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THE INFLUENCE OF ASYMMETRIC FLEXURE HINGES ON THE AXIS OF ROTATION

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ABSTRACT

Material coherent flexure hinges allow a specific geometrical design of monolithic compliant mechanisms and their deformation behavior. In precision engineering especially prismatic flexures with lumped compliance are used. In this contribution the potential of simple asymmetric flexure hinges to minimize the shift of rotational axis is shown. Therefore the notch contour is described by different and with respect to the transverse height axis asymmetric hinge geometries based on circular fillets. For the FEM-based comparison with common contours a low stress-deflection ratio is considered as an additional criterion to the precision of rotation.

Index Terms - Compliant mechanisms, flexure hinges, precision of rotation, asymmetric notch contours

1. INTRODUCTION

Due to their advantages, compliant mechanisms are state of the art in many fields of application, such as microsystems technology (because of the frictionless and wearless function), precision engineering (because of the absence of clearance and stick-slip) or metrology (because of their repeatability). In plane compliant mechanisms material coherent hinges fulfill the function of conventional revolute pairs but limited to small angular deflections of a few degrees (usually < ±5°).

In addition to the notch contour the distribution of compliance and the symmetry properties of the flexure hinges are crucial for the realization of guiding and transfer tasks in precise motion systems (Figure 1). In high precision guides and deformation elements (e.g. in precision balances) as well as in monolithic mechanisms for micromanipulation hinges with lumped compliance ($l \approx H$, cf. [1]) are used in particular.

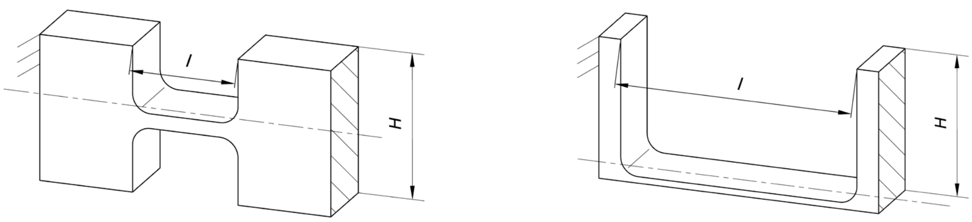

Figure 1: *Prismatic flexure hinges with different distribution of compliance and symmetry properties - (a) symmetric hinge with lumped compliance $l \approx H$, (b) asymmetric hinge with distributed compliance $l >> H$*

Flexure hinges have a limited motion range, which is defined i.a. by resulting stress. Furthermore, there is an in often cases undesirable contour and load-dependent shift of the rotational axis during deflection. The design of the notch contour aims for a (high) specific compliance as well as a small ratio of stress to deflection and the research aims to influence the deformation behavior of flexure hinges by geometrical design (e.g. [2]), choice of materials (e.g. [1]) or a combination of both (e.g. [3]).

Particularly in precision engineering, there are many studies on increasing the motion range as well as improving the precision of rotation during deformation. These resulting in complex types of hinges, which feature only a small shift (in the single-digit micrometer range) of the rotational axis [4], (cf. Figure 2).

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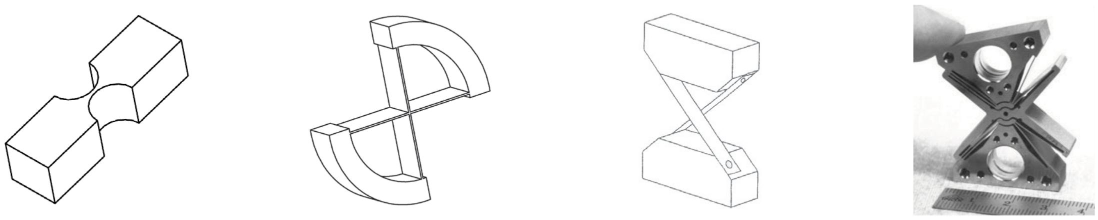


(a) (b) (c) (d)

Figure 2: *Development of more complex flexure hinges: (a) prismatic notch hinge [5], (b) leaf-type hinge [6], (c) cross-axis hinge [7] und (d) butterfly hinge [4]*

Because of the higher manufacturing effort caused by these complex hinges, the minimization of the shift of rotational axis by optimal notch design of simple prismatic hinges (cf. Figure 2a) is an on-going issue. Notch contours based on geometric primitives (e.g. circular, elliptical or corner filleted contours) and as a result of an optimization process, complex mathematical functions (e.g. conic sections, splines) are increasingly used to describe the hinge geometry ([8], [9], [10], [11]). The realization of these complex, sometimes completely numerical optimized and in some cases not in mechanisms and hinges differentiable structures is only possible by modern beam cutting methods (such as laser or waterjet cutting but also wire-EDM).

However, the transfer of the different notch contour optimization results is difficult. In most cases, the stress-deflection ratio and rarely the shift of rotational axis are objects of investigation. In almost all studies symmetric contours with different geometries and load cases are considered and interpreted in a different manner.

The aim of this contribution is to show the influence of specific ratios of geometrical characteristics of symmetric notch contours and compare them (as far as possible) to references of known literature. Furthermore asymmetric contours with respect to the longitudinal axis (equivalent to the central axis of the hinge) or the height axis (equivalent to the transverse axis in height direction) are examined, which has been done only in particular cases [12], [13] before. The potential of pivot optimized hinge geometries based on asymmetrically undercut contours will be shown first time.

