexure hinge depending on the coefficient a_4 (based on: $l = H$, $h = 0.1H$ and a constant moment M_{bz}). The notch contour $a_4 = 0.9$ provides the maximum deflection.*

In the following considerations, the polynomial with the coefficients according to Table 1 (*contour 3*) will be considered.

2.2. Polynomials of higher order

By simplifying the polynomial of 4th order, it can be shown that any symmetrical polynomial depends on the semi hinge parameters h, l and H and the order n of the polynomial (eq. 4).

$$h(x) = h + \left(\frac{1-h}{l^n}\right)(x-l)^n \tag{4}$$

An increase of the order of the polynomial produces a smoother slope of the notch contour and an approximation of corner-filleted notches with radii values of the fillet inversely proportional. Very small radii result in high notch stresses, so that with regard to the stress-deflection ratio an optimal order (between circular notch contour with a low order and quasi rectangular notch with a very high order) can be expected. Based on the chosen assumptions, this optimum is $n = 16^{th}$ order and is independent of the notch length l_G (cf. Figure 4).

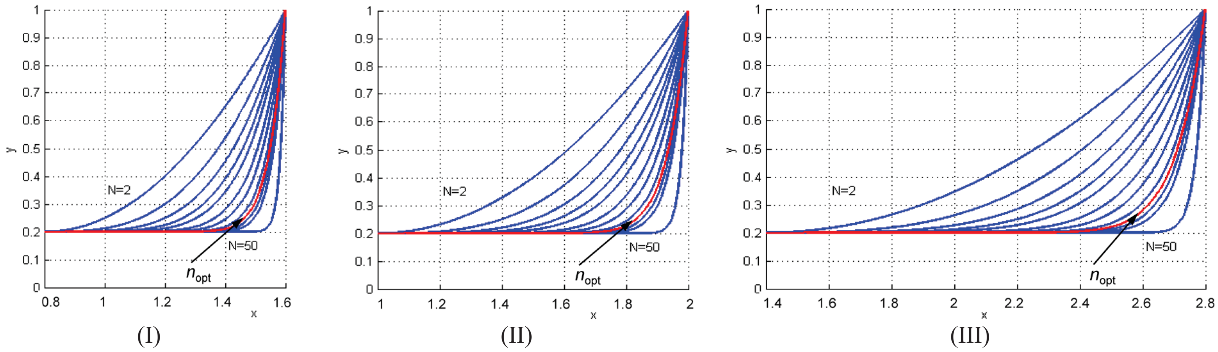


Figure 4: *Optimal order of polynomial functions for a minimal ratio of stress to deflection in dependence of the notch length l_G (exemplified for $l = H$ and $h = 0.2H$ with $H = 1$ and a displacement $v = 0.05H$ at the hinge end) - (I) length $l_G = 0.8H_G$, (II) length $l_G = H_G$ and (III) length $l_G = 1.4H_G$.*

For the further considerations the 16^{th} order polynomial hinge geometry *(contour 4)* with $h_G = 0.1H_G$ and $l_G = H_G$ is used (eq. 5).

$$h(x) = 0.1 + 0.9\,(x-1)^{16} \tag{5}$$

The resulting contours for polynomials of 16^{th} order are geometrically similar to corner filleted notch contours. The main difference to the latter is the slope at the intersection to the rigid segments of the hinge. The simplification by approximating the polynomial contours by different radii will be further regarded.

3. SIMPLIFICATION OF COMPLEX CONTOURS AND RESULTS

The further considerations will relate to the hinge geometries with biquadratic polynomial notch contour and 16^{th} order polynomial notch contour considering the possibility of *simplification*. The aim is to create hinge geometries that can be realized by milling without any jet-cutting or wire-EDM manufacturing processes. The observations therefore relate particularly to precision engineering applications.

3.1. Concept of simplification

For this the polynomial contour will be substituted by radii for the tool and its movement (cf. Figure 5). At this R_1 represents the maximum radius of the tool and the radii R_2 and R_3 are related to the milling trajectories that have to be realized by the control of the machine tool. The substitution shown in Figure 5 is normalized to H and has to be scaled to adapt to different contour dimensions.