2. MATERIAL AND METHOD

This contribution presents flexure hinges which are described by – with respect to the transverse height axis (y-axis) – symmetric and asymmetric notch contours based on radii. These contours can be defined by straight and circular segments and they are easy to machine. Furthermore, a widespread adaption as well as optimization is possible with regard to the geometrical parameters.

The aim of the geometrical design presented here is a bending-loaded prismatic flexure hinge with a notch contour that results in a minimal ratio of stress to deflection as well as a minimal shift of the rotational axis. Subject of the model-based FEM investigations are flexure hinges with the following characteristics (Figure 3):

- a planar motion,
- a prismatic contour with rectangular cross section,
- hinge dimensions: notch length l, notch height H, minimal residual height h and width $B = 0.5H$,
- a lumped distribution of compliance ($l = H$),
- with respect to the longitudinal axis (x-axis) a symmetric notch contour.

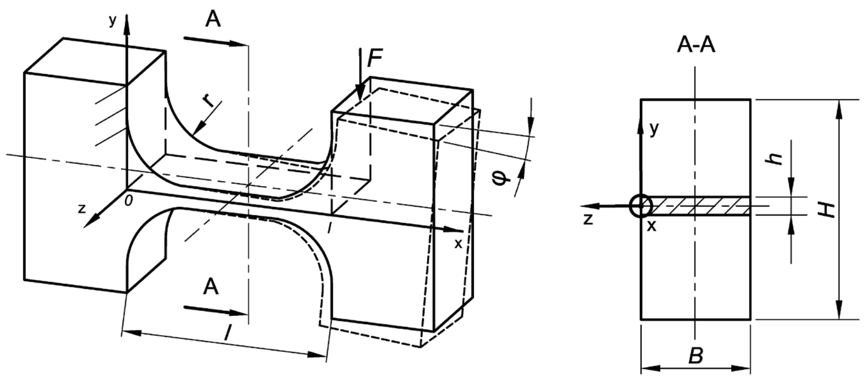


Figure 3: *Model and parameters of the investigated flexure hinge with a symmetric notch contour based on quadrant fillets (radius r) and the two attached rigid segments*

Under the assumption of rigid terminal segments, the investigation is exclusively focused on the notch contour. One end of the hinge is therefore considered as fixed and the free end is loaded with the single force F at $x = l$. Due to the shift of the axis of rotation during motion it is difficult to apply discrete angles of deflection on the moved segment. Therefore, the deflection is realized with a displacement $v = 0.05H$ in this investigation. This corresponds to an angle in the order of $\varphi \approx 5°$. However, the resulting angle of deflection will be evaluated. The material which is used for all studies is steel with linear elastic material behavior and the following properties: $E = 200$ GPa, $\mu = 0.3$ und $\rho = 7.85$ gcm^{-3}.

To reduce the stress-deflection ratio the radius of rectangular contours with corner fillets is investigated at first in terms of the notch length and height. Furthermore, asymmetric hinge geometries are presented whose optimal radii are determined with regard to minimal maximum stress and a reduced shift rotational axis. Therefore different approaches for modeling the position of the rotational axis are considered. The influences of the notch contour as well as the minimal notch height on the shift of the rotational axis are examined. In addition, the potential of novel contours with undercuts will be discussed and the resulting contours are compared with typical hinge geometries by utilizing FEM simulations.

3. STRESS-OPTIMAL SYMMETRIC CORNER-FILLETED CONTOURS

On the basis of a design of experiments the optimal radius r of a symmetric contour with corner fillets is determined depending on the notch length l (for a constant height h and H) and the minimal notch height h (for a constant length l and height H).

3.1. Investigation of notch length

The investigation of the influence of the notch length is exemplified for a minimal notch height $h = 0.2H$ with $H = 1$ mm. The correlation of the resulting relative maximum stress σ_{max}/E and the relative fillet radius r/H for eight different notch lengths is shown in Figure 4.

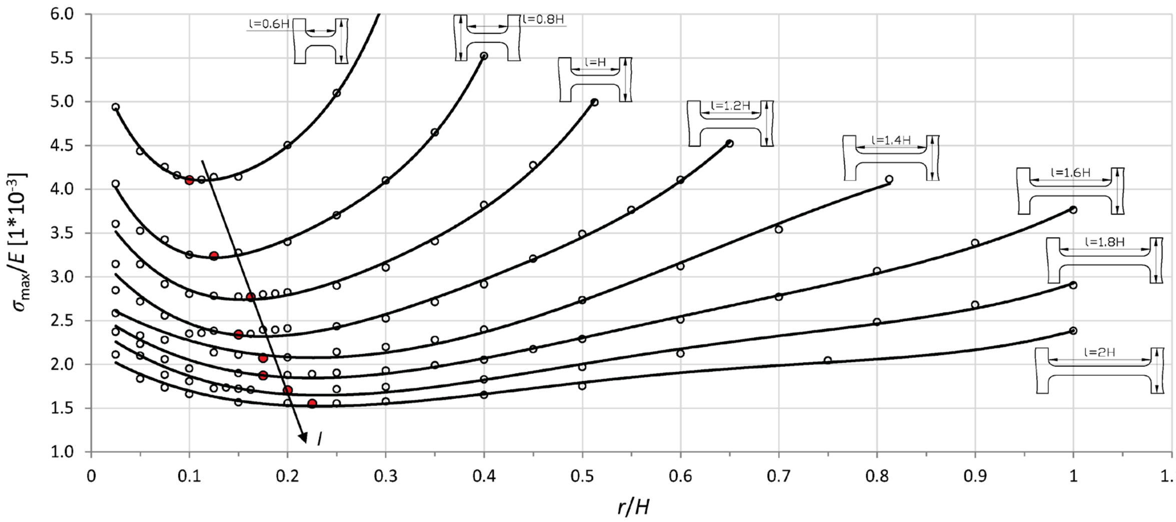


Figure 4: *Resulting relative maximum stress as a function of the relative fillet radius under variation of the notch length for a displacement v = 0.05H (optimal radius value is highlighted as a solid point)*

Depending on the fillet radius, different maximum stress values result for different notch lengths. Very small radii cause a stress concentration at the corners of the notch. A large fillet radius leads to a significant increase of the stress as well which appears rather in the center of the notch. In general a suitable ratio of radius to notch length $r/l = 0.1$ can be identified for flexure hinges with lumped compliance and the studied minimal notch height.

3.2. Investigation of minimal notch height

The investigation of the influence of the minimal notch height is exemplified for a notch length $l = H$ with $H = 1$ mm. The correlation of the resulting relative maximum stress σ_{max}/E and the relative fillet radius r/H for six different minimal notch heights is shown in Figure 5.

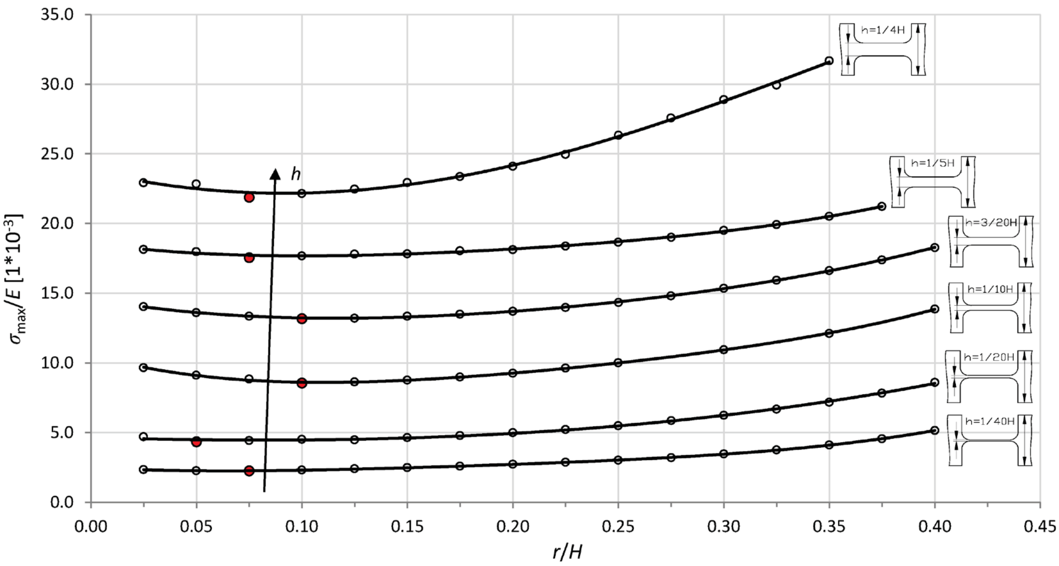


Figure 5: *Resulting relative maximum stress as a function of the relative fillet radius under variation of the minimal notch height for a displacement v = 0.05H (optimal radius value is highlighted as a solid point)*

The studies, carried out according to the mentioned assumptions, pointing out only a marginal trend and not a significant dependence of the optimal fillet radius to the minimal notch height. In comparison, a suitable ratio of $r/h < 2$ is mentioned in literature to avoid stress concentrations [4]. This ratio is satisfied for all investigated radii. The determined stress optimal ratio of fillet radius to notch length $r/l = 0.1$ is confirmed.

4. ANALYSIS AND INVESTIGATION OF THE PRECISION OF ROTATION

The rotational axis of a flexure hinge is not consistently defined in literature. There are different approaches to model the instantaneous center of rotation, which will result in different values for the shift of the rotational axis (cf. Figure 6). The following approaches are most frequently used and are selected for the further investigation using FEM simulations.