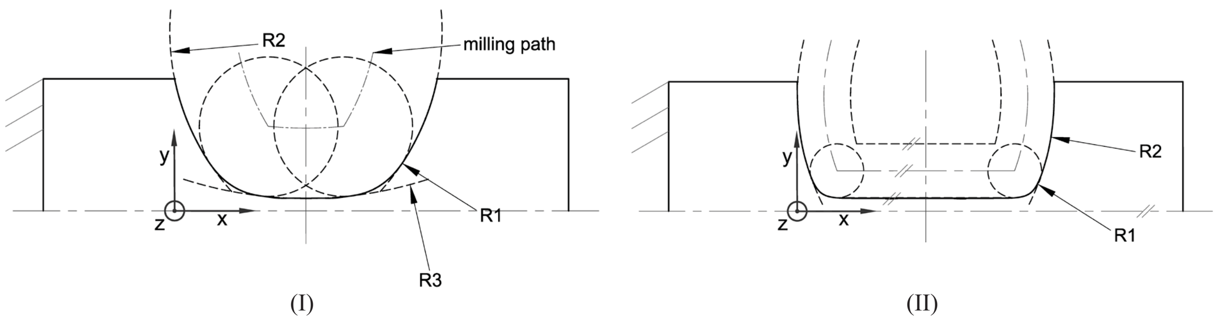


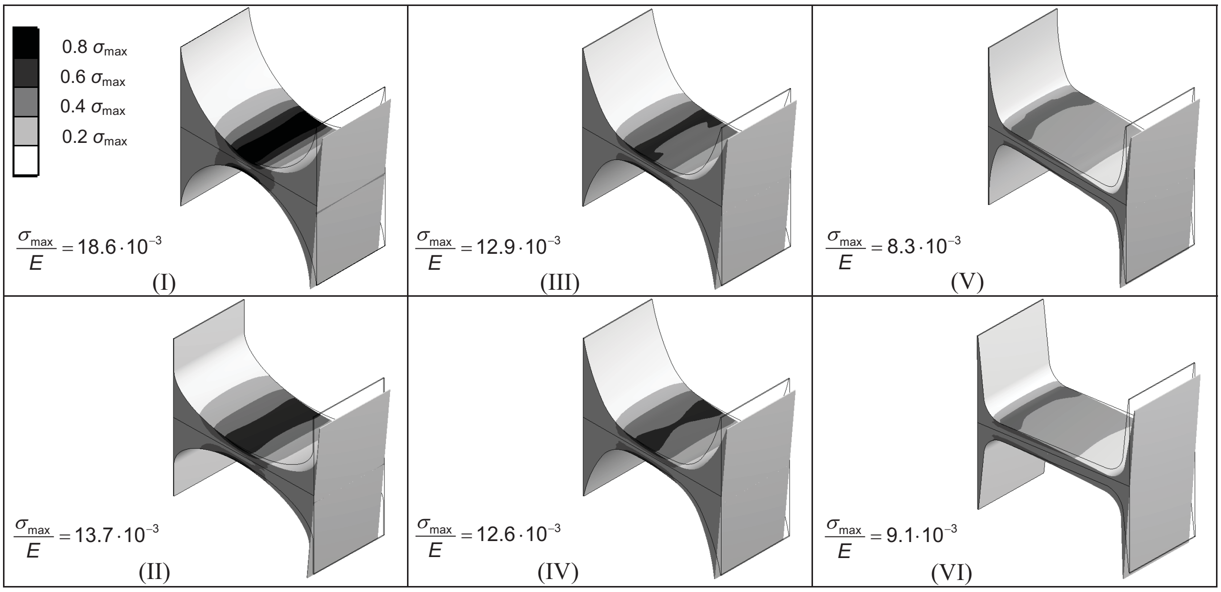
Figure 5: *Substitution of polynomial notch contours with radii - (I) contour 3 and (II) contour 4. Both, the original and the simplified contour are represented.*

The chosen radii represent the maximum permissible values. To avoid unpleasant fractional tool diameters, smaller diameters may be chosen with the demand to adapt milling trajectories accordingly.