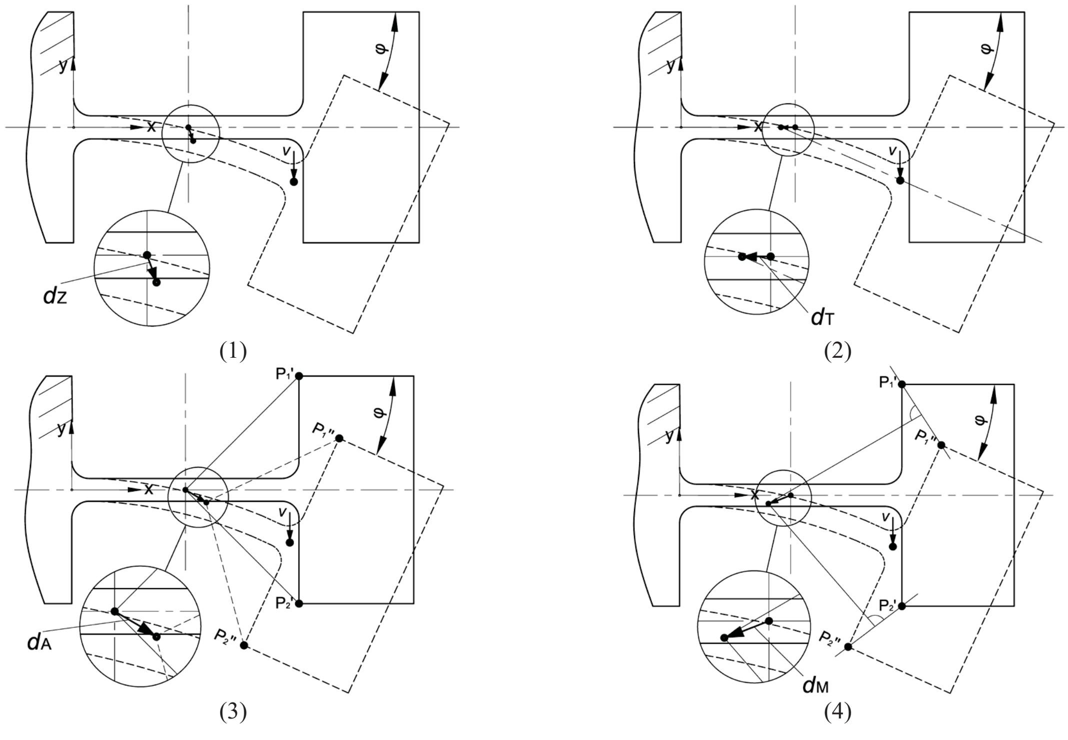


Figure 6: *Models of the shift of rotational axis - (1) center node of contour, (2) intersection of tangents at the bending axis, (3) moved fixed center and (4) the instantaneous center according to the deflected segment*

4.1. Modeling the rotational axis

The position of the rotational axis is described in literature through the geometric center of the flexible hinge segment (notch contour) or by calculation with the help of the position of the deflected rigid segment. In this contribution four frequently used approaches are examined and compared for different influences (Figure 6).

A simple approach (1) to determine the instantaneous center of rotation is the calculation of the shift d_Z of the actual center respectively middle point of the notch contour from one position to another [14].

Another approach (2) considers the intersection of the tangents at the bending axis, and – for the assumption of a rigid fixed segment – the shift d_T only in x-direction [15].

Based on the pseudo-rigid-body model [7], which allows the deviation of any point on the moved segment (e.g. P_1) to the ideal circular path position only as an absolute value, with the method mentioned in [15] the relative shift of rotational axis can be determined. From position P_1‘ to P_1‘‘ the approach (3) uses the method to calculate the shift d_A of the virtual center point which is fixed on the deflected segment.

Another approach (4) is the approximated calculation of the fixed centrode with the help of two points P_1 and P_2 [16], [17]. In this case the shift d_M describes the motion of the instantaneous center of the deflected segment.

4.2. Influences on the shift of rotational axis

Corresponding to the four mentioned approaches the trajectories of the rotational axis positions are exemplified for two different hinge geometries (circular and rectangular contour with stress optimal corner fillets) with the minimal notch height $h = 0.2H$ and $H = l = 1$ mm (cf. Figure 7).

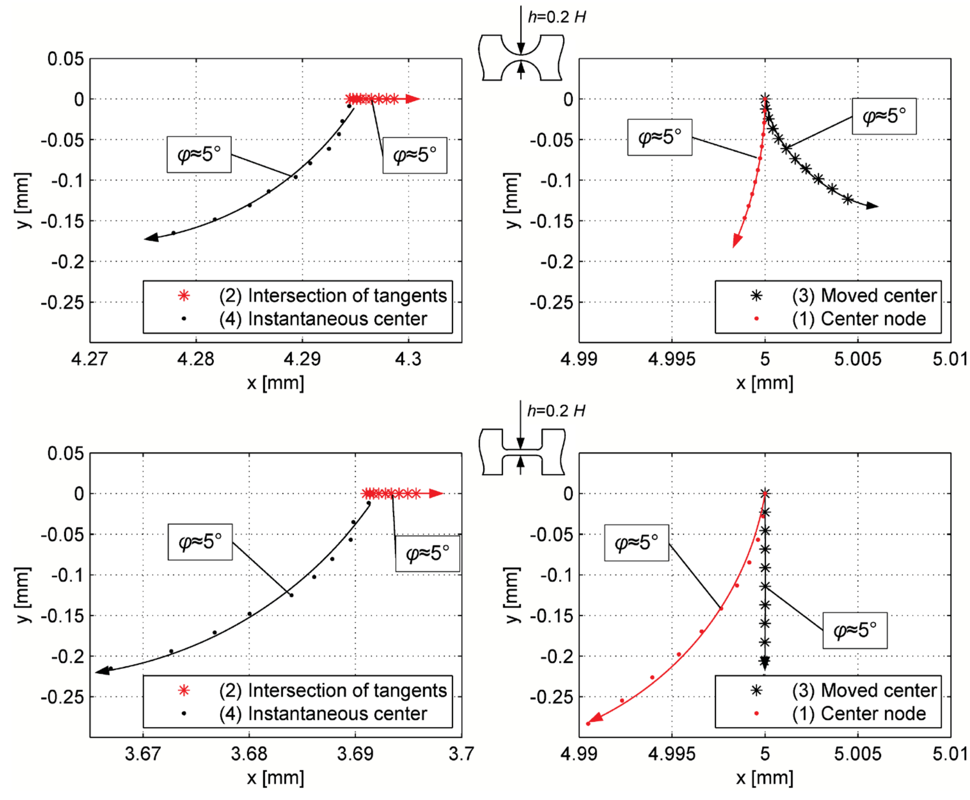


Figure 7: *Influence of the notch contour on the shift of rotational axis for the considered approaches and a maximum deflection of 10° (the values of the shift of rotational axis for $\varphi \approx 5°$ are marked)*

The FEM simulations point out, that the different definitions lead to different rotational axis coordinates. Furthermore, the absolute position of the axis of rotation and the direction and magnitude of the shift of the rotational axis depend on the chosen model as well as the notch contour. However, it can be recognized that the deviation of the rotational axis in direction of deflection (y) is more distinctive.

Thus, the approach (2) which determines the shift d_T only in x-direction is disadvantageous. The approach (1) is due to the difficult accessible reference point after deflection (d_Z) not suitable for a metrological investigation. The accuracy of model-based description of the shift of rotational axis d_M according to approach (4) depends on the given step size and the load levels (infinitesimal analysis).

Therefore, the approach (3) using a moved fixed center is chosen for further investigations of the shift d_A. This approach is comparably easy to implement in the FEM model too.

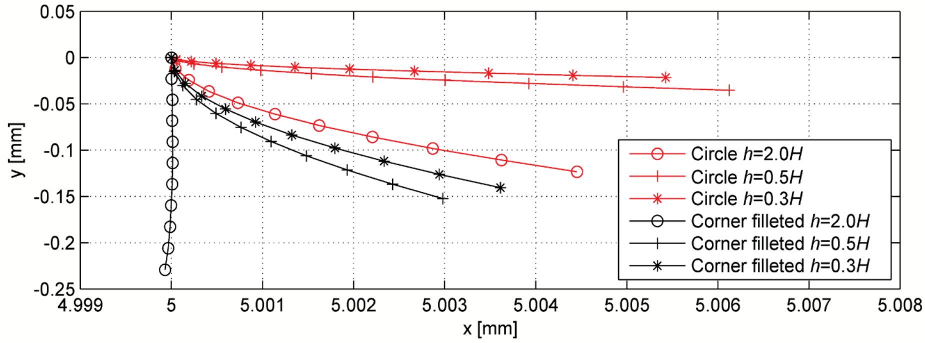


Figure 8: *Influence of the minimal notch height on the shift of the rotational axis for different notch contours and a maximum deflection of 10° according to approach (3)*

The basic dimensions of the flexure hinge and the minimum notch height h also influences the precision of rotation (cf. Figure 8). Further it can be shown again, that the curve of the axis of rotation is dependent on the notch contour. In conclusion thicker minimal notch heights in principle lead to a major shift of the rotational axis in the direction of deflection.

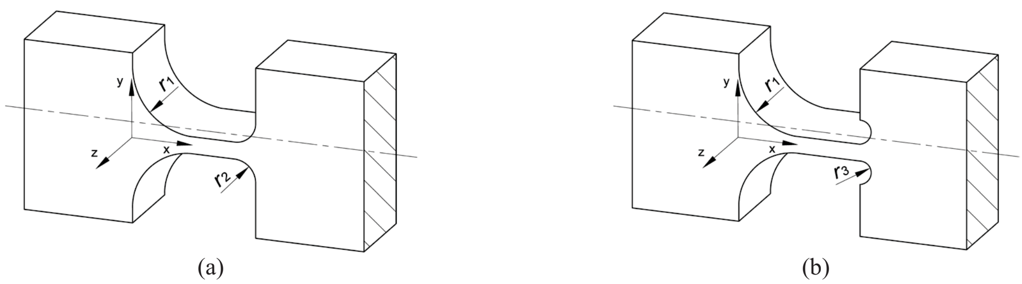
4.3. Summary of the approach for modeling the center of rotation

The chosen approach (3), which calculates the center shift of a geometry fixed on the deflected segment, is suitable to model the position of rotational axis as well as to detect the effects caused by different hinge geometries. For the resulting shift the y-deviation is dominant. The characteristic of the motion of the axis of rotation is difficult to predict and depends on many factors, such as the hinge dimensions, the type of load and load working point as well as the notch contour.

Using the chosen approach, various asymmetrical notch contours were investigated regarding the shift of the rotational axis and the maximum stress.

5. STRESS AND PIVOT OPTIMIZED ASYMMETRIC CONTOURS

Flexure hinges with semi-circular contours are often used and realize a small shift of rotational axis, but they show also a high stress-deflection ratio (see results in Table 1). Stress optimal symmetric corner-fillets cause on the other hand a high shift of rotational axis. Combining positive aspects of both in order to minimize axis shift and the stress-deflection ratio, asymmetric corner-filleted contours with respect to the height axis are investigated (Figure 9). These hinge geometries allow an optimization with respect to both objectives by the independent variation of the two radii r_1 and r_2. Furthermore a semi-circular undercut with the radius r_3 can be considered, offering additional freedom of design. The conflict between resulting maximum stress and shift of rotational axis can be solved at the expense of the more complex arrangement of geometrical features.



(a) (b)

Figure 9: *Models of the investigated asymmetric flexure hinges (h = 0.1H with H = l = 10) - (a) asymmetric contour with different corner fillets and (b) asymmetric contour with a corner fillet and a semi-circular undercut*

The optimal parameter combination of the two radii r_1/r_2 respectively r_1/r_3 is calculated by using a deterministic optimization method (adaptive response surface method, ARSM). Starting from an initial design, which was calculated by design of experiments, the resulting optimal values depend on the variation range of parameters and the weighting of the two objectives. For the investigation, the following assumptions were made:

- variation of the quadrant radii r_1 and r_2 from 0.5..4 mm,
- variation of the semi-circular radius r_3 from 0.2..2 mm,
- minimization of the maximum stress under satisfaction of the constraint of a shift of the rotational axis smaller than 45 microns (corresponding to the ellipse contour).