3.2. Result of the simplification in terms of stress-deflection

The effects of the contour simplification with respect to the stress-deflection ratio were evaluated by FEM analyses and are shown in Table 2 compared to the reference and the original polynomial contours.

Table 2: *FEM results with stress distribution and ratio of relative maximum (exemplified for v = 0.1H): (I) circular reference contour 1, (II) elliptical reference contour 2, (III) bi-quadratic polynomial contour 3 and (IV) its approximation, (V) 16th order polynomial contour 4 and (VI) its approximation.*



It turns out that the contours obtained by the polynomial approach showed:

- compared to the reference contours a lower (substantially cf. circular contour 1), respectively slightly (cf. contour 2) stress-deflection ratio,
- the 16th order polynomial contour 4 and its approximation with the best stress-deflection ratio at all,
- the particular more uniform stress state of polynomial contours in comparison to the reference contours,
- the simplification of the contour resulting only to a marginal change in the stress-deflection ratio and stress distribution.

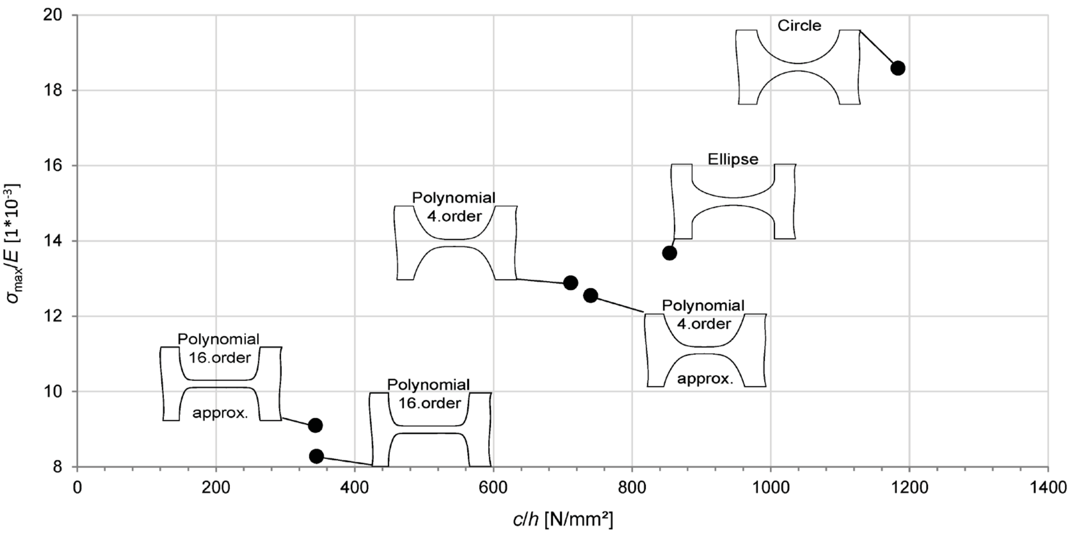


Figure 6: *Comparison of the determined polynomial contours and their approximations with common reference contours for the relation of the two criteria relative maximum stress σ_{max}/E and relative stiffness c/h_G*

Furthermore, due to the smaller amount of material (with comparable minimum notch height h_G) an increasing elasticity of hinges with high order polynomial notch contours can be noticed (cf. Figure 6).

4. DISCUSSION

The presented description of flexure hinges using polynomial notch contours is a simple and basic approach and can be used in a variety of analytical models and programs for modeling the hinge geometry. Most notch contours can be approximated by the order and coefficients of the polynomial. The only requirement is that the hinge geometries can be described by a function (undercut contours are difficult).

Furthermore for the study presented in this paper only the criterion of stress-deflection ratio was considered, which, depending on the application is not the only or even the dominant object of interest. Besides this limitation, the presented approach provides further starting points for improvement, such as the variation of l_G and H_G relative to h_G. The considered polynomials of higher order beyond the biquadratic are of a simple type and all coefficients differ from zero. A further possible optimization due to the variation of the coefficients has not been investigated.