5.1. Asymmetric corner-filleted contours

In Figure 10 the relative maximum stress (σ_{max}/E) and the shift of rotational axis (d_A) of the examined combinations of parameters are shown in comparison with common contours and one particularly suitable polynomial contour [18].

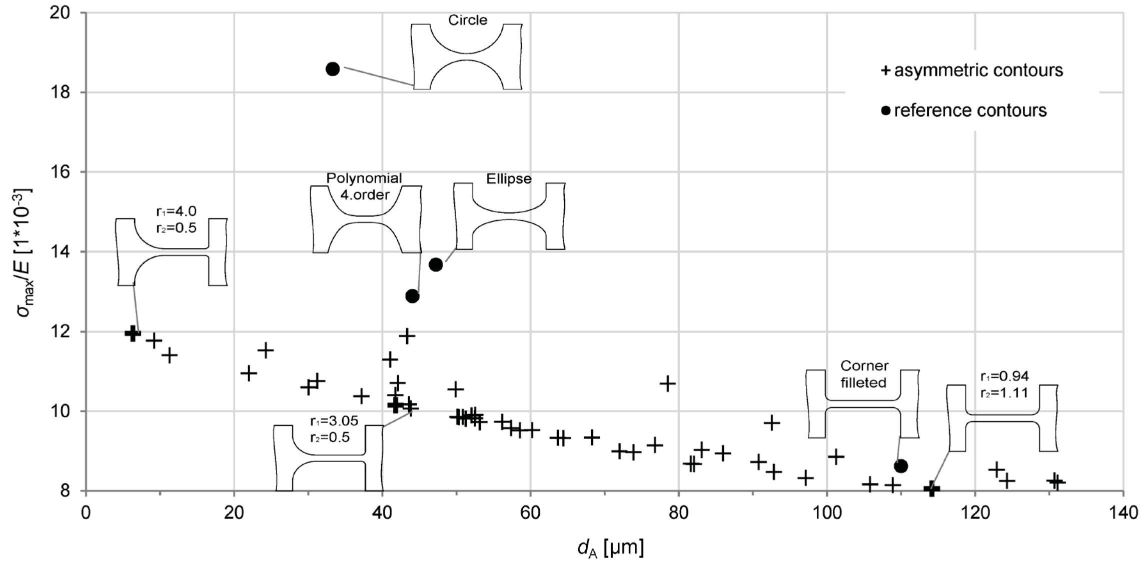


Figure 10: *Comparison of asymmetric corner-filleted contours with common contours for the relation of the two objectives maximum stress and shift of rotational axis*

According to the chosen approach it is obvious that asymmetric corner-filleted contours allow a significant improvement with regard to both objectives, the minimization of the maximum stress as well as the shift of rotational axis compared to common contours. Furthermore, there are parameter combinations that allow the fulfillment of only one objective in particular. A significant dependence on the weighting of the objectives can be concluded.

For further evaluation the stress and pivot optimal contour with the radii r_1 = 3.05 mm and r_2 = 0.50 mm is chosen.

5.2. Asymmetric contours with a semi-circular undercut

The introduction of undercuts confirms the advantages of asymmetric contours to reduce maximum stress and to improve precision of rotation compared to common shapes (Figure 11). Compared to asymmetric contours without undercut the improvement is insignificant for the predetermined semi-circular geometry.

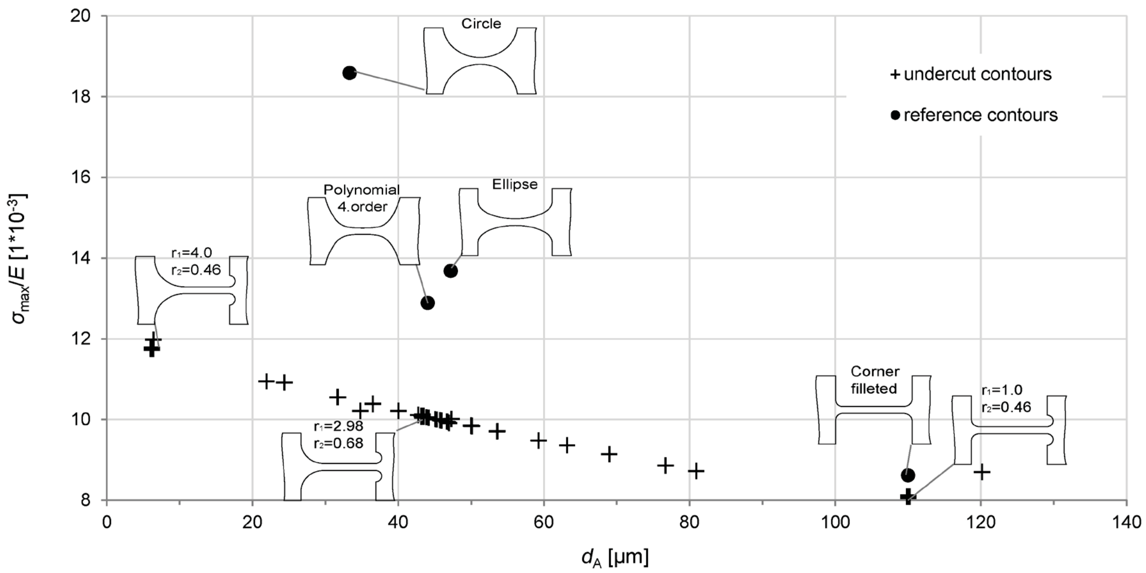


Figure 11: *Comparison of asymmetric undercut contours with common contours for the relation of the two objectives maximum stress and shift of rotational axis*

For the discussion of the results and the comparison of different hinge geometries the stress and pivot optimal undercut contour with the radii r_1 = 2.98 mm and r_3 = 0.68 mm is chosen.

6. DISCUSSION

To verify the suitability of the determined asymmetric contours, single flexure hinges with a minimum notch height $h = 0.1H$ and $H = l = 10$ mm are investigated for a given deflection of a, see Figure 12.

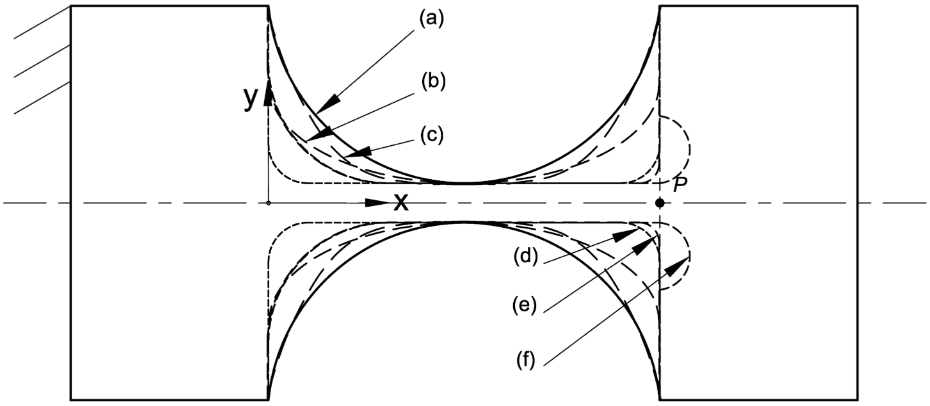
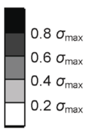
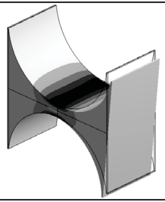
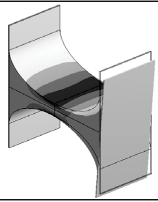
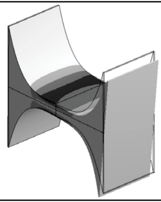
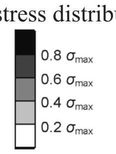
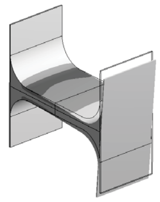
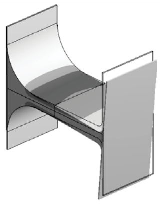
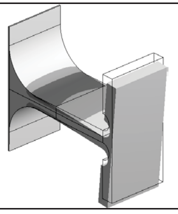


Figure 12: *Flexure hinge with investigated notch contours - (a) semi-circular contour (R = H/2) as reference contour, (b) elliptical contour ($r_a = 2r_b = H/2$), (c) bi-quadratic polynomial contour ([18]), (d) stress optimal symmetric corner-filleted contour (r = 0.1l), (e) asymmetric corner-filleted contour ($r_1 = 3.05$, $r_2 = 0.5$) and (f) asymmetric contour with semi-circular undercut ($r_1 = 2.98$, $r_3 = 0.68$)*

Table 1 shows the comparison of the six contours (cf. Figure 12) in regard to the stress-deflection ratio, the shift of rotational axis, the resulting angle of deflection of the moved segment and the elasticity of the hinge is exemplified for a displacement applied at point *P*.

***Table 1**: FEM results of model-based comparison of the optimized asymmetric contours with common contours for a deflection v = 0.5H (note the different angle of deflection for the same lateral displacement)*

Contour	**(a) Semi-circular**	**(b) Elliptical**	**(c) Bi-quadratic polynomial**
stress distribution 			
σ_{max}/E [1*10^{-3}]	18.6	13.7	12.9
d_A [µm]	33.3	47.2	44.1
φ [°]	5.35	5.19	5.23
rotational stiffness [Nm/°]	1.10	0.82	0.68
Contour	**(d) Symmetric corner-filleted**	**(e) Asymmetric corner-filleted**	**(f) Asymmetric undercut**
stress distribution 			
σ_{max}/E [1*10^{-3}]	8.6	10.3	10.1
d_A [µm]	110.2	40.1	44.3
φ [°]	4.47	5.28	5.23
rotational stiffness [Nm/°]	0.32	0.46	0.45

According to [4] for semi-circular contours a suitable ratio of $R/h \leq 5$ exists. The contour comparison shows further that optimal corner-filleted contours are better suited with regard to resulting maximum stress than semi-circular contours which are subjected to this ratio.

With consideration of reducing both objectives, maximum stress and shift the rotational axis, conventional elliptical contours and contours based on biquadratic polynomials showed particularly suitable properties so far. The novel asymmetric contours studied in this contribution, however, offer more potential for optimization. Compared to literature they allow similar suitable characteristics for the precision of rotation as the leaf-type hinges mentioned in [6]. The radii can be adjusted in a desired direction depending on the weighting of the objectives. The introduction of undercuts allows for the investigated fixed, predetermined semi-circle geometry only a marginal improvement of the stress value at a comparable shift of rotational axis, since the fillet radius r_1 is dominant.