The notch contours determined under the constraints adopted in this paper are very similar to corner-filleted notch contours and demonstrate, in comparison with circular and elliptical notch contours, a low stress-deflection ratio.

However, it can be shown (for a more precise analysis refer to [9]) that biquadratic polynomial hinge geometries are a good compromise in terms of minimizing both criteria, the maximum stress as well as the shift of the rotational axis (cf. Figure 7).

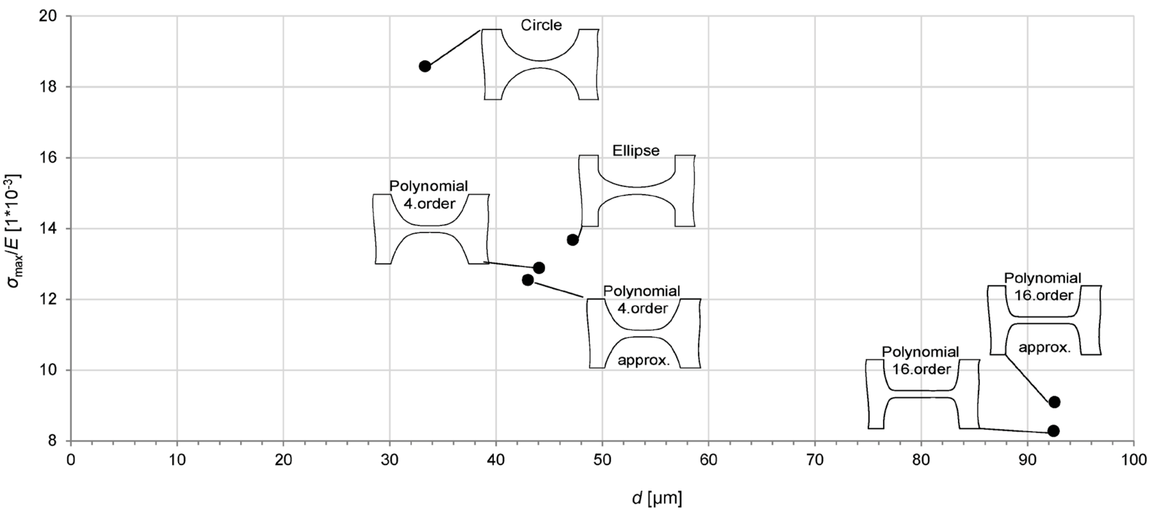


Figure 7: *Comparison of the determined polynomial contours and their approximations with common reference contours for the relation of the two criteria relative maximum stress σ_{max}/E and shift of rotational axis d*

The approach for simplification of the hinge geometry is based on the assumption of milling as manufacturing technology. This is a restriction which does not necessarily apply (e.g. lithographic fabrication of MEMS) and which requires a further examination of the influences of manufacturing deviations as well.

Nevertheless the presented simplification approach is advantageous compared to complex optimization approaches and the resulting complicated notch contours, because a suitable compromise can be made by relatively simple hinge geometries with respect to the most common optimization criteria (elasticity, stress-deflection ratio and shift of rotational axis), see Figure 6 and 7.

5. CONCLUSION

In this contribution, the potential of polynomial geometries as alternative notch contours for prismatic flexure hinges is described. The advantages of an increased motion range of a notch contour based on an analytically determined bi-quadratic polynomial function are shown. Furthermore, FEM-Simulations show that 16^{th} order polynomials are especially suited to minimize the stress-deflection ratio of flexures as a result of deformation.

Regarding the simplified manufacturing of complex hinge geometries by machining, the possibility to approximate any optimized contour with radii is exemplified for both determined polynomial contours. The model-based comparison of original and approximated contours with conventional circular and elliptical contours confirms the substitution as sufficiently accurate. The improvement of the ratio of maximum stress to deflection is also shown.

Particularly with regard to the use in precision engineering applications, the shift of the axis of rotation is also relevant. Here it is shown that biquadratic polynomials are well suited in terms of satisfying both objective criteria, a reduced stress level and a small shift of the rotational axis.

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