The analysis of the resulting angle values points out that the design of flexure hinges subjected to the criterion of a low stress-deflection ratio for precision engineering applications should not be made without the consideration of precision of rotation. If the hinge design is made without a special application task, then the exact consideration of the load situation is important, because it is obvious that superimposed tensile and compressive stresses strongly influence the results.

Also in terms of rotational stiffness the determined asymmetric contours allow a specific high compliance, while the shift of rotational axis is small too.

7. CONCLUSION

In this contribution, the potential of symmetric and asymmetric contours based on radii as alternative notch contours for prismatic flexure hinges is described.

In particular symmetric rectangular contours with optimal corner fillets can be noted as being already well suited for low stress values. A ratio of fillet radius to notch length of $r/l = 0.1$ can be generalized.

The advantages of asymmetric notch contours regarding a reduced shift of rotational axis are shown compared to conventional contours. Therefore different approaches to define the rotational axis have been compared. Based on the analysis of modeling the position of the rotational axis the optimal radii of asymmetric corner-filleted and undercut contours in terms of minimizing the stress-deflection ratio and obtaining an increased precision of rotation were determined. For asymmetric corner-filleted contours a stress and pivot optimal ratio of the radii $r_1 = 6r_2$ (with $r_1 = 0.3l$) can be specified.

The investigations show that an influence of the deformation behavior of simple prismatic flexure hinges through the optimization of the notch contour for one or more objectives according to predefined or freeform hinge geometries is possible respectively promising. The estimated contours are geometrically simple and can be manufactured by machining.

8. REFERENCES

[1] Böttcher, F., Christen, G., and Pfefferkorn, H., "Structure and function of joints and compliant mechanisms," 2001. In *Motion systems 2001: collected short papers of the Innovationskolleg "Bewegungssysteme",*, 30–35.

[2] Christen, G. and Pfefferkorn, H., "Nachgiebige Mechanismen: Aufbau, Gestaltung, Dimensionierung und experimentelle Untersuchung," 1998. In *VDI-Berichte Nr. 1423 1998*, 309–29.

[3] Modler, K.-H. and Modler, N., "Aktive textilverstärkte Nachgiebigkeitsmechanismen (A-TNM)," 2008. In *Getriebetechnik 2009*, 197–208.

[4] Henein, S., *Tutorial on the design of flexure-mechanisms: Flexures: simply subtle.* Neuchâtel, 2011.

[5] Jungnickel, U., "Miniaturisierte Positioniersysteme mit mehreren Freiheitsgraden auf der Basis monolithischer Strukturen," Dissertation, Universität Darmstadt, 2004.

[6] Xu, P., Yu, J., Zong, G., Bi, S., and Yu, Z., "Analysis of Rotational Precision for an Isosceles-Trapezoidal Flexural Pivot," *Journal of mechanical design*, vol. 130, no. 5, p. 52302, 2008.

[7] Jensen, B. D. and Howell, L. L., "The modeling of cross-axis flexural pivots," *Mechanism and Machine Theory*, vol. 37, no. 5, pp. 461–476, 2002.

[8] Bona, F. De and Munteanu, M. G., "Optimized Flexural Hinges for Compliant Micromechanisms," *Analog Integrated Circuits and Signal Processing*, vol. 44, no. 2, pp. 163–174, 2005.

[9] Vallance, R. R., Haghighian, B., and Marsh, E. R., "A unified geometric model for designing elastic pivots," *Precision Engineering*, vol. 32, no. 4, pp. 278–288, 2008.

[10] Zelenika, S., Munteanu, M. G., and Bona, F. De, “Optimized flexural hinge shapes for microsystems and high-precision applications,” *Mechanism and Machine Theory*, vol. 44, no. 10p, pp. 1826–1839, 2009.

[11] Linß, S., Erbe, T., Grinevich, D., and Zentner, L., “An approach on model-based shape optimization of compliant mechanisms,” 2010. In *Proceedings of 15th International Conference. Mechanika*, 271–76.

[12] Zhang, Z. and Hu, H., “Comparison of Single-Notch Circular Flexure Hinge Rotational Stiffness Equations with FEA Results and Derivation of Empirical Formulations,” 2009. In *Proceedings of the International Joint Conference on Computational Sciences and Optimization (CSO), 2009*, 286–88.

[13] Chen, G., Jia, J., and Li, Z.-W., “On Hybrid Flexure Hinges,” 2005. In *Proceedings of the 2005 IEEE networking, sensing and control*, 709-704.

[14] Lobontiu, N., Paine, J. S. N., Garcia, E., and Goldfarb, M., “Corner-Filleted Flexure Hinges,” *Journal of mechanical design / publ. bimonthly by the American Society of Mechanical Engineers*, vol. 123, no. 3, pp. 346–352, 2001.

[15] Zelenika, S. and Bona, F. De, “Analytical and experimental characterization of high-precision flexural pivots subjected to lateral loads,” *Precision Engineering*, vol. 26, no. 4, pp. 381–388, 2002.

[16] Raatz, A., “Stoffschlüssige Gelenke aus pseudo-elastischen Formgedächtnislegierungen in Parallelrobotern,” Dissertation, TU Braunschweig, 2006.

[17] Dirksen, F. and Lammering, R., “On mechanical properties of planar flexure hinges of compliant mechanisms,” 2011. In *Second International Symposium on Compliant Mechanisms (CoMe)*.

[18] Linß, S., Erbe, T., and Zentner, L., “On polynomial flexure hinges for increased deflection and an approach for simplified manufacturing,” 2011. In *Proceedings of 13th World Congress in Mechanism and Machine Science*.